UNIVERSITY OF BRADFORD

Coding

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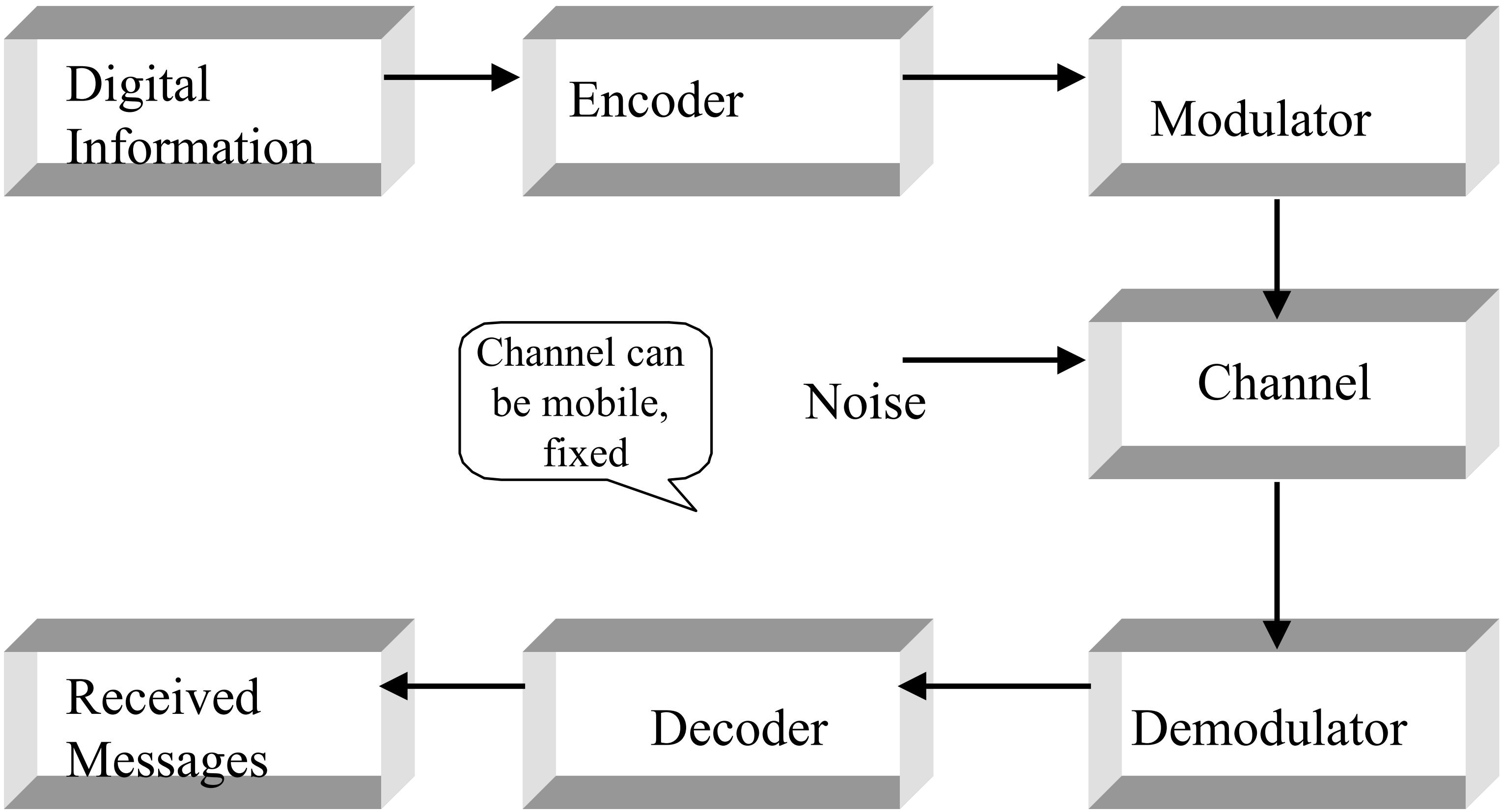
Contents

- Fundamentals of Coding
- Forward Error Correction
  - Block Codes
  - Convolutional Codes
  - Interleavers
- ARQ

