Inductor and Capacitor

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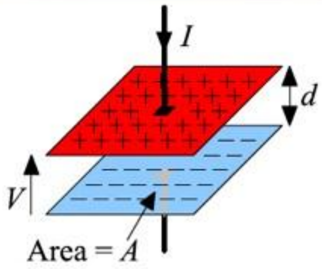
Capacitors & Capacitance

- A capacitor is formed from two conducting plates separated by a thin insulating layer called a **dielectric**.
- If a current i flows, positive change, q, will accumulate on the upper plate. To preserve charge neutrality, a balancing negative charge will be present on the lower plate.



- There will be a potential energy difference (or voltage v) between the plates proportional to q.

$$v = \frac{d}{A\epsilon}q$$

where A is the area of the plates, d is their separation and ε is the permittivity of the insulating layer ($\varepsilon_0 = 8.85\ pF/m$ for a vacuum).

- The quantity $C = A\varepsilon/d$ is the capacitance and is measured in Farads (F). Hence $q = Cv$, and the current i is the rate of charge on the plate.

The Capacitor Equations:

$$q = Cv,\ \ therefore\ \ \frac{dq}{dt} = i = C\frac{dv}{dt}\ \ and\ \ v = \frac{1}{C}\int i\,dt$$

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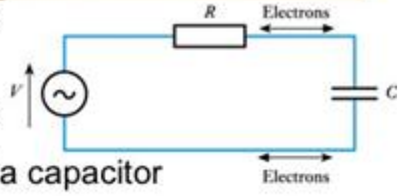
Capacitance is measured in Farads.

DC, AC and Capacitors

- A constant current (DC) cannot flow through a capacitor
  - There is an insulator between the two terminals



- An alternating current (AC) can "flow through" a capacitor
  - Since the voltage across a capacitor is proportional to the charge on it, an alternating voltage must correspond to an alternating charge
  - This can give the impression that an alternating current flows through the capacitor

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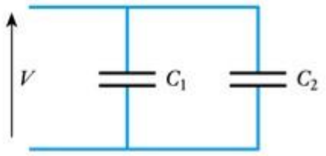
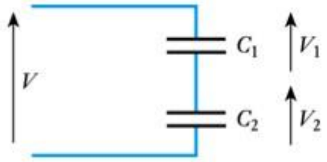
Since there is an insulating layer between the two conducting plates of a capacitor, DC current cannot flow through a capacitor. So always remember: **A CAPACITOR IN SERIES BLOCKS DC part of a signal**. However, alternating or changing current can flow through a capacitor.

Capacitors in Series and in Parallel

- **Capacitors in parallel**
  - consider a voltage V applied across two capacitors
  - then the charge on each is
    $Q_1 = VC_1$ and $Q_2 = VC_2$
  - if the two capacitors are replaced with a single capacitor C which has a similar effect as the pair, then
    Charge stored on combined $C = Q_1 + Q_2$
    $\Rightarrow VC = VC_1 + VC_2$
    $\Rightarrow C = C_1 + C_2$





- **Capacitors in series**
  - consider a voltage V applied across two capacitors in series
  - the only charge that can be applied to the lower plate of C_1 is that supplied by the upper plate of C_2. Therefore the charge on each capacitor must be identical.
  - Let this be Q, and therefore if a single capacitor C has the same effect as the pair, then:
    $V = V_1 + V_2 \Rightarrow Q/C = Q/C_1 + Q/C_2$
    $\Rightarrow 1/C = 1/C_1 + 1/C_2$

