AC Bridges:-

Bridge Circuit:-

A bridge circuit is a special electric circuit consisting of 4 arms and 4 nodes arranged in an predefined manner.

→ In those 4 arms, if parameters of 3 arms are known, then we can easily find the parameters of the 4th arm.

There are basically 2 types of bridges:-

(1):- DC Bridge

(2):- AC Bridge

(1):- DC Bridge:-

(a):- It contain DC Supply only.

(b):- parameter included is only (R)

(c):- phasor is not included.

(2):- AC Bridge:-

(a):- It contain AC Supply

(b):- parameter included are R, L and C

(c):- Phasor's are included.

Maxwell Inductance bridge :-

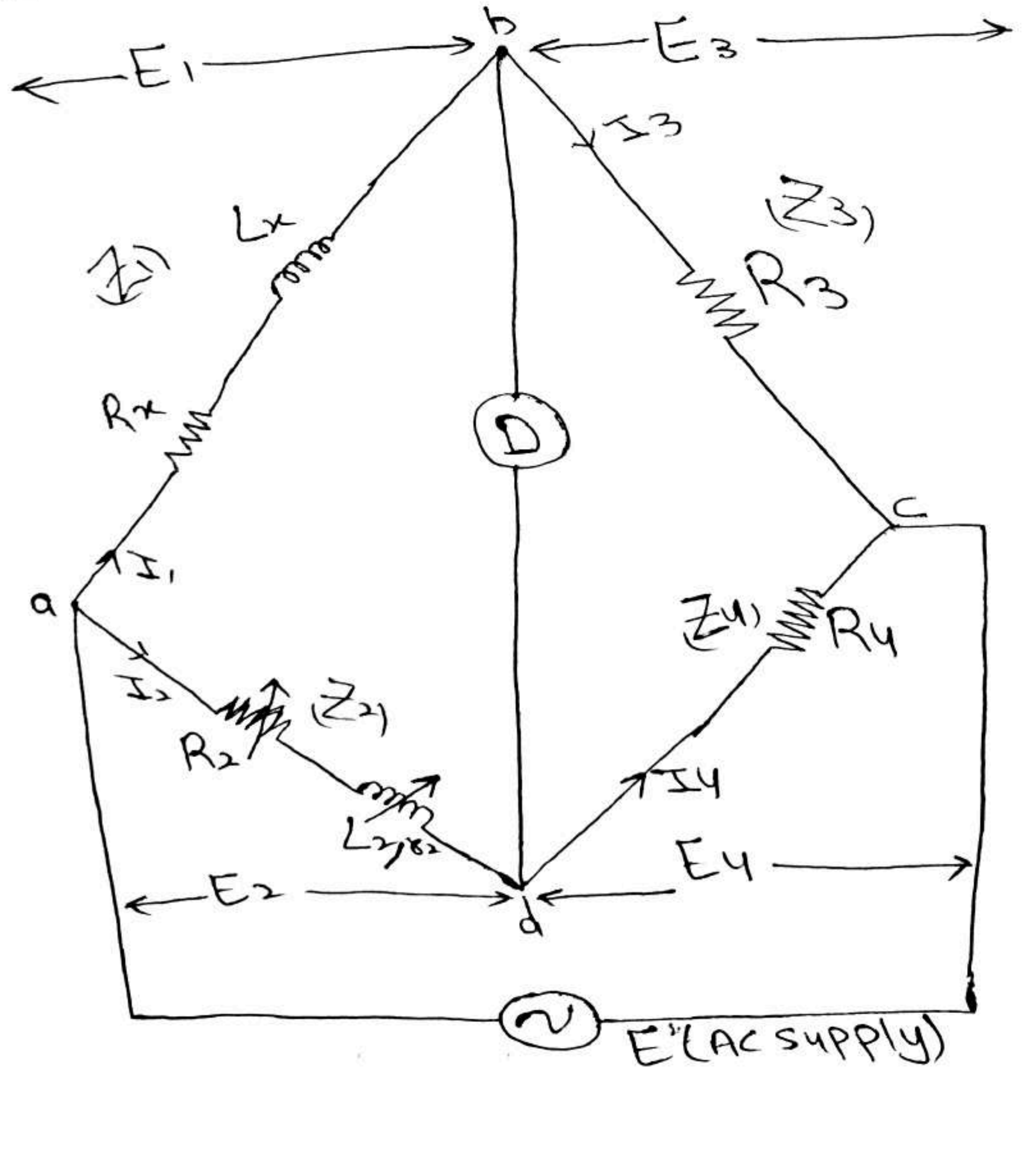
→ This bridge was invented by maxwell.

→ This is used for the measurement of inductance.

→ It measures an unknown inductance by comparing it with a known inductance.

→ It is an AC bridge which consists of 4 arms containing 4 impedances in which 3 are known and one is unknown.

