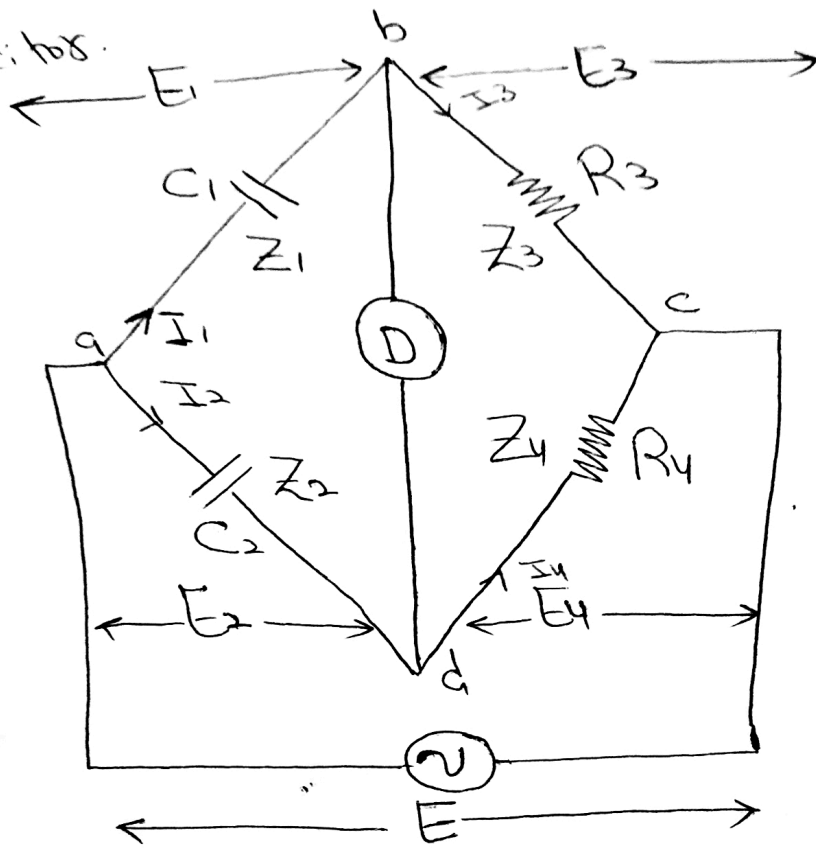


# De Sauty's Bridge :-

→ This bridge is used for the measurement of capacitance of a capacitor by comparing it with a standard capacitor.



Arm a-b :- unknown capacitance ( $C_1$ )

$$Z_1 = \frac{1}{j\omega C_1}$$

Arm a-d :- Standard capacitor ( $C_2$ )

$$Z_2 = \frac{1}{j\omega C_2}$$

Arm b-c :- Known non-inductive resistance ( $R_3$ )

$$Z_3 = R_3$$

Arm c-d :- Known non-inductive resistance ( $R_4$ )

$$Z_4 = R_4$$

At balance condition:-

$$Z_1 Z_4 = Z_2 Z_3$$

putting the values in above equation:-

$$\left(\frac{1}{j\omega C_1}\right) R_4 = \left(\frac{1}{j\omega C_2}\right) R_3$$

$$\left(\frac{1}{C_1}\right) R_4 = \left(\frac{1}{C_2}\right) R_3$$

$$\text{or } \frac{R_4}{R_3} = \frac{C_1}{C_2}$$

$$\boxed{C_1 = C_2 \cdot \frac{R_4}{R_3}}$$

where  $C_1 =$  unknown capacitance  
 $C_2 =$  standard capacitance

→ Note that these bridge can only measure capacitance of those capacitors which are free from dielectric losses or which are ideal.

