Signals

Signals are functions of independent variables that carry information about the behaviour or nature of some phenomenon.

Systems

A system is an entity that responds to particular signals by producing other signals or some desired behaviour.

Examples:

1. Voltages and currents as a function of time in an electrical circuit are examples of signals. Circuit itself is an example of system, which in this case responds to applied voltages and currents.
2. When an automobile press driver press the accelerator, the speed of auto mobile increases. Here the automobile is the system and the pressure on the accelerator is the signal.
3. A camera is a system and the signal in this case is light from different sources and reflected from objects to produce image.
4. Acoustic signals ---audio or speech signals (analogue or digital). A very common example is a mobile phone. The cell phone is a system and which can respond to both transmitting and receiving signals.
5. A robot arm is a system, and the control inputs given are signals.

The scope of Signals and Systems

The concept of signals and systems arise in wide variety of fields and play important role in diverse areas of sciences and technology such as communications, circuit design, energy generation, signal processing, control engineering, etc. In many contexts in which the concept of signals and systems arise, a wide variety of problems are of importance:

- Characterizing a system in detail to understand how it will respond to various inputs. For e.g. analysis of a circuit in order to quantify its response to different voltages and current sources, the determination of an air craft's response characteristics both to pilot commands and to wind gusts.

- Designing systems to process signals in particular way. For e.g. a very common context in which such problem arise is in the design of systems to enhance or restore the signals that have been degraded in some way. For example, when a pilot is communicating with an air traffic control tower, the communication can be degraded by the high level of background noise in the cockpit. In this and many similar cases, it is possible to design systems that will retain the desired signal, in this case the pilot's voice, and reject (at least approximately) the unwanted signal, i.e. the noise. Another example in which it has been useful to design a system for restoration of a degraded signal is in restoring old recordings.

A similar set of objectives can also be found in the general area of image restoration and image enhancement. In receiving- images from deep space probes, the image is typically a degraded version of the scene being photographed because of limitations on the imaging equipment, possible atmospheric effects, and perhaps errors in signal transmission in returning the images to earth. Consequently, images returned from space are routinely processed by a system to compensate for some of these degradations. In addition, such images are usually processed to enhance certain features, such as lines (corresponding, for example, to river beds or faults) or regional boundaries in which there are sharp contrasts in color or darkness. The development of systems to perform this processing then becomes an issue of system design.

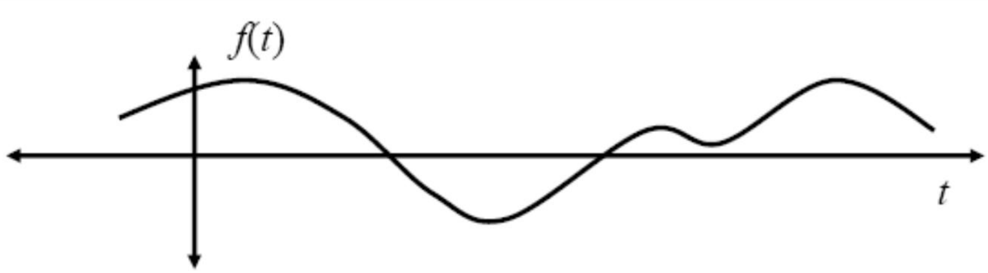
- In many applications there is a need to design systems to extract specific pieces of information from signals. For e.g. the estimation of heart rate from an electrocardiogram, economic forecasting in which we may wish to analyze history of economic time series to make prediction about future behavior of stock market.

- Designing of signals with specific properties. For e.g. in communications applications considerable attention is paid to designing signals that meets the constraints and requirements for successful transmission. (The constraints can be (i) long distance communication through atmosphere requires the use of signals with frequencies in particular EM spectrum (ii) Interference by other transmitted signals and distortion due to transmission must also be taken into account while designing communication signals)

<u>CLASSIFICATION OF SIGNALS:</u>

1-Continuous time signals:

A C-T signal is specified for any value of t ($t \in R$).Here we have used the symbol ‘t’ to denote the CT independent variable. Continuous time signals are denoted by parenthesis i-e x(t).



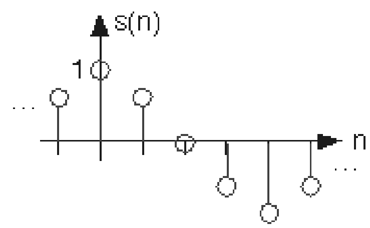
$t \in R$ where $f(t) \in R$

Example:

Telephone and video camera output, speech signal as a function of time, atmospheric pressure as a function of altitude.

2-Discrete time signals:

A DT signal is specified only at integer/discrete values of independent variable. We use symbol ‘n’ to denote DT independent variable. Discrete time signals are denoted by square brackets i-e x[n].



Example:

Weekly Dow Jones stock market index.

Representation of number of family members living in a house of a particular community in the form of graph.

A DT signal may represent a phenomenon which is inherently discrete. Otherwise we use sampling to convert a CT signal into DT signal.

3-Analog signals:

A signal whose amplitude can take on any value in a continuous range. It means that there can be infinite number of possible values between minimum and maximum of an analog signal.

