Lecture # 10

Discrete Structure

Venn Diagram

Example: A number of computer users are surveyed to find out if they have a printer, modem or scanner. Draw separate Venn diagrams and shade the areas, which represent the following configurations.

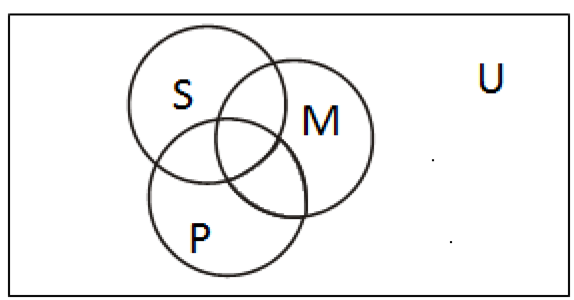
(i) modem and printer but no scanner

(ii) scanner but no printer and no modem

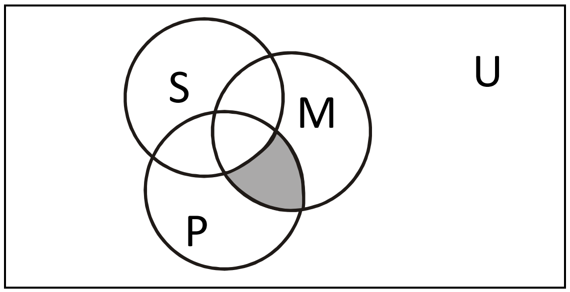
(iii) scanner or printer but no modem.

(iv) no modem and no printer.

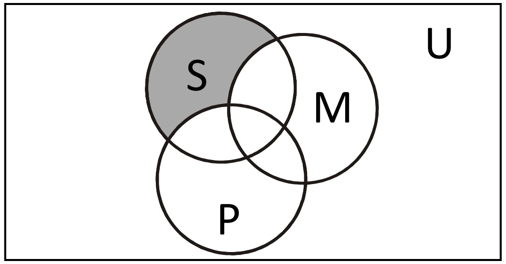
Solution: Let P, M, and S represent the set of computer users having printer, and scanner respectively as shown below:



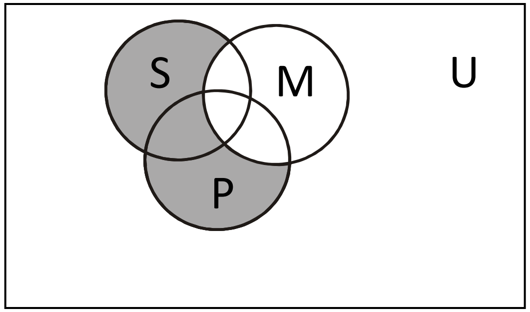
Venn Diagram (Cont.)



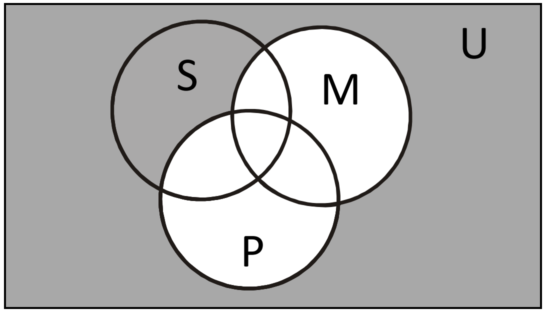
(i) Modem and printer but no scanner



(ii) Scanner but no printer and no modem



(iii) Scanner or printer but no modem



(iv) no modem and no printer

Venn Diagram (Cont.)

Example: Of 21 typists in an office, 5 use all manual typewriters (M), electronic typewriters (E) and word processors (W); 9 use E and W; 7 use M and W; 6 use M and E; but no one uses M only.

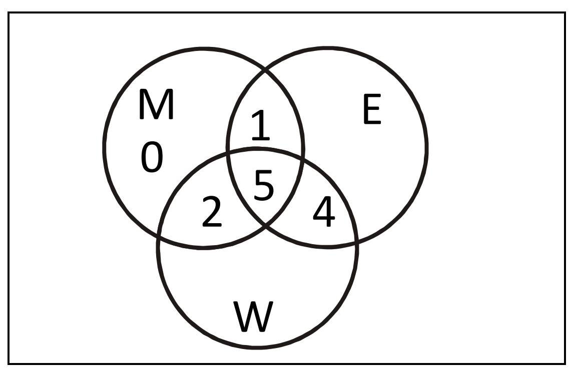
(i) Represent this information in a Venn Diagram

(ii) If the same number of typists use electronic as use word processors, then

  (a) How many use word processors only,

  (b) How many use electronic typewriters?

