EEE113
CIRCUIT ANALYSIS I

Chapter 3
Methods of Analysis

Materials from Fundamentals of Electric Circuits, Alexander & Sadiku 4e, The McGraw-Hill Companies, Inc.

Methods of Analysis - Chapter 3

3.1 Introduction
3.2 Nodal analysis *(without voltage sources)*
3.3 Nodal analysis with voltage sources
3.4 Mesh analysis *(without current sources)*
3.5 Mesh analysis with current sources.

3.1 Introduction (1)

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There are two ways to write the minimum number of simultaneous equations to solve a given circuit, to obtain required current or voltage values.

Analysis Methods

Nodal

- -Define & label **voltage drop/rise** at nodes
- -Use **KCL** to analyze it

Mesh

- -Define & label **current flow** in a loop
- -Use **KVL** to analyze it

3.1 Introduction (2)

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Methods of analysis in solving any resistive circuit with current and voltage sources requires knowledge in: **KCL** , **KVL** , Ohm's Law

recap: By passive sign convention:
Current flows from hi potential to lo potential in a resistor.

$$i = \frac{v_{higher} - v_{lower}}{R}$$

How to apply these laws?

3.2 Nodal Analysis (1)

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- Based on application of **KCL**.
- Have 2 types:
  1. Circuit without voltage source
  2. Circuit with voltage source
- Use node voltages as circuit variables.
- Need to define node voltage.

How to define node voltage?

3.2 Nodal Analysis (2)

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Steps to determine the node voltages:

1. Select a node as the reference node (usually ground).
2. Assign voltages $v_1, v_2, \ldots, v_{n-1}$ to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.
3. Apply KCL to each of the n-1 non-reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
4. Solve the resulting simultaneous equations to obtain the unknown node voltages.

3.2 Nodal Analysis (3)

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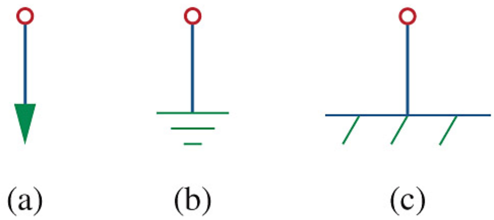
Common symbols for reference nodes:

(a) common ground, (b) ground, (c) chassis ground



Methods to solve simultaneous equations:

1. Elimination
2. Substitution
3. Cramer's Rule *(Appendix A in textbook)*

3.2 Nodal Analysis (4)

(without voltage sources)

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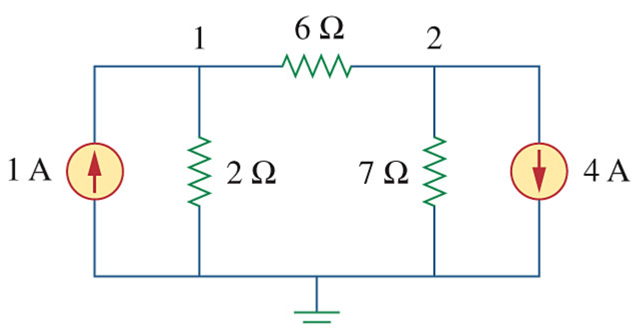
Practice Problem 3.1

Obtain the node voltages in the circuit given below.



3.2 Nodal Analysis (5)
(without voltage sources)

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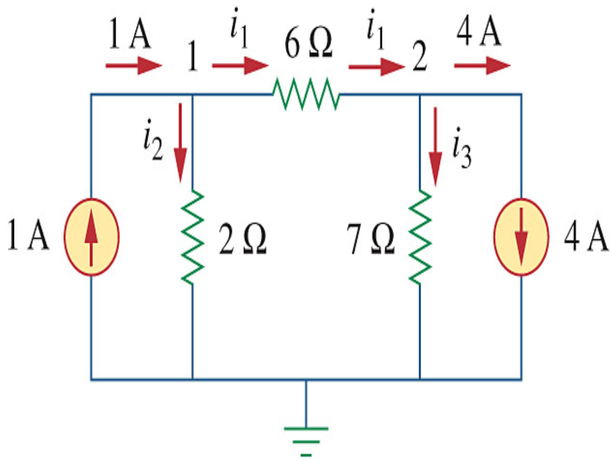
Soln. Prac. Prob. 3.1