Application

- Error correcting techniques are used to improve the bit error rate performance of a digital signal

Digital Transmission Chain



Channel Capacity

- Shannon-Hartley Law

$$R = \log_2\left(1 + \frac{P}{N_0 B}\right) \quad bit/s$$

  - P = Received Carrier Power, Watts
  - B = Bandwidth of channel, Hz
  - R = Capacity, bit/s
  - N_0 = Single sided noise power spectral density, W/Hz

Power & Bandwidth Limits

- If $R < B$, link is said to be power limited
  - Inefficient use of bandwidth
- If $R > B$, link is said to be bandwidth limited
  - Could increase capacity by using available transmit power in wider bandwidth

Shannon Limit

- Shannon limit defines E_b/N_0 below which link cannot operate at capacity
- This is equal to -1.6 dB
- This is a theoretical link as E_b/N_0 is dependent upon modulation and coding scheme

Coding Techniques

- Techniques can be divided into two broad categories
- Forward Error Correction (FEC)
  - Errors are detected and corrected for at the receiver
- Automatic Repeat Request (ARQ)
  - Used when high degree of integrity is required and latency is not a problem

Channel Coding using FEC

- Involves adding *r* redundancy bits to source information
- Two mechanisms employed in satellite communications
  - Block Coding
  - Convolutional Coding

Block Codes

- Information is divided into blocks of *k* symbols
- These are then coded into blocks of *n* symbols (n > k)
  - This is known as a (n, k) block code
- Coding of one block is entirely independent of another
- Convenient method of coding information that is naturally divided into blocks

Linear Block Codes

- 2^K possible message blocks to which are added (n-k) redundant check bits
- The redundant check bits are generated from the *k* message bits by a pre-determined rule

Code Generation

$$\mathbf{c} = \mathbf{m}\, G$$

where:

- c = code sequence
  - $[c_1, c_2, c_3 \ldots c_n]$
- m = message sequence
  - $[m_1, m_2, m_3 \ldots, m_n]$
- G = *Generator Matrix*

Example

$$G = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad m = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$c = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

This is a (3, 2) code. Note modulo-2 arithmetic used.

Systematic Linear Block Codes

- First k bits of codeword are the message and the remaining $(n-k)$ bits are the check bits
- General form of G

$$G = \begin{bmatrix} I_k & P_k \end{bmatrix}_{k \times n}$$

  - I_k = Identity matrix of order k
  - P = arbitrary k x (n-k) matrix

Error Detection

- Parity check matrix is used to detect errors

$$H = \left[P^T : \quad I_{n-k} \right]_{(n-k)\times n}$$

- P^T = Transpose of matrix P
  - (Interchange of Rows and Columns of matrix P)

Implementation of Error Correction

- Error detection achieved by multiplying received codeword *R* by the ***transpose*** of the parity check matrix H^T
- If received correctly
  - $R = C$
  - $C \times H^T = 0$

Example of Error Detection

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Occurrence of Error

- If codeword *R* is in error
  - R = C + E
  - E = Error vector
- Errors are detected by finding the ***error syndrome S***
- $S = RH^T$
  - of length (n-k), where (n-k) is the number of parity check bits in a codeword

Example (continued)

- **c** = **m**G

  = 101001 0

- Syndrome S = cH^T = 0
  - Implies error free transmission
- If an error is introduced into the 2nd bit

  R = 1110010

Then S = RH^T = 110

Non-zero implies an error

Hamming Distance

- Hamming distance, *d*, between two code vectors C_1 and C_2 is the number of components by which they differ,
- For a block code, this corresponds to the smallest distance between any pairs of codeword in the entire code

Error Detection and Correction

- Number of errors that can be detected
  - $= d_{min} - 1$
- No of errors that can be corrected
  - $= 1/2(d_{min} - 1)$
- Let
  - $C_1 = [1, 0, 0, 1, 0, 1]$
  - $C_2 = [1, 0, 1, 0, 1, 1]$
  - d = 3, therefore 2 errors can be detected and 1 error can be corrected.

