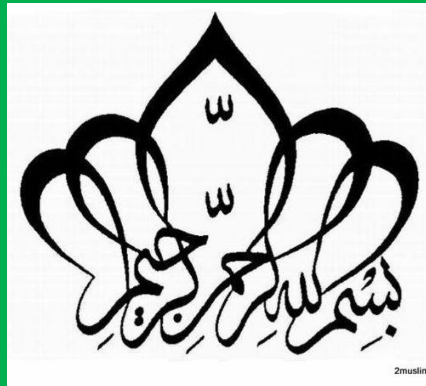

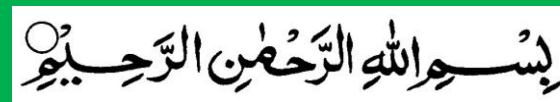
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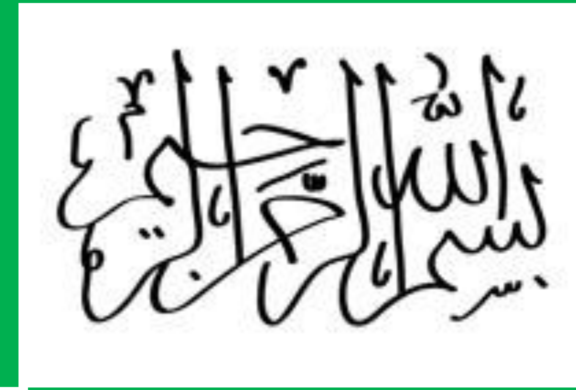
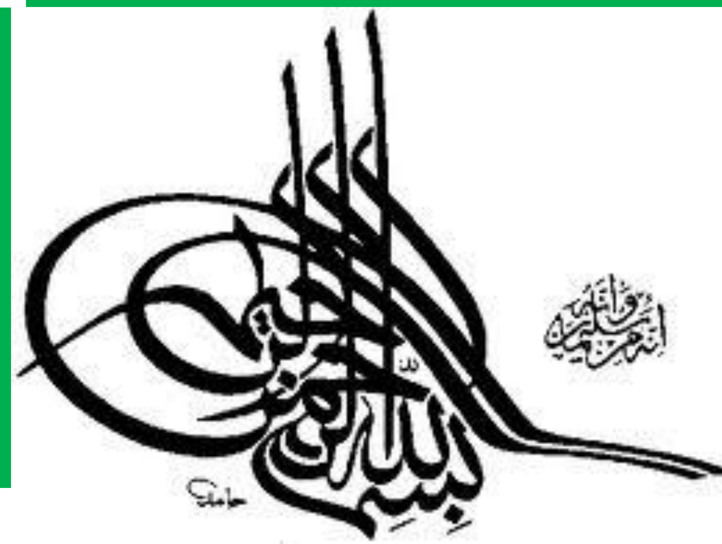


