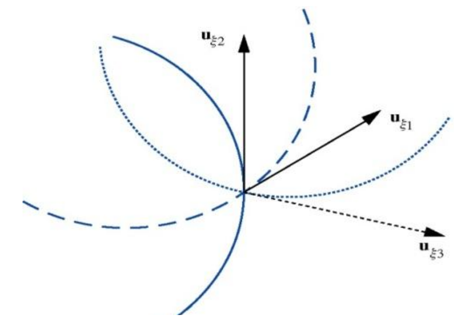
CHAPTER 2

Coordinate Systems and Transformation

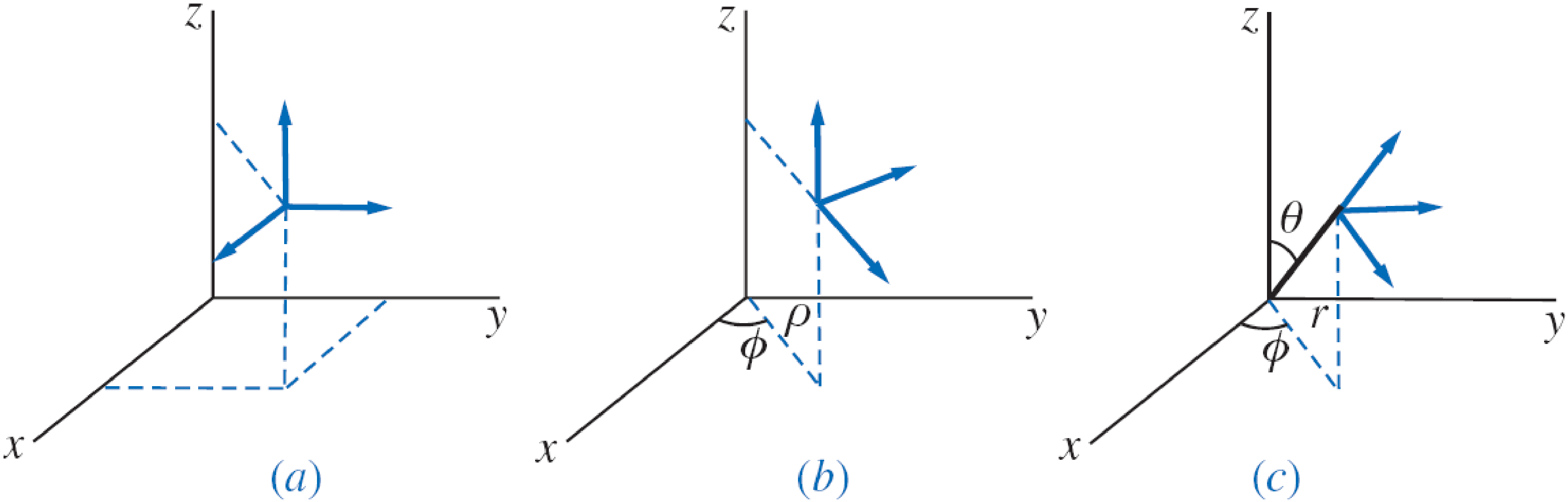
Special Thank to Eng :Mohammed EL Asmar

2.1: Coordinate systems

- In a 3D space, a coordinate system can be specified by the intersection of 3 surfaces. An orthogonal coordinate system is defined when these three surfaces are mutually orthogonal at a point .



- Most commonly used coordinate systems



(a)– Cartesian; (b) – Cylindrical; (c) – Spherical.

Why we need a new coordinate systems ?

It is more easy to analysis some problems that has a cylindrical or spherical symmetry with spherical and cylindrical coordinate .

2.2: Cartesian Coordinates (x,y,z)

An intersection of 3 planes:

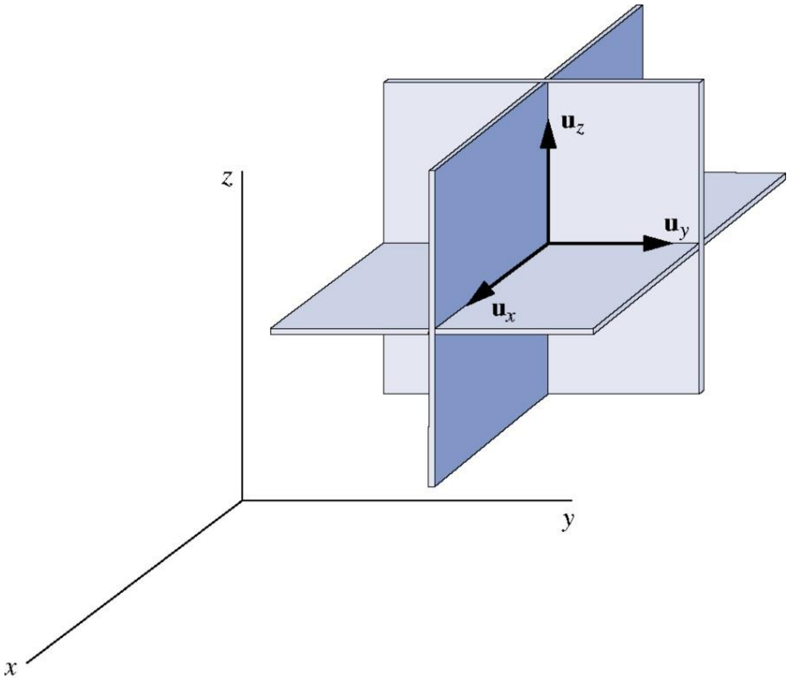
x = const; y = const; z = const

$$-\infty < x < \infty$$

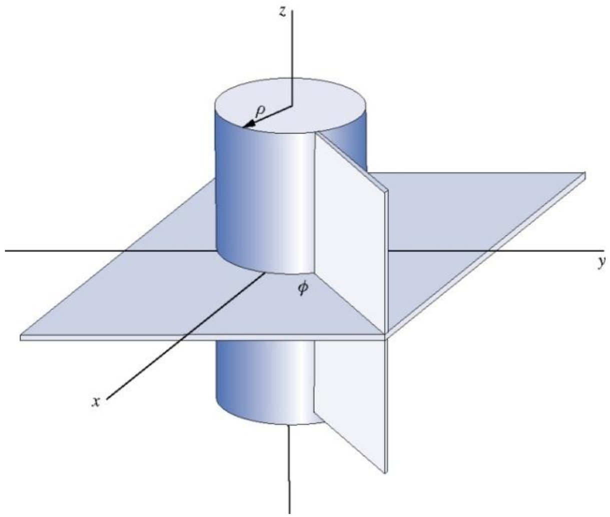
$$-\infty < y < \infty$$

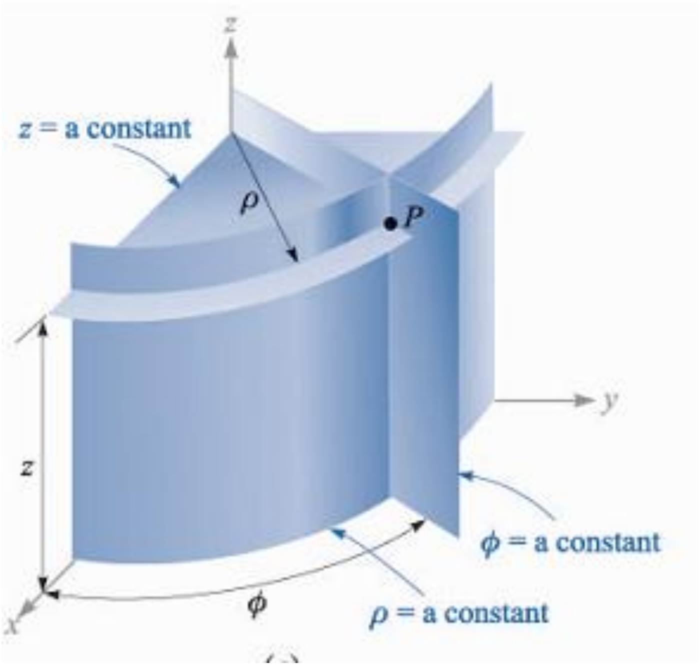
$$-\infty < z < \infty$$

$$A = A_x a_x + A_y a_y + A_z a_z$$



2.3: Cylindrical Coordinates (ρ ,ø,z)