Syndrome and Error Correction

- To detect 4 errors and correct 2 of them requires a code with a minimum distance of 5
- Syndrome points to position of error when 1 error detected
- Syndrome is used to suggest most likely codeword when more than 1 error detected

Cyclic Codes

- These are of the form (n, k)
- If
  - $(v_0, v_1, v_2, v_3, \ldots. V_{n-1})$
- is a codeword, then so is
  - $(v_{n-1}, v_0, v_1, v_2 \ldots v_{n-2})$
- Can be thought of as a shift to the right register with feedback

Popular Forms of Block Codes

- Hamming Codes
  - Minimum distance of 3
- BCH
  - Most powerful of all codes
- Reed-Solomon (RS)
  - Used for correcting bursty errors in mobile satellite communications
- Golay M
  - Minimum distance of 7
- Code selection is dependent generally on channel characteristics

Convolutional Codes

- Information is presented to the coder in frames of k_0 bits
- Encoder output consists of frames of n_0 bits ($n_0 > k_0$)
- Encoder retains some memory of previous frames and this is used in the coding process
- The memory order of the code, *m*, is the number of previous frames remembered
- Code can be termed as either (n_0, k_0, m) code or a (n, k) where $n = (m+1)n_o$ and $k = (m+1)k_0$
- The larger *m*, the more powerful the code

Code Generation

- Generated by a tapped shift register and two or more modulo-2 adders
- Can be represented pictorially as for example:

