

* State space method:

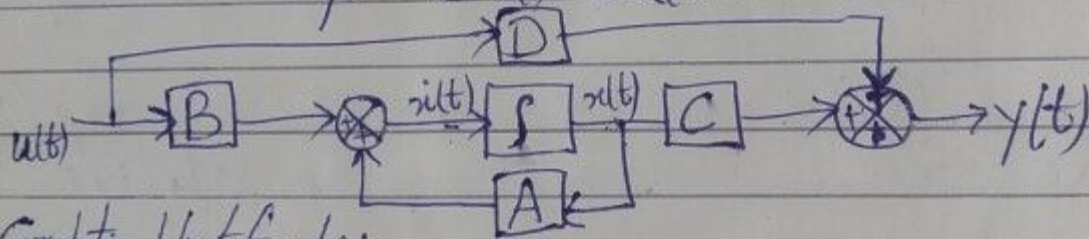
- i) For MIMO systems representation
- ii) Linear, non-linear, time variant and invariant systems

{ A pole is always an eigenvalue but not the other way round. In t.f, a zero can cancel out a pole so that not all e-values occur in t.f as poles }

Linear t.i,

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$



Correlation b/w t.f and ss

$$sX(s) - x(0) = AX(s) + BU(s) \rightarrow (i)$$

$$Y(s) = CX(s) + DU(s) \rightarrow (ii)$$

Take $x(0) = 0$ in (i).

$$sX(s) = AX(s) + BU(s)$$

$$sX(s) - AX(s) = BU(s)$$

$$(sI - A)X(s) = BU(s)$$

$$X(s) = (sI - A)^{-1} BU(s)$$

Put in (ii)

$$Y(s) = [C(sI - A)^{-1}B + D] U(s)$$

$$G(s) = Y(s)/U(s)$$

$$= [C(sI - A)^{-1}B + D]$$

$$G(s) = C(sI - A)^{-1}B + D$$

$$Q \quad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u$$

$$\text{Soln} \quad G(s) = C(sI - A)^{-1}B + D$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \left[s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \right]^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \right]^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{1}{s(s+3)+2} \times \begin{bmatrix} s+3 & +1 \\ -2 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+3s+2} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ s \end{bmatrix}$$

$$= \frac{1}{s^2+3s+2} \times \begin{bmatrix} 1 \end{bmatrix}$$

$$G(s) = \frac{1}{s^2+3s+2}$$

$$[\text{num}, \text{den}] = \text{ss2tf}(A, B, C, D)$$

$$[A, B, C, D] = \text{tf2ss}(\text{num}, \text{den})$$

State var \rightarrow The smallest set of variables that completely determine the state of system

State vector \rightarrow A vector having n number of set var's that completely determine dynamic behvraf system