→ It also contains a detector in order to detect the balance condition of the bridge and an AC supply.



Arm a-b :- unknown inductance (L_x) connected in series with a resistance (R_x).

Arm b-c :- Known non-inductive resistance (R_3)

Arm c-d :- Known non-inductive resistance (R_4).

Note :- non-inductive resistance means that (R_4) is a pure resistor which doesnot contain any inductance.

Arm d-a :- variable inductance (L_2) of fixed internal resistance (r_2) connected in series with variable resistance (R_2).

Balance equation :-

$$Z_1 Z_4 = Z_2 Z_3$$

where $Z_1 = R_x + j\omega L_x$

$$Z_2 = R_2 + r_2 + j\omega L_2$$

$$Z_3 = R_3$$

$$Z_4 = R_4$$

$$(R_x + j\omega L_x) R_4 = (R_2 + r_2 + j\omega L_2) R_3$$

$$R_x R_4 + j\omega L_x R_4 = R_2 R_3 + R_3 r_2 + j\omega L_2 R_3$$

$$R_x R_4 + j\omega L_x R_4 = (R_2 + r_2) R_3 + j\omega L_2 R_3$$

proof of balance Equation

when the bridge is balanced potential at point B = potential at D

this means that :-

$E_1 = E_2$ & $E_3 = E_4$

$I_1 Z_1 = I_2 Z_2$ & $I_3 Z_3 = I_4 Z_4$

$\frac{I_1}{I_2} = \frac{Z_2}{Z_1}$ & $\frac{I_3}{I_4} = \frac{Z_4}{Z_3}$

as $I_1 = I_3$ & $I_2 = I_4$

Hence

$$\frac{I_1}{I_2} = \frac{Z_2}{Z_1} = \frac{Z_4}{Z_3}$$

or

$$\frac{Z_2}{Z_1} = \frac{Z_4}{Z_3}$$

$$Z_1 Z_4 = Z_2 Z_3$$

Here $R_x R_4$ & $(R_2+r_2)R_3$ are real terms while $j\omega L_x R_4$ & $j\omega L_2 R_3$ are imaginary terms.

Hence:-

$$R_x R_4 = (R_2+r_2)R_3 \quad \& \quad \cancel{j\omega} L_x R_4 = \cancel{j\omega} L_2 R_3$$

$$\Rightarrow \boxed{R_x = (R_2+r_2)\frac{R_3}{R_4}} \qquad \boxed{L_x = L_2\frac{R_3}{R_4}}$$

- From this equation, we can see that the unknown inductance (L_x) can be calculated by comparing it with known inductance (L_2).

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Maxwell Inductance Capacitance Bridge:-

→ It is a type of AC Bridge.

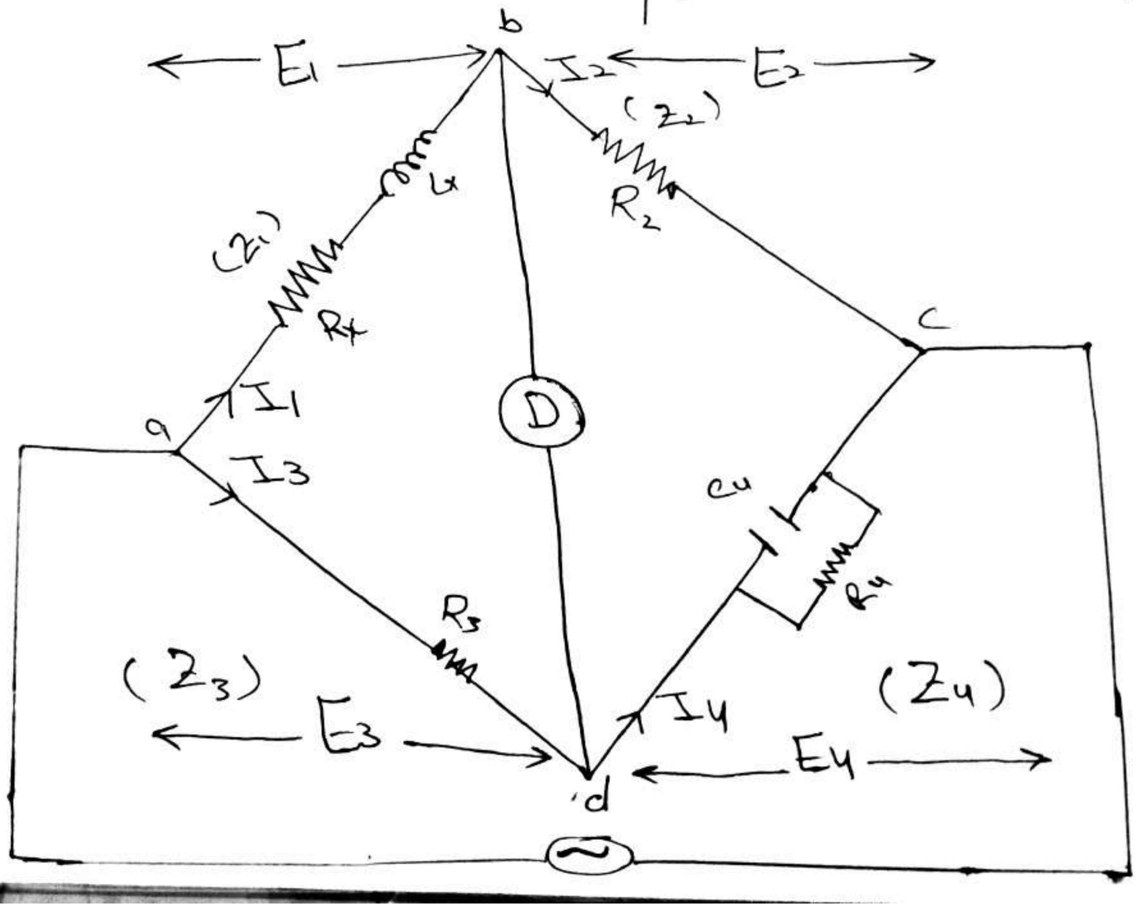
→ It measures inductance by comparing it with a standard capacitance.