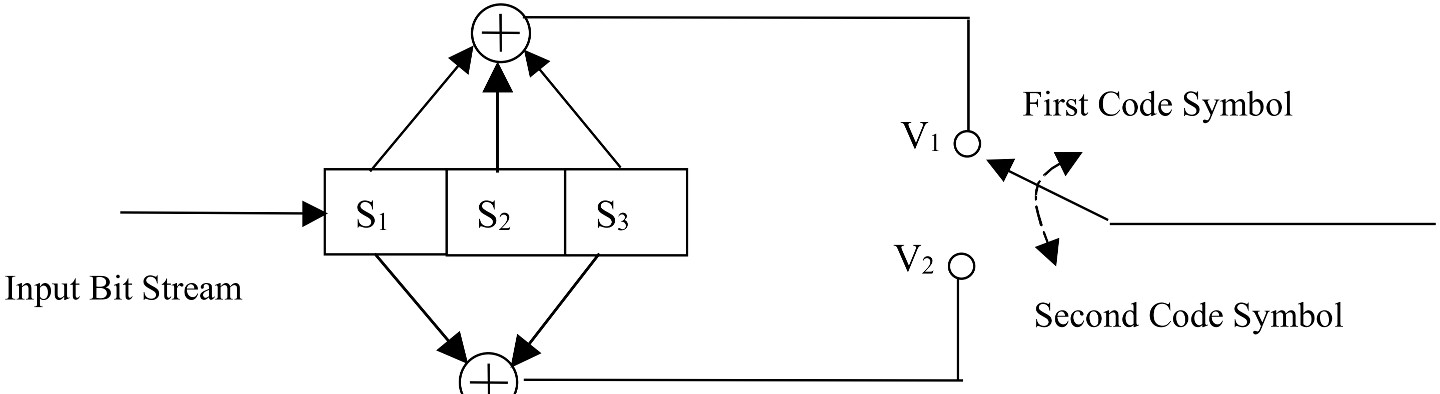


Diagram shows 1/2 rate encoder of constraint length 3

Code Rate Relationship

- If ρ = Code Rate
- The encoded output rate is increased to R_c according to

$$R_c = \frac{R_b}{\rho} \qquad \rho = \frac{\eta}{\eta + r_b}$$

  - Where