CT analog signal $x(t) \in R$

DT analog signal $x(n) \in R$

4-Digital signals:

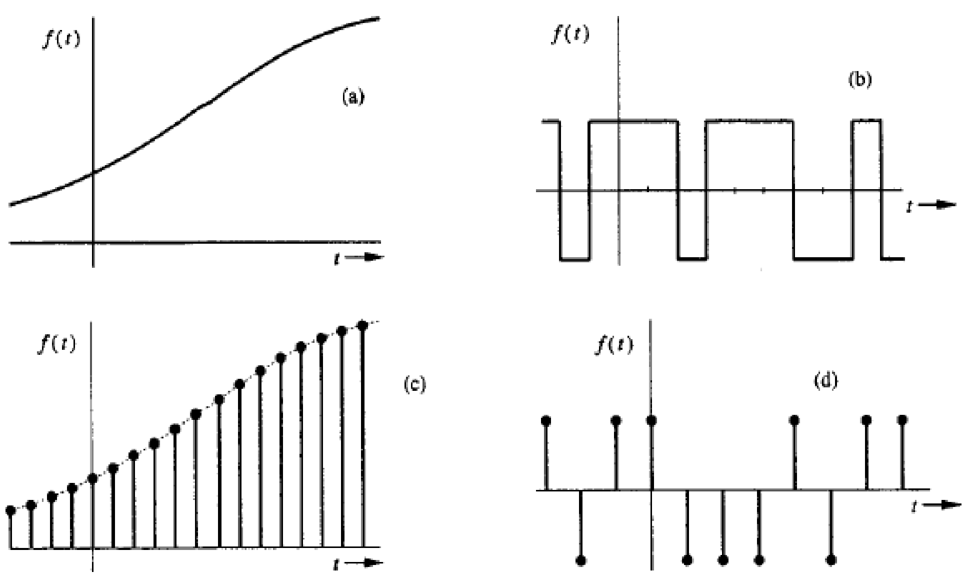
A signal whose amplitude can take only a finite number of values between minimum and maximum point of a signal.

CT digital signal $x(t) \in Z$

DT digital signal $x(n) \in Z$

Comments.

The terms continuous-time and discrete-time qualify the nature of a signal along the time (horizontal) axis. The terms analog and digital, on the other hand, qualify the nature of the signal amplitude (vertical axis). Figure below shows examples of various types of signals. It is clear that analog is not necessarily continuous-time and digital need not be discrete-time. Figure (c) below shows an example of an analog discrete-time signal. An analog signal can be converted into a digital signal [analog-to-digital (AID) conversion] through quantization (rounding off).



Examples of Signals: (a) analog, continuous-time (b) digital, continuous-time (c) analog, discrete-time (d) digital, discrete-time.

5-Periodic signals:

A CT signal is said to be periodic signal if for some positive constant :

$$x(t) = x(t + T_0) \qquad for\ all\ 't'$$

To = smallest integral such that the signal amplitude starts repeating.

= period of a periodic signal x(t).

The smallest value of To that satisfies the periodicity conditions is called period of a periodic signal. Due to periodicity it is also true that

$$x(t) = x(t + mT_0) \ \forall \ t; \ \text{for any integer m.}$$

Because, a periodic signal x(t) remains unchanged when time shifted by one period. It starts at $t = -\infty$ and continues forever. If it starts at t=0 then the condition $x(t) = x(t + mTo)$ cannot be fulfilled for all values of m.

In above discussion , we have defined periodicity of a CT signal. For DT signals we can define the periodicity in similar fashion.

$$x[n] = x[n + N_0]$$

$$= x[n + mN_0]$$

N_0 = period of DT periodic signal.

If No is the period of DT signal, 2N,3N will also be period of DT periodic signal just like CT signals (2To,3To).

Both DT & CT periodic signals:

$$x(t) = t\,(t + mT_0)$$

$$x[n] = x[n + mN_0]$$

can be generated by periodic extension of any segment of duration of the fundamental period (To or No).

6-Aperiodic signals

A signal is aperiodic if it is not periodic signal .Or in other words , a CT signal is said to be aperiodic if there is no such positive constant To ,for which the following equation gets satisfied:

$$x(t) = x(t + T_0)$$

For e.g.

x (t) = constant , is an aperiodic signal.

(fundamental period, T_0, not defined).

Exercise.

Check the following signals for periodicity:

(i) x[n] $= \cos 2n\pi$

$(ii) x[n] = [\frac{2n\pi}{20} + 4\pi]$

$(iii) x[n] = \cos 2n$

$(iv) y(t) = \sin(\frac{2\pi t}{5})$

$(v) x(t) = cos^2 \ 2\pi t$

$(vi) x(t) = e^{-2t} \ . \ \cos 2\pi t$

$(vii) x(t) = sint + \cos\sqrt{3}\, t$

7-Real and Complex value signals

A signals is real valued if its dependent variable can take only real values. For e.g. x(t) = $x(t) = \sin \ wt$

A signals is complex valued if its dependent variable can take on complex values. For e.g. $x(t) = e^{-j2t}$

8- Energy and Power Signals

A signal with finite energy is an energy signal. The mathematical expression for a energy signal is:

$$\boldsymbol{E = \int_{-\infty}^{\infty} |x^2(t)| dt}$$

A signal is an energy signal if its amplitude → 0 as t → ∞, otherwise , we have to define a "power" signal.

When the amplitude does not → 0 as t → ∞, the signal energy become infinite. Then a more meaningful measure is the time average of energy. This measure is called *Power* of the signal. Mathematically:

$$P = Lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|x^2(t)|dt$$

Comments .

- If a signal is a power signal, it has Infinite Energy.
- If a signal is an energy signal, it has Zero power.
- All practical signals have finite energies, and are therefore energy signals. It's impossible to generate a true power signal in practice because such a signal has infinite duration and infinite energy.
- Because of periodic repetitions, Periodic signals are normally power signals.
- Sometimes, a signal may neither be a power signal nor an energy signal, e.g. e^{-at}.

Exercise .