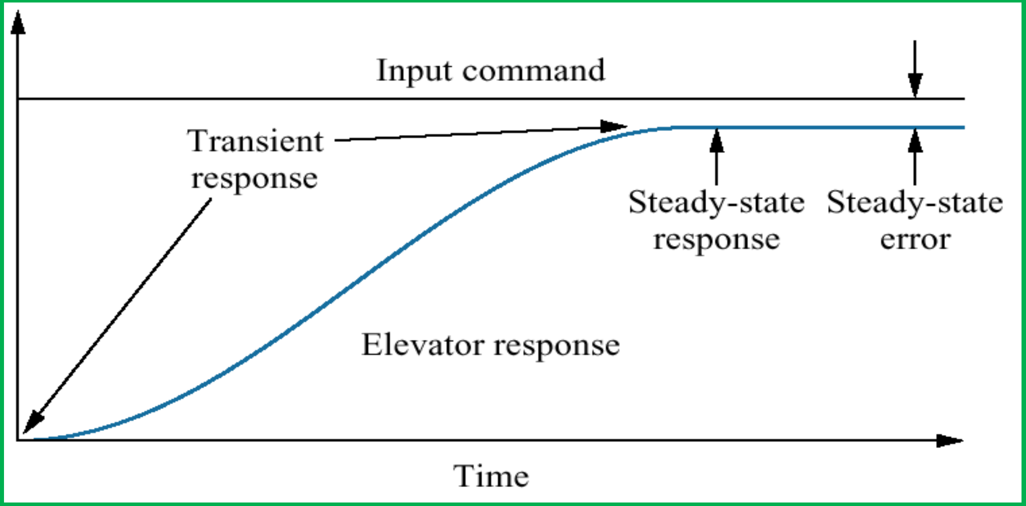


# Design Aspects (Performance)

- Transient response
- Steady-state response
- Stability
- Robustness
  - Disturbance rejection
  - Sensitivity

# System Responses

- Transient response
- Steady-state response




A distinct advantage of f/back control system is the ability to adjust transient and steady-state response

# Laplace Transforms of Test Inputs

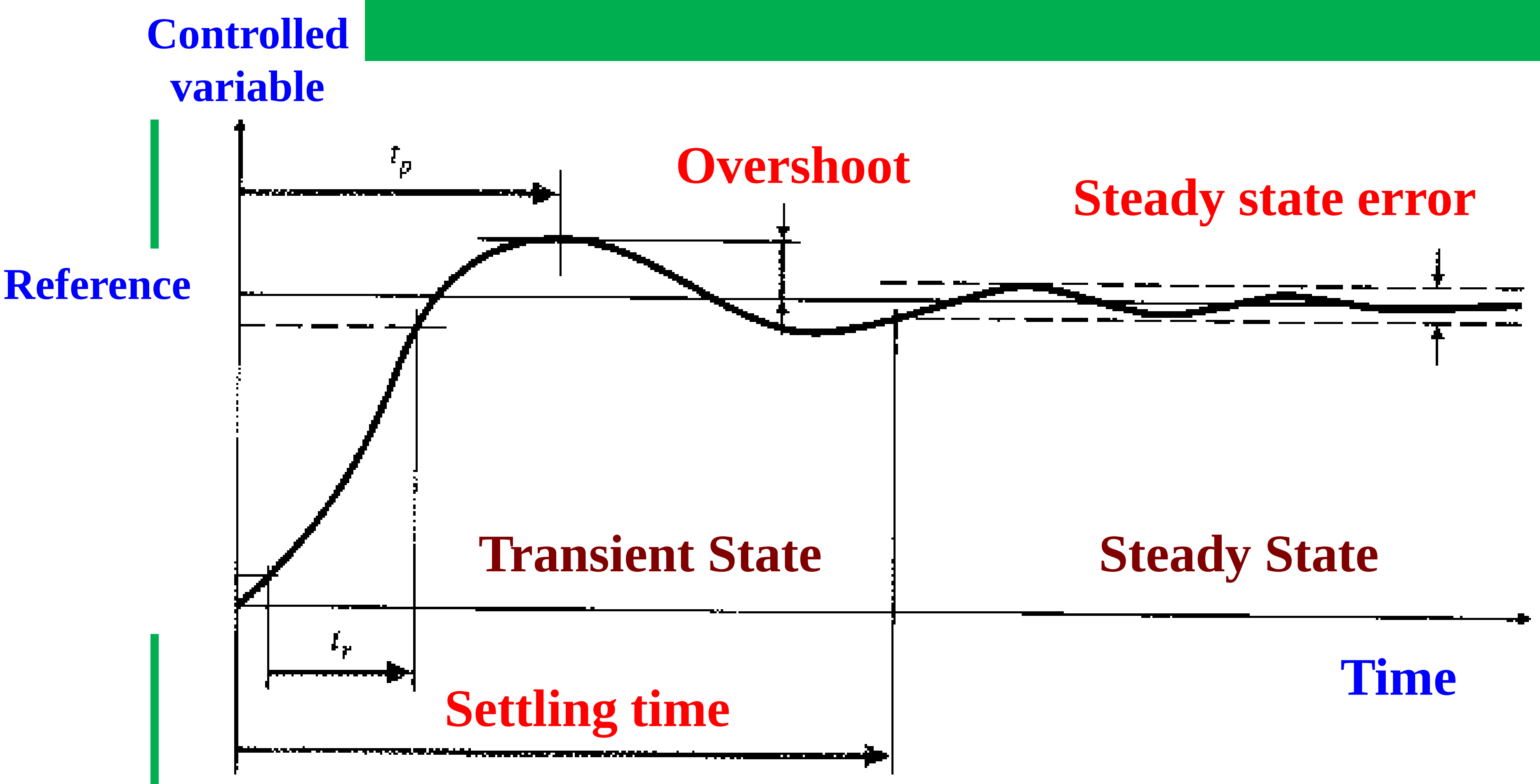
$$r(t) = t^n \qquad R(s) = \frac{n!}{s^{n+1}}$$