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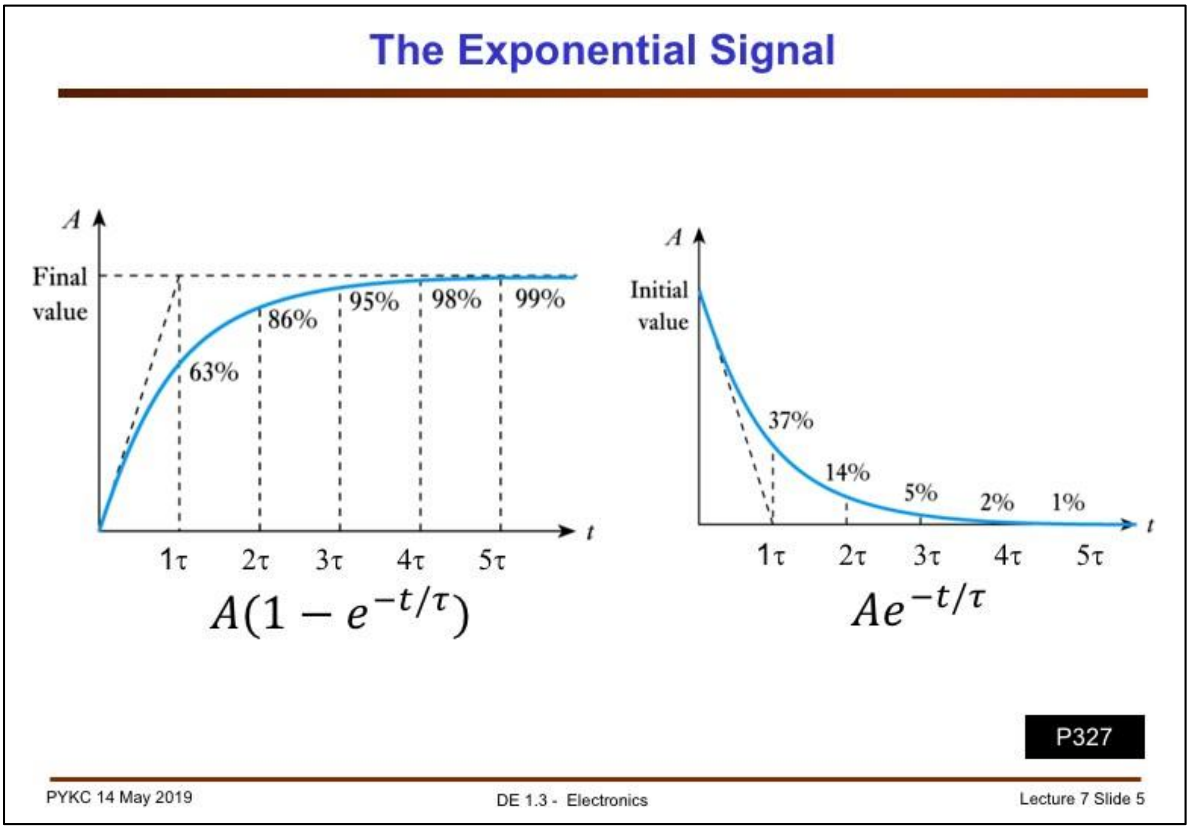
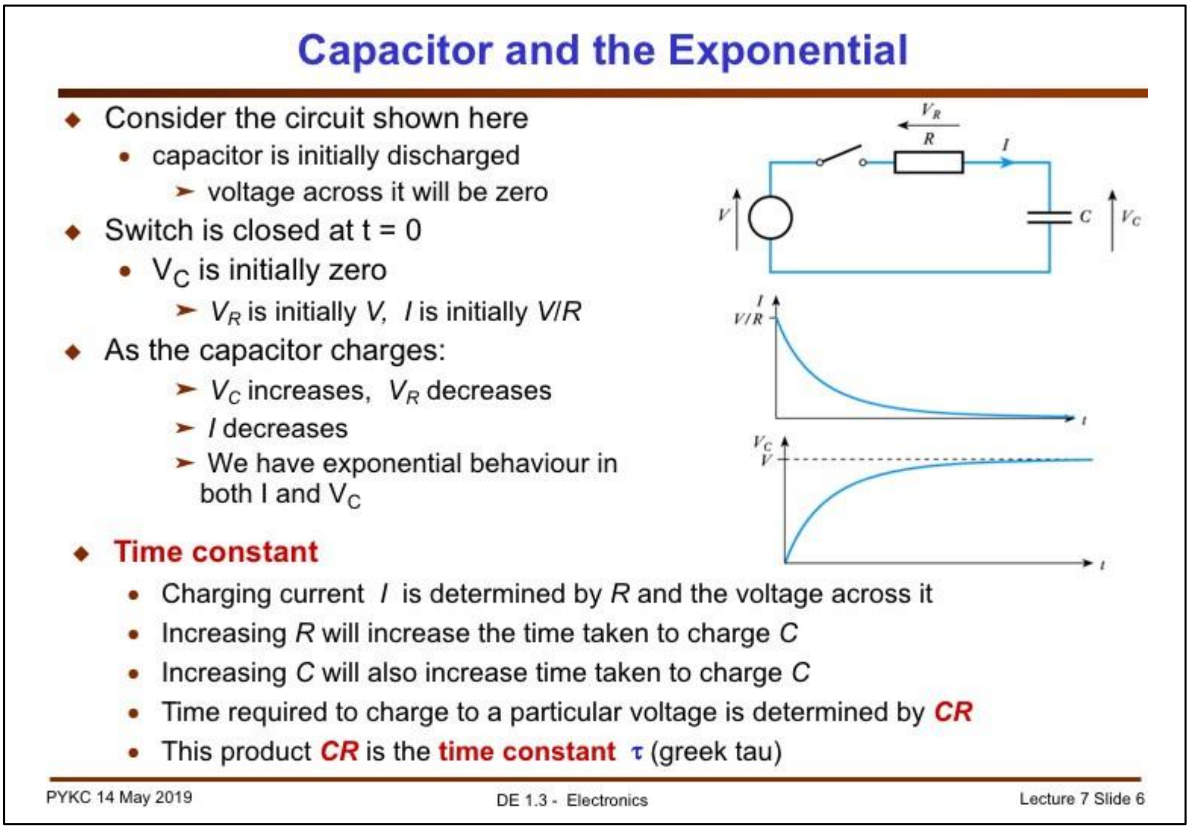