(i) Determine whether the following signal is Energy or power Signal

$$x(t) = \{5\cos\pi t, -0.5 \le t < +0.5\}$$

Solution :

$$E = 25\int_{-0.5}^{0.5}\cos^2 \pi t dt$$

$$= 25\int_{-0.5}^{0.5} 1 + \cos 2\pi t / 2dt$$

$$= 25/2\int_{-0.5}^{0.5} 1 + \cos 2\pi t dt$$

$$= 12.5[t_{-0.5}^{..5} + \sin 2\pi t / 2\pi_{-0.5}^{+0.5}]$$

$$= 12.5[(.5 + .5) + 1/2\pi[\sin\pi - \sin(-\pi)]$$

$$x(t) = 12.5[1 + 0] = 12.5$$

(ii) Determine whether the following signal is Energy or power Signal

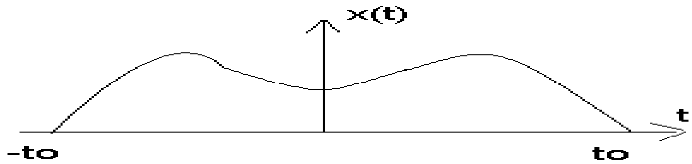
$x(t) = \cos\omega .t$

9-Even and odd Signals

- A signal $x(t)$ is an even signal if:

$$\boldsymbol{x(t) = x(-t)}$$

For e.g. Cosine Function. Another example of an even function is given below**:**

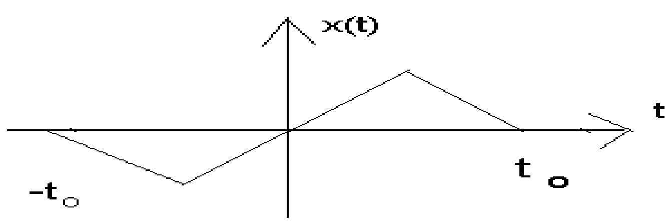


Even functions are symmetrical about vertical axis.

- A signal $x(t)$ is an odd signal if:

$$\boldsymbol{x(t) = -\,x(-t)}$$

For e.g Sine Function**.** Another example of an even function is given below:



Odd functions are anti symmetric about vertical axis.

Comments.

Any signal can be broken into a sum of two signals, one of which is odd and one of which is even.

Even part is given by:

$$\boldsymbol{x_e(t) = \frac{1}{2}[x(t) + x(-t)]}$$

Odd part is given by:

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

Then the complete signal wil be :

$$x(t) = x_o(t) + x_e(t)$$

10-Deterministic and Random Signals

A deterministic signal is the one about which there is no uncertainty w.r.t its value at any time. For e.g a square wave.

A random signal is a signal about which there is uncertainty before its occurrence. For e.g electrical noise generated in the amplifier of radio or tv.

Some Useful Signal Operations

We discuss here three useful signal operations: shifting, scaling, and reflection(reversal).Since the independent variable in our signal description is time, these operations are discussed as time shifting, time scaling, and time reversal. However, this discussion is valid for functions having independent variables other than time (e.g., frequency or distance).

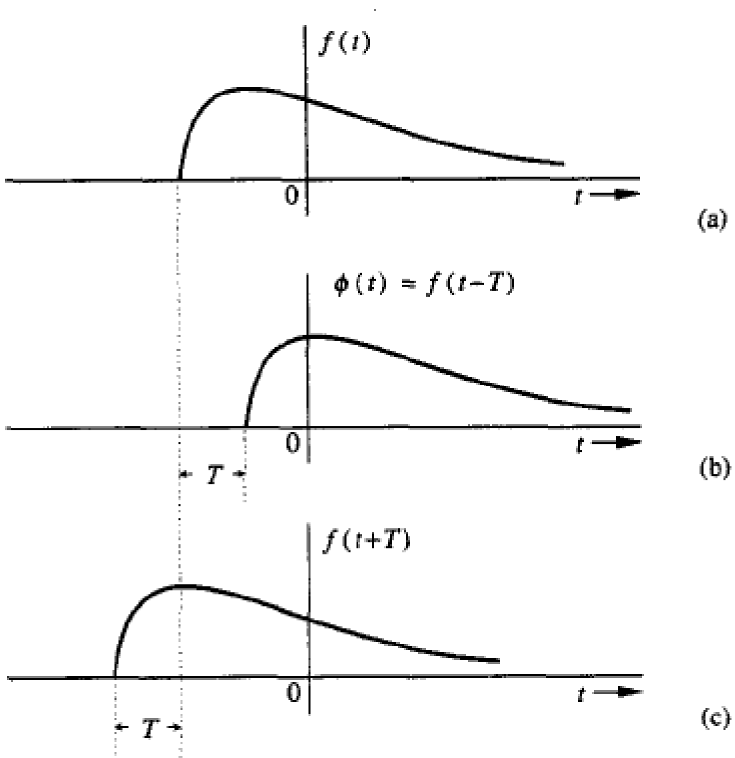
1. Time Shifting

Consider a signal f *(t)* as shown in fig(a) below and the same signal delayed by T seconds (Fig. b), which we shall denote by $\emptyset(t)$. Whatever happens in f (t) at

some instant *t* also happens in $\emptyset(t)$, T seconds later at the instant *t* + T. Therefore:

$\emptyset(t+T) = f(t)$ or

$\emptyset(t) = f(t-T)$



Therefore, to time-shift a signal by T, we replace t with t - T. Thus f (t - T) represents *f* (t) time-shifted by T seconds.

- If T is positive, the shift is to the right (delay).
- If T is negative, the shift is to the left (advance).

Thus, f (t - 2) is f *(t)* delayed (right-shifted) by 2 seconds, and f (t + 2) is f (t) advanced (left-shifted) by .2 seconds.