3.2 Nodal Analysis (5)
(without voltage sources)

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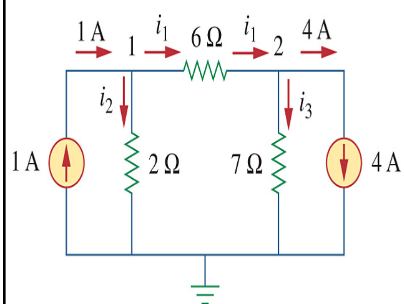
Soln. Prac. Prob. 3.1



At node 1,

$$1 = i_1 + i_2 \longrightarrow 1 = \frac{v_1 - v_2}{6} + \frac{v_1 - 0}{2}$$

or $6 = 4v_1 - v_2$ (1)

At node 2,

$$i_1 = 4 + i_3 \longrightarrow \frac{v_1 - v_2}{6} = 4 + \frac{v_2 - 0}{7}$$

or $168 = 7v_1 - 13v_2$ (2)

Solving (1) and (2) gives

$v_1 =$ **<u>–2 V</u>**, $v_2 =$ **<u>–14 V</u>**

3.2 Nodal Analysis (6)

(without voltage sources)

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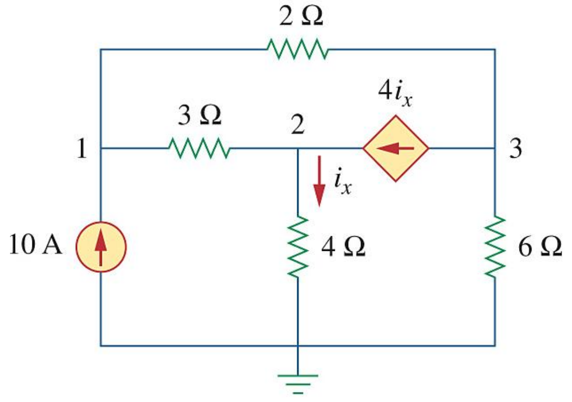
Practice Problem 3.2

Find the voltages at the three non-reference nodes in the circuit given below.



3.2 Nodal Analysis (7)

(without voltage sources)

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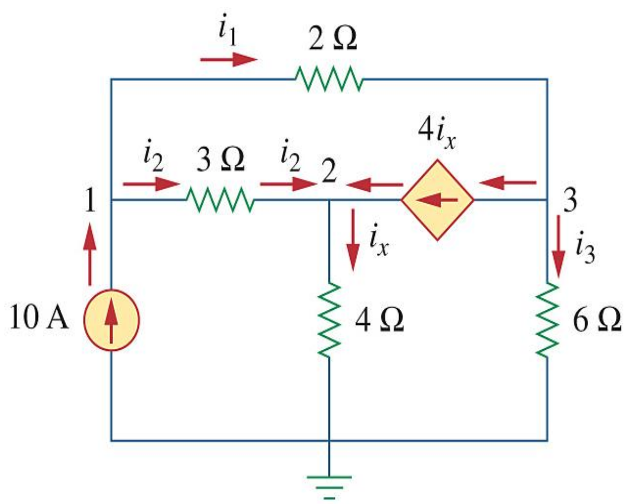
Soln. Prac. Prob. 3.2



3.2 Nodal Analysis (7)

(without voltage sources)

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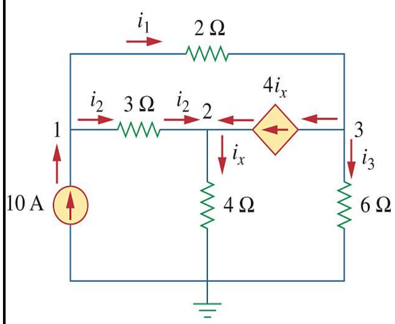
Soln. Prac. Prob. 3.2



At node 1,

$$10 = i_1 + i_2 = \frac{v_1 - v_3}{2} + \frac{v_1 - v_2}{3}$$

$$\text{or } 60 = 5v_1 - 2v_2 - 3v_3 \qquad (1)$$

At node 2,

$$i_2 + 4i_x = i_x \longrightarrow \frac{v_1 - v_2}{3} + 3\frac{v_2}{4} = 0$$

$$\text{or } \; 4v_1 + 5v_2 = 0 \qquad (2)$$

3.2 Nodal Analysis (8)

(without voltage sources)

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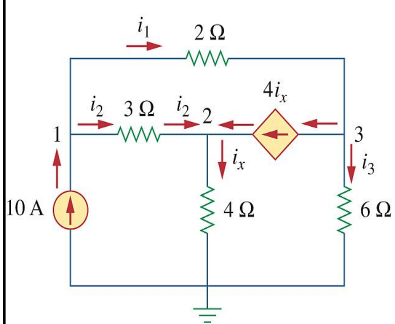
cont. Soln. Prac. Prob. 3.2