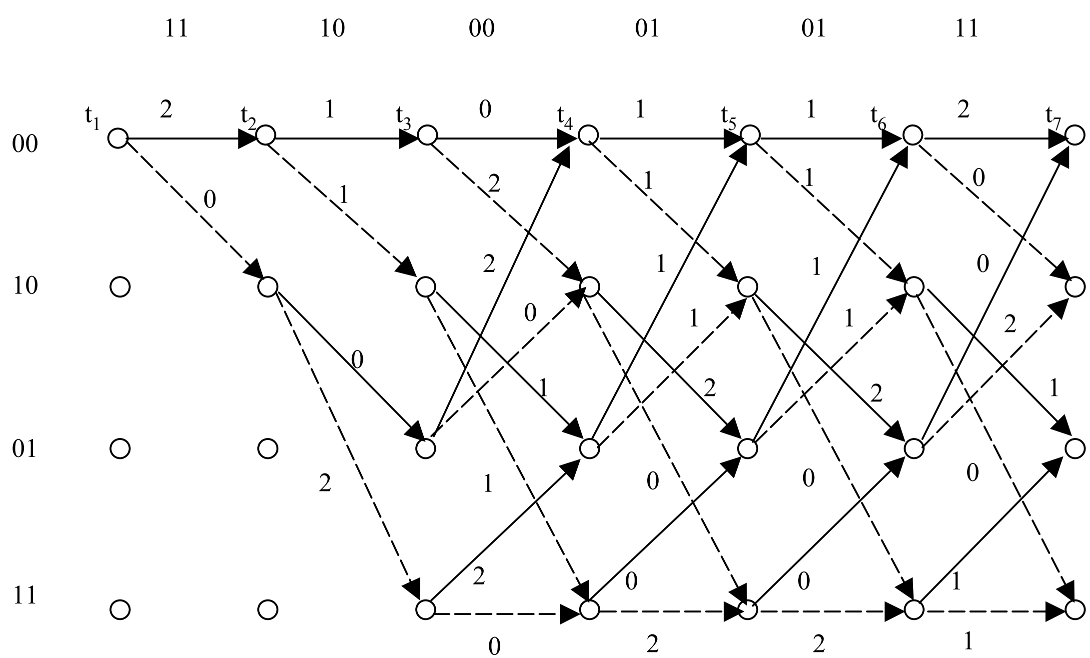
    η= No of Source bits

    r_b = added redundancy bits

- Typically ρ = 1/2, 3/4, 7/8, etc

Decoder

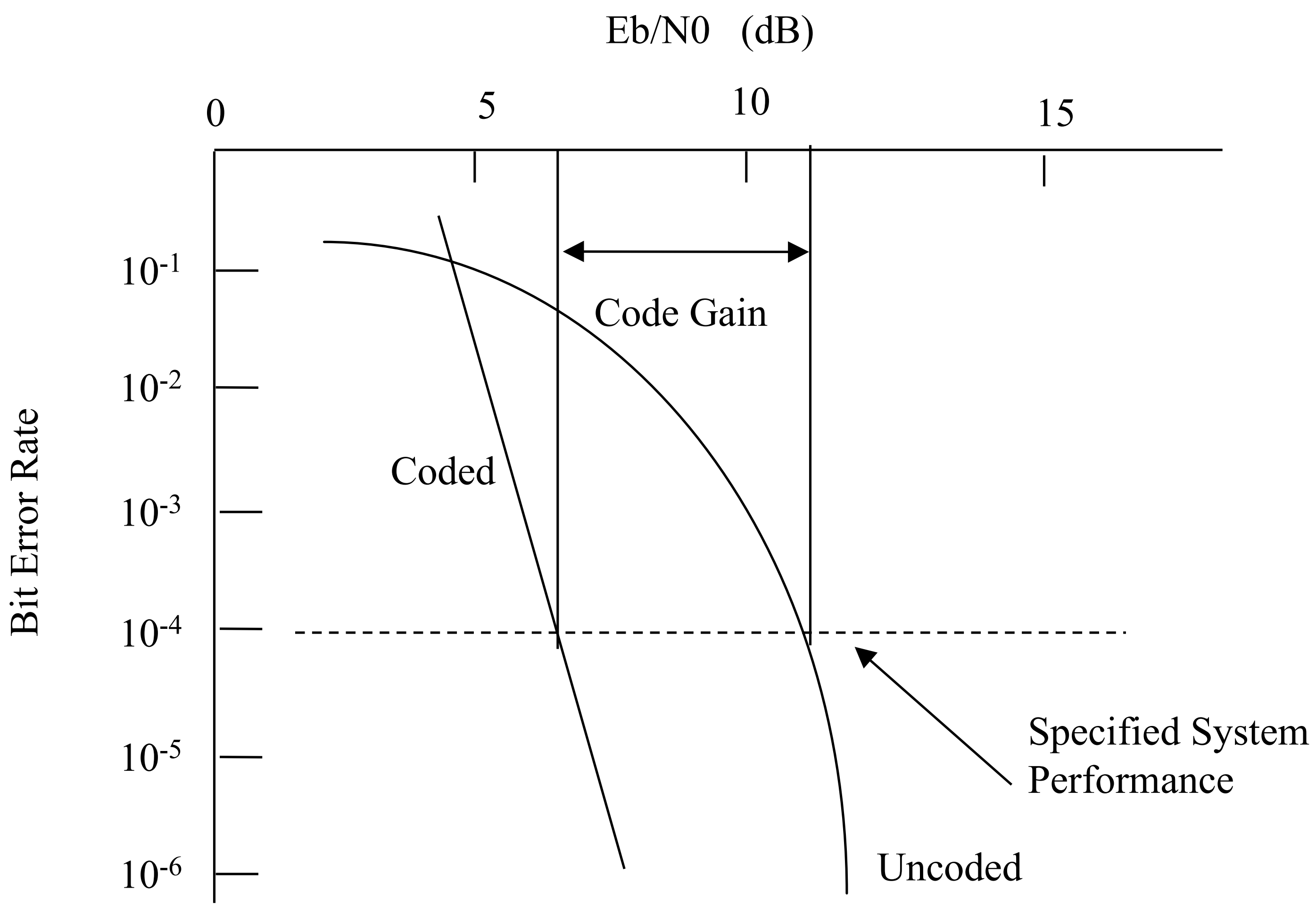
- Viterbi Decoding is generally used to decode receive bits
- Works on the principle of trellis decoding