→ It is named after the Scientist Maxwell

→ It consists of 4 arms having 4 impedances in which 3 impedances are known and one is unknown.

→ It also consists of a detecter which is used to obtain the balance condition of the bridge.

→ It also consists of an AC power supply which is used to operate the bridge.



Now:-

$Z_1 = R_x + j\omega L_x$

$Z_2 = R_2$

$Z_3 = R_3$

$Z_4 = R_4 \parallel \frac{1}{j\omega C_4}$

$\frac{1}{Z_4} = \frac{1}{R_4} + \frac{1}{X_C} \Rightarrow \frac{1}{Z_4} = \frac{1}{R_4} + j\omega C_4 \quad (\because X_C = \frac{1}{j\omega C})$

$\frac{1}{Z_4} = \frac{1 + j\omega R_4 C_4}{R_4} \Rightarrow Z_4 = \frac{R_4}{1 + j\omega R_4 C_4}$

Balance equation:-

$Z_1 Z_4 = Z_2 Z_3$

putting valuesin above equation

$(R_x + j\omega L_x)\left(\frac{R_4}{1 + j\omega C_4 R_4}\right) = R_2 R_3$

$R_x R_4 + j\omega L_x R_4 = R_2 R_3 + j\omega R_2 R_3 C_4 R_4$

Here $R_x R_4$ & $R_2 R_3$ are Real terms while $j\omega L_x R_4$ & $j\omega R_2 R_3 C_4 R_4$ are imaginary terms.

$R_x R_4 = R_2 R_3$ & $\cancel{j\omega} L_x \cancel{R_4} = \cancel{j\omega} R_2 R_3 C_4 \cancel{R_4}$

$L_x = R_2 R_3 C_4$

$\Rightarrow \boxed{R_x = \frac{R_2 R_3}{R_4}}$ $\boxed{L_x = R_2 R_3 C_4}$

Hence $Z_1 = \frac{R_2 R_3}{R_4} + j\omega R_2 R_3 C_4$

Quality factor of the coil (Lx):-

- Every inductor possesses a small resistance in addition to its inductance

→ The lower the value of this resistance (R), the better the quality of the coil

→ The quality factor or Q factor of an inductor is defined as "the ratio of the coil reactance to its resistance".

- Thus for an inductor, quality factor is expressed as

$$Q = \frac{wL}{R} \text{ or } \frac{X_L}{R} \quad \text{—— i)}$$

For the given bridge, Lx is unknown and we want to find its Q.

So equation i) can be written as:-

$$Q = \frac{wLx}{Rx} \quad \text{—— 2)}$$

putting the value of (Lx) and (Rx) in equation 2)