At node 3,

$$i_1 = i_3 + 4i_x \longrightarrow \frac{v_1 - v_3}{2} = \frac{v_3 - 0}{6} + 4\frac{v_2}{4}$$

$$\text{or } \; -3v_1 + 6v_2 + 4v_3 = 0 \qquad (3)$$

Solving (1) to (3) gives

$v_1 = \underline{\mathbf{80\ V}}$, $v_2 = \underline{\mathbf{-64\ V}}$, $v_3 = \underline{\mathbf{156\ V}}$

3.3 Nodal Analysis (1)

(with voltage sources)

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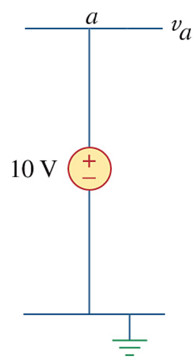
Case 1 : A voltage source connected between reference node and non-reference node.



3.3 Nodal Analysis (2)

(with voltage sources)

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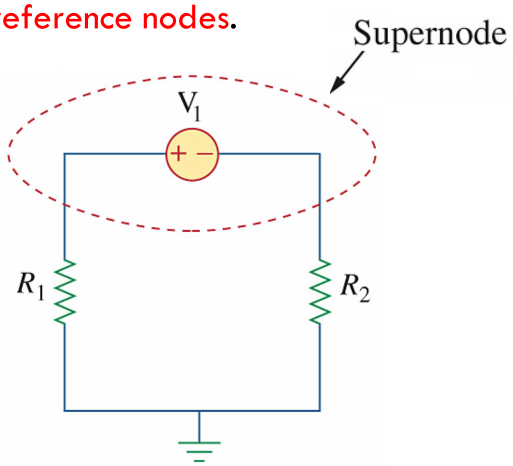
Case 2 : A voltage source connected between 2 non-reference nodes.



3.3 Nodal Analysis (3)

(with voltage sources)

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Supernode is formed by enclosing a (dependent or independent) voltage source connected between 2 non-reference nodes & any elements connected in parallel with it.

Properties:

1. Voltage source inside supernode provides a constraint equation needed to solve for node voltages.
2. Supernode has no voltage of its own.
3. Supernode requires application of both **KCL** & **KVL**.

3.3 Nodal Analysis (4)

(with voltage sources)

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How to determine node voltage?

Basic Steps:

1. Take off all voltage sources (and any element in parallel with it) in supernodes & apply **KCL** to supernodes.
2. Put voltage sources back to the nodes and apply **KVL** to relative loops (supernodes loops).

3.3 Nodal Analysis (5)

(with voltage sources)

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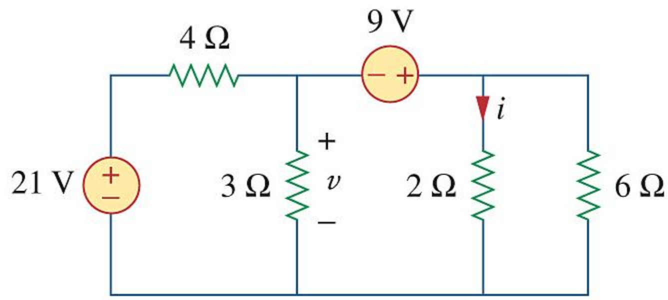
Practice Problem 3.3

Find v an i in the circuit given below.



3.3 Nodal Analysis (6)

(with voltage source)

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Soln. Prac. Prob. 3.3

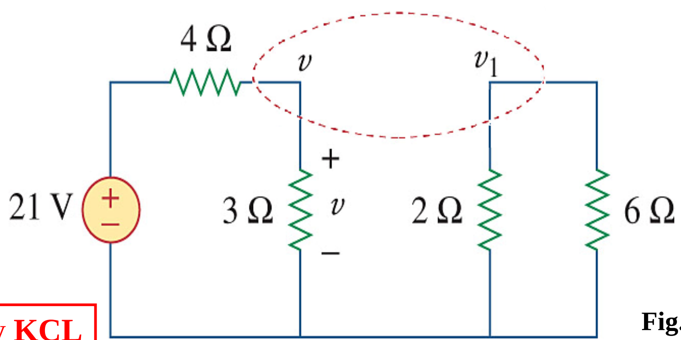

Fig. (a)