- An intersection of a cylinder and 2 planes

$$0 \le \rho < \infty$$

$$0 \le \varphi \le 2\pi$$

$$-\infty < z < \infty$$

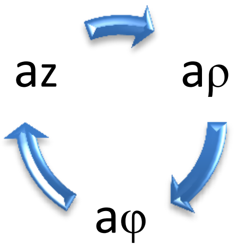
$$A = A_\rho a_\rho + A_\varphi a_\varphi + A_z a_z$$

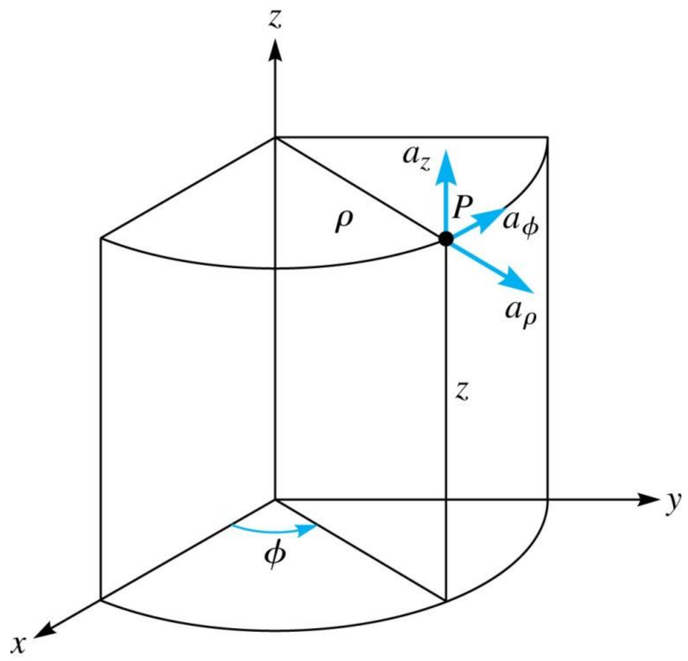
Properties:

$$a_\rho . a_\rho = 1 \;,\quad a_\varphi . a_\varphi = 1 \;,\quad a_z . a_z = 1$$

$$a_\rho . a_\varphi = 0 \;,\quad a_\varphi . a_z = 0 \;,\quad a_\rho . a_z = 0$$

$$a_\rho \text{x}\, a_\varphi = a_z \;,\quad a_\varphi \text{x}\, a_z = a_\rho \;,\quad a_z \text{x}\, a_\rho = a_\varphi$$





$$a\rho \perp a\varphi \perp az$$

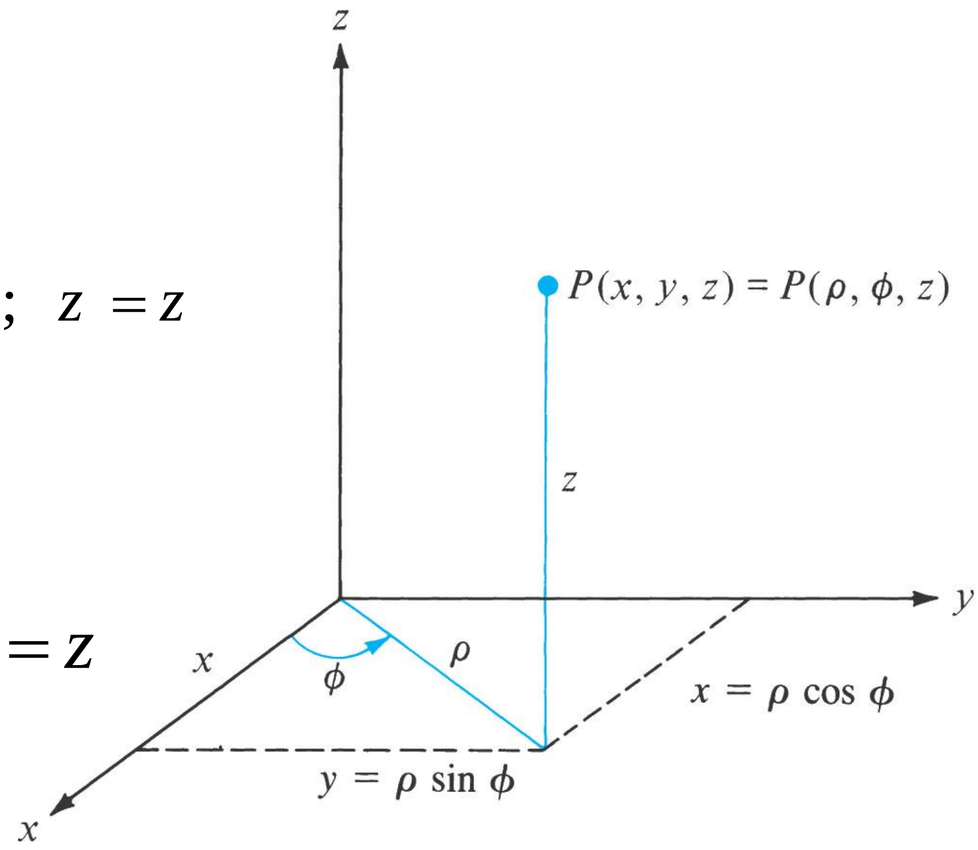
Transformation points $(x, y, z) \Leftrightarrow (\rho, \phi, z)$

1. Cartesian to Cylindrical:

$$\rho = \sqrt{x^2 + y^2}; \quad \phi = \tan^{-1}\left(\frac{y}{x}\right); \quad z = z$$

2. Cylindrical to Cartesian:

$$x = \rho\cos\phi; \quad y = \rho\sin\phi; \quad z = z$$



Transformation vectors

$$A = A_x a_x + A_y a_y + A_z a_z \Leftrightarrow A = A_\rho a_\rho + A_\phi a_\phi + A_z a_z$$

$$(x, y, z) \Leftrightarrow (\rho, \phi, z)$$

- To find any desired component of a vector, we recall from the discussion of the dot product that a component in a desired direction may be obtained by taking the dot product of the vector and a unit vector in the desired direction

Hence,

$$A_\rho = A \bullet a_\rho \quad and \quad A_\phi = A \bullet a_\phi$$

Expanding these dot products, we have

$$A_\rho = (A_x \mathbf{a}_x + A_y \mathbf{a}_y + A_z \mathbf{a}_z) \cdot \mathbf{a}_\rho = A_x \mathbf{a}_x \cdot \mathbf{a}_\rho + A_y \mathbf{a}_y \cdot \mathbf{a}_\rho$$

$$A_\phi = (A_x \mathbf{a}_x + A_y \mathbf{a}_y + A_z \mathbf{a}_z) \cdot \mathbf{a}_\phi = A_x \mathbf{a}_x \cdot \mathbf{a}_\phi + A_y \mathbf{a}_y \cdot \mathbf{a}_\phi$$

$$A_z = (A_x \mathbf{a}_x + A_y \mathbf{a}_y + A_z \mathbf{a}_z) \cdot \mathbf{a}_z = A_z \mathbf{a}_z \cdot \mathbf{a}_z = A_z$$