Solution (i): Venn diagram representation



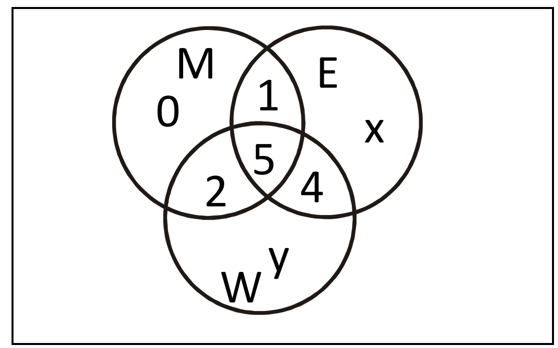
Venn Diagram (Cont.)

Solution (ii): Let the no. of typists using E only be x, and the no. of typists using W only be y

Total no. of typists using E = Total no. of typists using W

$$1 + 5 + 4 + x = 2 + 5 + 4 + y$$

or, $x - y = 1$ ....(1)



Also, total no. of typists = 21

or, $0 + x + y + 1 + 2 + 4 + 5 = 21$

$x + y = 9$ .... (2)

Solving (1) & (2), we get

$$x = 5, \quad y = 4$$

(a) No. of typists using word processor only is $y = 4$

(b) Typists using electronic typewriters $= 1 + 5 + 4 + x$

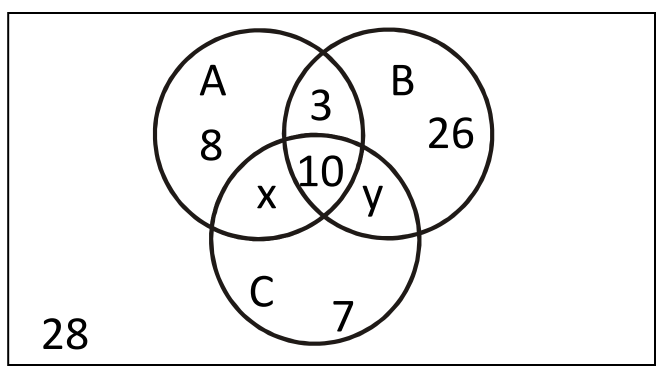
$$= 1 + 5 + 4 + 5 = 15$$

Venn Diagram (Cont.)

Example: In a school, 100 students have access to three software packages: A, B, and C. 28 did not use any software, 8 used only packages A, 26 used only packages B, 7 used only packages C, 10 used all three packages, and 13 used both A and B.

(i) Draw a Venn diagram with all sets enumerated as for as possible. Label the two subsets which cannot be enumerated as x and y, in any order.

Solution:



Venn Diagram (Cont.)

(ii) If twice as many students used B as A, write down a pair of simultaneous equations in x and y

Solution: No. of students using B = 2 X No. of students using A

$\Rightarrow$ 3 + 10 + 26 + y = 2 (8 + 3 + 10 + x)

$\Rightarrow$ 39 + y = 42 + 2x

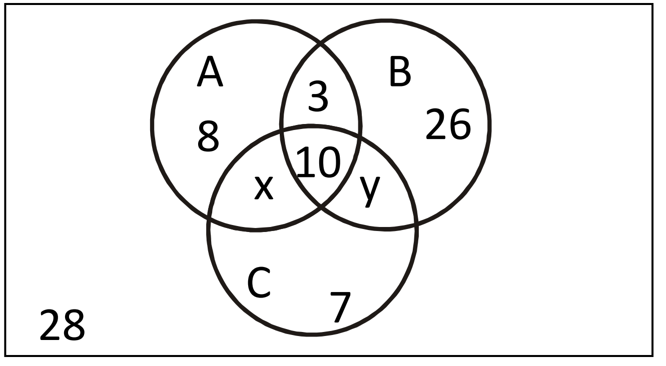
Or, y = 2x + 3 ....(1)

Total number of students = 100

$\Rightarrow$ 8 + 3 + 26 + 10 + 7 + 28 + x + y = 100

$\Rightarrow$ 82 + x + y = 100

Or, x + y = 18 ....(2)



Venn Diagram (Cont.)

