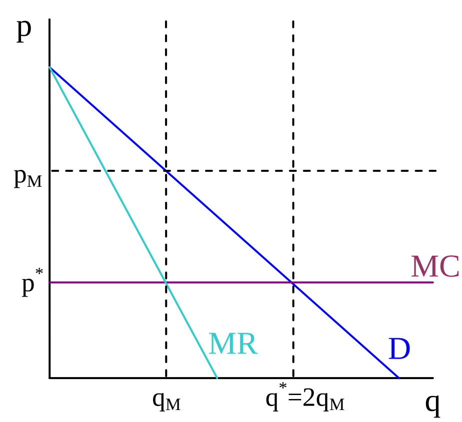
Week 7 - Game Theory and Industrial Organisation

The Cournot and Bertrand models are the two basic templates for models of **oligopoly**; industry structures with a small number of firms. There are a number of similarities in that they are both game theoretic models of oligopoly (i.e. both assume a small number of firms and use the same equilibrium concepts). In the simplest form of both models, we assume that the firms' products are perfect substitutes. The difference is that in the Cournot model firms compete by setting quantities as the strategic variable whereas in the Bertrand model they compete by setting prices. This is a key conceptual distinction, which leads to widely differing predictions.

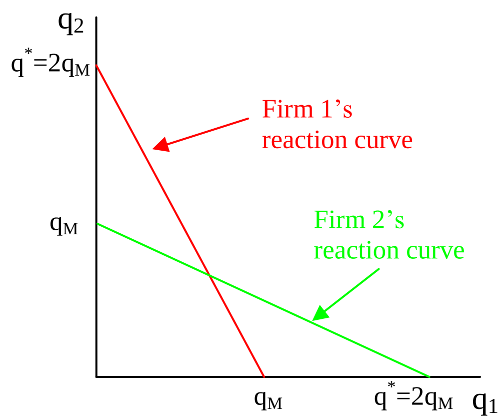
When there are only a small number of firms in an industry, we must model the **strategic interaction** between them. (This is in contrast with the assumptions of the perfect competition model, where we assume that each firm is so small relative to the total industry that they cannot affect each others' profits, and so we connect them together purely via the industry equilibrium price.) We model situations of strategic interaction using **game theory**. Game theoretic models require a number of components. Firstly, we have **players**. In the case of oligopoly models, these are the firms, so note that we are treating firms as if they are rational individual decision makers. Players rationally act so as to maximize their **payoffs**. In the case of oligopoly models, the payoffs are the firms' profits. The payoffs received by the players depend on their choice of **move**. The type of move is also referred to as the **strategic variable**. A player's **strategy** is a distinct and broader concept than that of a move because it prescribes the move that the player will make in every possible circumstance. In simple single phase simultaneous move games (like the basic Bertrand and Cournot models), however, each strategy is simply a move because players have no information upon which to condition their move. This is what we mean when we say that these are games with **imperfect information**. When we are looking at sequential games, we must think more carefully about defining the players' strategies.

In order to predict an outcome in game theoretic models, we use the concept of **Nash equilibrium**. A Nash equilibrium has the property that each player chooses their strategy in order to maximize their payoff *given* the strategies of the other players. The intuitive justification for this in a simultaneous move game is that it is only at a Nash equilibrium that each player's expectations about the other player's behaviour could be fulfilled. At a point which is not a Nash equilibrium, players would be making implicit assumptions about the behaviour of the other player which would not be borne out in reality (i.e. given what the other player actually does, the players would not be maximizing their profit).

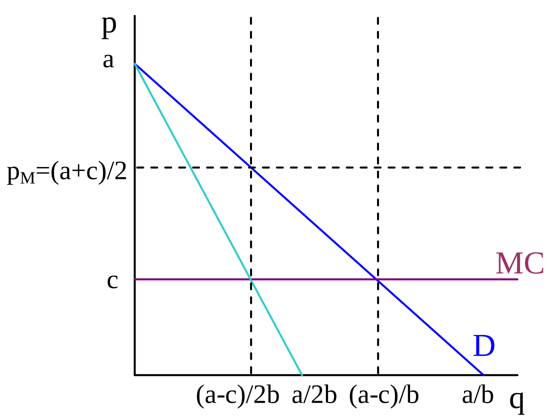
The starting points for the game theoretic analysis of oligopoly are the Cournot and Bertrand models of **duopoly**. These are 2 player, single phase, simultaneous move games. To keep things simple, we assume a linear demand curve, constant marginal costs and 0 fixed costs in the following discussion. This means that the perfectly competitive socially optimal quantity q^* is equal to $2q_M$, where q_M is the quantity chosen to maximize their profits by a monopolist facing this demand curve. The price chosen by the monopolist is p_M. The perfectly competitive price p^* is simply the same as the constant MC.



