

The Harmonic Mean :=

The Harmonic mean is a type of numerical average. It is denoted by " H " and defined for a set of n values x_1, x_2, \dots, x_n as the reciprocal of the arithmetic mean of the reciprocal of the values.

FORMULA :=

$$H = \text{Reciprocal of } \frac{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}}{n}$$

$$H = \frac{n}{\sum \frac{1}{x_i}}, \text{ where } x \neq 0$$

Example :=

Consider 2, 3, 5, 7 and 60 as the number of observations. Calculate the Harmonic mean.

Here $n = 5$

$$\begin{aligned} \text{So } H &= \frac{5}{\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{60}\right)} \\ &= \frac{5}{(0.5 + 0.33 + 0.2 + 0.14 + 0.017)} \\ &= \frac{5}{1.187} \\ &= 4.21 \end{aligned}$$

HARMONIC MEAN FOR GROUPED DATA :=

For data organised into a frequency distribution having " K " classes with classmarks $x_1, x_2, x_3, \dots, x_K$ and the corresponding frequencies $f_1, f_2, f_3, \dots, f_K$ ($\sum f_i = n$) the harmonic mean of the distribution is given as (P.T.O)

Formula :=

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$$H = \text{Reciprocal of } \frac{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \dots + \frac{f_k}{x_k}}{f_1 + f_2 + \dots + f_k}$$

$$= \frac{\sum f_i}{\sum f_i \frac{1}{x_i}} \quad \text{OR} \quad \frac{n}{\sum f_i \frac{1}{x_i}}$$

Weighted Harmonic Mean :=

Similarly, the weighted Harmonic mean is defined as

Formula :=

$$H_w = \frac{w_1 + w_2 + \dots + w_n}{w_1 \left(\frac{1}{x_1}\right) + w_2 \left(\frac{1}{x_2}\right) + \dots + w_n \left(\frac{1}{x_n}\right)}$$

$$= \frac{\sum w_i}{\sum w_i \left(\frac{1}{x_i}\right)}$$

Example :=

Find the Harmonic mean from the following frequency distribution table.

Class	Frequency
0-9	5
10-19	8
20-29	4
30-39	3
40-49	2

Sol:

Classes	frequency (f)	x	$\frac{1}{x}$	$f \cdot \frac{1}{x}$
0-9	5	4.5	0.222	1.11
10-19	8	14.5	0.068	0.544
20-29	4	24.5	0.0401	0.1604
30-39	3	34.5	0.0289	0.0867
40-49	2	44.5	0.0224	0.0448

$$n = \sum f = 22$$

$$\sum f \cdot \frac{1}{x} = 1.9459$$

$$H.M = \frac{n}{\sum f \cdot \frac{1}{x}} = \frac{22}{1.9459} = 11.305$$

Example := For Students to Practice

Calculate the H.M from the following frequency distribution.

Classes	frequency (f)
65-84	9
85-104	10
105-124	17
125-144	10
145-164	5
165-184	4
185-204	5

Median :=

The median is a simple measure of central tendency. The median is defined as a value which divides a data set that have been ordered into two equal parts. OR

The median is a value at or below which 50% of the ordered data lie.

Odd values :=

When the number of observations i.e. " n " is odd, the formula to find median is

$$\text{median} = \left(\frac{n+1}{2} \right)^{\text{th}}$$

Even values :=

When the number of observations i.e. " n " is even, the formula to find the median is

$$\text{median} = \frac{1}{2} \left[n/2, n/2 + 1 \right]^{\text{th}}$$

Examples :=

Compute the median

3, 31, 5, 16, 7, 9, 21

Arrange the data in ascending order first

3, 5, 7, 9, 16, 21, 31

$$\text{median} = \left(\frac{n+1}{2} \right)^{\text{th}}$$

$$= \left(\frac{7+1}{2} \right)^{\text{th}}$$

$$= \left(\frac{8}{2} \right)^{\text{th}} = 4^{\text{th}} = 9$$

Example::

3, 5, 7, 9, 16, 21, 31, 33

here $n = 8$ (even)

$$\begin{aligned} & \frac{1}{2} \left[n/2, n/2 + 1 \right]^{th} \\ &= \frac{1}{2} \left[8/2, 8/2 + 1 \right]^{th} \\ &= \frac{1}{2} \left[4, \frac{10}{2} \right]^{th} \\ &= \frac{1}{2} [4, 5]^{th} \\ &= \frac{1}{2} [9, 16] \\ &= \frac{25}{2} = 12.5 \end{aligned}$$

Median for Grouped data

In order to calculate the median for grouped data consider the following formula.

$$\text{Median} = l + \frac{\frac{n}{2} - c}{f} (m - l)$$

OR
$$\text{Median} = l + \frac{\frac{n}{2} - c}{f} (h)$$

where

- l = lower class boundary LCB of the median group
- h = width of the median group
- f = frequency of the median group
- m = size of the
- c = next lower c.f of the median group

Example :=

Consider the following frequency distribution table

Class-Interval	frequency	C.B	x	$f \cdot x$	C.f
1-5	6	0.5-5.5	3	18	6
6-10	10	5.5-10.5	8	80	16
11-15	<u>18</u>	<u>10.5-15.5</u>	13	234	34
16-20	10	15.5-20.5	18	180	44
21-25	6	20.5-25.5	23	138	50

here $m = \frac{n}{2} = \frac{50}{2} = 25$

$$l = 10.5$$

$$h = 5$$

$$f = 18$$

$$c.f = 16 \text{ ("c" in this case)}$$

So put the values

$$\text{Median} = l + \frac{\frac{h}{f}(m-c)}$$

$$= 10.5 + \frac{5}{18}(25-16)$$

$$= 10.5 + 0.277(9)$$

$$= 10.5 + 2.493$$

$$= 12.993 \approx 13$$

Exercise := For Students to Solve

Find Mean and Median from the following data

3.1	1.8	6.4	10.2	11.2	7.2	10.3
6.8	1.5	2.9	3.4	8.4	5.5	7.7
0.7	3.9	8.2	8.0	11.6	12.1	3.9
0.9	4.3	15.0	0.5			

Quantiles :=

In most situations we come across number of observations which is quite large. There are some principles and rules according to which a data set can be divided into two equal parts, or it may be extended to any number of divisions. The different techniques of dividing a data set are as follows:

i) Quartiles :=

Quartiles are values that divide the distribution or data set into four equal parts. These are Q_1 , Q_2 and Q_3 respectively.

Q_1 is called the first or lower quartile

Q_3 is known as the third or upper quartile.

ii) Deciles :=

Deciles are values that divide the data set or distribution into ten equal parts. Deciles are denoted by $D_1, D_2, D_3, \dots, D_9$

iii) Percentiles :=

Percentiles are values that divide the data set or distribution into hundred equal parts. These are represented by $P_1, P_2, P_3, \dots, P_{99}$

NOTE :=

The Second Quartile or the fifth decile or the fiftieth percentile is identical with the median.

Example :=

Given below are the marks obtained by 9 students.

45, 32, 37, 46, 39, 36, 41, 48 and 36

Find median and quartiles.