(iii) Solve these equations to find x and y

Solution: So we have two simultaneous equations for x and y

$$y = 2x + 3 \text{ ………………(1)}$$

$$x + y = 18 \text{ ………………(2)}$$

Solving (1) in (2), we get,

$$x + (2x + 3) = 18$$

$$3x + 3 = 18$$

$$3x = 15$$

$$x = 5$$

and $y = 13$

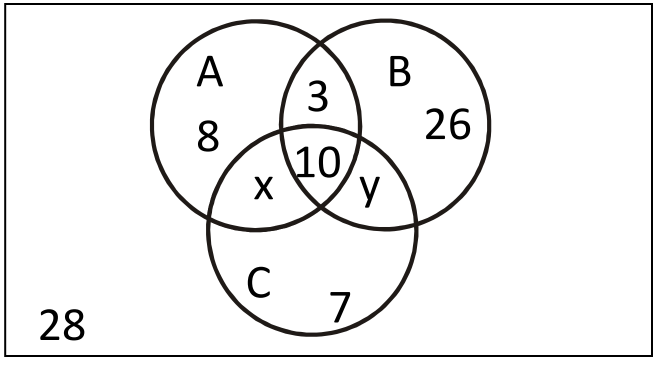
Venn Diagram (Cont.)

(iv) How many students used package C?

Solution: No. of students using package C = x + y + 10 + 7

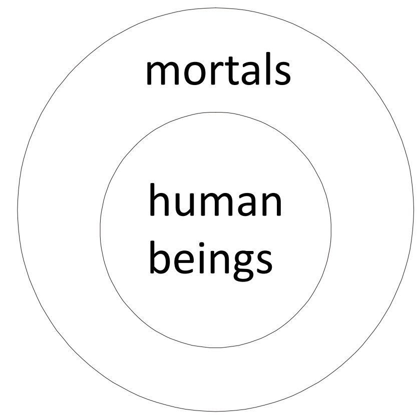
= 5 + 13 + 10 + 7

= 35



Testing Validity of Arguments

Example: Use diagrams to show the validity of the following argument:

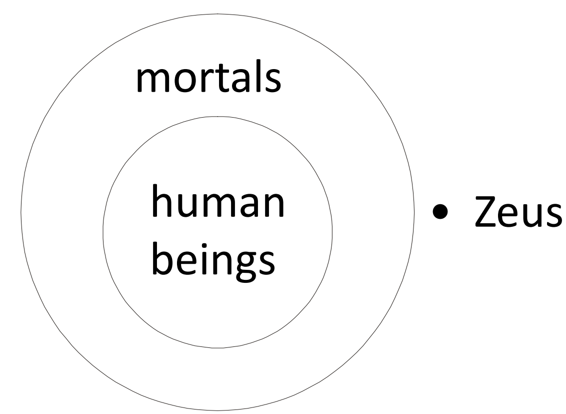


All human beings are mortal

Zeus is not mortal

∴ Zeus is not a human being

Solution: The Disk labeled "human beings" is inside a disk labeled "mortals", so the set of human beings is subset of set of Mortals. A dot labeled "Zeus" is outside the disk labeled "mortals". Accordingly, the conclusion follows necessarily from the truth of the premises; hence the argument is valid.



Testing Validity of Arguments (Cont.)

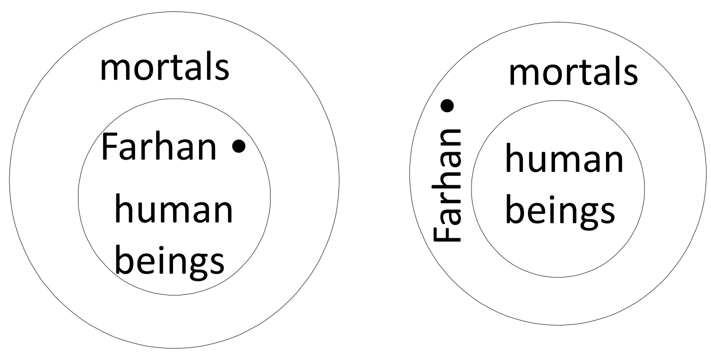
Example: Use a diagram to show the invalidity of the following argument:

All human beings are mortal

Farhan is mortal

∴ Farhan is a human being

Solution: “Farhan is mortal” is represented by a dot labeled “Farhan” inside the mortal disk in either of the following two ways:



The conclusion “Farhan is a human being” is true in the first case but not in the second case as it doesn't necessarily follow from the premises, so the argument is invalid.

Testing Validity of Arguments (Cont.)

Example: Shows the validity of the following argument:

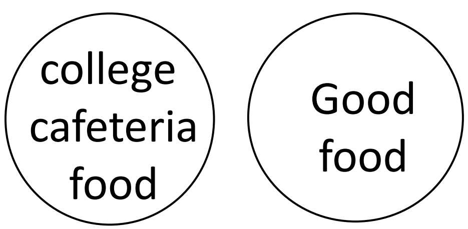
No college cafeteria food is good.