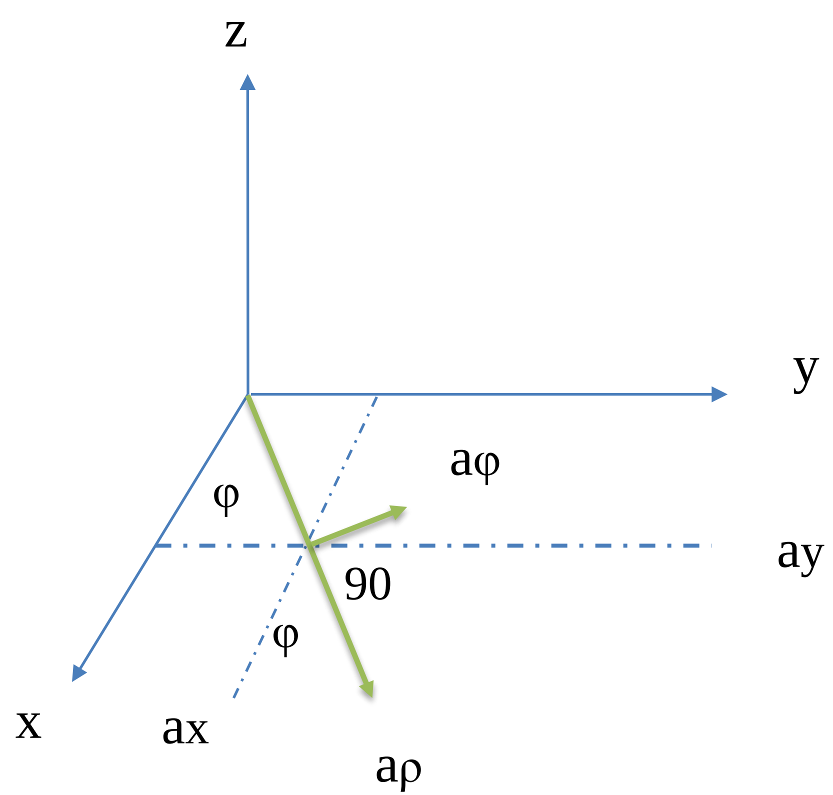
The angel between a_x and a_ρ is ϕ thus $a_x . a_\rho = \cos(\phi)$

but the angel between a_x and a_ϕ is $90 + \phi$ thus $a_x . a_\rho = \cos(90 + \phi) = -\sin(\phi)$

$$A_\rho = A_x \cos\phi + A_y \sin\phi$$

$$A_\phi = -A_x \sin\phi + A_y \cos\phi$$

$$A_z = A_z$$



طريقة اخري للفهم :

$$\because A = A_x a_x + A_y a_y + A_z a_z \qquad (1)$$

The relation between $(\mathrm{a}_x, \mathrm{a}_y, \mathrm{a}_z)$ and $(\mathrm{a}_\rho, \mathrm{a}_\phi, \mathrm{a}_z)$ are obtained geometrically from

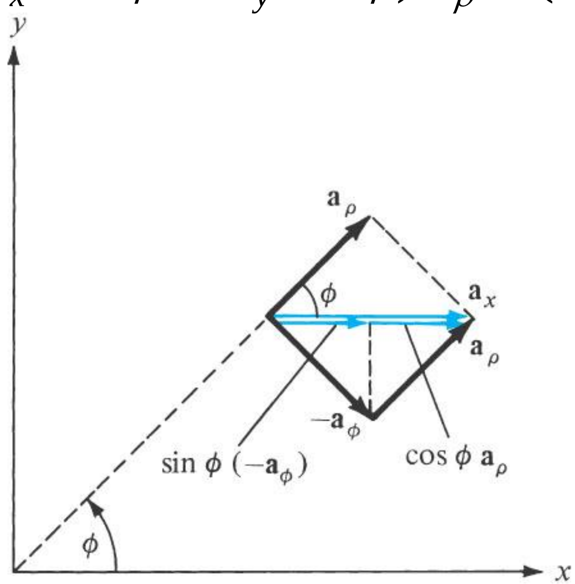
$$\mathrm{a}_x = \cos\phi\, \mathrm{a}_\rho - \sin\phi\, \mathrm{a}_\phi$$

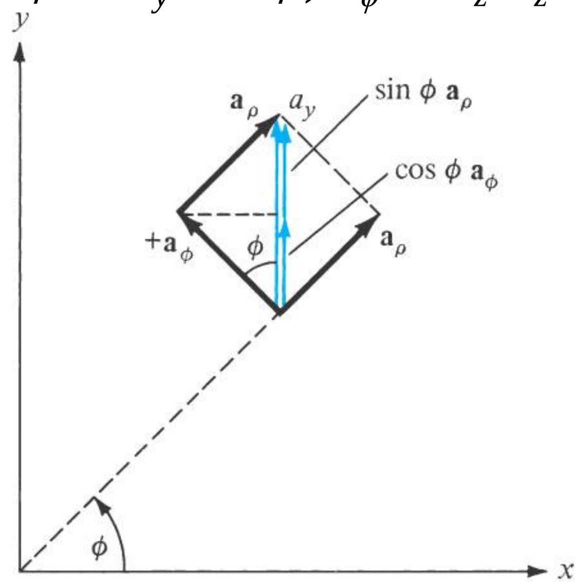
الان عوض عن ay, ax, في المعادلة1 ثم رتب الحدود

$$\mathrm{a}_y = \sin\phi\, \mathrm{a}_\rho + \cos\phi\, \mathrm{a}_\phi$$

$$a_z = a_z$$

$$A = (A_x \cos\phi + A_y \sin\phi)\mathrm{a}_\rho + (-A_x \sin\phi + A_y \cos\phi)\mathrm{a}_\phi + A_z\, a_z$$





$$A_\rho = A_x \cos\phi + A_y \sin\phi$$

$$A_\phi = -A_x \sin\phi + A_y \cos\phi$$

$$A_z = A_z$$

الخلاصة :

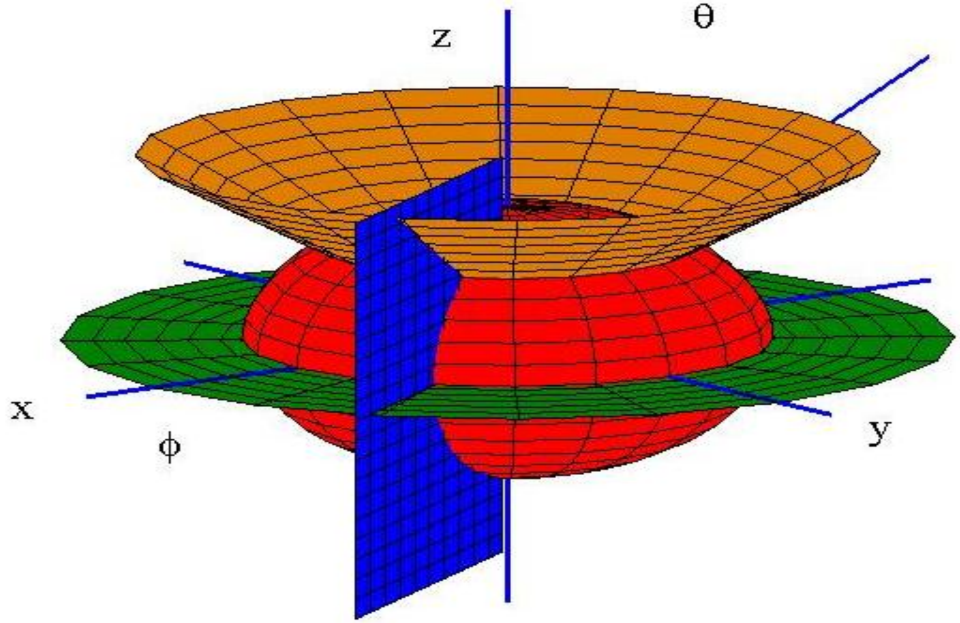
$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

