

Example: For students to practice
Calculate the variance and standard deviation from
the following sample data of 9 students.

45, 32, 37, 46, 39, 36, 41, 48, 36

Hint:

$$\text{Use } \bar{x} = \frac{\sum x_i}{n} \quad s^2 = \frac{\sum (x_i - \bar{x})^2}{n} \quad s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

OR Alternative formula's

$$s^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 \quad s = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

Grouped Data Example

Calculate the variance and Standard Deviation from the
following data

class	f_i	x_i	$f_i x_i$	x_i^2	$f_i x_i^2$
65-84	9	74.5	670.5	5550.25	49952.25
85-104	10	94.5	945.0	8930.25	89302.50
105-124	17	114.5	1946.5	13110.25	222874.25
125-144	10	134.5	1345.0	18090.25	180902.50
145-164	5	154.5	772.5	23870.25	119351.25
165-184	4	174.5	698.0	30450.25	121801.00
185-204	5	194.5	972.5	37830.25	189151.25

$$\sum f_i = 60 \quad \sum f_i x_i = 7350.0 \quad \sum f_i x_i^2 = 973335.00$$

$$s^2 = \frac{\sum f_i x_i^2}{n} - \left(\frac{\sum f_i x_i}{n}\right)^2$$

$$\frac{973335.00}{60} - \left(\frac{7350}{60}\right)^2 = 16222.25 - 15006.25$$

$$= 1216$$

$$\text{and } s = \sqrt{\frac{\sum f_i x_i^2}{n} - \left(\frac{\sum f_i x_i}{n}\right)^2} = \sqrt{1216} = 34.87$$

$$M.D = \frac{\sum |x_i - \bar{x}|}{n}, \text{ for sample data}$$

$$M.D = \frac{\sum |x_i - \mu|}{N}, \text{ for population data}$$

For data to be organised in groups i.e. grouped frequency distribution having k classes with midpoints $x_1, x_2, x_3, \dots, x_k$ and corresponding frequencies $f_1, f_2, f_3, \dots, f_k$ the mean deviation of the sample is given by

$$M.D = \phi_n = \frac{\sum f_i |x_i - \bar{x}|}{n}$$

OR

$$= \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$$

Mean deviation is absolute measure of dispersion, its relative measure known as the co-efficient of mean deviation is obtained by

$$\rightarrow \text{Co-efficient of M.D from Mean} = \frac{\phi_n}{\text{Mean}} \text{ OR } \frac{\phi_n}{\text{Mean}} \times 100$$

$$\rightarrow \text{Co-efficient of M.D from Median} = \frac{\phi_m}{\text{Median}} \text{ OR } \frac{\phi_m}{\text{Median}} \times 100$$

Example:

Consider the following observations to find mean deviation from i) the mean ii) the median

45, 32, 37, 46, 39, 36, 41, 48, 36

also calculate co-efficient of mean deviation.

Sol:

Arrange the data in ascending order

32, 36, 36, 37, 39, 41, 45, 46, 48

$$\begin{aligned}
 \text{Median} &= \left(\frac{n+1}{2}\right)^{\text{th}} \\
 &= \left[\frac{n}{2} + 1\right]^{\text{th}} \\
 &= \left[\frac{9}{2} + 1\right]^{\text{th}} \Rightarrow \left[\frac{9+2}{2}\right]^{\text{th}} = \left[\frac{11}{2}\right]^{\text{th}} = [5.5]^{\text{th}} \\
 &= 5^{\text{th}} = 39
 \end{aligned}$$

$$\begin{aligned}
 \bar{x} &= \frac{\sum x}{n} = \frac{32+36+36+37+39+41+45+46+48}{9} \\
 &= \frac{360}{9} \\
 &= 40
 \end{aligned}$$

x_i	$x_i - \bar{x}$	$ x_i - \bar{x} $	$ x_i - \text{median} $
32	$32 - 40 = -8$	8	$ 32 - 39 = 7$
36	$36 - 40 = -4$	4	$ 36 - 39 = 3$
36	$36 - 40 = -4$	4	$ 36 - 39 = 3$
37	$37 - 40 = -3$	3	$ 37 - 39 = 2$
39	$39 - 40 = -1$	1	$ 39 - 39 = 0$
41	$41 - 40 = 1$	1	$ 41 - 39 = 2$
45	$45 - 40 = 5$	5	$ 45 - 39 = 6$
46	$46 - 40 = 6$	6	$ 46 - 39 = 7$
48	$48 - 40 = 8$	8	$ 48 - 39 = 9$
Σ 360	0	40	39

$$\text{M.D (from mean)} = \sum |x_i - \bar{x}|$$

$$= \frac{40}{9} = 4.4$$

$$\text{M.D (from median)} = \sum |x_i - \text{median}|$$

$$= \frac{39}{9} = 4.3$$

$$\text{Co-efficient of M.D} = \frac{M.D}{\bar{x}} \text{ or } \frac{M.D}{\text{median}}$$

$$= \frac{4.4}{40} \text{ or } \frac{4.3}{39}$$

$$= 0.11 \text{ or } 0.11$$

Example:

Calculate the mean deviation from mean of the following frequency distribution, and mean deviation from median as well. Also find the co-efficients.

