K Nearest Neighbors

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2010

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KNN - Definition

KNN is a simple algorithm that stores all available cases and classifies new cases based on a similarity measure.

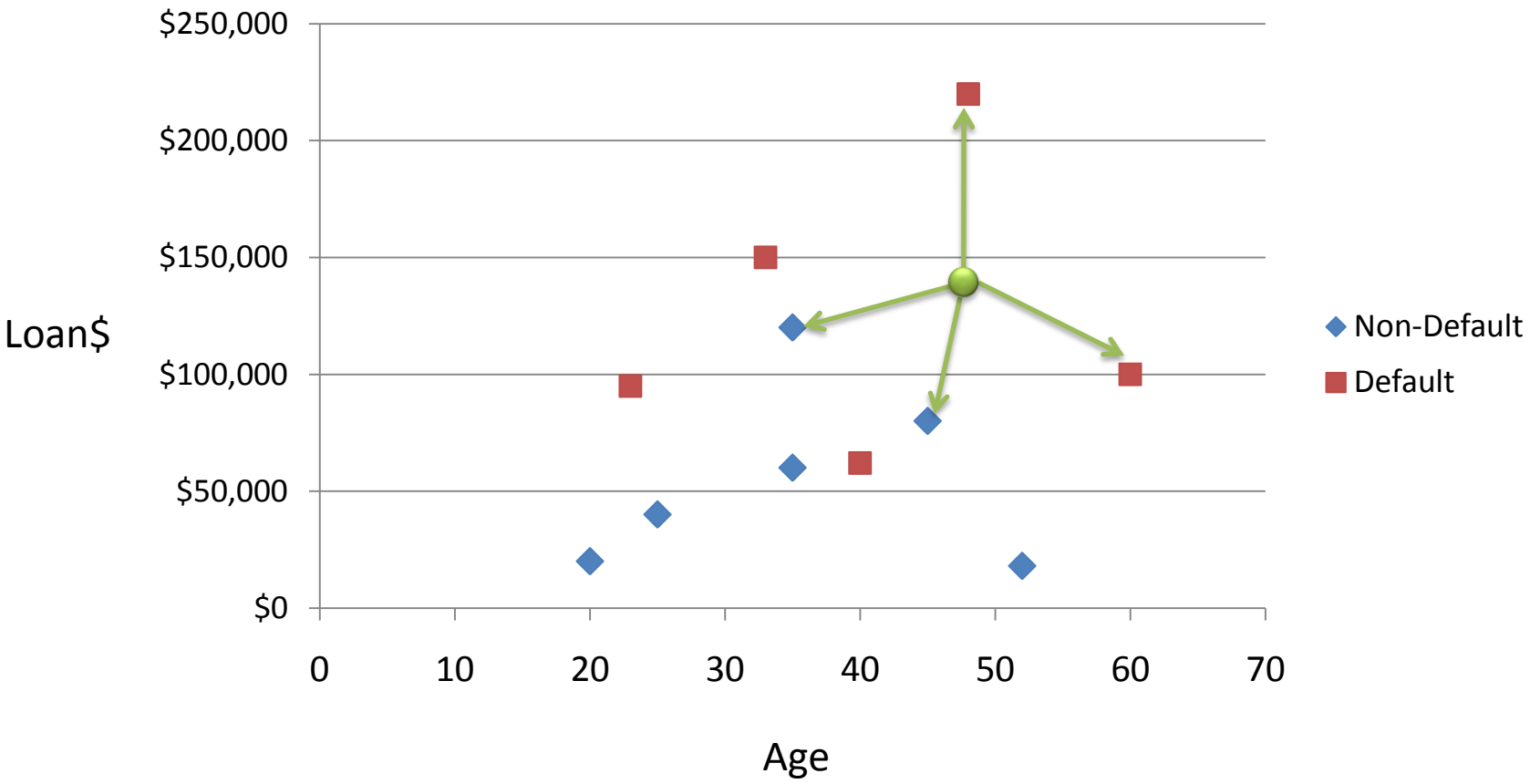
KNN – different names

- K-Nearest Neighbors
- Memory-Based Reasoning
- Example-Based Reasoning
- Instance-Based Learning
- Case-Based Reasoning
- Lazy Learning

KNN – Short History

- Nearest Neighbors have been used in statistical estimation and pattern recognition already in the beginning of 1970's (non-parametric techniques).
- Dynamic Memory: A theory of Reminding and Learning in Computer and People (Schank, 1982).
- People reason by remembering and learn by doing.
- Thinking is reminding, making analogies.
- Examples = Concepts???