$$Q = \frac{w R_2 R_3 C_4}{\frac{R_2 R_3}{R_4}}$$

$$Q = (w \cancel{R_2 R_3} C_4)\left(\frac{R_4}{\cancel{R_2 R_3}}\right)$$

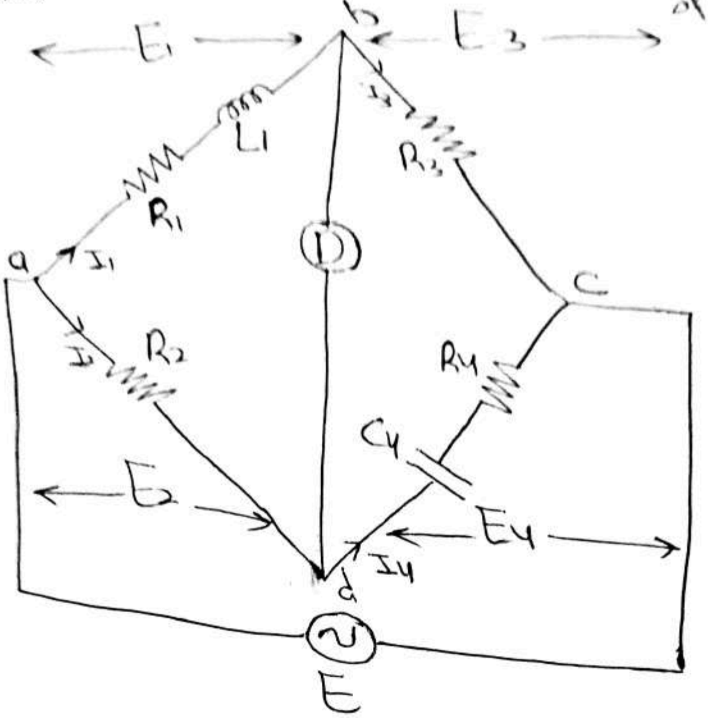
$$\boxed{Q = w R_4 C_4}$$

Hay's Bridge:

→ It is used for the measurement of inductance

→ modification of maxwell's Bridge

→ It consists of series combination of R_4 & C_4 in the arm c-d.



→ Here Arm a-b conteins the unknown inductance (L_1) having resistance (R_1).

$$Z_1 = R_1 + j\omega L_1$$

→ Arm b-c :- Known non-inductive resistance R_3

$$Z_3 = R_3$$

→ Arm c-d:- Standard Capacitor (C_4) is connected in series with (R_4).

$$Z_4 = R_4 + X_c = R_4 + \frac{1}{j\omega C_4} = R_4 - \frac{j}{\omega C_4} \quad \left(\because \frac{1}{j} = -j\right)$$

→ Arm d-a:- Known non-inductive resistance (R_2):-

$$Z_2 = R_2$$

$Z_1 = R_1 + j\omega L_1$, $Z_2 = R_2$

$Z_3 = R_3$, $Z_4 = R_4 - \frac{j}{\omega C_4}$

Balance equation

$Z_1 Z_4 = Z_2 Z_3$

putting values in the above equation

$$(R_1 + j\omega L_1)\left(R_4 - \frac{j}{\omega C_4}\right) = R_2 R_3$$

$$R_1 R_4 - \frac{jR_1}{\omega C_4} + j\omega L_1 R_4 + \frac{L_1}{C_4} = R_2 R_3$$

equating the real and imaginary parts:

$$R_1 R_4 + \frac{L_1}{C_4} = R_2 R_3 \quad \text{—— (a)}$$

$$j\omega L_1 R_4 - \frac{jR_1}{\omega C_4} = 0 \Rightarrow \cancel{j}\omega L_1 R_4 = \frac{\cancel{j}R_1}{\omega C_4}$$

$$R_1 = \omega^2 L_1 R_4 C_4 \quad \text{—— (b)}$$

putting R_1 in this equation (a)

$$\Rightarrow \omega^2 L_1 R_4^2 C_4 + \frac{L_1}{C_4} = R_2 R_3$$

$$L_1\left(\frac{\omega^2 R_4^2 C_4^2 + 1}{C_4}\right) = R_2 R_3$$

$$\boxed{L_1 = \frac{R_2 R_3 C_4}{1 + \omega^2 R_4^2 C_4^2}}$$ put this value in eq (b)

$$\text{(b)} \Rightarrow R_1 = \omega^2\left(\frac{R_2 R_3 C_4}{1 + \omega^2 R_4^2 C_4^2}\right) R_4 C_4 \Rightarrow \boxed{R_1 = \frac{\omega^2 R_2 R_3 R_4 C_4^2}{1 + \omega^2 R_4^2 C_4^2}}$$

For L_1, quality factor can be expressed as:-

$$Q = \frac{\omega L_1}{R_1} \quad \text{——} \ (1)$$

putting the values of (L_1) & (R_1) in above equation;-

$$Q = \frac{\omega \dfrac{R_2 R_3 C_4}{\cancel{1+\omega^2 R_4^2 C_4^2}}}{\dfrac{\omega^2 R_2 R_3 R_4 C_4^2}{\cancel{1+\omega^2 R_4^2 C_4^2}}}$$

$$= \frac{\cancel{\omega} \cancel{R_2} \cancel{R_3} \cancel{C_4}}{\omega^{\cancel{2}} \cancel{R_2} \cancel{R_3} R_4 C_4^{\cancel{2}}}$$

$$\boxed{Q = \frac{1}{\omega R_4 C_4}}$$