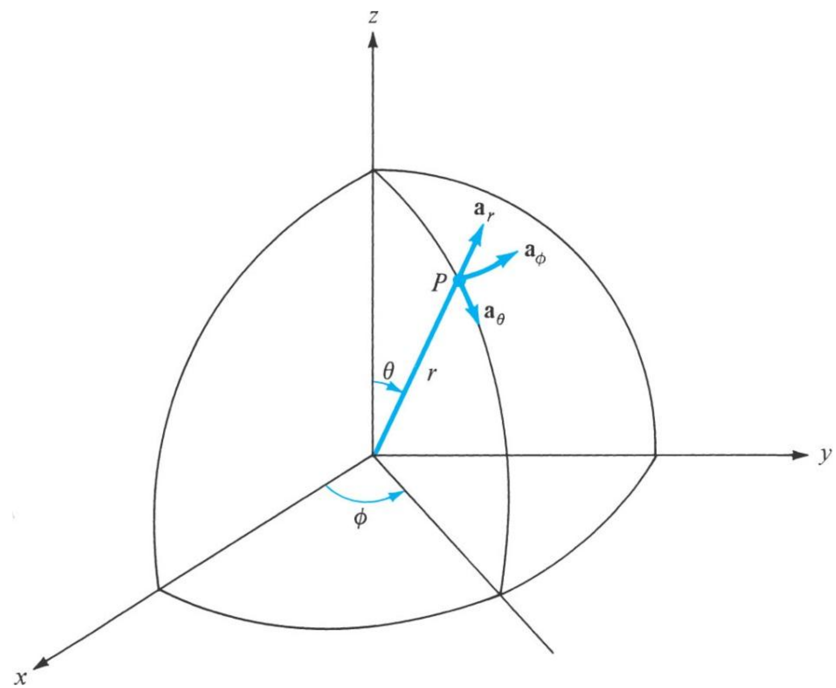
$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$A = A_x a_x + A_y a_y + A_z a_z \Leftrightarrow A = A_\rho a_\rho + A_\phi a_\phi + A_z a_z$$

2.4: Spherical Coordinates(r, θ, ϕ)

An intersection of a sphere of radius r, a plane that makes an angle ϕ to the x axis, and a cone that makes an angle θ to the z axis.





$$0 \leq r < \infty$$

$$0 \leq \theta \leq \pi$$

$$0 \leq \varphi \leq 2\pi$$

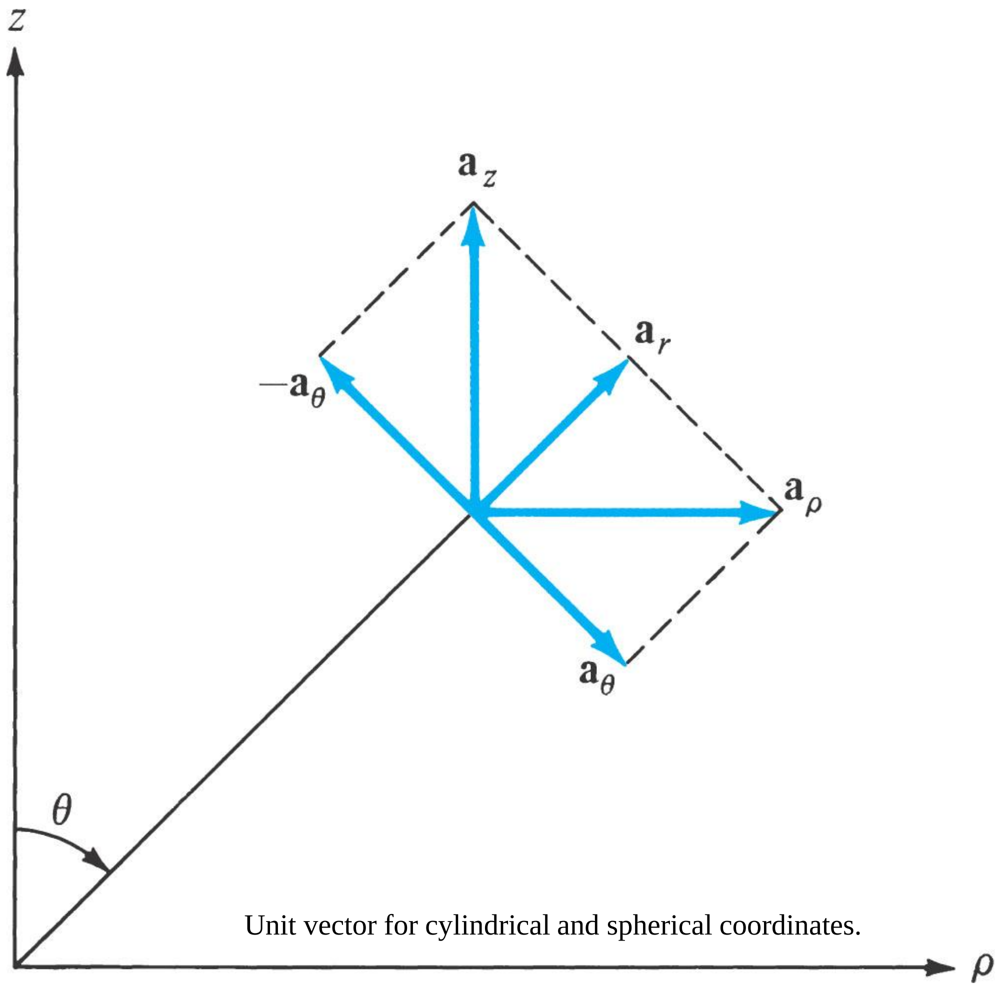
$$A = A_r\, a_r + A_\theta\, a_\theta + A_\varphi\, a_\varphi$$

$$a_r . a_r = 1 \ , \ a_\theta . a_\theta = 1 \ , \ a_\varphi . a_\varphi = 1$$

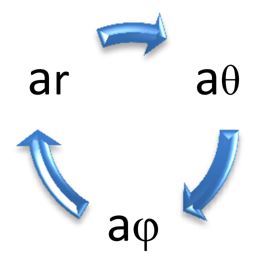
$$a_r . a_\theta = 0 \ , \ a_\theta . a_\varphi = 0 \ , \ a_r . a_\varphi = 0$$

$$a_r \text{x} \, a_\theta = a_\varphi \ , \ a_\theta \text{x} \, a_\varphi = a_r \ , \ a_\varphi \text{x} \, a_r = a_\theta$$

$$a_r \perp a_\theta \perp a_\varphi$$



Unit vector for cylindrical and spherical coordinates.



Transformation points $(x, y, z) \Leftrightarrow (r, \theta, \phi)$

$$z = r\cos\theta$$

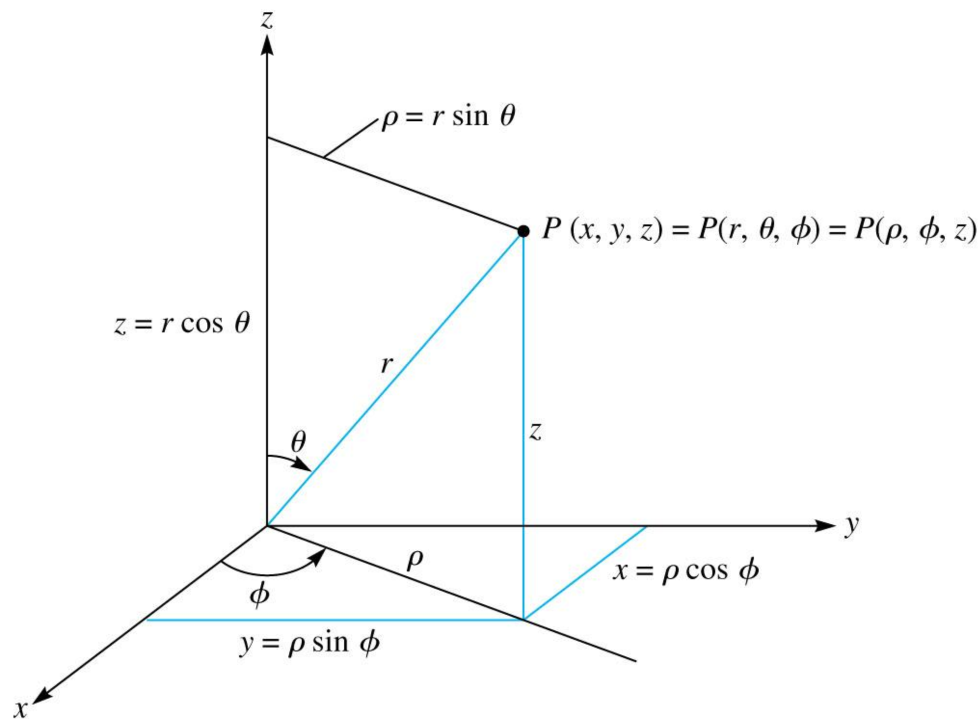
$$\rho = r\sin\theta$$

$$x = \rho\cos\varphi = r\sin\theta\cos\varphi$$

$$y = \rho\sin\varphi = r\sin\theta\sin\varphi$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\tan\varphi = \frac{y}{x}$$