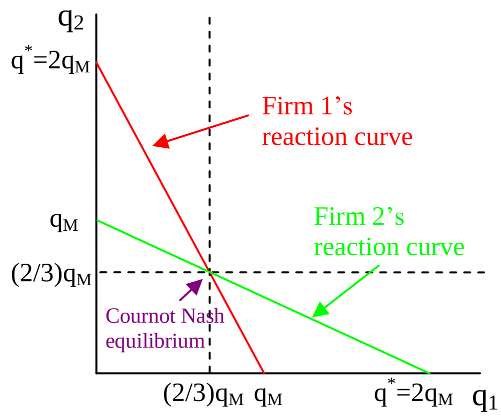
In the Cournot model, quantity is the strategic variable. Each firm chooses their quantity simultaneously, and then the market demand curve determines the price that they will both receive per unit of output. To find the Nash equilibrium, we need first to determine the **reaction curves** of the two firms. We plot these in (q_1, q_2) space, where the two variables are the quantities produced by the two firms respectively. We can sketch firm 1's reaction curve by thinking about how much it will choose to produce if firm 2 produces 0 units (this will be the monopoly output level q_M and will determine the intercept with the x-axis) and how much firm 2 would have to produce before firm 1 will choose to produce 0 (this will be where firm 2 produces $q^*=2q_M$ units and will determine the intercept of firm 1's reaction curve with the y-axis). The equation for firm 1's reaction curve is therefore: $q_1=q_M-(1/2)q_2$. By a symmetrical reasoning process, $q_2=q_M-(1/2)q_1$.



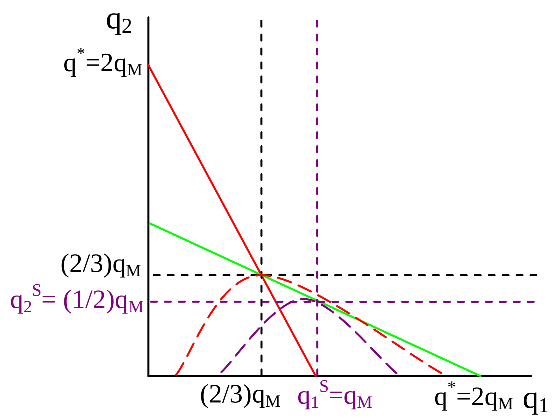
We can prove that the two firms' reaction curves are straight lines by maximizing their profit functions. For example, if we make the inverse demand curve $p=a-b(q_1+q_2)$ and the marginal cost c then firm 1's profit function will be $\pi_1=q_1(a-bq_1-bq_2)-q_1c$. The derivative with respect to q_1 is $\partial\pi_1/\partial q_1=a-2bq_1-bq_2-c=0$. When we set this equal to 0 we can rearrange to find the optimal quantity, which is $q_1=(a-c)/(2b)-(1/2)q_2$. We can see from the graph below that (a-c)/2b is indeed the optimal monopoly output level, leading to an optimal monopoly price of $p_M=a-b((a-c)/(2b))=(a+c)/2$:



The **Cournot Nash equilibrium** is where the two reaction curves cross. This is where $q_1=q_2=(2/3)q_M$. We can prove this mathematically by rearranging firm 2's reaction curve so that q_1 is the subject to give $q_1=2q_M-2q_2$ and then equating with the reaction curve for firm 1 to give $2q_M-2q_2=q_M-(1/2)q_2$. Solving this for q_2 gives $q_2=(2/3)q_M$. By symmetry, the same applies for q_1. The total industry output is therefore $(4/3)q_M$. The Cournot duopoly therefore results in a higher output than the monopolist but lower than the competitive market output of $q^*=2q_M$. It also turns out that as the number of firms in the Cournot oligopoly model increases, the total output gets closer and closer to the competitive output level (although each individual firm produces less and less because they get a smaller share of the total output when there are more firms).