Apply **KCL**

At the supernode in Fig. (a),

$$\frac{21-v}{4} = \frac{v}{3} + \frac{v_1}{2} + \frac{v_1}{6} \longrightarrow 63 = 7v + 8v_1 \qquad (1)$$

3.3 Nodal Analysis (7)

(with voltage source)

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Soln. Prac. Prob. 3.3

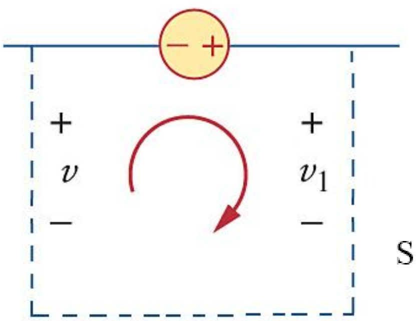

Fig. (b)

Apply **KVL**

Apply KVL to the loop in Fig. (b),

$$-v - 9 + v_1 = 0$$

$$\longrightarrow v_1 = v + 9 \qquad (2)$$

Solving (1) and (2), $v = \underline{\mathbf{-600\ mV}}$

$$v_1 = v + 9 = 8.4,\ i_1 = \frac{v_1}{2} = 4.2$$

$$i_1 = \underline{\mathbf{4.2\ A}}$$

3.3 Nodal Analysis (8)

(with voltage source)

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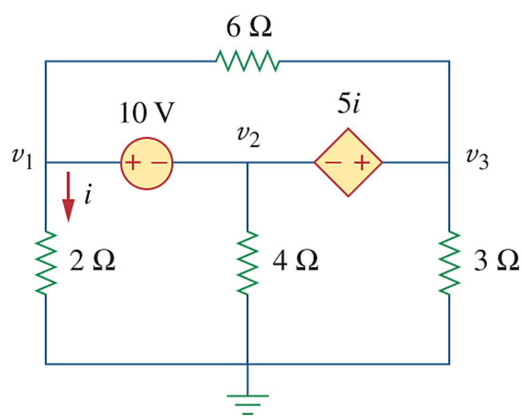
Practice Problem 3.4

Find v_1 , v_2 and v_3 in the circuit given using nodal analysis.



3.3 Nodal Analysis (9)

(with voltage source)

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Soln. Prac. Prob. 3.4

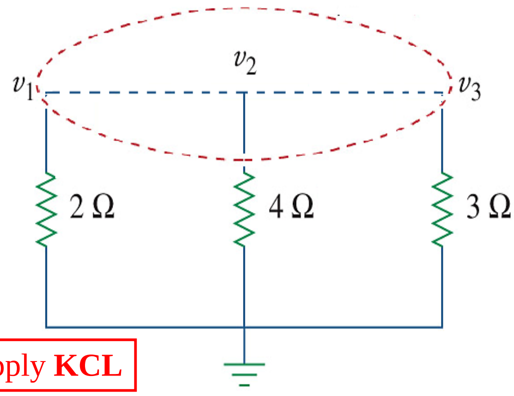

Fig. (a)

Apply **KCL**

From Fig. (a),

$$\frac{v_1}{2} + \frac{v_2}{4} + \frac{v_3}{3} = 0 \longrightarrow 6v_1 + 3v_2 + 4v_3 = 0$$

3.3 Nodal Analysis (10)

(with voltage source)

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Soln. Prac. Prob. 3.4

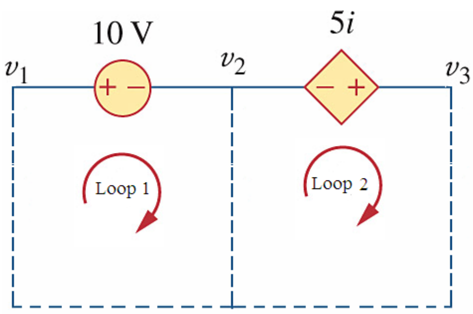

Fig. (b)

Apply **KVL**

From Fig. (b),

Loop 1 $- v_1 + 10 + v_2 = 0$

$\longrightarrow v_1 = v_2 + 10$ (2)

Loop 2 $- v_2 - 5i + v_3 = 0$

$\longrightarrow v_3 = v_2 + 5i$ (3)

Solving (1) to (3), we obtain

v_1 = **3.043V**, v_2 = **–6.956 V**, v_3 = **652.2 mV**

3.3 Nodal Analysis (11)

(with voltage source)