Class	f	C.B	n	f.x	x-x̄	f(x-x̄)	C.f	x-x̄	f x-x̄
5-7	15	4.5-7.5	6	90	-5	75	15	5	75
8-10	18	7.5- 10.5 ^{10.5}	9	162	-2	36	33	2	36
11-13	27	10.5-13.5	12	324	-1	27	60	1	27
14-16	10	13.5-16.5	15	150	4	40	70	4	40
17-19	6	16.5-19.5	18	108	7	42	76	7	42

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{834}{76} = 10.97 = 11$$

$$M.D = \phi_n = \frac{\sum f|x-\bar{x}|}{\sum f} = \frac{220}{76} = 2.89$$

$$\text{Co-efficient of M.D from mean} = \frac{\phi_n}{A.M} \text{ or } \frac{\phi_n}{A.M} \times 100$$

$$= \frac{2.89}{11} \text{ or } \frac{2.89}{11} \times 100$$

$$= 0.2627 \text{ or } 26.27$$

Now

$$\text{Median} = l + \frac{h}{f} (m - c) \quad m = \frac{n}{2} = \frac{76}{2} = 38$$

$$\text{where } l = 10.5 \quad h = 3 \quad f = 27 \quad m = 38 \quad c = 33$$

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find

$$\text{Median} = 10.5 + \frac{3}{27} (38 - 33)$$

$$= 10.5 + 0.11(5)$$

$$= 11.05 \cong 11$$

M.D from Median

$$\phi_m = \frac{\sum |x - \bar{x}|}{\sum f}$$

$$= \frac{220}{76}$$

$$= 2.89$$

Co-efficient of M.D from median

$$\frac{\phi_m}{\text{median}} \quad \text{or} \quad \frac{\phi_m}{\text{median}} \times 100$$

$$\frac{2.89}{11} \quad \text{or} \quad \frac{2.89}{11} \times 100$$

$$0.2627 \quad \text{or} \quad 26.27$$

Example: For students to practice

Calculate the mean deviation of the following frequency distribution.

Classes:	65-84	85-104	105-124	125-144	145-164	165-184	185-204
	9	10	17	10	5	4	5

Find Quartile Deviation from Grouped Data

Classes	f	c.f
21-24	4	4
25-28	13	17
29-32	20	37
33-36	30	67
37-40	18	85
41-44	10	95
45-48	5	100

$$Q.D = \frac{Q_3 - Q_1}{2}$$

$$Q_1 = l + \frac{h}{f} (r_1 - c) \quad r_1 = \frac{n}{4} = \frac{100}{4} = 25$$

$$l = 29 \quad f = 20 \quad h = 4 \quad c = 17$$

$$= 29 + \frac{4}{20} (25 - 17)$$

$$= 29 + 1.6$$

$$= 30.6$$

$$Q_3 = l + \frac{h}{f} (r_3 - c) \quad r_3 = \frac{3n}{4} = \frac{300}{4} = 75$$

$$l = 37 \quad h = 4 \quad f = 18 \quad c = 67$$

$$= 37 + \frac{4}{18} (75 - 67)$$

$$= 37 + 1.77$$

$$= 38.77$$

Now

$$Q.D = \frac{Q_3 - Q_1}{2} = \frac{38.77 - 30.6}{2} = 4.085$$

The Variance:

The variance of a set of observations is defined as the mean of the squares of deviations of all the observations from their mean.

Population Variance:

When it is calculated from the entire population, the variance is called the population variance, traditionally ~~denoted~~ denoted by σ^2 .

σ is a greek lower case letter pronounced as sigma

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$$

Sample Variance:

If instead the data from the sample are used to calculate the variance, it is referred to as sample variance and is denoted by S^2

$$S^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

The variance is also denoted by $\text{Var}(X)$

Standard Deviation:

The positive square root of the variance is called standard deviation. Symbolically it is written as

$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}} \Rightarrow \text{For Population data}$$

$$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} \Rightarrow \text{For Sample data}$$