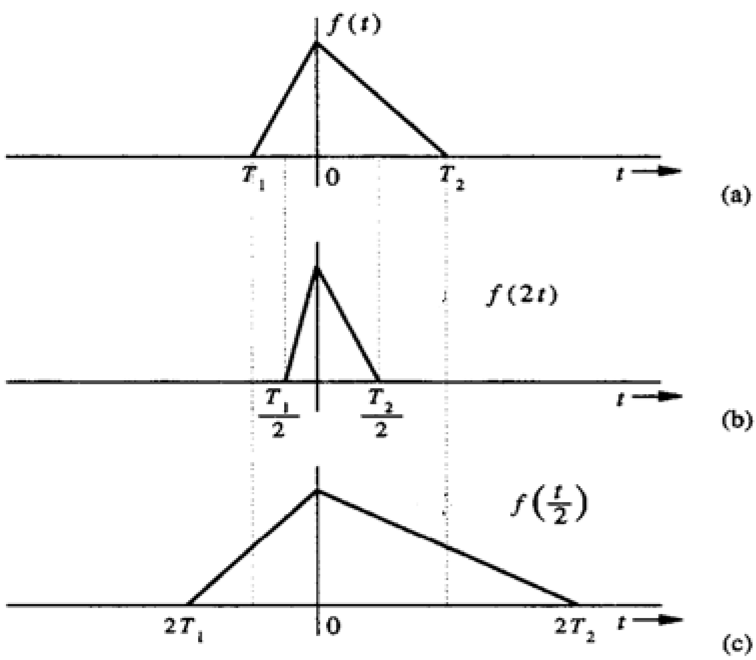
2. Time Scaling

The compression or expansion of a signal in time is known as time scaling. Let $f(t)$ denote a continuous time signal, then the signal $y(t)$, obtained by scaling the independent variable't' , by a factor 'a' , is given by:

$$y(t) = f(at)$$

- If a>1; $y(t)$ will be a compressed version of $f(t)$
- If 0<a<1; $y(t)$ will be an expanded version of $f(t)$

Consider the figure below. Fig (b) is an example of compression by a factor of 2 and fig (c) is an example of expansion by a factor of ½.



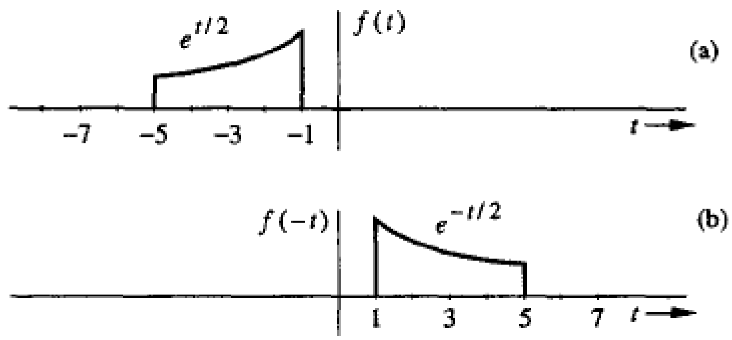
It should be noted that the origin t=0 is the anchor point, which remains unchanged under the scaling operation because at t=0, x (t) = x (at) = x (0)

3. Time Reversal

A signal $y(t)$ will be the reflected version of $f(t)$, about the origin t=0 , if:

$$y(t) = f(-t)$$

i-e just replace t by –t.



- Even signals are same as their reflected signals.
- Odd signals are negative of their reflected signals.

Combined Operations

Certain complex operations require simultaneous use of more than one of the above operations. The most general operation involving all the three operations is

$$y(t) = x(at - b)$$

The easiest way to realize this is in the following sequence of operation:

1. Time-shift x*(t)* by b to obtain v = x *(t* - b).
2. Now time-scale the shifted signal x (t - b). i-e $y(t) = v(at)$

For instance, the signal x (2t - 6) can be obtained by first delaying x(t) by 6 to obtain x(t - 6) and then time-compressing this signal by factor 2 (replace t with 2t) to obtain x (2t - 6).

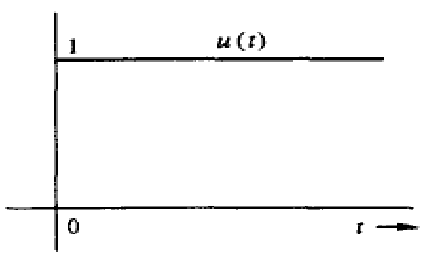
Some Useful Signal Models

In the area of signals and systems, the step, the impulse, and the exponential functions are very useful. They not only serve as a basis for representing other signals, but their use can simplify many aspects of the signals and systems.

1. Continuous time Unit Step Function u (t).

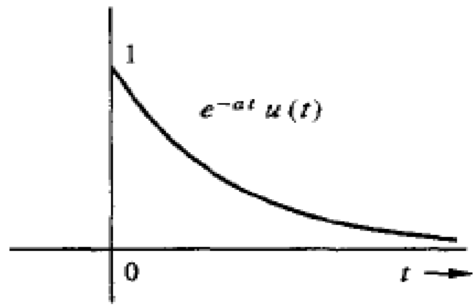
In much of our discussion, the signals begin at t = 0 (causal signals). Such signals can be conveniently described in terms of unit step function u(t) shown in Fig (a). This function is defined by:

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



(a)

If we want a signal to start at t = 0 (so that it has a value of zero for t < 0), we only need to multiply the signal with u(t). For instance, the signal e^{-at} represents an everlasting exponential that starts at t = ∞. The causal form (t>0) of this exponential illustrated in Fig. b can be described as $e^{-at}\,u(t)$.

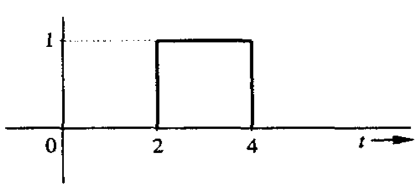


(b)

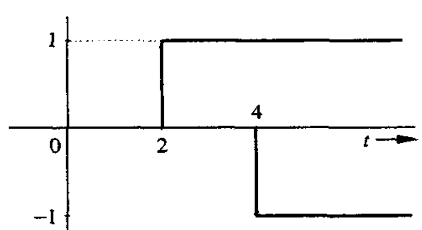
exponential $e^{-at}u(t)$.