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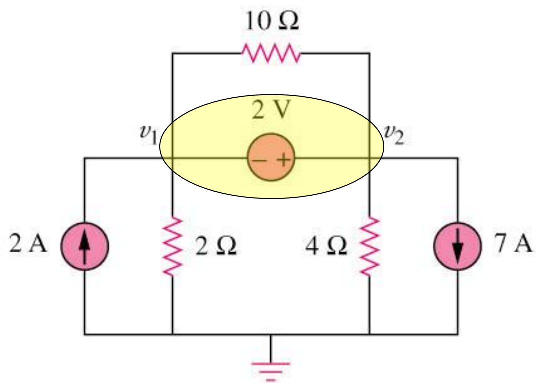
Example 1 – circuit with independent voltage source



How to handle the 2V voltage source?

3.3 Nodal Analysis (12)

(with voltage source)

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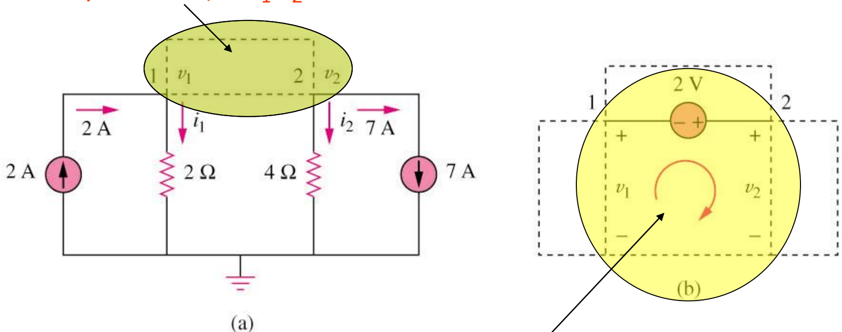
Solution 1 – circuit with independent voltage source

Super-node, $2\text{-}i_1\text{-}i_2\text{-}7 = 0$



Apply KVL, $v_1+2-v_2 = 0$

3.3 Nodal Analysis (13)

(with voltage source)

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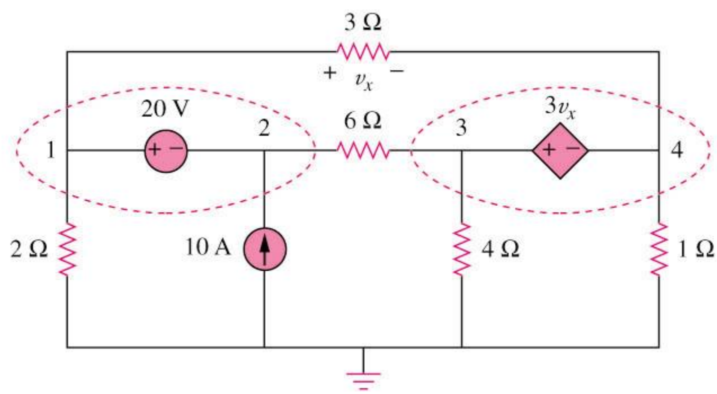
<u>Example 2</u> – circuit with two independent voltage sources



3.3 Nodal Analysis (14)

(with voltage source)

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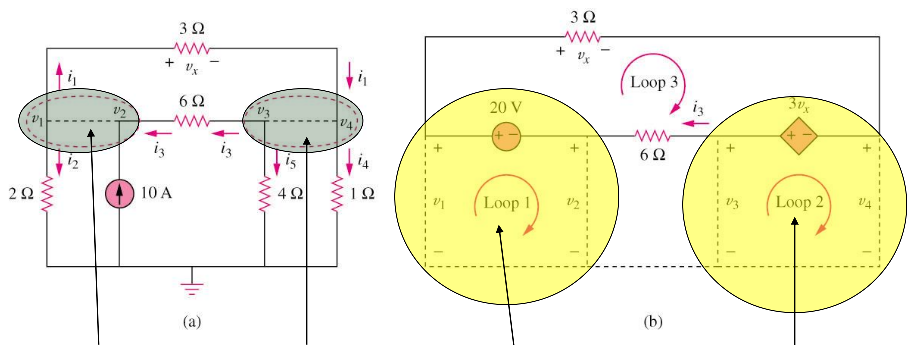
<u>Solution 2</u> – circuit with two independent voltage sources



$-i_1-i_2+i_3=0$ $-i_3-i_5-i_4+i_1=0$ $v_1-20-v_2=0$ $v_3-3v_x-v_4=0$

3.4 Mesh Analysis (1)

(without current sources)