Before we embark on circuits using capacitors, let us examine one of the signals that you explored in Lab 1 in the past two weeks – the exponential signal.

Exponential signals are interesting. Here the rate of change is shown in terms of the time constant t (tau).

The following facts are worth remembering:

1. For exponential rise, the signal reaches 63% at one t, and 95% at 3t.
2. For exponential fall, the signal reaches 37% at one t, and 5% at 3t.



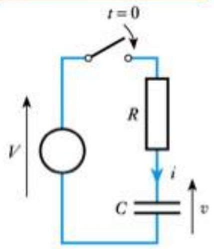
For the circuit shown here, assume the capacitor has zero charge (and 0v) at t = 0. The switch is closed, connecting the circuit to the constant voltage source Vs. Initially the voltage drop across the resistor is Vs. A current of Vs/R flows from the source to capacitor. However, a Vc increases, the current I decreases. This results in the exponential drop of changing current and an exponential rise of the capacitor voltage. We will examine mathematically how i and Vc changes over time in the next lecture.

For now, it is important to consider the parameter known as the time constant. If R is large, the charging current I is small, and it takes longer to charge the capacitor. For a given R, if C is large, it can store more charge for a given voltage, therefore the time needed to charge a capacitor to a certain voltage is proportional to the produce R x C. RC is known as the time constant of this circuit.

Step Response of a RC circuit

- Consider what happens to the circuit shown here as the switch is closed at t = 0.
- Apply KVL around the loop, we get:

$$iR+v=V,\ \ but\ \ i=C\frac{dv}{dt}\ \ therefore\ \ RC\frac{dv}{dt}+v=V$$

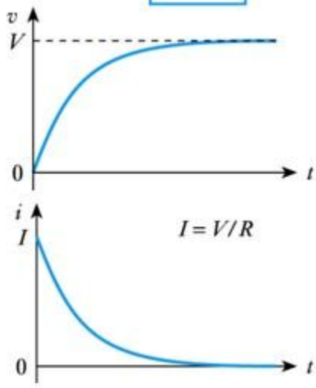


- This is a simple **first-order differential equation** with constant coefficients.
- Assuming V_C = 0 at t = 0, the solution to this is

$$v=V(1-e^{-\frac{t}{RC}})=V(1-e^{-\frac{t}{\tau}}),\quad \text{where } \tau=RC=\text{time-constant}$$

- Since $i=C\frac{dv}{dt}$ this gives (assuming V_C = 0 at t = 0):

$$i=I\times e^{-\frac{t}{RC}}=I\times e^{-\frac{t}{\tau}},\quad \text{where } I=\frac{V}{R}$$



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We have seen this circuit before in the last lecture. We will now derive the exponential equation formally. For that you need to be familiar with solving first-order differential equations from your maths lectures.

We want to solve: $RC\frac{dv}{dt}+v=V$

$$\frac{dv}{dt}=\frac{V-v}{RC}$$

$$\Rightarrow \frac{dt}{RC}=\frac{dv}{V-v}$$

Integrate both sides, we get:

$$\frac{t}{RC}=-\ln(V-v)+A,\quad \text{where A constant of integration}$$

Use boundary condition: when t= 0, v = 0:

$$\frac{0}{RC}=-\ln(V-0)+A$$

$$\Rightarrow A=\ln V$$

Therefore

$$\frac{t}{RC}=-\ln(V-v)+\ln V=\ln\frac{V}{V-v}$$

$$\Rightarrow e^{\frac{t}{RC}}=\frac{V}{V-v}\ \Rightarrow v=V(1-e^{-\frac{t}{RC}})$$