The unit step function also proves very useful in specifying a function with different mathematical descriptions over different intervals. Using the unit step function, we can describe such functions by a single expression that is valid for all *t*. Consider, for example, the rectangular pulse depicted in Fig (a). We can express such a pulse in terms of familiar step functions by observing that the pulse $f(t)$ can be expressed as the sum of the two delayed unit step functions as shown in Fig (b). The unit step function $u(t)$ delayed by T seconds is $u(t-T)$. From Fig (b), it is clear that :

$$f(t) = u(t-2) - u(t-4)$$



(a)



(b)

2. The Continuous time Unit Impulse Function $\delta(t)$

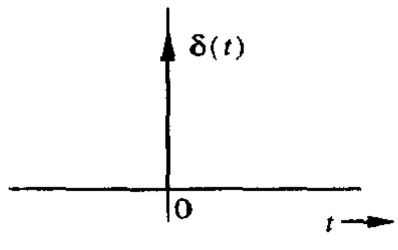
The unit impulse function is one of the most important functions in the

study of signals and systems. This function was first defined by P. A**.** M Dirac as:

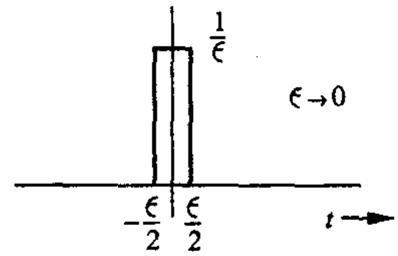
$$\delta(t) = 0 \qquad t \neq 0 \; \ldots\ldots\ldots\ldots\ldots (1)$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1 \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (2)$$

We can visualize an impulse as a tall, narrow rectangular pulse of unit area, as illustrated in Fig (b). The width of this rectangular pulse is a very small value $\in \to 0$. Consequently, its height is a very large value $1/\in$. The unit impulse therefore can be regarded as a rectangular pulse with a width that has become infinitesimally small, a height that has become infinitely large, and an overall area that has been maintained at unity. Thus $\delta(t)$= 0 everywhere except at t = 0, where it is undefined. For this reason a unit impulse is represented by the spear-like symbol in Fig (a).



(a)



(b)

From eq. 2 it follows that the function $k\delta(t) = 0$, for all t $\neq$ 0, and its area is k. Thus, $k\delta(t)$ is an impulse function whose area is k (in contrast to the unit impulse function, whose area is 1).

Sampling Property of the Unit Impulse Function

Let us now consider what happens when we multiply the unit impulse $\delta(t)$ by a function $\emptyset(t)$that is known to be continuous at t = 0. Since the impulse exists only at t = 0, and the value of $\emptyset(t)$at t = 0 is $\emptyset(0)$, we obtain :

$$\emptyset(t)\delta(t) = \emptyset(0)\delta(t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

Similarly, if $\emptyset(t)$is multiplied by an impulse $\delta(t-T)$ (impulse located at t = T), Then provided $\emptyset(t)$is continuous at t = T

$$\emptyset(t)\delta(t-T) = \emptyset(\mathrm{T})\delta(t-T) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

From Eq. (3) it follows that:

$$\int_{-\infty}^{\infty} \emptyset(t)\delta(t)\, dt = \emptyset(0)\int_{-\infty}^{\infty} \delta(t)\, dt = \emptyset(0) \ldots\ldots\ldots\ldots\ldots\ldots..(5)$$

provided $\emptyset(t)$ is continuous at t = 0. This result means that the area under the product of a function with an impulse $\emptyset(t)$is equal to the value of that function at the instant where the unit impulse is located. This property is very important and useful, and is known as the sampling or sifting property of the unit impulse.

From Eq. (4) it follows that

$$\int_{-\infty}^{\infty} \emptyset(t)\delta(t-T)\, dt = \emptyset(T) \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(6)$$

Equation (6) is just another form of sampling or sifting property. In the case

of Eq. (6), the impulse $\delta(t)$ is located at t = *T*. Therefore, the area under

$\emptyset(t)\delta(t-T)$ is $\emptyset(T)$, the value of $\emptyset(T)$ at the instant where the impulse is located

(at *t* = T). In these derivations we have assumed that the function is continuous

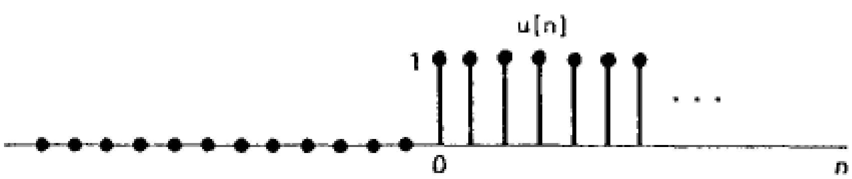
at the instant where the impulse is located.

3. Discrete time Unit Step Function u [n]

The counterpart of the continuous-time step function is the discrete-time unit step, denoted by u[n] and defined by:

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

The discrete time unit step function is shown below in the figure:



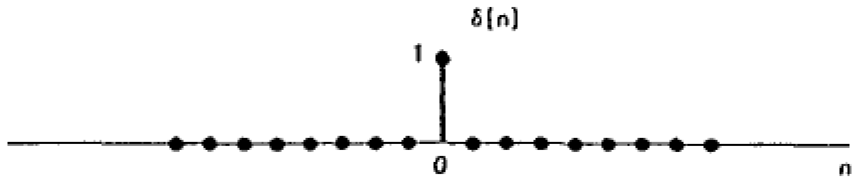
Unit step sequence.

4. Discrete time Unit Impulse Function δ[n]

Similarly, discrete-time impulse signal is denoted by $\delta[n]$ and defined by:

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

The discrete time unit impulse is shown below in the figure:



Unit sample (impulse).

Comment .