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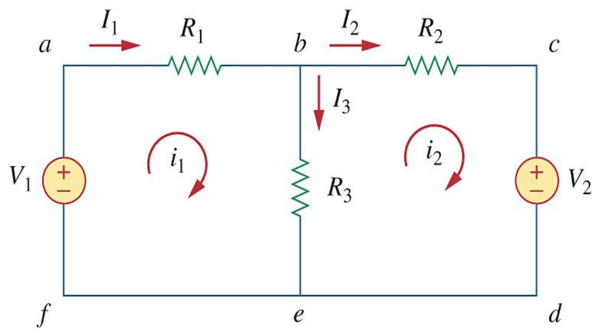
- A mesh is a loop which does not contain any other loops within it.



Example:

Paths ***abef*** & ***bcde*** are meshes.

Path ***abcdef*** is not a mesh.

3.4 Mesh Analysis (2)

(without current sources)

30

- Based on application of **KVL**.
- Have 2 types:
  1. Circuit without current source
  2. Circuit with current source
- Use mesh currents as circuit variables.

How to define mesh current?

3.4 Mesh Analysis (3)

(without current sources)

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Steps to determine the mesh currents:

1. Assign currents $i_1, i_2, \ldots, i_n$ to the n meshes.

2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of mesh currents.

3. Solve the resulting n simultaneous equations to obtain the unknown mesh currents.

3.4 Mesh Analysis (4)

(without current sources)

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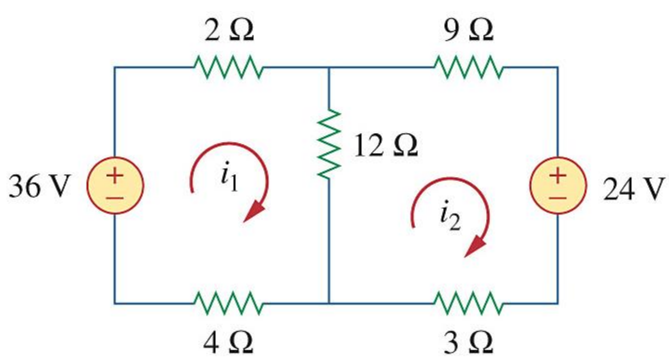
Practice Problem 3.5

Calculate the mesh currents i_1 and i_2 of the circuit given below.



3.4 Mesh Analysis (5)
(without current sources)

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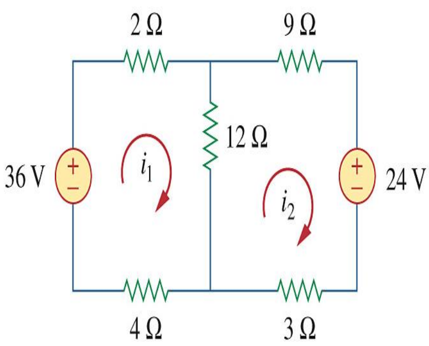
Soln. Prac. Prob. 3.5



Apply **KVL**

$$-36 + 18i_1 - 12i_2 = 0$$

Mesh 1 → $3i_1 - 2i_2 = 6 \quad (1)$

$$24 + 24i_2 - 12i_1 = 0$$

Mesh 2 → $-3i_1 + 6i_2 = -6 \quad (2)$

From (1) and (2) we get

$i_1 = \underline{\mathbf{2\ A}}$, $i_2 = \underline{\mathbf{0A}}$

3.4 Mesh Analysis (6)
(without current sources)

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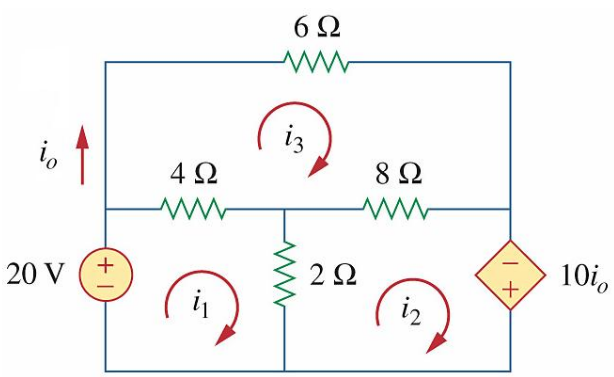
Practice Problem 3.6

Using mesh analysis, find i_o in the circuit given below.