Discharging Capacitor in a RC circuit

- Consider what happens to the circuit shown here as the left switch is open and the right switch closed at t = 0.
- At $t = 0$, $V_C = V$.
- Apply KVL around the right loop, we get:

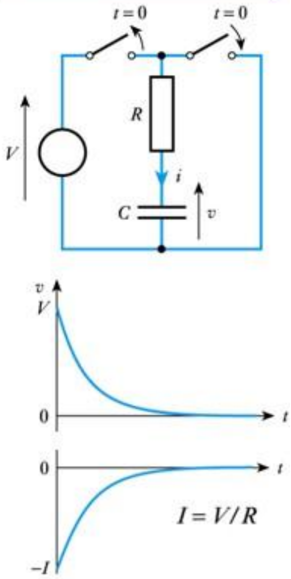
$$iR+v=0, \quad and \quad i=C\frac{dv}{dt} \quad therefore \quad RC\frac{dv}{dt}+v=0$$

- Solving this simple first-order differential equation gives:

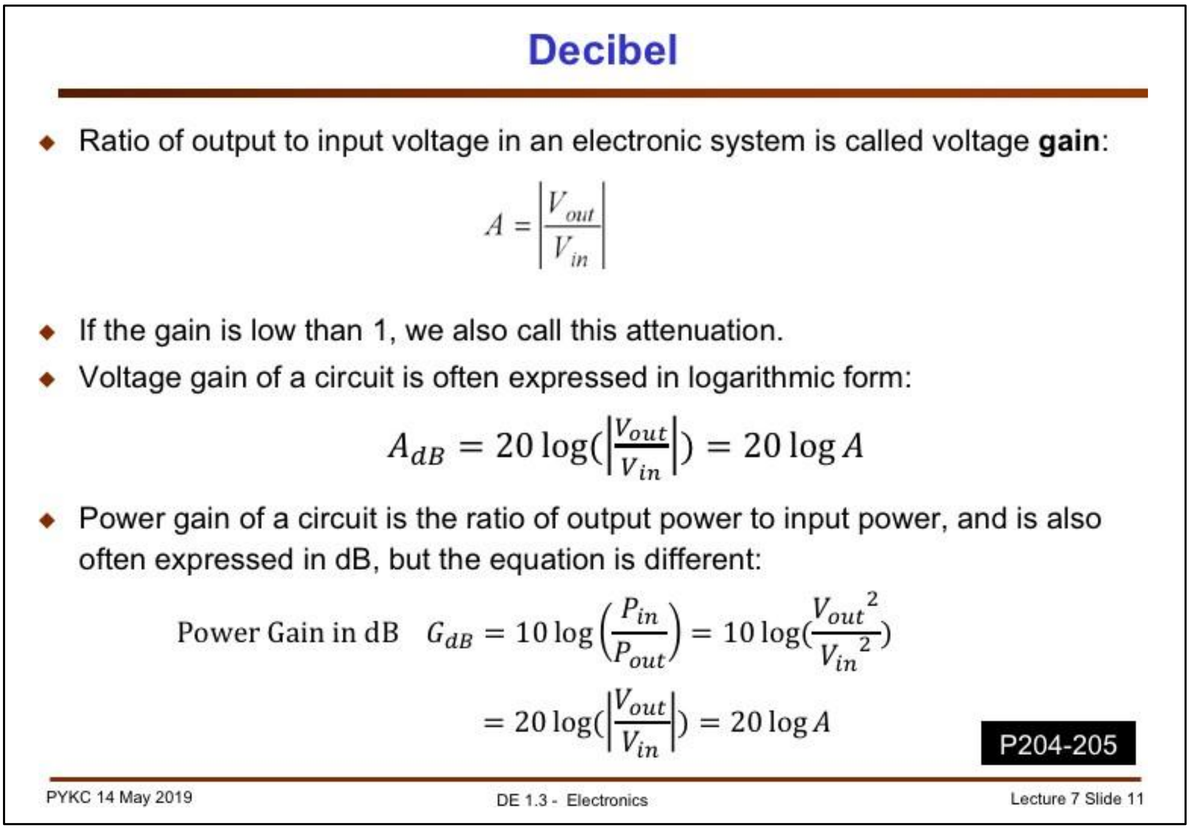
$$v=V\times e^{-\frac{t}{RC}}=V\times e^{-\frac{t}{\tau}}, \quad \text{where } \tau = RC = \text{time-constant}$$

- And:

$$i=-I\times e^{-\frac{t}{RC}}=-I\times e^{-\frac{t}{\tau}}, \quad \text{where } I=\frac{V}{R}$$



Let us now consider what happens if we charge up the capacitor, then at t = 0, discharges it. The equations is also easy to solve and it is clear that the discharge profiles in V and I also follow the exponential curves.