KNN Classification



KNN Classification – Distance

Age	Loan	Default	Distance
25	$40,000	N	102000
35	$60,000	N	82000
45	$80,000	N	62000
20	$20,000	N	122000
35	$120,000	N	22000
52	$18,000	N	124000
23	$95,000	Y	47000
40	$62,000	Y	80000
60	$100,000	Y	42000
48	$220,000	Y	78000
33	$150,000	Y	8000
48	**$142,000**	**?**	

Euclidean Distance

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Similarity - Distance Measure

Euclidean

$$\sqrt{\sum_{i=1}^{k}\left(x_i - y_i\right)^2}$$

Manhattan

$$\sum_{i=1}^{k}\left|x_i - y_i\right|$$

Minkowski

$$\left(\sqrt{\sum_{i=1}^{k}\left(\left|x_i - y_i\right|\right)^q}\right)^{1/q}$$

KNN Classification – Standardized Distance

Age	Loan	Default	Distance
0.125	0.11	N	0.7652
0.375	0.21	N	0.5200
0.625	0.31	N	0.3160
0	0.01	N	0.9245
0.375	0.50	N	0.3428
0.8	0.00	N	0.6220
0.075	0.38	Y	0.6669
0.5	0.22	Y	0.4437
1	0.41	Y	0.3650
0.7	1.00	Y	0.3861
0.325	0.65	Y	0.3771
0.7	**0.61**	**?**	

Standardized Variable

$$X_s = \frac{X - Min}{Max - Min}$$

KNN Regression - Distance

Age	Loan	House Price Index	Distance
25	$40,000	135	102000
35	$60,000	256	82000
45	$80,000	231	62000
20	$20,000	267	122000
35	$120,000	139	22000
52	$18,000	150	124000
23	$95,000	127	47000
40	$62,000	216	80000
60	$100,000	139	42000
48	$220,000	250	78000
33	$150,000	264	8000
48	**$142,000**	**?**	

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

KNN Regression – Standardized Distance

Age	Loan	House Price Index	Distance
0.125	0.11	135	0.7652
0.375	0.21	256	0.5200
0.625	0.31	231	0.3160
0	0.01	267	0.9245
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$$X_s = \frac{X - Min}{Max - Min}$$

KNN – Number of Neighbors

- If K=1, select the nearest neighbor
- If K>1,
  - For classification select the most frequent neighbor.
  - For regression calculate the average of K neighbors.