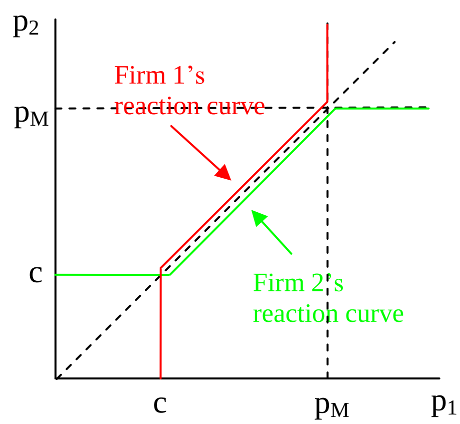
Since along their reaction curve each firm is maximizing their profits given their assumption of the quantity that the other player will set, there will be a tangency condition at each point along a reaction curve between each firm's isoprofit curve and the horizontal or vertical line representing the fixed quantity that the player is assuming is to be set by the other player. The diagram below illustrates this for firm 1 at the Cournot-Nash equilibrium. The isoprofit curve we are interested in is the one which firm 1 is on at the Cournot-Nash equilibrium. This is shown as the red dashed line. Since it is tangential to the horizontal line at $(2/3)q_M$, we know that part of this curve must lie above firm 2's reaction curve, which is shown in green.



Suppose we now make firm 1 the industry leader, so that they can set their quantity before firm 2. We must now solve the model using backwards induction by first considering which quantity firm 2 will optimally choose once firm 1 has set their quantity. This is given by firm 2's reaction curve. Firm 1 can treat this reaction curve as fixed, because firm 2 cannot credibly threaten to behave any other way, because once firm 1 has chosen their quantity, this curve represents firm 2's best response. Firm 1 wishes to reach their highest profit given firm 2's reaction curve. The fact that part of the red isoprofit line lies above firm 2's reaction curve means that firm 1 can reach a higher profit than at the Cournot Nash equilibrium. They will maximize their profits by creating a tangency condition, but this time between firm 2's reaction curve and their isoprofit curve, rather than the horizontal line as in the Cournot-Nash equilibrium. This is shown in purple, and results in a higher quantity q_1^S and profit level for firm 1 and a lower quantity q_2^S and profit level for firm 2 (lower isoprofit curves represent higher profits). This is called the **Stackelberg equilibrium**. It can in fact be shown (as we will see below) that if the demand curve is linear and marginal costs are identical and constant, then firm 1 will now produce the monopoly output level q_M and firm 2 will produce $(1/2)q_M$. The overall amount produced will be $(3/2)q_M$. This is greater than in the Cournot-Nash equilibrium with linear demand curves.

We can work out firm 1's optimal quantity in the above Stackelberg model by substituting firm 2's reaction curve into firm 1's profit function. Firm 2's reaction curve is $q_2=q_M-(1/2)q_1$. Firm 1's profit function is $\pi_1=q_1(a-bq_1-bq_2)-q_1c$. Combining these two equations gives: $\pi_1=q_1(a-(1/2)bq_1-bq_M)-q_1c$. Only once we have substituted in firm 2's reaction function into firm 1's profit function can we differentiate it to find the first order condition for firm 1's profit maximization. This is a reflection of the process of backwards induction. The derivative of firm 1's profit function is $\partial\pi_1/\partial q_1=a-bq_1-bq_M-c$. Equating this to 0 gives $q_1=(a-c)/b-q_M=2q_M-q_M=q_M$. Firm 2 then produces $q_2=q_M-(1/2)q_1=q_M-(1/2)q_M=(1/2)q_M$. The total industry output is therefore $(3/2)q_M$, which is greater than the total industry output of $(4/3)q_M$ at the Cournot-Nash equilibrium. Note that this is still less than the competitive quantity.