In electronics, we often ask the question: what is the ratio of the output to input signal? This ratio is important. If it is larger than 1, the electronic circuit provides gain (we also call this amplification as will be seen in a later lecture).

If the ratio is lower than 1, then the circuit provides attenuation (or suppression).

However, we often express this ratio or gain not just as such a ratio, but in logarithmic scale. Why? It turns out that expressing such ratio in log scale provide us with much higher **dynamic range**. For example, our human perception is generally in log scale, not linear scale. Our hearing and seeing sensitivity is not linear, but logarithmic.

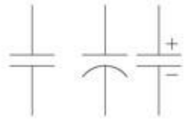
Log - base 10 of the ratio is known as a bel. Scaling this further up by a factor of 10 is known as decibel. That ratio is generally considering the ratio of output power to input power (not voltage). However, since power is voltage square, we found that the common equation to find voltage gain (ratio) in decibel is given by the equation:

$$A_{dB} = 20\log(\left|\frac{V_{out}}{V_{in}}\right|) = 20\log A$$

This is an equation that you must commit to memory – very useful for many things!

Types of Capacitors

- Capacitor symbol represents the two separated plates. Capacitor types are distinguished by the material used as the insulator.



- **Polystyrene**: Two sheets of foil separated by a thin plastic film and rolled up to save space. Values: 10 pF to 1 nF.

- **Ceramic**: Alternate layers of metal and ceramic (a few µm thick). Values: 1 nF to 1 µF.

- **Electrolytic**: Two sheets of aluminium foil separated by paper soaked in conducting electrolyte. The insulator is a thin oxide layer on one of the foils. Values: 1 µF to 10mF.

- Electrolytic capacitors are **polarised**: the foil with the oxide layer must always be at a positive **voltage** relative to the other (else **explosion**).
- Negative terminal indicated by a curved plate in symbol or "-".

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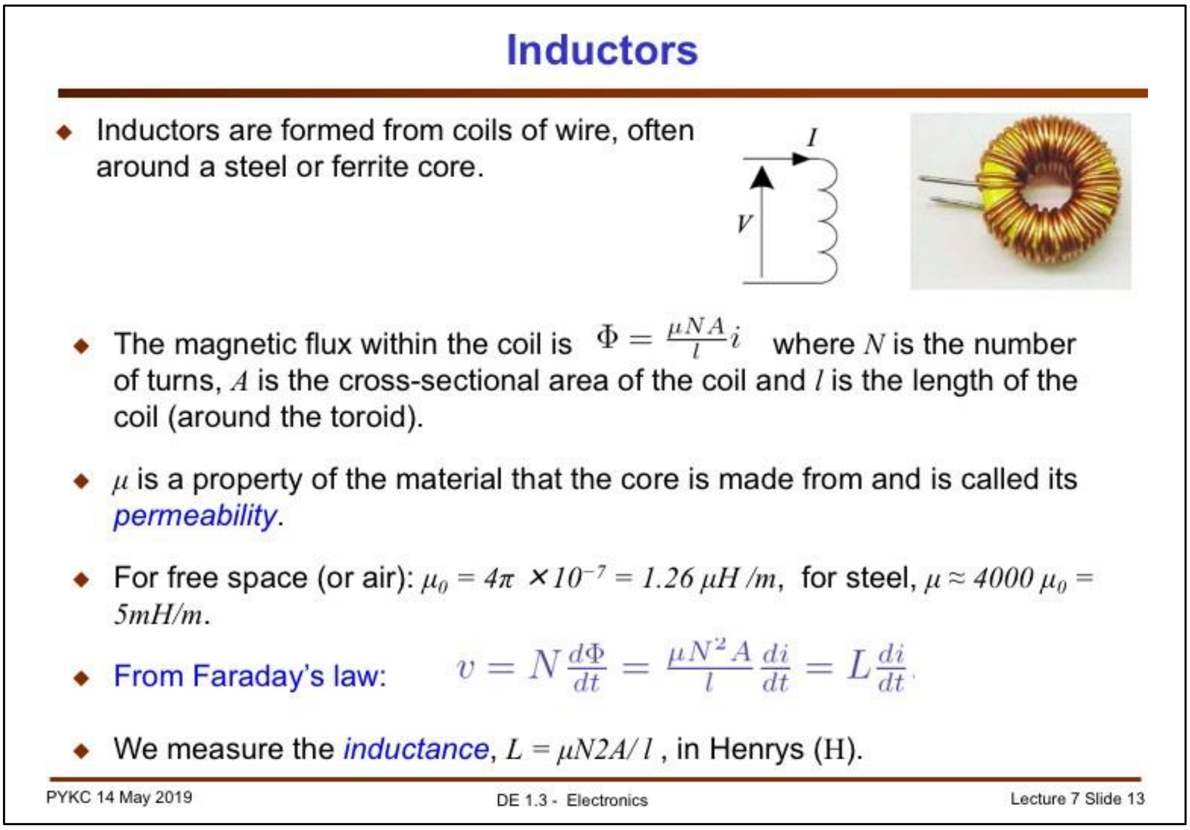
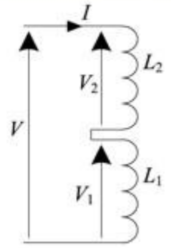