. Cartesian to Spherical:

$$r = \sqrt{x^2 + y^2 + z^2}; \quad \theta = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right); \quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$$

. Spherical to Cartesian:

$$x = r \sin\theta \cos\phi; \quad y = r \sin\theta \sin\phi; \quad z = r \cos\theta$$

$$\mathbf{A} = A_x \mathbf{a}_x + A_y \mathbf{a}_y + A_z \mathbf{a}_z \Leftrightarrow \mathbf{A} = A_r \mathbf{a}_r + A_\theta \mathbf{a}_\theta + A_\phi \mathbf{a}_\phi$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\cos\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

Coordinate Transformation Procedure

(1) Transform the component scalars into the new coordinate system.

(2) Insert the component scalars into the coordinate transformation matrix and evaluate.

PE 2.1 :A-Convert Point P(1,3,5) form Cartesian to cylindrical

$$\rho = \sqrt{x^2 + y^2} = \sqrt{1+9} = 3.162$$

$$\tan\varphi = \frac{y}{x} = \frac{3}{1} \rightarrow \varphi = \tan^{-1} 3 = 71.56$$

$$z = z = 5$$

$$P(\rho, \varphi, z) = \mathrm{P}(3.162, 71.56, 5)$$

B-Convert Point P(1,3,5) form Cartesian to spherical :

$$\rho = \sqrt{x^2 + y^2} = \sqrt{1+9} = 3.162$$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{1+9+25} = 5.91$$

$$\rho = r\sin\theta, z = r\cos\theta$$

$$\tan\theta = \frac{\rho}{z} \Rightarrow \theta = \tan^{-1}\frac{3.162}{5} = 32.31^o$$

$$\tan\varphi = \frac{y}{x} = \frac{3}{1} \rightarrow \varphi = \tan^{-1} 3 = 71.56$$

$$(r, \theta, \varphi) = (5.91, 32.31^o, 71.56^o)$$

Problem 2.1:A-Convert Point P form cylindrical to Cartesian P(2,30,5)

$$P(2,30,5) = P(\rho, \varphi, z)$$

$$x = \rho\cos\varphi = (2)\cos(30) = 1.732$$

$$y = \rho\sin\varphi = (2)\sin(30) = 1$$

$$z = z = 5$$

$$\mathrm{P(x, y, z) = P(1.732\ ,\ 1\ ,5)}$$

B- Convert Point T to Cartesian T(10,pi/4,pi/3)

$$P(10, \pi/4, \pi/3) = P(r, \theta, \varphi)$$

$$z = r\cos\theta = (10)\cos(\pi/4) = 7.07106$$

$$\rho = r\sin\theta = (10)\sin(\pi/4) = 7.07106$$

$$x = \rho\cos\varphi = 7.07106 * \cos(\pi/3) = 3.5355$$

$$y = \rho\sin\varphi = 7.07106 * \sin(\pi/3) = 6.123$$

$$(x, y, z) = (3.5355, 3.5355, 7.07106)$$

Problem 2.3: If V=xz-xy+yz , express V In cylindrical.

$$V = (\rho\cos\varphi)z - (\rho\cos\varphi)(\rho\sin\varphi) + (\rho\sin\varphi)z$$

$$V(\rho, \varphi, z) = z\rho\cos\varphi - \rho^2\cos\varphi\sin\varphi + \rho z\sin\varphi$$

Example 2.1:

Given point $P(-2, 6, 3)$ and vector $\mathbf{A} = y\mathbf{a}_x + (x + z)\mathbf{a}_y$, express P and $\mathbf{A}$ in cylindrical and spherical coordinates. Evaluate $\mathbf{A}$ at P in the Cartesian, cylindrical, and spherical systems.

At point P: $x = -2$, $y = 6$, $z = 3$. Hence,

$$\rho = \sqrt{x^2 + y^2} = \sqrt{4 + 36} = 6.32$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{6}{-2} = 108.43°$$

ارجو الانتباه الي ان الربع
الذي تقع به الزاوية هو
الربع الثاني بينما الالة
الحاسبة سيكون جوابها
زاوية في الربع الرابع لذلك
سنضيف 180 او نطرح
180

$$z = 3$$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{4 + 36 + 9} = 7$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z} = \tan^{-1}\frac{\sqrt{40}}{3} = 64.62°$$

Thus,

$$P(-2, 6, 3) = P(6.32, 108.43°, 3) = P(7, 64.62°, 108.43°)$$

For vector $\mathbf{A}$, $A_x = y$, $A_y = x + z$, $A_z = 0$. Hence, in the cylindrical system

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ x + z \\ 0 \end{bmatrix}$$

or

$$A_\rho = y\cos\phi + (x + z)\sin\phi$$

$$A_\phi = -y\sin\phi + (x + z)\cos\phi$$

$$A_z = 0$$

But $x = \rho\cos\phi$, $y = \rho\sin\phi$, and substituting these yields

$$\mathbf{A} = (A_\rho, A_\phi, A_z) = [\rho\cos\phi\sin\phi + (\rho\cos\phi + z)\sin\phi]\mathbf{a}_\rho + [-\rho\sin^2\phi + (\rho\cos\phi + z)\cos\phi]\mathbf{a}_\phi$$

2.4(a) Transform the vector (x+z)ay to cylindrical

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ x+z \\ 0 \end{bmatrix}$$

$$x = \rho\cos\varphi$$
$$y = \rho\sin\varphi$$
$$z = r\cos\theta$$
$$\rho = r\sin\theta$$
$$\rho = \sqrt{x^2 + y^2}$$
$$r = \sqrt{x^2 + y^2 + z^2}$$

$$A\rho = 0 * \cos\varphi + (x+z)\sin\varphi + 0$$
$$A\rho = (x+z)\sin\varphi$$
$$A\rho = (\rho\cos\varphi + z)\sin\varphi$$

$$A\varphi = 0 * (-\sin\varphi) + (x+z)\cos\varphi + 0$$
$$A\varphi = (x+z)\cos\varphi$$
$$A\varphi = (\rho\cos\varphi + z)\cos\varphi$$

$$Az = 0$$

2.5 :Convert the vector F to cylindrical

$$F = \frac{xax + yay + 4az}{\sqrt{x^2 + y^2 + z^2}}$$

$$x = \rho\cos\varphi$$
$$y = \rho\sin\varphi$$
$$z = r\cos\theta$$
$$\rho = r\sin\theta$$
$$\rho = \sqrt{x^2 + y^2}$$
$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$A\rho = \frac{x^*\cos\varphi + y^*\sin\varphi + 0}{\sqrt{x^2 + y^2 + z^2}} = \frac{\rho\cos\varphi^*\cos\varphi + \rho\sin\varphi^*\sin\varphi}{\sqrt{\rho^2 + z^2}} = \frac{\rho}{\sqrt{\rho^2 + z^2}}$$

$$A\varphi = \frac{x^*(-\sin\varphi) + y^*\cos\varphi + 0}{\sqrt{x^2 + y^2 + z^2}} = \frac{\rho\cos\varphi^* - \sin\varphi + \rho\sin\varphi^*\cos\varphi}{\sqrt{\rho^2 + z^2}} = 0$$

$$Az = \frac{4}{\sqrt{x^2 + y^2 + z^2}} = \frac{4}{\sqrt{\rho^2 + z^2}}$$

$$A_{cyl.} = \frac{\rho}{\sqrt{\rho^2 + z^2}}\, a_\rho + \frac{4}{\sqrt{\rho^2 + z^2}}\, a_z$$