3.4 Mesh Analysis (7)

(without current sources)

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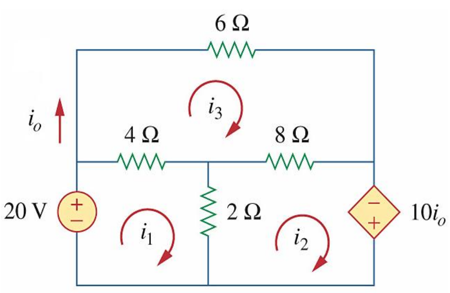
Soln. Prac. Prob. 3.6



Apply **KVL**

$$-20 + 6i_1 - 2i_2 - 4i_3 = 0$$

Mesh 1 → $3i_1 - i_2 - 2i_3 = 10 \quad (1)$

$$10i_2 - 2i_1 - 8i_3 - 10i_0 = 0$$

But $i_0 = i_3$,

Mesh 2 → $-i_1 + 5i_2 - 9i_3 = 0 \quad (2)$

$$18i_3 - 4i_1 - 8i_2 = 0$$

Mesh 3 → $-2i_1 - 4i_2 + 9i_3 = 0 \quad (3)$

3.4 Mesh Analysis (8)

(without current sources)

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cont. Soln. Prac. Prob. 3.6

From (1) to (3),

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 5 & -9 \\ -2 & -4 & 9 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ -1 & 5 & -9 \\ -2 & -4 & 9 \\ 3 & -1 & -2 \\ -1 & 5 & -9 \end{vmatrix} = 135 - 8 - 18 - 20 - 108 - 9 = -2$$

$$\Delta_1 = \begin{vmatrix} 10 & -1 & -2 \\ 0 & 5 & -9 \\ 0 & -4 & 9 \\ 10 & -1 & -2 \\ 0 & 5 & -9 \end{vmatrix} = 450 - 360 = 90$$

$$\Delta_2 = \begin{vmatrix} 3 & 10 & -2 \\ -1 & 0 & -9 \\ -2 & 0 & 9 \\ 3 & 10 & -2 \\ -1 & 0 & -9 \end{vmatrix} = 180 + 90 = 270$$

3.4 Mesh Analysis (9)

(without current sources)

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cont. Soln. Prac. Prob. 3.6

$$\Delta_3 = \begin{vmatrix} 3 & -1 & 10 \\ -1 & 5 & 0 \\ -2 & -4 & 0 \\ 3 & -1 & 10 \\ -1 & 5 & 0 \end{vmatrix} = 40 + 100 = 140$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{90}{-28} = -3.214$$

$$i_2 = \frac{\Delta_2}{\Delta} = \frac{270}{-28} = -9.643$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{140}{-28} = -5A$$

$$i_0 = i_3 = \underline{\mathbf{-5A}}$$

3.5 Mesh Analysis (1)

(with current sources)

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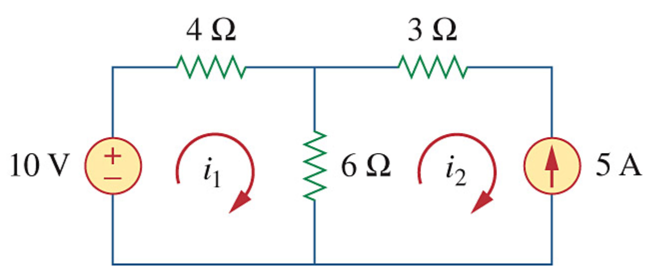
Case 1 : A current source exists only in one mesh.



i_2 = -5A

3.5 Mesh Analysis (2)

(with current sources)

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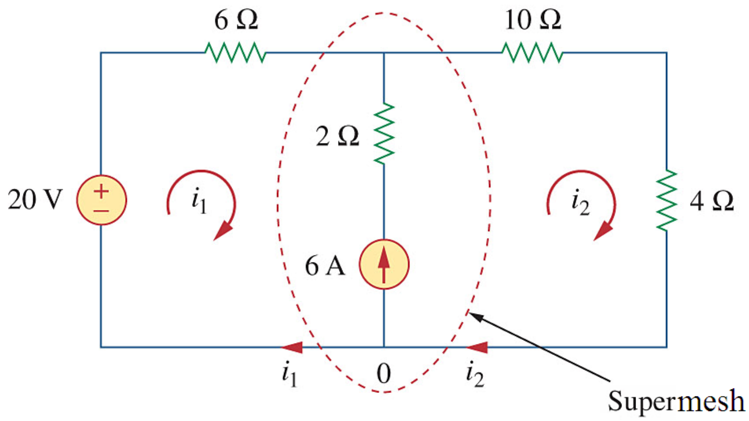
Case 2 : A current source exists between 2 meshes.