Inductors are the complementary component to the capacitor. They are not commonly found in electronic circuits because they are bulky and expensive, and practical inductors are far from ideal. However, they are found in motors, transformers and other electrical mechanisms. They are also found as stray effects (undesirable side effects) with interconnecting wires (such as wires that you use to connect circuits on the breadboard). Inductors are used as antennae for sending and receiving radio signals, and form part of transformers used in wirelesscharging.

Here are some basic equations governing an inductor. The most important is v = IL di/dt. Note the similarity to the capacitor equations.

Series and Parallel Inductors

Series inductors:

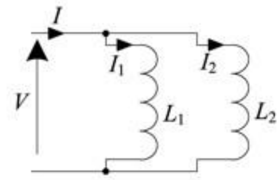


$$v = v_1 + v_2 = L_1\frac{di}{dt} + L_2\frac{di}{dt}$$
$$= (L_1 + L_2)\frac{di}{dt}$$

- Same equation as a single inductor of value $L_1 + L_2$

Parallel inductors:

$$\frac{di}{dt} = \frac{d(i_1+i_2)}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$
$$= \frac{v}{L_1} + \frac{v}{L_2} = v\left(\frac{1}{L_1} + \frac{1}{L_2}\right)$$
$$v = \frac{1}{\frac{1}{L_1}+\frac{1}{L_2}}\frac{di}{dt}$$



- Same as a single inductor of value $\frac{1}{\frac{1}{L_1}+\frac{1}{L_2}} = \frac{L_1 L_2}{L_1+L_2}$
- Inductors combine just like resistors.

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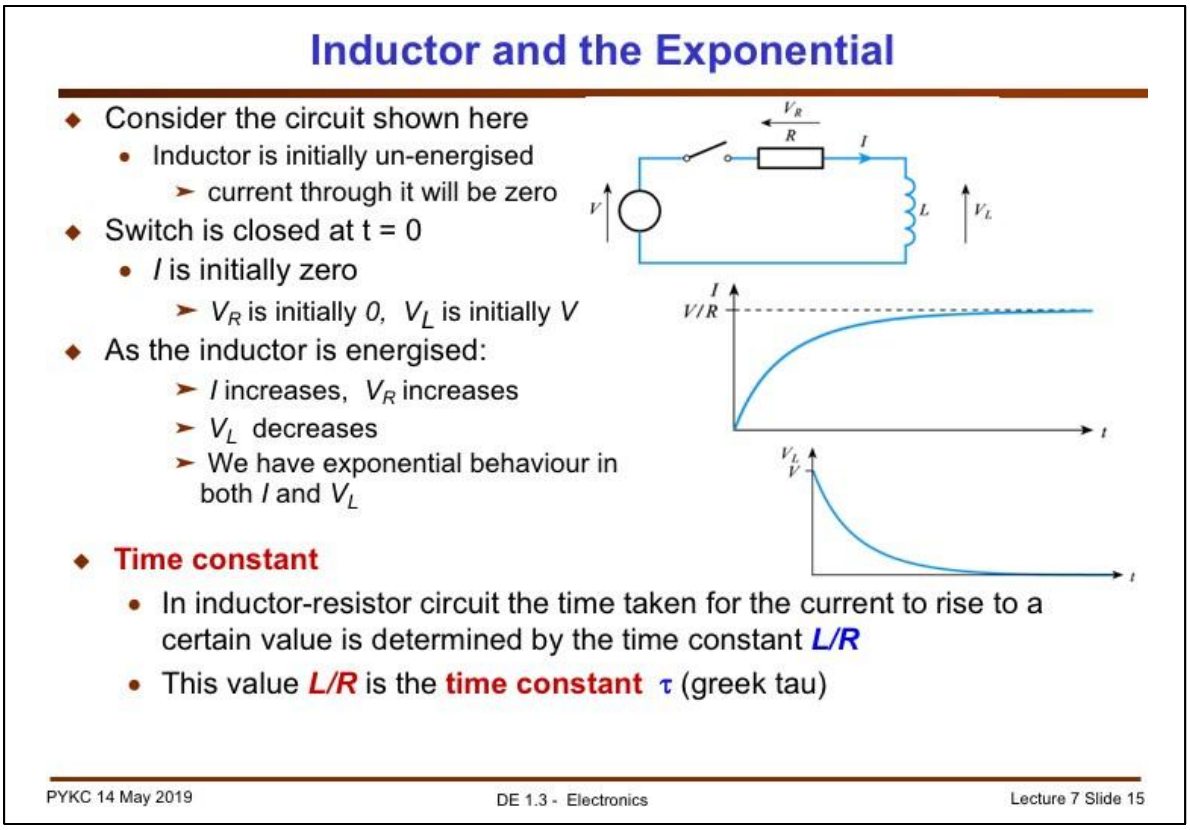
Series and parallel inductors combine just like resistorsdo.