The distance between tow points :

$$d = |r_2 - r_1|$$

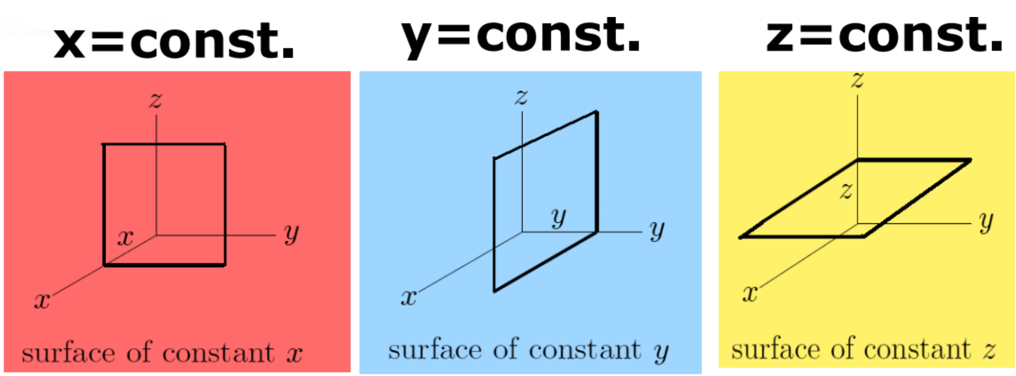
Cartesian $\Rightarrow d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$

Cylindrical $\Rightarrow d^2 = \rho_2^2 + \rho_1^2 - 2\rho_2\rho_1 \cos(\phi_2 - \phi_1) + (z_2 - z_1)^2$

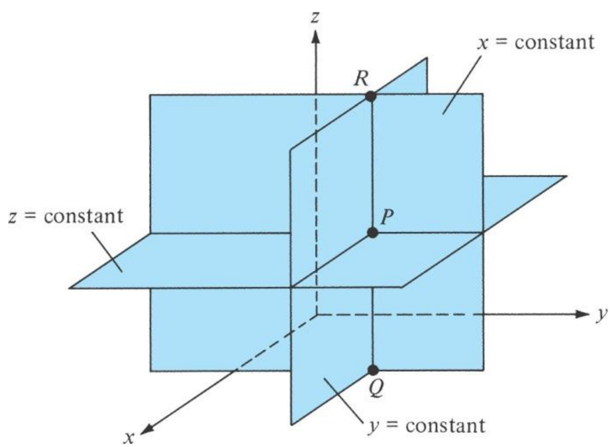
Spherical $\Rightarrow d^2 = r_2^2 + r_1^2 - 2r_2 r_1 \cos\theta_2 \cos\theta_1 - 2r_2 r_1 \sin\theta_2 \sin\theta_1 \cos(\phi_2 - \phi_1)$

- **Constant coordinate surface**

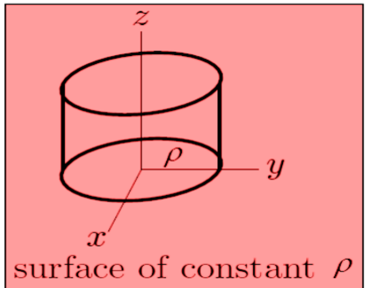
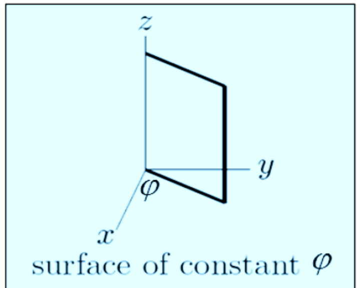
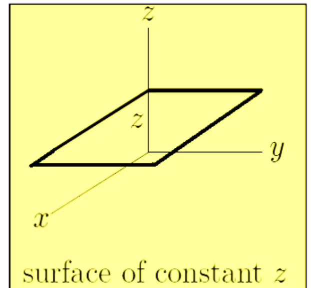
- Cartesian coordinate

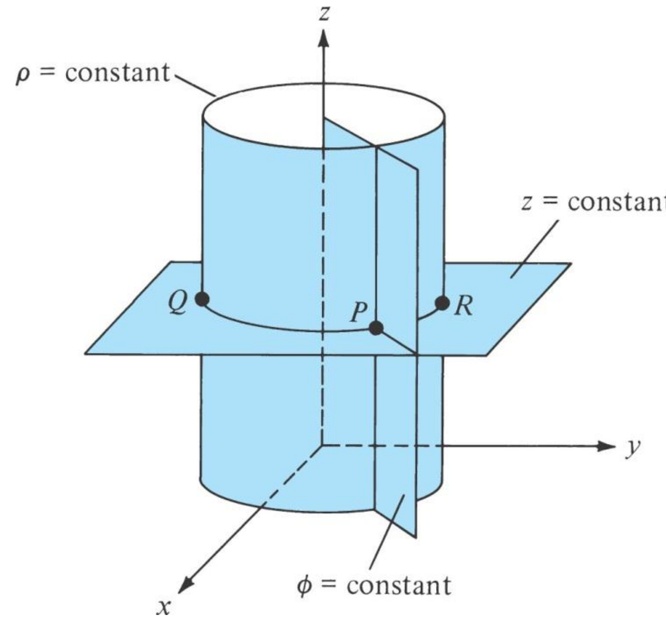


x=const.	y=const.	z=const.
$-\infty < y < \infty$	$-\infty < x < \infty$	$-\infty < x < \infty$
$-\infty < z < \infty$	$-\infty < z < \infty$	$-\infty < y < \infty$



- Cylindrical

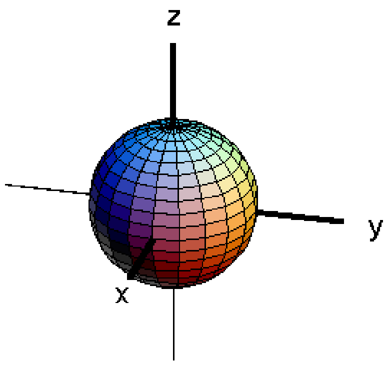
ρ=const.	φ=const.	z=const.
		
$0 \le \varphi \le 2\pi$	$0 \le \rho < \infty$	$0 \le \rho < \infty$
$-\infty \le z \le \infty$	$-\infty < z < \infty$	$0 \le \varphi \le 2\pi$
Circular Cylindrical	semi infinite plane	infinite plane