Proof of Balance Equation:-

when the bridge is balanced:-

this means that:-

$$E_1 = E_2$$

$$I_1 Z_1 = I_2 Z_2$$

$$\frac{I_1}{I_2} = \frac{Z_2}{Z_1}$$

$$\text{as } I_1 = I_3 \quad E_1$$

$$I_2 = I_4$$

Hence

$$E_3 = E_4$$

$$I_3 Z_3 = I_4 Z_4$$

$$\frac{I_3}{I_4} = \frac{Z_4}{Z_3}$$

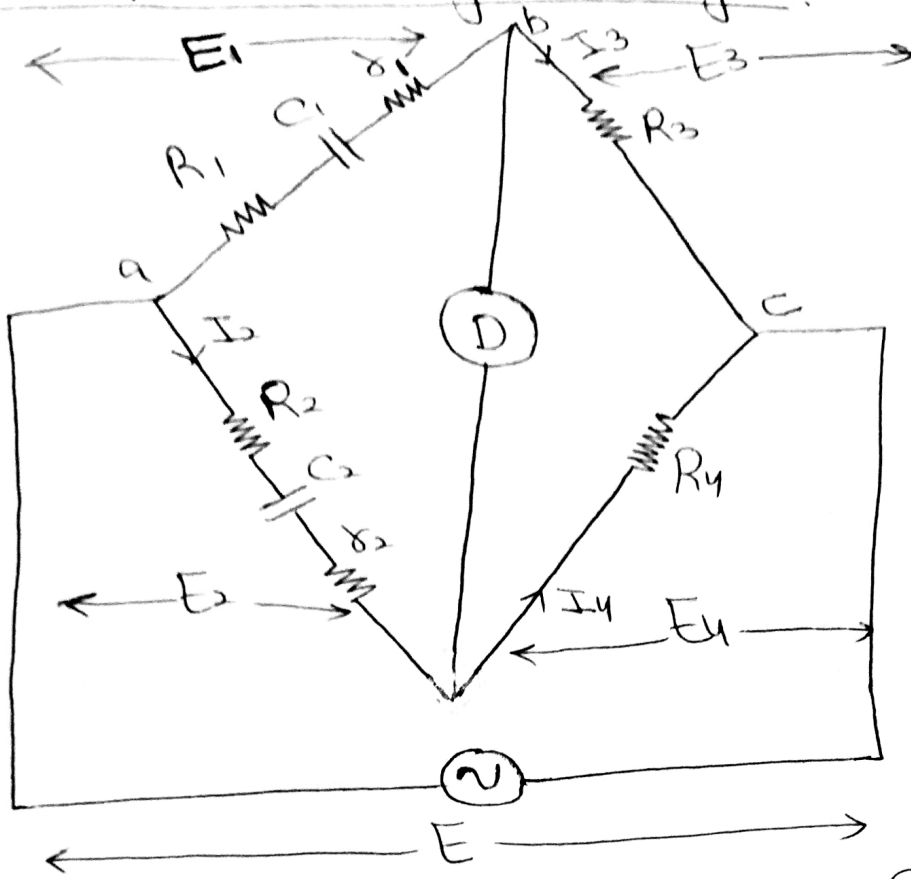
$$\frac{I_1}{I_2} = \frac{Z_2}{Z_1} = \frac{Z_4}{Z_3}$$

$$\frac{Z_2}{Z_1} = \frac{Z_4}{Z_3}$$

$$\boxed{Z_2 Z_3 = Z_1 Z_4}$$

potential at point b = potential at point d

# Modified De-Sauty's Bridge :-



Arm a-b :- unknown capacitance ( $C_1$ ) connected in series with resistance ( $R_1$ ) and loss component ( $\delta_1$ ) representing loss.

Arm a-d :- Standard capacitor ( $C_2$ ) connected in series with resistance ( $R_2$ ) and loss component ( $\delta_2$ ) representing loss.

$R_3, E_1, R_4$  are known non-inductive resistances.

Now :-

$$Z_1 = R_1 + \delta_1 + \frac{1}{j\omega C_1}$$

$$Z_2 = R_2 + \delta_2 + \frac{1}{j\omega C_2}$$

$$Z_3 = R_3$$

$$Z_4 = R_4$$

at balance condition:-

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left( R_1 + \delta_1 + \frac{1}{j\omega C_1} \right) R_4 = \left( R_2 + \delta_2 + \frac{1}{j\omega C_2} \right) R_3$$

$$\underbrace{(R_1 + \delta_1) R_4}_{\text{Real part}} + \underbrace{\frac{R_4}{j\omega C_1}}_{\text{imaginary part}} = \underbrace{(R_2 + \delta_2) R_3}_{\text{Real part}} + \underbrace{\frac{R_3}{j\omega C_2}}_{\text{Imaginary part}}$$

equating Real part:-  $(R_1 + \delta_1) R_4 = (R_2 + \delta_2) R_3$

$$\Rightarrow \frac{R_4}{R_3} = \frac{R_2 + \delta_2}{R_1 + \delta_1} \quad \text{--- (1)}$$