In the Bertrand model of duopoly, we keep the assumption of two players moving simultaneously in a single phase game. We also maintain the assumptions of identical constant marginal costs and a linear demand curve with the firms producing perfect substitutes. The strategic variable is now, however, price rather than quantity. We assume that the firm which charges the lowest price gains the entire market demand at that price. This means that the two firms' reaction curves will be as illustrated in the diagram below. If firm 1 sets a certain price which is greater than marginal cost but less than the monopoly price p_M, then firm 2 will want to just undercut that price by a tiny amount to take the entire market demand and all the profits. So, firm 1's demand curve will lie slightly to the left of the 45 degree line for all prices above c and below p_M. By the same token, firm 2's reaction curve will always lie slightly to the right of the 45 degree line for all prices higher then c and below p_M. Note also that if either firm charges less than c, the other firm will keep their price at c. (Although there are other prices which would lead to 0 profits provided they are greater than the price set by the other firm, keeping the price at c is a natural choice of best response since a price of c weakly dominates any price below c.) If the other firm sets a price above p_M, the firm will wish to set the price at p_M in order to get the monopoly profits. This means that the only place where the two reaction curves cross (i.e. the only Nash equilibrium) is where both firms price at marginal cost. If either firm had a price higher than marginal cost, the firms could not be making mutual best responses because whatever price set by each firm the other would want to just undercut it by a slight amount (or undercut to p_M if the other firm's price is greater than p_M). When both firms are pricing at marginal cost, however, they are making 0 profits and cannot make it any higher by changing their price given the price chosen by the other firm, and so this constitutes a Nash equilibrium.



If we change the Bertrand model so that firm 1 sets its price before firm 2, but still assume that the firm which charges the lower price takes the entire market demand, then this does not make any difference to the analysis because firm 2 will always choose to undercut firm 1's price in the same manner as described above unless firm 1 sets price at marginal cost (and vice versa for firm 1), and so both firm's setting their price equal to marginal cost is again the only Nash equilibrium.

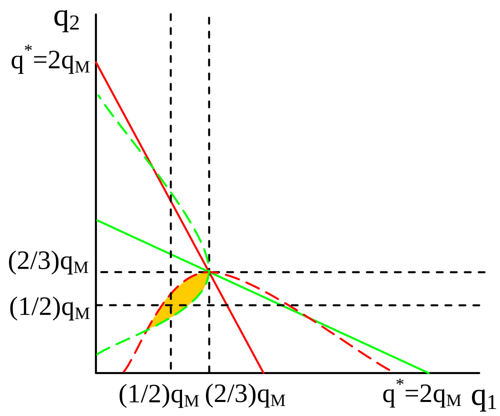
Which model is likely to best fit oligopolists' behaviour in a particular market depends on the empirical features of that market. If commitments must be made to output capacity in advance of price competition between firms, then it is likely that the Cournot model will be more applicable. Since capacity takes a long time to develop, whereas prices can usually quickly be changed, this provides a presumption in favour of the Cournot model in big industrial situations. This also fits our intuitive feeling that the number of firms in an oligopoly should influence the competitiveness of their behaviour.

On the other hand, if firms find it difficult to change their price after they have set it then the Bertrand model is more likely to be realistic. For example, in an auction where firms are bidding the price downwards, a firm must choose at what price to drop out of the bidding. Both firms will choose to do this only when price equals marginal cost (because otherwise they could still make a profit if the other firm dropped out, and so would be irrational to drop out themselves), so the price would be bid all the way down to marginal cost. So, by carefully designing the rules of auctions for various licenses (e.g. licenses for wavelengths for mobile phones and TV channels) the government can try to create a market situation in which firms will play the Bertrand equilibrium, which is obviously beneficial to the government because it raises as much revenue as possible (and therefore, hopefully, to society).

The Bertrand model may also be useful for modelling the behaviour of small firms. For example, if there are two cheese stalls at the local market then, assuming they cannot collude to keep their prices up, we would expect that neither would run the risk of losing all its customers to the other by putting their prices above marginal cost on a particular day (we would think here of the length of the game being a single morning's trading, whereas with the Cournot model in an industrial situation the single phase capacity building game could be played over a period of months or years). Another situation is which the Bertrand model would be more applicable would be if firms have excess capacity. For example, in the sunny season hotels in a particular area would be expected to decide their maximum capacities in line with the Cournot model. The rationing of hotel rooms between lots of potential tourists will keep the price up. However, once we get into the winter, it is likely that competition between hotels for the small number of customers will push the price of staying in a room right down to marginal cost, since each hotel has excess supply at all prices it chooses, so none will run the risk of being undercut. The key point to take away from the Bertrand model is that we do not *necessarily* have to have a large number of firms in a market for their behaviour to be competitive; it depends on other features of the market situation in which they interact.