Code Gain

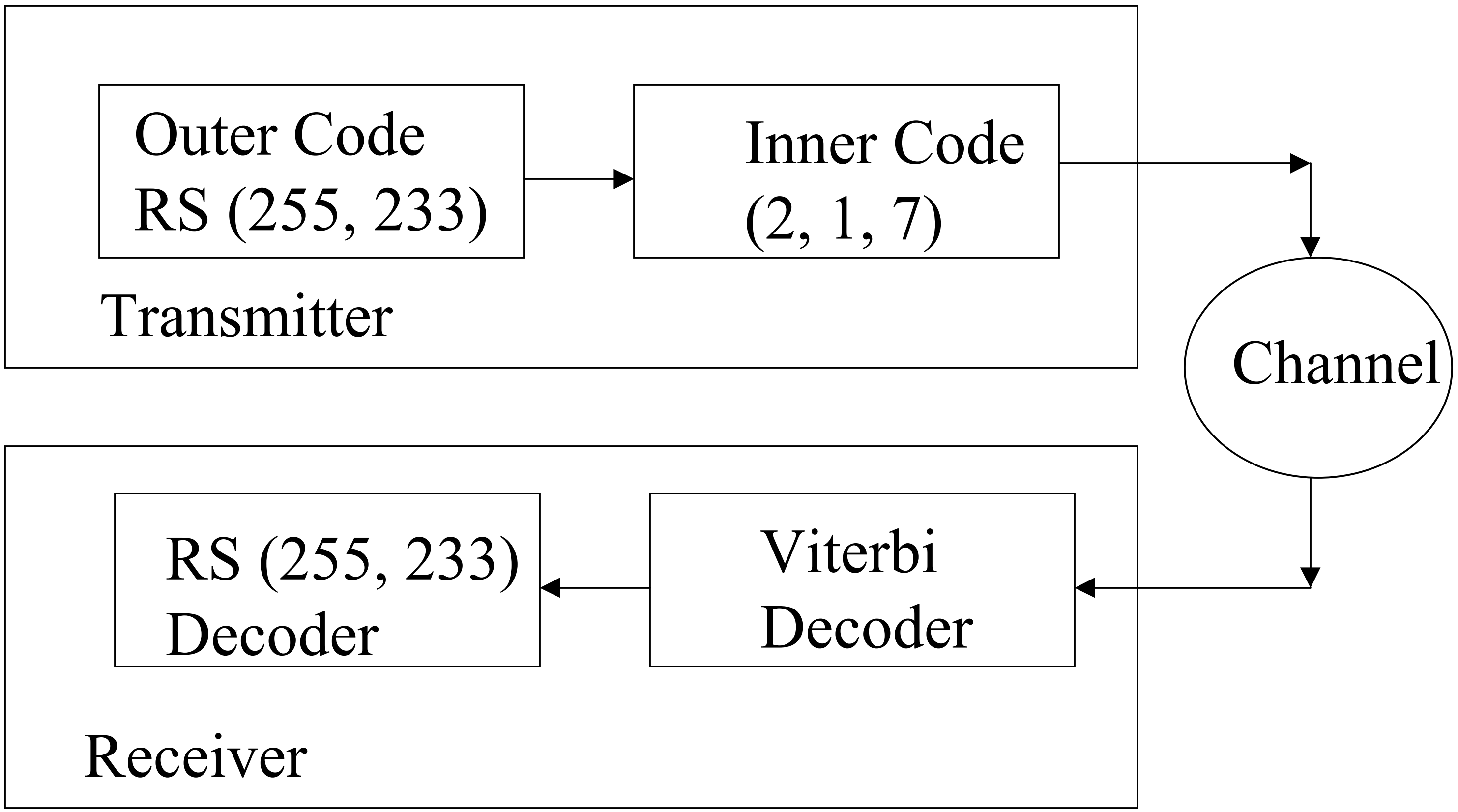
- Difference in dB between E_b/N_0 for a given BER in the case of ideal transmission and that of a particular coding scheme
- Coding allows improvement in BER performance for the same transmit power
- The improvement is measured in terms of the coding gain
- Coding gain is typically 3 - 6 dB