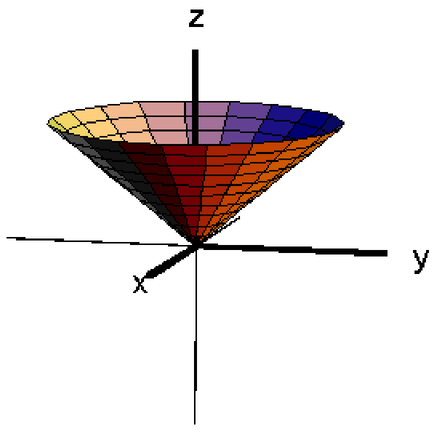


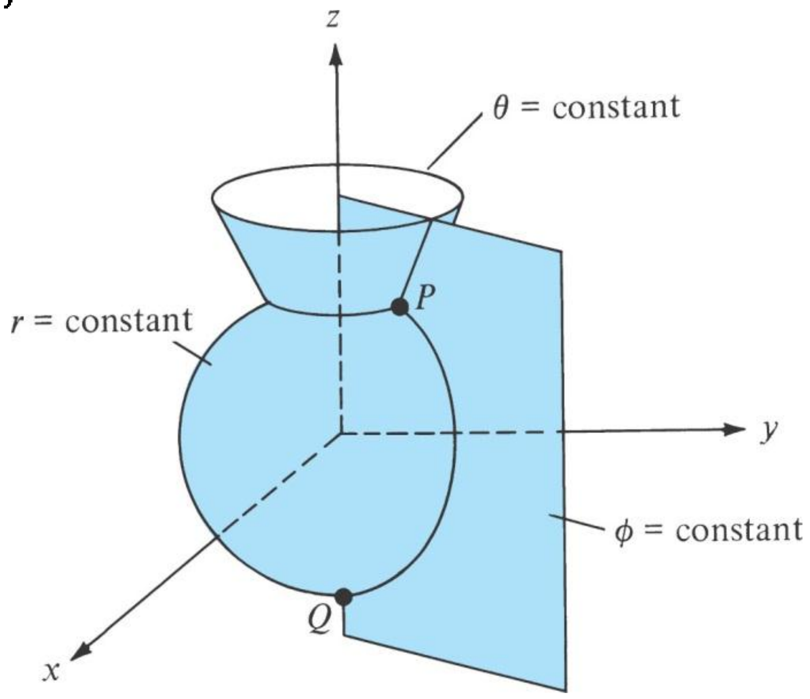
- Spherical

r:const :sphere

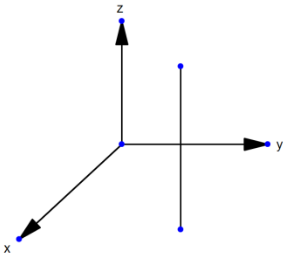
θ:const : cone



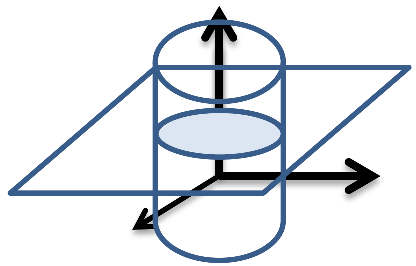




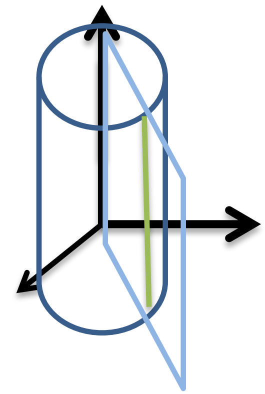
x ,y const



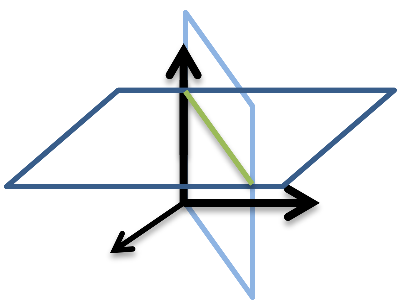
ρ ,z const(circle)

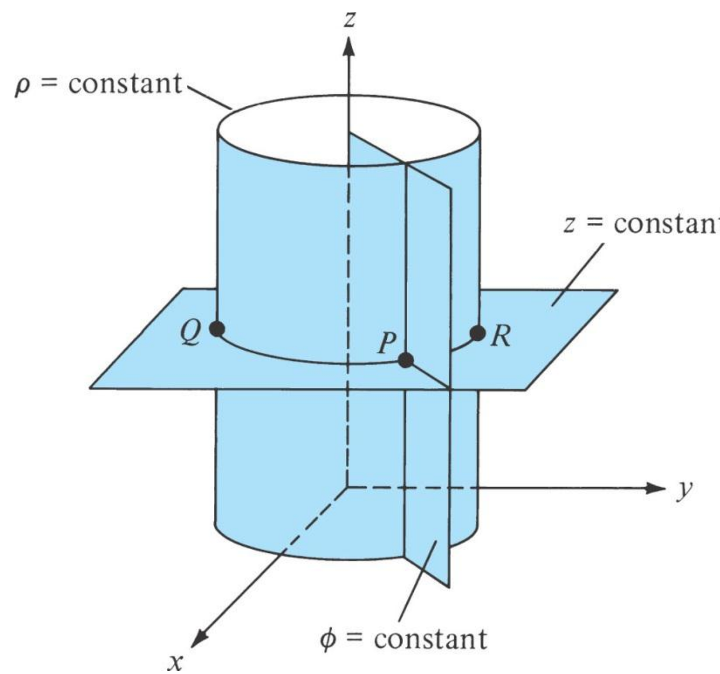


ρ, ϕ const(line)



ϕ,z const

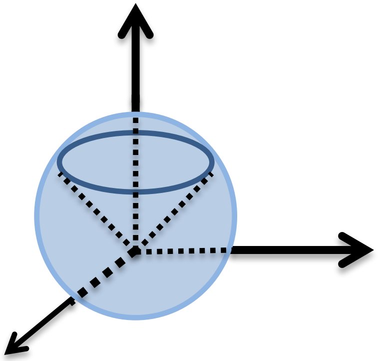


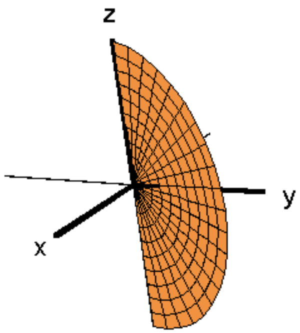


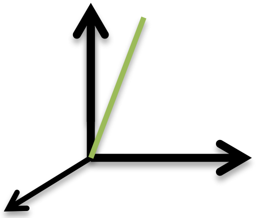
Θ,r const (circle)

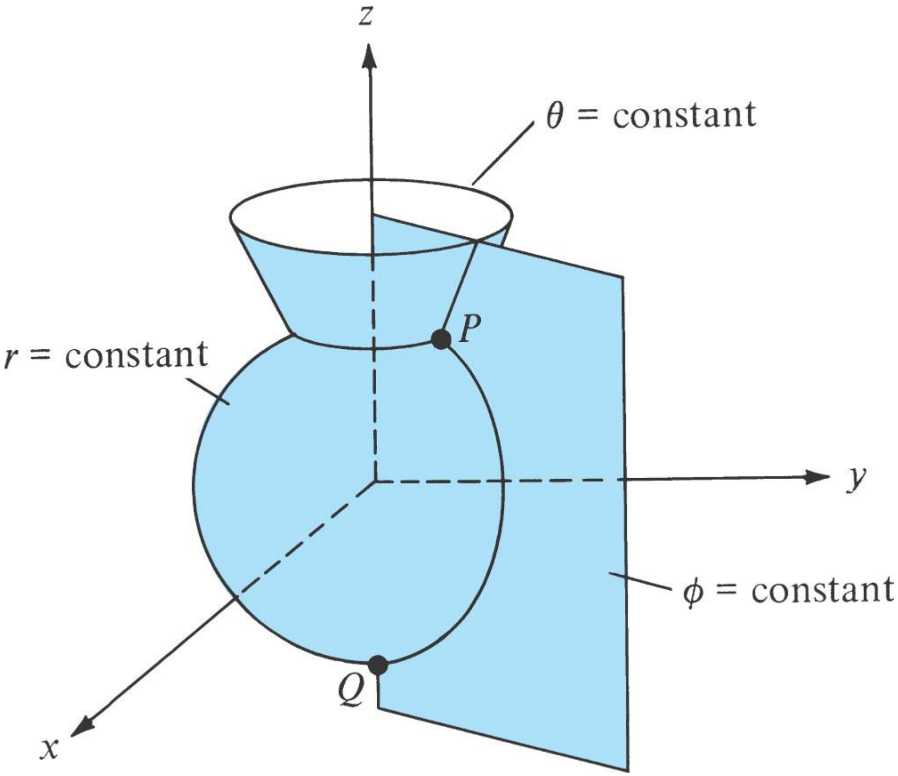
r, ϕ const(bow)