(iii) If the two firms have different marginal costs then in the Bertrand case, the firm with the lower costs will want to charge just under the marginal cost of the firm with the higher costs, so that it captures the whole market. In the Cournot case, the firm with the higher cost produces less. Suppose that the demand curve is $p=a-b(q_1+q_2)$ and the two firms' profit functions are $\pi_1=q_1(a-bq_1-bq_2)-q_1c_1$ and $\pi_2=q_2(a-bq_1-bq_2)-q_2c_2$ respectively. The two firms' best response functions are therefore $q_1=(a-c_1)/(2b)-(1/2)q_2$ and $q_2=(a-c_2)/(2b)-(1/2)q_1$. Solving these two equations simultaneously gives us: $q_1=(1/3)(2a-2c_1+c_2)$ and $q_2=(1/3)(2a-2c_2+c_1)$. Each firm's output is decreasing in its own cost but increasing in the other firm's cost. This is because when the other firm has a higher cost, it decreases its output in the Cournot-Nash equilibrium, thus allowing the first firm to increase its output to take up part of the market that is vacated.

2. If the two players in the Cournot or Bertrand games discussed in question 1 could coordinate their actions by making an agreement which they could trust each other to keep, they could both get higher profits than in the Nash equilibrium in the single game. For example, they could both agree to produce $(1/2)q_M$ and split the monopoly profits 50-50. This would clearly be better for the Bertrand duopolists, who get 0 profits in the single phase Bertrand game. It is also better for the Cournot oligopolists, since they together produce more than the monopoly output in the Cournot equilibrium. They could increase the amount of profit they have to share between them by both restricting their quantity. Producing half of the monopoly output each is a natural point to agree upon because it maximizes the combined profits and splits them equally between the two identical firms. Any of the points in the yellow shaded area in the diagram below illustrate those points where both firms would be better off than in the Cournot Nash equilibrium, *provided* both firms stick to the agreed output quantities. However, we know that they would not optimally choose to do this because both are below their reaction curve. This means that there will be an incentive for both firms to cheat and raise their profit by raising their quantity above the agreed level. This will of course hurt the other firm. We can therefore see that there is a prisoners' dilemma structure to the problem of sustaining collusion; both firms would prefer to produce a high quantity whether or not the other firm produces a high quantity. However, the two firms have a joint interest in reducing their combined quantity in order to keep the price and therefore their profits high.



The easiest way to think about the prisoners' dilemma game played by oligopolists is to take the Bertrand duopoly game with constant and identical marginal costs. If the firms collude, we assume that they split the monopoly profits 50-50 and so get $(1/2)\pi_M$ each (e.g. they both set price p_M and half the demand goes to each firm). If either firm defects, they slightly undercut the other and get π_M whilst the other firm gets 0. If they both defect, we assume that they end up at the Bertrand Nash equilibrium in the standalone game, where they both get 0 profits. The strategic form of the game is therefore:

Firm 2's move / *Firm 1's move*	**Co-operate**	**Defect**
Co-operate	$(1/2)\pi_M$, $(1/2)\pi_M$	0, π_M
Defect	π_M, 0	0, 0

Since both firms know that they will do at least as well by defecting as they will by co-operating, whatever the other firm does (i.e. defection is a weakly dominant strategy) then they will both rationally always choose to defect. Even if the firms made a verbal agreement before the game begins, they would still both always rationally choose to break the agreement, and both would know this, and so any kind of agreement would be worthless. It is true that if the firms could be legally or physically forced to keep the agreement, it could be sustained. However, it is likely that the legal system will be designed if anything to make it harder rather than easier to force firm's to keep collusive agreements (e.g. competition policy).

Does all this mean that this kind of collusion cannot occur? No, because what the above analysis is missing out is the fact that firms do not interact in a single isolated period. They will in reality play the game many times. This means that it would be possible to punish firms who defect in a certain period by giving them a lower payoff in future periods by doing so. For example, firm 2 can say to firm 1 that if they ever break the agreement, they will play the Nash equilibrium in all future periods (i.e. price equal to marginal cost). This is called the threat of **Nash reversion**. This is a credible threat because firm 2 knows that firm 1 will optimally choose to price at marginal cost if firm 2 carries out the threat, and this would mean that firm 2 pricing at marginal cost would be a best response to firm 1's best response to firm 2's threat. To put it another way, the fact that marginal cost pricing is a Nash equilibrium in the subgame played in each period means that either player can always force the other player back to that equilibrium by declaring that they will play the Bertrand equilibrium from then on in the repeated game. However, although this threat is always credible, it is not always effective. In particular, it cannot work if the game is only repeated a finite number of times. The effectiveness of the threat also depends on the degree to which players discount their payoffs in future periods.