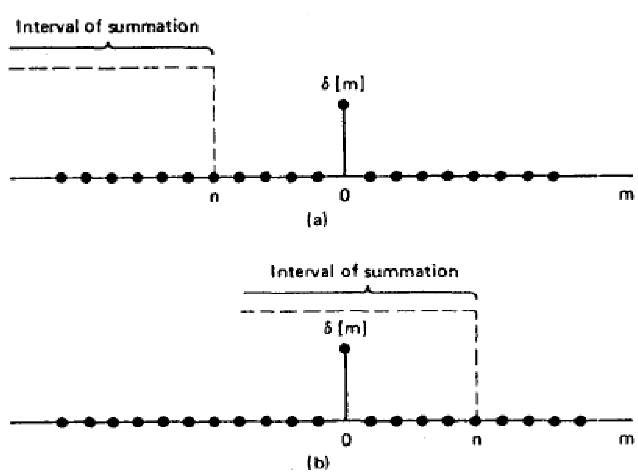
There is a close relationship between discrete time Unit Impulse and unit step. In particular, the discrete-time unit impulse is the first difference of the discrete-time step:

$$\delta[n] = u[n] - u[n-1] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

Similarly, the discrete-time unit step is the running sum of the unit sample. That is

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \,(8)$$

Eq. 8 is illustrated graphically in the figure below :



(a) $n < 0$; (b) $n > 0$.

Since the only nonzero value of the unit sample is at the point at which its argument is zero, we see from the figure that the running sum in eq. 8 is 0 for $n < 0$ and 1 for $n \geq 0$.

5. Continuous-Time Complex Exponential and Sinusoidal Signals

The continuous-time *complex exponential signal* is of the form:

$$\mathrm{x}(t) = C\, e^{at} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

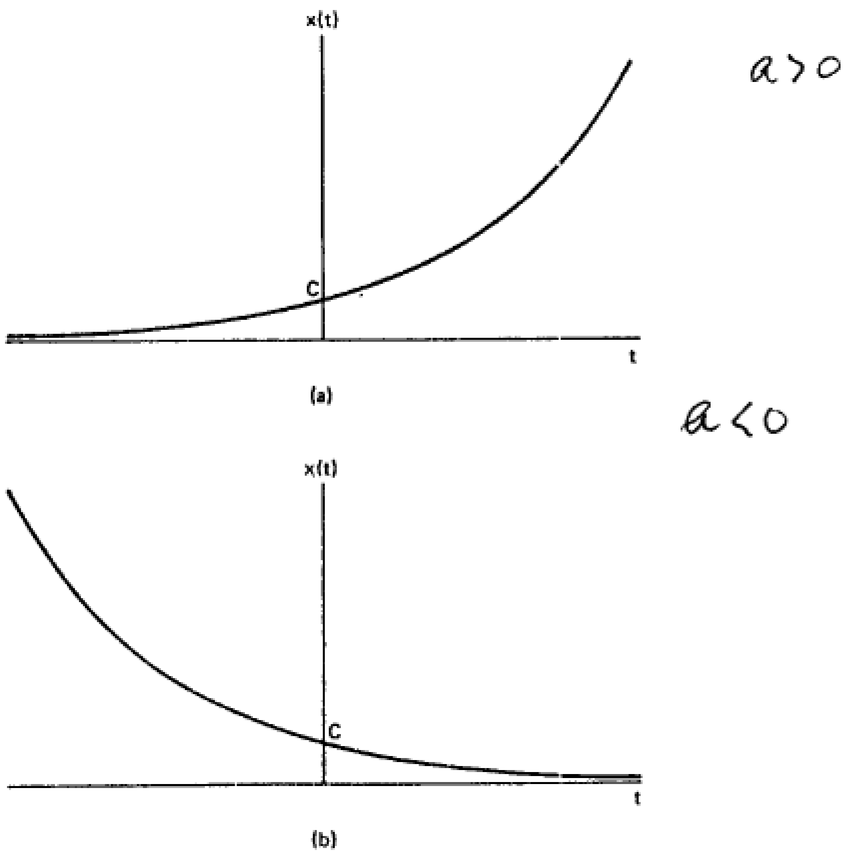
where C and *a* are, in general, complex numbers. Depending upon the values of these

parameters, the complex exponential can take on several different characteristics. As

illustrated in Figure below, if C and a are real [in which case x(t) is called a real exponential], there are basically two types of behaviour:

- If *a* is positive, then as 't' increases *x(t)* is called as a growing exponential,
- If a is negative, then x(t) is a decaying exponential.

This is shown in the figure below:



Continuous-time real exponential $x(t) = Ce^{at}$: (a) $a > 0$; (b) $a < 0$.

A second important class of complex exponentials is obtained by constraining a

to be purely imaginary. Specifically, consider:

$$\mathrm{x(t)} = \mathrm{e}^{\mathrm{j}w_0 \mathrm{t}}$$

An important property of this signal is that it is **periodic.** To verify this, we recall

From that x(t) will be periodic with period T if :

$$e^{jw_0 t} = e^{jw_0 (t+T)}$$

or, since

$$e^{jw_0 (t+T)} = e^{jw_0 t} . e^{jw_0 T} \dots\dots\dots\dots\dots\dots\dots .. (10)$$

It follows that for periodicity, we must have that

$$e^{jw_0 T} = 1 \dots\dots\dots\dots\dots\dots .. (11)$$

If $w_0 = 0$, then eq. (11) gets satisfied easily. If $w_0 \neq 0$, then the fundamental period T_o of x(t), that is, the smallest positive value of T for which eq. (11) holds, is given by:

$$T_0 = \frac{2\pi}{|w_0|} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots . (12)$$

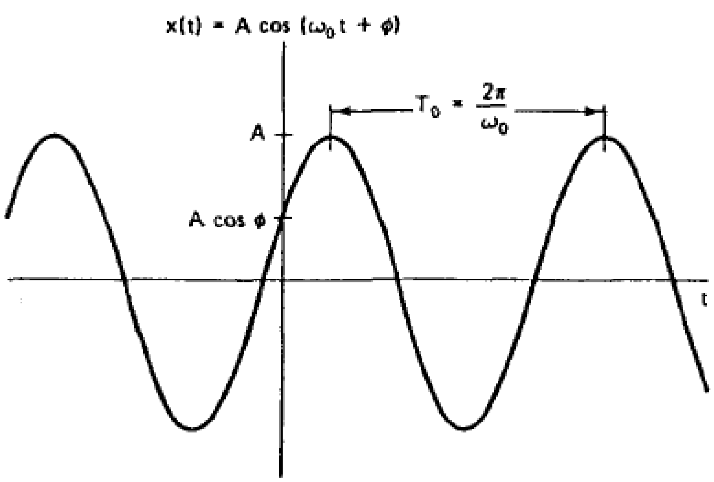
Thus $e^{jw_0 t}$ and $e^{-jw_0 t}$ have the same fundamental period.