Energising an inductor is similar to that of charging a capacitor. Except that the inductor CURRENT (as suppose to the capacitor voltage) is rising exponentially. The inductor voltage is decreasing exponentially.

The time constant is L/R as suppose to RC.

Step Response of a LR circuit

- Consider what happens to the circuit shown here as the switch is closed at t = 0.
- Apply KVL around the loop, we get:

$$iR + v = V, \ \ but \ \ v = L\frac{di}{dt} \quad therefore \ \ iR + L\frac{di}{dt} = V$$

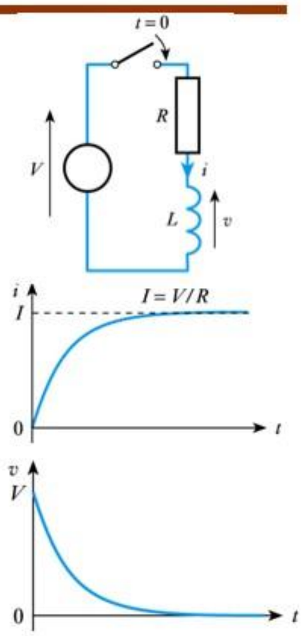
- This is a simple first-order differential equation with constant coefficients.
- Assuming $i_L = 0$ at $t = 0$, the solution to this is

$$i = I(1 - e^{-\frac{t}{L/R}}) = I(1 - e^{-\frac{t}{\tau}}), \quad \text{where } \tau = \frac{L}{R} = \text{time-constant}$$

$$\text{and} \quad I = V/R$$

- Since $v = L\frac{di}{dt}$ this gives (assuming $V_L = 0$ at $t = 0$):

$$v = V \times e^{-\frac{t}{L/R}} = V \times e^{-\frac{t}{\tau}}, \quad \text{where } V = i \times R$$



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We can perform the same analysis with an inductor being energised (we don’t call this charging). At t=0, when the switch is first closed, NO CURRENT FLOWS, since the current through an inductor cannot change instantaneously. Since no current flows, voltage across the inductor must be V, the same as the voltage source. Therefore as soon as the switch is closed, v goes from 0 to V instantaneously! This is a characteristic of a LR circuit.

The current rises from 0, therefore the voltage drop across the resistor R increases, decreasing the inductor voltage. Solving the first-order differential equation provides the exact equations for i_L and v_L.

De-energising Inductor in a LR circuit

- Consider what happens to the circuit shown here as the left switch is open and the right switch closed at t = 0.
- At $t = 0$, $i_L = V/R$.
- Apply KVL around the right loop, we get:

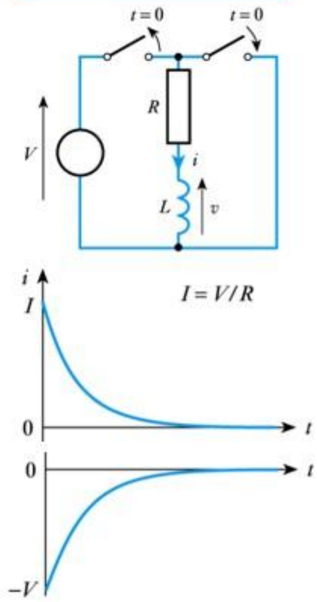
$$iR + v = 0, \quad and \quad v = L\frac{di}{dt} \quad therefore \quad iR + L\frac{di}{dt} = 0$$