Θ, ϕ const (line)









P2.6: Express B in Cartesian

$$B = 2r\sin\theta\cos\varphi\, ar + r\cos\theta\cos\varphi\, a\theta - r\sin\varphi\, a\varphi$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\cos\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

$$\text{Ax} = (\sin\theta\cos\varphi) * 2r\sin\theta\cos\varphi + (\cos\theta\cos\varphi) * r\cos\theta\cos\varphi$$
$$+ (-\sin\varphi) * -r\sin\varphi$$

$$\text{Ax} = (2r\sin^2\theta\cos^2\varphi) + (r\cos^2\theta\cos^2\varphi) + (r\sin^2\varphi)$$

$$\text{Ax} = \frac{r^2}{r^2}(2r\sin^2\theta\cos^2\varphi) + (\frac{r}{r}\text{rcos}^2\theta\ \frac{\rho^2}{\rho^2}\cos^2\varphi) + (r\frac{\rho^2}{\rho^2}\sin^2\varphi)$$

$$\text{Ax} = \frac{2}{r}(r^2\sin^2\theta\cos^2\varphi) + (\frac{1}{r\rho^2}\text{r}^2\cos^2\theta\ \ \rho^2\cos^2\varphi) + (\frac{r}{\rho^2}\rho^2\sin^2\varphi)$$
$$= \frac{2}{r}(x^2) + (\frac{1}{r\rho^2}z^2x^2) + (\frac{r}{\rho^2}y^2)$$
$$= \frac{2x^2}{\sqrt{x^2+y^2+z^2}} + \frac{zx}{(x^2+y^2)\sqrt{x^2+y^2+z^2}} + \frac{\sqrt{x^2+y^2+z^2}}{x^2+y^2}y^2$$

2.11:Transform A to rectangular

$$A = \rho \cos\varphi \; a\rho + \rho z^2 \sin\varphi \;\; az$$

$$Ax = \rho\cos^2\varphi = \frac{\rho}{\rho}\rho\cos^2\varphi = \frac{1}{\rho}\rho^2\cos^2\varphi = \frac{x^2}{\rho} = \frac{x^2}{\sqrt{x^2+y^2}}$$

$$Ay = \sin\phi \; \rho\cos\phi = \frac{\rho}{\rho}\rho\sin\phi\cos\phi = \frac{1}{\rho}\rho\sin\phi \quad \rho\cos\phi = \frac{xy}{\sqrt{x^2+y^2}}$$

$$Az = \rho\, z^2 \sin\varphi = z^2 y$$

$$A = \frac{x^2}{\sqrt{x^2+y^2}}\, ax + \frac{xy}{\sqrt{x^2+y^2}}\, ay + z^2 y az$$

Given

Wanted

			Rectangular			Cylindrical			Spherical	
	•	A_x	A_y	A_z	A_ρ	A_φ	A_z	A_r	A_θ	A_φ
Rectangular	A_x	1	0	0	$\cos\phi$	$-\sin\phi$	0	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
	A_y	0	1	0	$\sin\phi$	$\cos\phi$	0	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
	A_z	0	0	1	0	0	1	$\cos\theta$	$-\sin\theta$	0
Cylindrical	A_ρ	$\cos\phi$	$\sin\phi$	0	1	0	0	$\sin\theta$	$\cos\theta$	0
	A_φ	$-\sin\phi$	$\cos\phi$	0	0	1	0	0	0	1
	A_z	0	0	1	0	0	1	$\cos\theta$	$-\sin\theta$	0
Spherical	A_r	$\sin\theta\cos\phi$	$\sin\theta\sin\phi$	$\cos\theta$	$\sin\theta$	0	$\cos\theta$	1	0	0
	A_θ	$\cos\theta\cos\phi$	$\cos\theta\sin\phi$	$-\sin\theta$	$\cos\theta$	0	$-\sin\theta$	0	1	0
	A_φ	$-\sin\phi$	$\cos\phi$	0	0	1	0	0	0	1

Transform A to spherical and Find the value of A at point(3,-4,0)

$$A = \rho\cos\varphi\, a\rho + \rho z^2 \sin\varphi \;\; az$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\varphi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \rho\cos\phi \\ 0 \\ \rho z^2 \sin\phi \end{bmatrix}$$

$$Ar = \rho\cos\phi\sin\theta + \rho z^2 \sin\phi\cos\theta$$

$$= (r\sin\theta)\cos\phi\sin\theta + (r\sin\theta)(r\cos\theta)^2 \sin\phi\cos\theta$$

$$= r\sin^2\theta\cos\phi + r^3\sin\theta\cos^3\theta\sin\phi$$

$$A_\theta = r\sin\theta\cos\theta\cos\phi - r^3\sin^2\cos^2\theta\sin\phi$$

$$A\phi = 0$$

$$\Rightarrow A = A_r\, a_r + A_\theta\, a_\theta + A_\phi\, a_\phi$$

$$(x, y, z) = (3,-4,0) \Rightarrow (r,\theta,\varphi) = (5, \pi/2, -53.13^o)$$

$$A_{\downarrow (5,\pi/2,-53.13^o)} = 3a_r$$

$|A| = 3$ as in part(a)

2.14:Calculate the distance between the points:

(a) P1=(2,1,5) and P2=(6,-1,2)

$$P1P2 = P2 - P1 = 4ax - 2ay - 3az$$

$$|P1P2| = \sqrt{16+4+9} = 5.38$$

(b) P1=(3,pi/2,-1) and P2=(5,3pi/2,5)

$$d^2 = \rho_2^2 + \rho_1^2 - 2\rho_2\rho_1 \cos(\phi_2 - \phi_1) + (z_2 - z_1)^2$$

$$d^2{}_{\downarrow cyl.} = 5^2 + 3^2 - 2(5)(3)\cos(\pi) + (6)^2 = 100$$

$$d = 10$$

Or convert all points to Cartesian coordinates :

$(3,\text{pi}/2,\text{-}1) \Rightarrow (0,3,\text{-}1)$

$(5,3\text{pi}/2,5) \Rightarrow (0,\text{-}5,5)$

$$\text{d} = \sqrt{0+64+36} = 10$$

P.E:2.3:

$$H = \rho z \cos\varphi \ \ a\rho \ + \sin\frac{\varphi}{2} a_{\varphi} + \rho^2 \ az$$

At point $(1, \pi/3, 0)$ find :

(a) H.ax : first we must convert [H to Cartesian] or [A to cylindrical

$$\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho z \cos\varphi \\ \sin\frac{\varphi}{2} \\ \rho^2 \end{bmatrix}$$

$$H = \rho z \cos\varphi \ a\rho \ + \sin\frac{\varphi}{2} a_{\varphi} + \rho^{2} \ az \Big|$$

At point $(1, \pi/3, 0)$ find :

(a) Hxaθ

first we must convert [H to sph] or [A to cyl].

$$A\rho = \cos\theta$$
$$A_{\varphi} = 0$$
$$Az = -\sin\theta$$
$$\theta = \tan^{-1}(\frac{\rho}{z}) = \tan^{-1}(\frac{1}{0}) = \pi / 2$$
$$A\rho = \cos 90 = 0$$
$$A_{\varphi} = 0$$
$$Az = -\sin 90 = -1$$
$$H \downarrow (1, pi/3, 0) = 0.5 \ a\varphi + az$$
$$H \text{ x } A = H \text{ x az} = (0.5 \ a\varphi + az)\text{x}(az)$$

$$\Rightarrow \begin{vmatrix} a\rho & a\varphi & az \\ 0 & 0.5 & 1 \\ 0 & 0 & -1 \end{vmatrix} = -0.5 \ a_{\rho}$$