A signal closely related to the periodic complex exponential is the sinusoidal

Signal which is defined by the equation:

$$\boldsymbol{x(t) = A\, cos(w_0 t + \phi)}$$

as shown in Figure below:



Continuous-time sinusoidal signal.

Where,

T = seconds

w_0 = radians

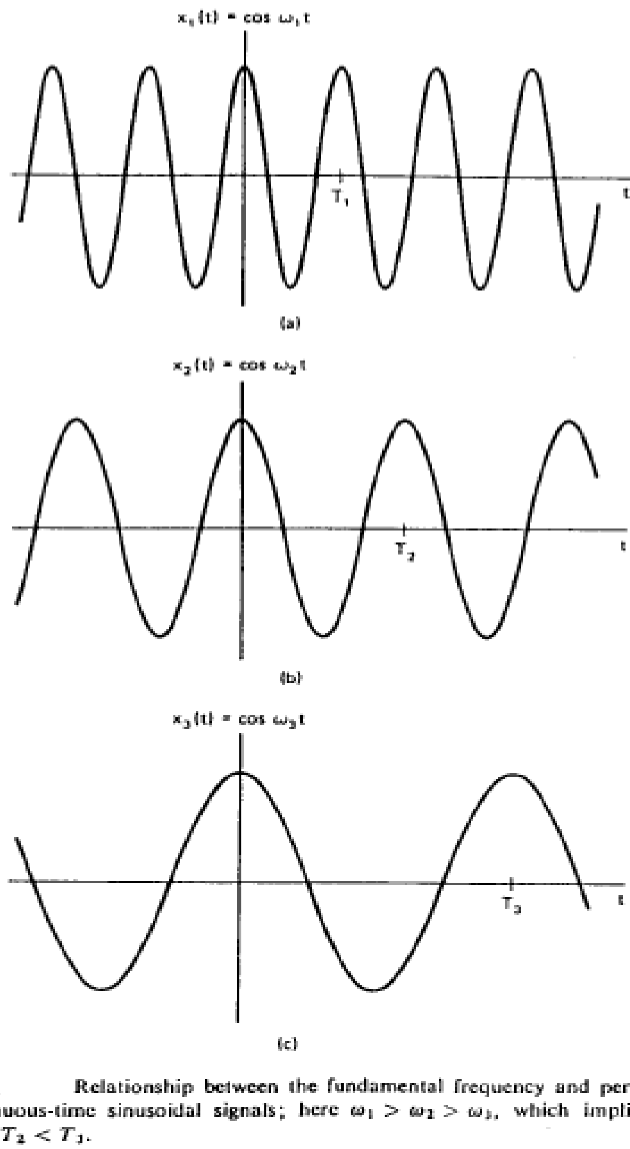
ϕ = radians per second.

$w_0 = 2\pi f_0$ where f, has the units of cycles per second or Hertz (Hz).

The sinusoidal signal is also periodic with fundamental period T_o given by eq. (12).

Comments.

1. From eq. (12) we see that the fundamental period T, of a continuous-time Sinusoidal signal or a periodic complex exponential is inversely proportional to $|w_0|$, which we will refer to as the fundamental frequency. From Figure below we see graphically what this means:



Relationship between the fundamental frequency and period for continuous-time sinusoidal signals; here $\omega_1 > \omega_2 > \omega_3$, which implies that $T_1 < T_2 < T_3$.

If we decrease the magnitude of w_0, we slow down the rate of oscillation and therefore increase the period. Exactly the opposite effects occur if we increase the magnitude of a,. Consider now the case w_0, = 0. In this case, x(t) is constant and therefore is periodic with period T for any positive value of T. Thus, the fundamental period of a constant signal is undefined. On the other hand, there is no ambiguity in defining the fundamental frequency of constant signal to be zero. That is, a constant signal has a zero rate of oscillation.

2. Periodic complex exponentials play a central role in a substantial part of treatment of signals and systems. On several occasions its useful to consider the notion of harmonically related complex exponentials, that is, sets of periodic exponentials with fundamental frequencies that are all multiples of a single positive frequency w_o :

$$\phi_k(t) = e^{jkw_0 t} \ , \ k = 0, \pm 1, \pm 2, \pm 3, \dots ..$$

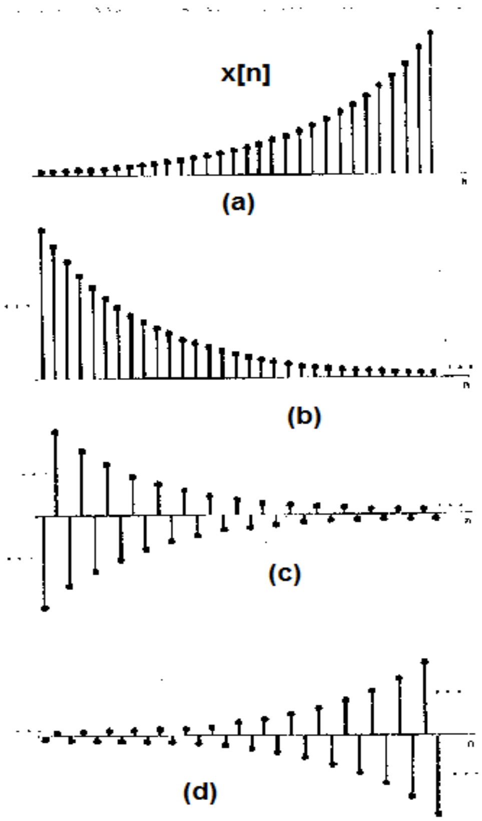
For ***k*** = 0, $\phi_k(t)$ is a constant, while for any other value of k, $\phi_k(t)$is periodic with fundamental period $\frac{2\pi}{|k|w_0}$ or fundamental frequency $|k|w_0$ **.** Since a signal that is periodic with period T is also periodic with period mT for any positive integer m, we see that all of the $\phi_k(t)$ have a common period of $\frac{2\pi}{w_0}$. The use of the term "harmonic" is consistent with its use in music, where it refers to tones resulting from variations in acoustic pressure at frequencies which are harmonically related.