equating imaginary part:-

$$\frac{R_4}{j\omega C_1} = \frac{R_3}{j\omega C_2}$$

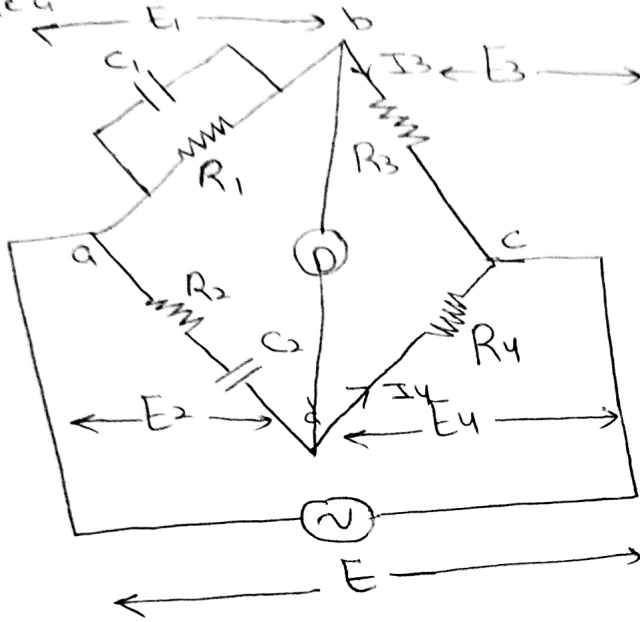
$$\frac{C_1}{C_2} = \frac{R_4}{R_3} \quad \text{--- (2)}$$

Comparing eq (1) & (2)

$$\boxed{\frac{C_1}{C_2} = \frac{R_4}{R_3} = \frac{R_2 + \delta_2}{R_1 + \delta_1}}$$

# Wein's Bridge

- It is used for the measurement of frequency in the range from 100Hz to 100kHz.
- Its accuracy is between 0.1% to 0.5%.
- named after the scientist wein who invented it.



Arm a-b :- Capacitance ( $C_1$ ) connected in parallel with ( $R_1$ )

$$\frac{1}{Z_1} = \frac{1}{R_1} + \frac{1}{X_C} = \frac{1}{R_1} + j\omega C_1$$

$$\left( X_C = \frac{1}{j\omega C_1} \right)$$

$$\frac{1}{Z_1} = \frac{1 + j\omega C_1 R_1}{R_1}$$

$$Z_1 = \frac{R_1}{1 + j\omega C_1 R_1}$$

Arm b-c :- Known inductive Resistance ( $R_3$ )

$$Z_3 = R_3$$

Arm c-d :-  $Z_4 = R_4$

Arm a-d :-  $Z_2 = R_2 + \frac{1}{j\omega C_2}$

at balance condition:-

$$Z_1 Z_4 = Z_2 Z_3$$

putting the values in above equation:-

$$\left( \frac{R_1}{1 + j\omega C_1 R_1} \right) R_4 = \left( R_2 + \frac{1}{j\omega C_2} \right) R_3$$

$$\frac{R_4}{R_3} = \left( \frac{j\omega R_2 C_2 + 1}{j\omega C_2} \right) \left( \frac{1 + j\omega C_1 R_1}{R_1} \right)$$

$$\frac{R_4}{R_3} = \frac{j\omega R_2 C_2 + j^2 \omega^2 C_2 R_2 C_1 R_1 + 1 + j\omega C_1 R_1}{j\omega R_1 C_2}$$

$$\frac{R_4}{R_3} = \frac{j\omega R_2 C_2}{j\omega R_1 C_2} + \frac{j^2 \omega^2 C_2 R_2 C_1 R_1}{j\omega R_1 C_2} + \frac{1}{j\omega R_1 C_2} + \frac{j\omega C_1 R_1}{j\omega R_1 C_2}$$

now equating real & imaginary parts:-

Real parts:-

$$\frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2}$$

imaginary parts:-

$$j\omega R_2 C_1 + \frac{1}{j\omega R_1 C_2} = 0$$

$$\begin{aligned} j^2 \omega^2 R_1 C_1 R_2 C_2 + 1 &= 0 \\ + \omega^2 R_1 C_1 R_2 C_2 &= +1 \end{aligned}$$

$$\omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

As we know that

$$\omega = 2\pi f$$

$$\text{or } f = \frac{1}{2\pi \sqrt{R_1 R_2 C_1 C_2}}$$