Code Gain



Eb/N0 (dB)
0
5
10
15
10-1
10-2
10-3
10-4
10-5
10-6
Bit Error Rate
Code Gain
Coded
Specified System
Performance
Uncoded

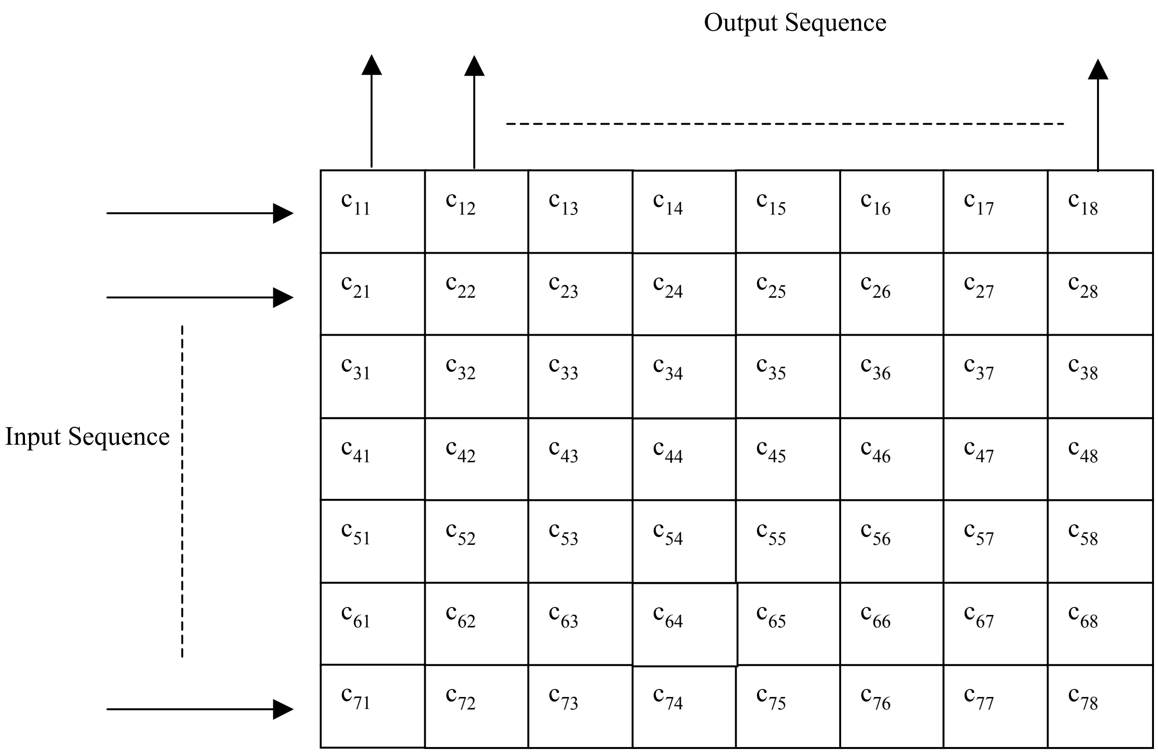
Concatenated Codes



Interleaving

- Used to reduce the effect of bursty errors
- Interleaver re-orders sequence of transmission bits to minimise effect of error burst
- De-interleaving is performed at the receiver to re-order into original sequence

Example Interleaver



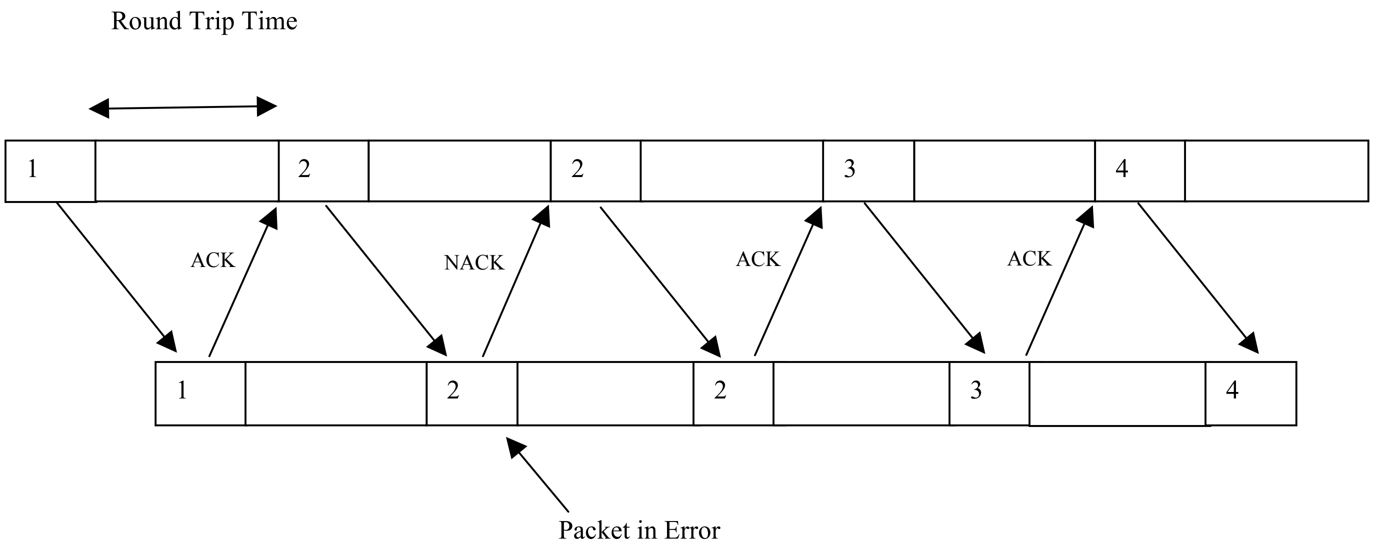
Automatic Repeat Request

- FEC Schemes cannot always correct for errors at the receiver
- ARQ schemes operate based on a re-transmission protocol, whereby an ACK is used to acknowledge correctly received data