3.5 Mesh Analysis (3)

(with current sources)

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Supermesh is formed when two meshes have a (dependent or independent) current source in common & include any elements connected in series with it.

Properties:

1. Current source inside supermesh provides a constraint equation needed to solve for mesh currents.
2. Supermesh has no currents of its own.
3. Supernode requires application of both **KVL** & **KCL**.

3.5 Mesh Analysis (4)

(with current sources)

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How to determine mesh currents?

Basic Steps:

1. Take off all current sources (and any element in series with it) in the supermesh & apply **KVL** to supermesh.
2. Put current sources back to the nodes and apply **KCL** to supermesh nodes.

3.5 Mesh Analysis (5)

(with current sources)

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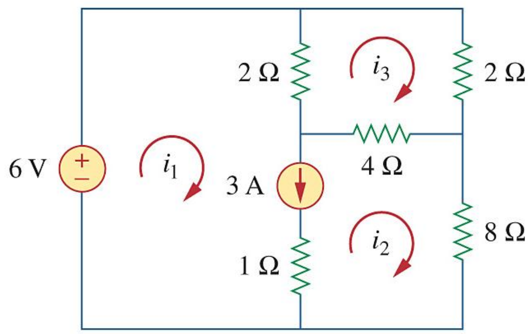
Practice Problem 3.7

Use mesh analysis to determine i_1 , i_2 and i_3 in the circuit given below.



3.5 Mesh Analysis (6)

(with current sources)

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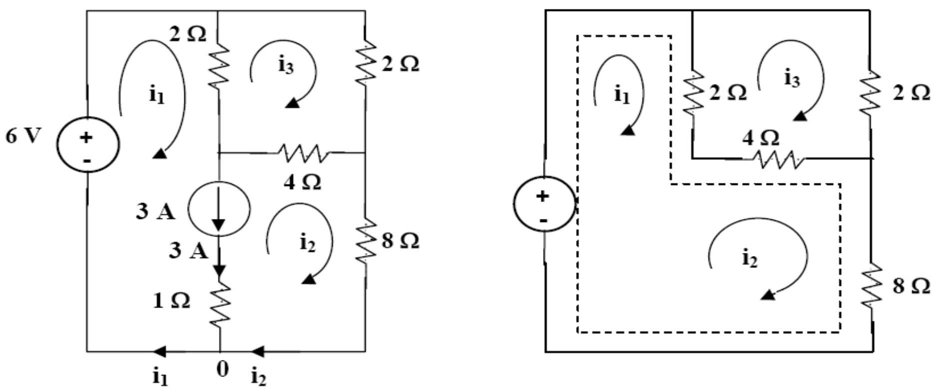
Soln. Prac. Prob. 3.7



3.5 Mesh Analysis (7)

(with current sources)

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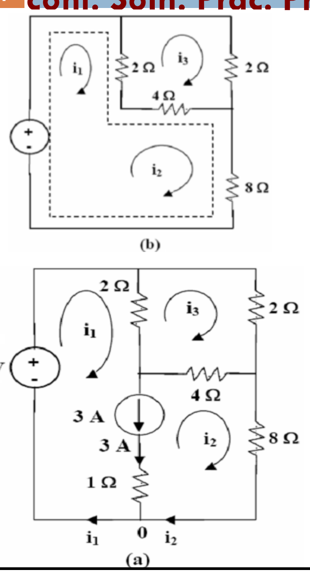
cont. Soln. Prac. Prob. 3.7



Apply **KVL**

For the supermesh,

$$-6 + 2i_1 - 2i_3 + 12i_2 - 4i_3 = 0$$

$$\longrightarrow \quad i_1 + 6i_2 - 3i_3 = 3 \qquad (1)$$

For mesh 3,

$$8i_3 - 2i_1 - 4i_2 = 0$$

$$\longrightarrow \quad -i_1 - 2i_2 + 4i_3 = 0 \qquad (2)$$

At node 0 in Fig. (a), Apply **KCL**

$$i_1 = 3 + i_2 \longrightarrow i_1 - i_2 = 3 \qquad (3)$$

Solving (1) to (3) yields

i_1 = **3.474A**, i_2 = **473.7 mA**, i_3 = **1.1052A**