6. Discrete -Time Complex Exponential and Sinusoidal Signals

As in continuous time, an important signal in discrete time is the *complex exponential signal* or *sequence*, defined by:

$$x[n] = C\,\alpha^n \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.. \quad (13)$$

If C and α are real, we can have one of several types of behaviours, as illustrated below:



$x[n] = C\alpha^n$: (a) $\alpha > 1$; (b) $0 < \alpha < 1$; (c) $-1 < \alpha < 0$; (d) $\alpha < -1$.

Basically if $|\alpha|> 1$, the signal grows exponentially with n, while if $|\alpha|< 1$we have a decaying exponential. Furthermore, if α is positive, all the values of $C\ \alpha^n$ are of the same sign, but if α is negative, then the sign of x[n] alternates. Note also that if α = 1, then x[n] is a constant, whereas if α = -1, x[n] alternates in value between +C and -C. Real discrete-time exponentials are often used to describe population growth as a function of generation and return on investment as a function of day, month, or quarter.

→ **As in the continuous-time case, the discrete exponential signal is closely related to the sinusoidal signal. The <u>discrete time sinusoidal signal</u> defined by:**

$$x[n] = A\ cos\ (\ \Omega_0 n + \Phi)$$

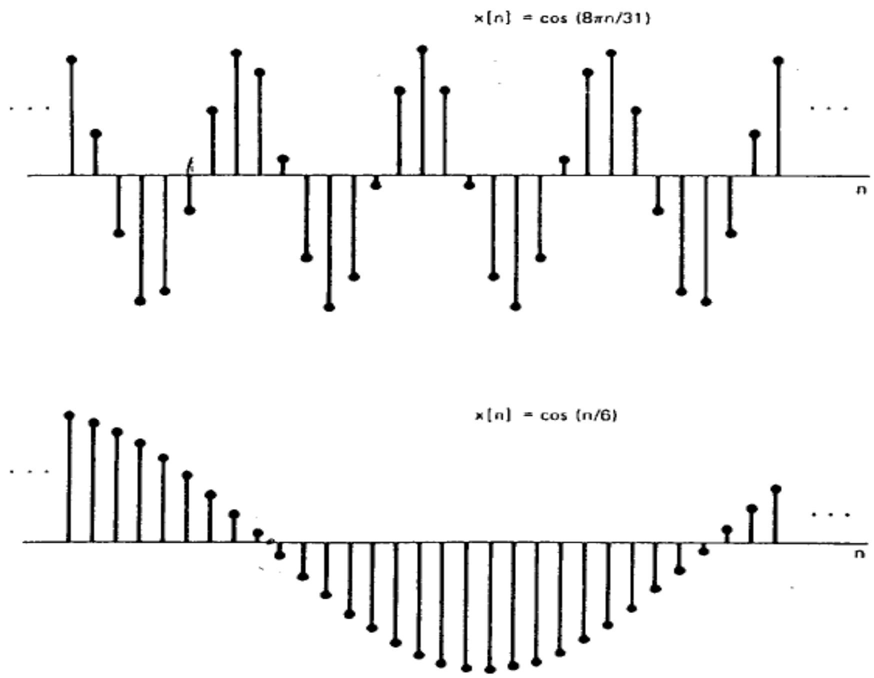
Where,

n = dimensionless

Ω_0 = radians

ϕ = radians

Two examples of sinusoidal sequences are shown in Figure:



<u>Periodicity Properties Of DT-Complex Exponentials</u>

While there are many similarities between CT and DT signals, there are also number of important differences between them.

(1) CT signals $e^{j\omega 0t}$complex all district for distinct values of w but this is not the case of DT exponential signal $e^{j\omega 0n}$. In order to clear this difference, let us consider a DT complex exponential signal with frequency ω+2π.

$$x[n] = e^{j(\omega 0+2\pi)n} = e^{j2\pi n}\, e^{j\omega 0n} = e^{j\omega 0n} \qquad \text{since } e^{j2\pi n} = 1$$

Or more general,

$$e^{j(\omega 0+2\pi k)n} = e^{j2\pi kn}\, e^{j\omega 0n} = e^{j\omega 0n}$$

Therefore DT complex exponential sequence at frequency ω_0 is the same as that ($\omega_0 \pm 2\pi$), ($\omega_0 \pm 4\pi$)and so n. In case of CT experimental $e^{j\omega 0t}$ are all distinct for distinct values of ω_0. Therefore, for DT exponential, we need only consider an instinct of length 2π in which to choose ω,

$$(\ 0 \ \leq \ \omega i8 \ \leq \ 2\pi \quad or \quad -\pi \ \leq \ \omega 0 \ \leq \ \pi \ )$$

Explain figure 1.27, page 27

Slow varying frequencies : ω0 near 0 or 2π and even multiples of π.

High varying frequencies : ω0 near ±π and other odd multiples of π.

(2) CT complex exponential signals $\mathrm{x(t)} = e^{\omega 0t}$ are periodic for any value of w where as DT complex exponential signals are only periodic for certain frequencies.

CT complex experimental -> always periodic

sDT complex experimental -> may be periodic for certain signals