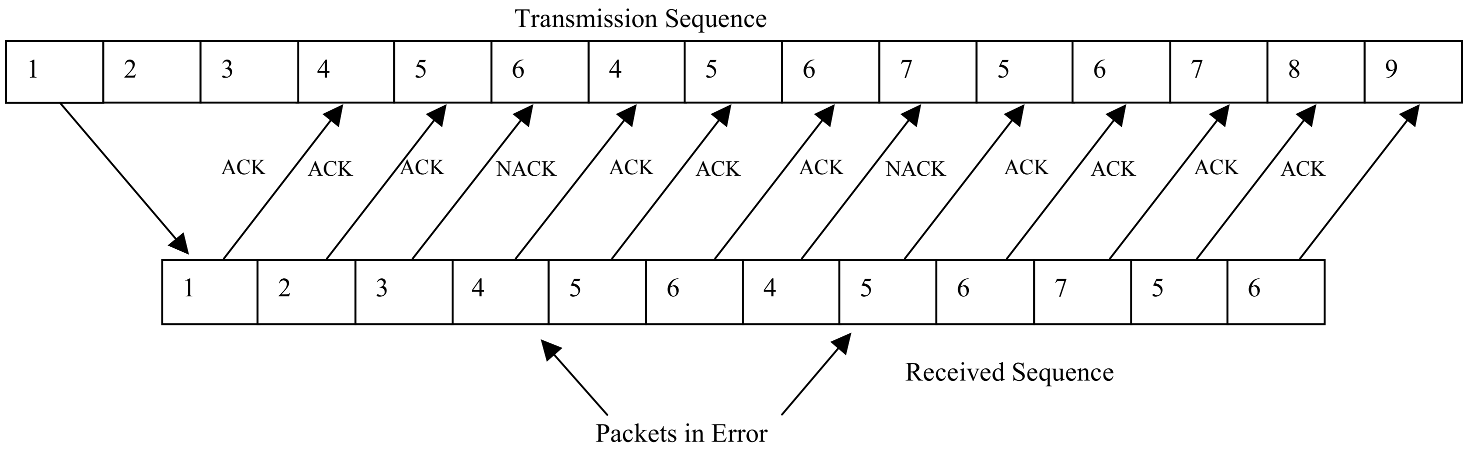
Three classes of ARQ

- Stop and Wait
- Continuous ARQ with repeat
- Continuous ARQ with selected repeat

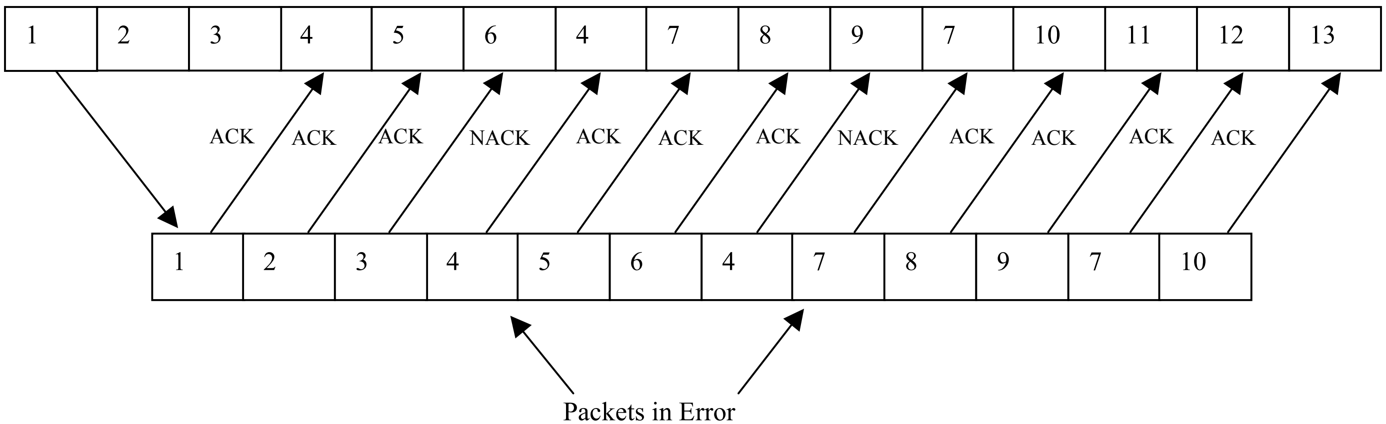
Stop and Wait



Continuous ARQ with repeat



Continuous ARQ with selected repeat



Directed Reading

- Chapter 5, Mobile Satellite Communication Networks, Sheriff & Hu
- Blackboard multiple choice revision questions