Distance – Categorical Variables

X	Y	Distance
Male	Male	0
Male	Female	1

$$x = y \Rightarrow D = 0$$

$$x \neq y \Rightarrow D = 1$$

Similarity – Hamming Distance

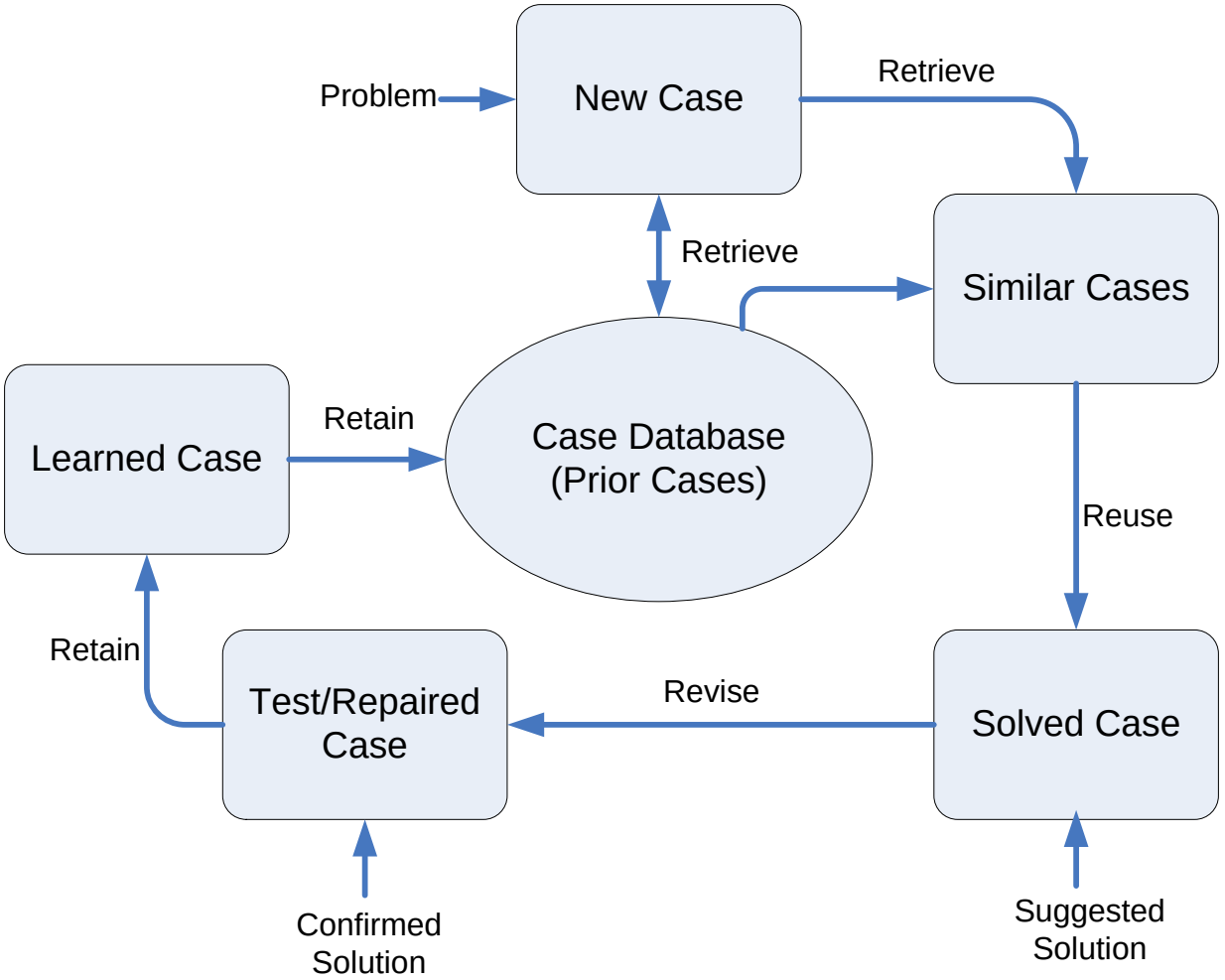
$$D_H = \sum_{i=1}^{k} |x_i - y_i|$$

Gene 1	A	A	T	C	C	A	G	T
Gene 2	T	C	T	C	A	A	G	C
Hamming Distance	1	1	0	0	1	0	0	1

Instance Based Reasoning

- **IB1** is based on the standard KNN
- **IB2** is incremental KNN learner that only incorporates misclassified instances into the classifier.
- **IB3** discards instances that do not perform well by keeping success records.

Case Based Reasoning



KNN - Applications

- Classification and Interpretation
  - legal, medical, news, banking
- Problem-solving
  - planning, pronunciation
- Function learning
  - dynamic control
- Teaching and aiding
  - help desk, user training

Summary

- KNN is conceptually simple, yet able to solve complex problems
- Can work with relatively little information
- Learning is simple (no learning at all!)
- Memory and CPU cost
- Feature selection problem
- Sensitive to representation

QUESTIONS?