- Solving this simple first-order differential equation gives:

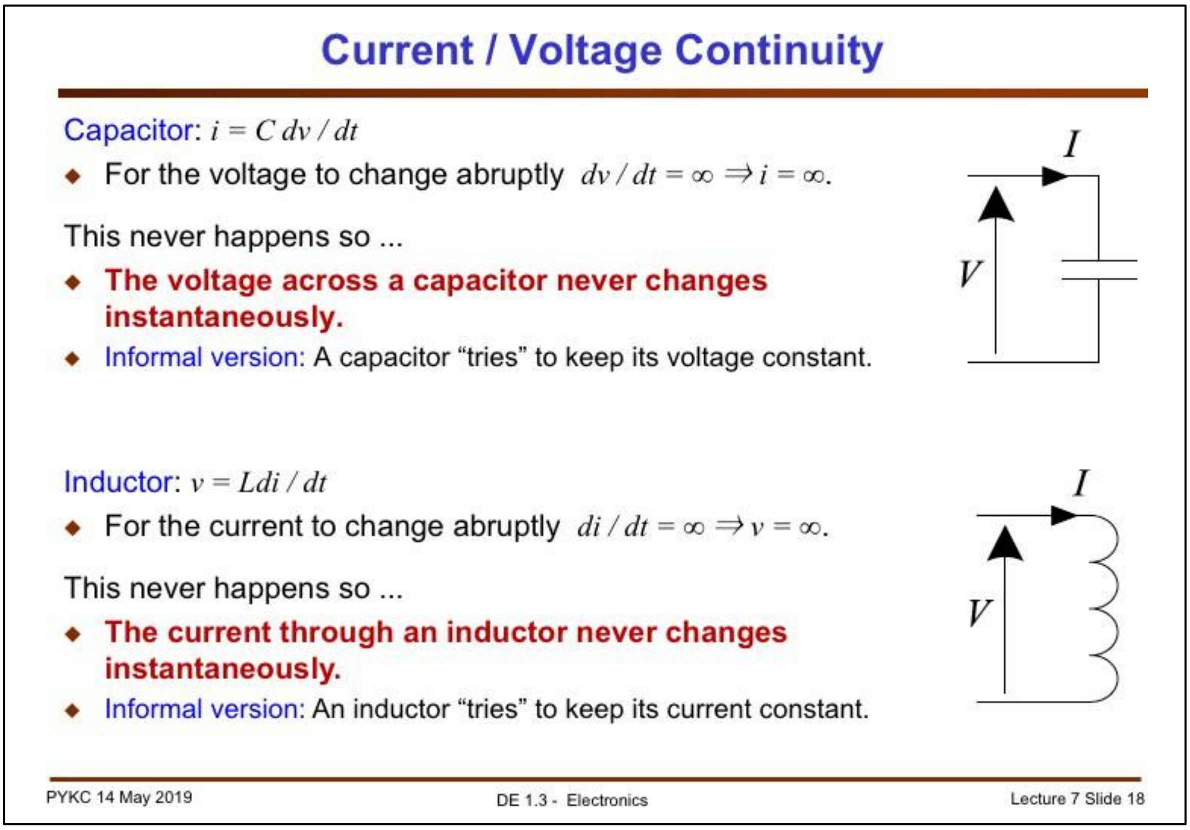
$$i = I \times e^{-\frac{t}{L/R}} = I \times e^{-\frac{t}{\tau}}, \quad \text{where } I = \frac{V}{R} \text{ and } \tau = \frac{L}{R}$$

- And:

$$v = -V \times e^{-\frac{t}{L/R}} = -V \times e^{-\frac{t}{\tau}},$$



Similarly when we de-energise the inductor, we get the exponential characteristics as we did for discharging the capacitor.



The take-home message that you must remember is that:

Capacitor tries to keep its voltage constant.
Inductor tries to keep its current constant.

Summary

- Capacitor:
  - $i = C\,dv/dt$
  - parallel capacitors add in value
  - v across a capacitor never changes instantaneously.
- Inductor:
  - $v = L\,di/dt$
  - series inductors add in value (like resistors)
  - i through an inductor never changes instantaneously.