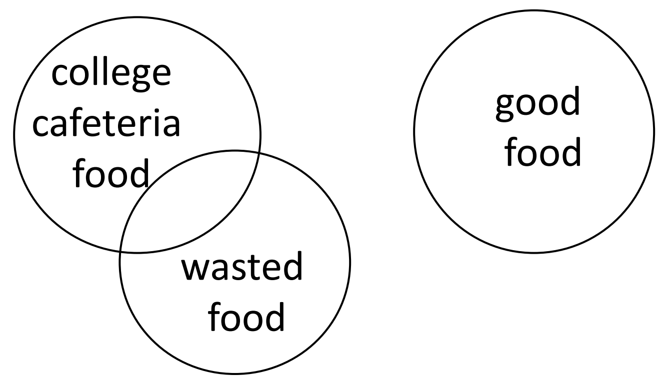
No good food is wasted.

∴ No college cafeteria food is wasted.

Solution: “No college food is good” could be represented by two disjoint disks given below

“No good food is wasted” is represented by disk “wasted food” that does not overlap the disk “good food”, but intersect with the disk “college cafeteria food”





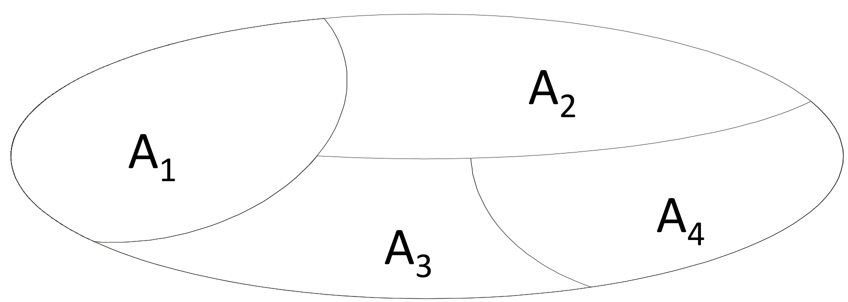
The conclusion does not necessarily follow from the premises, so argument is invalid.

Partition of a Set

- A set may be divided up into its disjoint subsets
- Such division is called a partition
- Partition of set A is a collection of non-empty subsets $\{A_1, A_2 ..., A_n\}$ of A, such that

1. $A = A_1 \cup, A_2 \cup \ldots \cup A_n$
2. $A_1, A_2, \ldots, A_n$ are mutually disjoint (or pair wise disjoint),

   i.e., $\forall$ i, j = 1, 2, …, n $A_i \cap A_j = \varnothing$ whenever $i \neq j$



A partition of a set

Partition of a Set (Cont.)

Example:

Let $A = \{1, 2, 3,4, 5, 6\}$

$A_1 = \{1, 2\}$, $A_2 = \{3, 4, 5\}$, $A_3 = \{6\}$

Then $A_1 \cup A_2 \cup A_3 = A$

and $A_1 \cap A_2 = \varnothing$, $A_1 \cap A_2 = \varnothing$ and $A_2 \cap A_3 = \varnothing$

i.e. A_1, A_2, A_3 are mutually disjoint

Accordingly, $\{A_1, A_2, A_3\}$ is a partition of A

Partition of a Set (Cont.)

Example: Is {E, O} a partition of Z? Justify.

Solution: $Z = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots\}$

$E = \{0, \pm 2, \pm 4, \pm 8, \ldots\}$

& $O = \{\pm 1, \pm 3, \pm 5, \pm 7, \ldots\}$

clearly $E \cup O = Z$ and $E \cap O = \varnothing$

Hence {E, O} is a partition of Z

For any nonempty set A, $\{A, A^c\}$ forms a partition of the universal set U, since

1. $A \cup A^c = U$

2. $A \cap A^c = \varnothing$

Power Set

- Power set denoted by P(A)of a set A is the set of all subsets of A
- If A has n elements then P(A) has 2^n elements

Example: Let A = {1, 2}, then P(A) = {∅, {1}, {2}, {1, 2}}

Example:

a. Find $P(\varnothing)$ b. Find $P(P(\varnothing))$ c. Find $P(P(P(\varnothing)))$

Solution:

a. ∅ contains no element, so $P(\varnothing)$ contain $2^0 = 1$ element

$$P(\varnothing) = \{\varnothing\}$$

b. $P(\varnothing)$ contains one element (∅), so $P(\varnothing)$ contain $2^1 = 2$ elements

$$P(P(\varnothing)) = \{\varnothing,\{\varnothing\}\}$$

c. $P(P(\varnothing))$ contains two elements, ∅ and {∅}, so $P(P(P(\varnothing)))$ contain $2^2 = 4$ elements

$$P(P(P(\varnothing)))= \{\varnothing,\{\varnothing\},\{\{\varnothing\}\},\{\{\varnothing\}\}, \{\varnothing,\{\varnothing\}\}\}$$