Suppose the duopolists knew that they were going to play the simple Bertrand game described above a certain number of times. Both players would then always know how many periods were left to play before the end of the repeated game. This means we would have to solve the model by backwards induction. Imagine we are in the final period. Firm 1 can no longer threaten firm 2 with anything if they defect, and likewise for firm 2, because the game ends after this period. The Nash equilibrium concept therefore requires that both firms defect, just as they would in a single standalone game. Suppose now we are in the penultimate period. Both firms know that they will both defect in the next period anyway, thus rendering the punishment from defecting ineffective. There is therefore nothing to stop both firms from defecting in the penultimate period anyway. We can carry on this chain of reasoning right up to the first period, and thus show that in a finitely repeated game, both firms will always defect in every period.

The problem of collusion *can* be solved when firms are playing an **infinitely repeated game**. This means that there is always a chance that the threat of Nash reversion will be effective and prevent a firm from ever defecting. For this to occur, it must be case that the firm values the benefit from defecting in the current period less than the loss incurred by Nash reversion in all the future subgames. If the future is discounted, then the sum of the loss of profit in all future periods will be finite. To take the Bertrand example, the gain in profits from defecting in the current period is $(1/2)\pi_M$ (equal to the profits which a firm gets when it defects whilst the other cooperates minus the profits it gets when both cooperate). Nash reversion means that the firm will lose $(1/2)\pi_M$ in every future period (this is the difference between the half of the collusive monopoly profit going to that firm and the Bertrand equilibrium profit of 0). If we let j be the number of periods into the future and r be the interest rate then the total present discounted value of the utility loss in all future periods will be given by:

$$\sum^{\infty}(1+r)^{-j}((1/2)\pi_M)$$

This is an infinite geometric series of the form:-

$$\sum^{\infty}(\delta^j M)$$

(where in this case $\delta=1/(1+r)$)

The sum of an infinite geometric series is given by $M(1/(1-\delta))$. Since the infinite punishment stream starts one period into the future, we must multiply this again by δ to capture the discounting one period into the future. So, our formula for the net gain from defecting is:

$$(1/2)\pi_M-(1/2)\pi_M(\delta/(1-\delta))$$
$$=(1/2)\pi_M(1-(1/(1+r))/(1-1/(1+r)))$$
$$=(1/2)\pi_M(1-1/(1+r)/(r/1+r))$$
$$=(1/2)\pi_M(1-1/r)$$

Note that $(1/r)M$ is the present discounted value of a perpetuity which pays M per period starting from next period. For the threat of Nash reversion to be effective (i.e. for defection to be deterred) we require the net gain from defecting to be negative. We therefore require that:

$$(1-1/r)<0$$
$$\Rightarrow r<1$$

So, under the assumptions made in this model, provided the interest rate is less than 100%, collusion will be sustained. Note that the significance of the interest rate is that it is the interest rate between this period and the next period when defection has been detected and Nash reversion enforced. So, if the annual interest rate is 5% but each period for the oligopoly game is 1 week (i.e. defection from the collusive agreement can be detected and Nash reversion enforced after one week) then the relevant interest rate will be the *weekly* interest rate, which will be less than 5% (approximately 5/52, ignoring compounding). So, the longer the period until defection can be detected, the higher the interest rate, and therefore the harder it is to sustain collusion. However, given that defection can usually be detected fairly quickly relative to the magnitude of the interest rates required to make it profitable to defect (i.e. more than a 100% interest rate in this case), it is generally found that collusion can be sustained indefinitely at realistic interest rates in an infinitely repeated oligopoly game regardless of the modelling assumptions made (e.g. Cournot or Bertrand in the single period game).

What does all this mean from the point of view of economic policy? Well, it means that game theory suggests that there is a real problem for public policy that oligopolistic firms can make tacit collusive agreements which are self-supporting in infinitely repeated games. This is bad for consumers and for society because it pushes up prices, reduces consumers' surplus and creates a deadweight loss. This creates a rationale for competition policy, which aims to make it as difficult as possible for oligopolists to make and enforce collusive agreements of this type. Ideally, competition policy aims to get every oligopolistic market to behave as much like the competitive market as possible. It does this by making any kind of communication between firms over price fixing illegal, and by making firms' pricing policies in principle open to legal challenge if economists can show that they harm consumers and therefore usually society as a whole by reducing total surplus.