| Input | Time domain | Frequency domain |
|---|---|---|
| Step: | $1(t), \quad t \geq 0$ | $\frac{1}{s}$ |
| Ramp: | $t, \quad t \geq 0$ | $\frac{1}{s^2}$ |
| Parabolic: | $\frac{1}{2}t^2 \quad t \geq 0$ | $\frac{1}{s^3}$ |
| Sinusoidal: | $A\sin \omega t$ | $\frac{A\omega}{s^2 + \omega^2}$ |

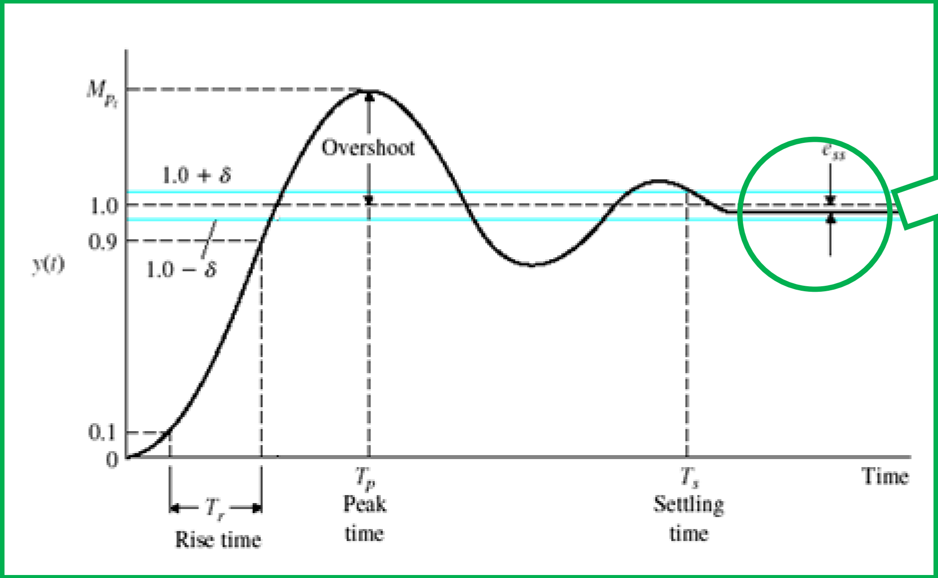
# Performance Indices

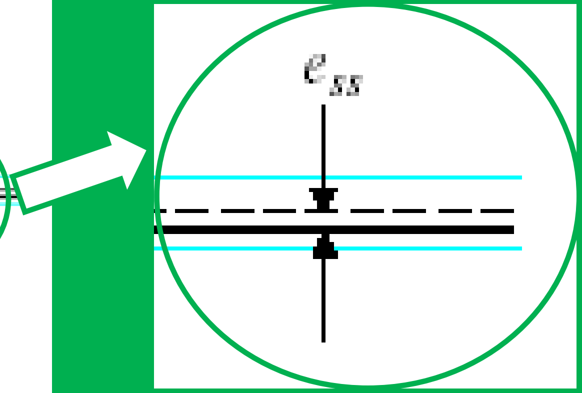
- Transient Performance:
  - Time delay ($t_d$)
  - Rise time ($t_r$)
  - Peak time ($t_p$)
  - Settling time ($t_s$)
  - Percent overshoot (σ%)
- Steady-state Performance:
  - Steady-state error

# Transient Response Specifications



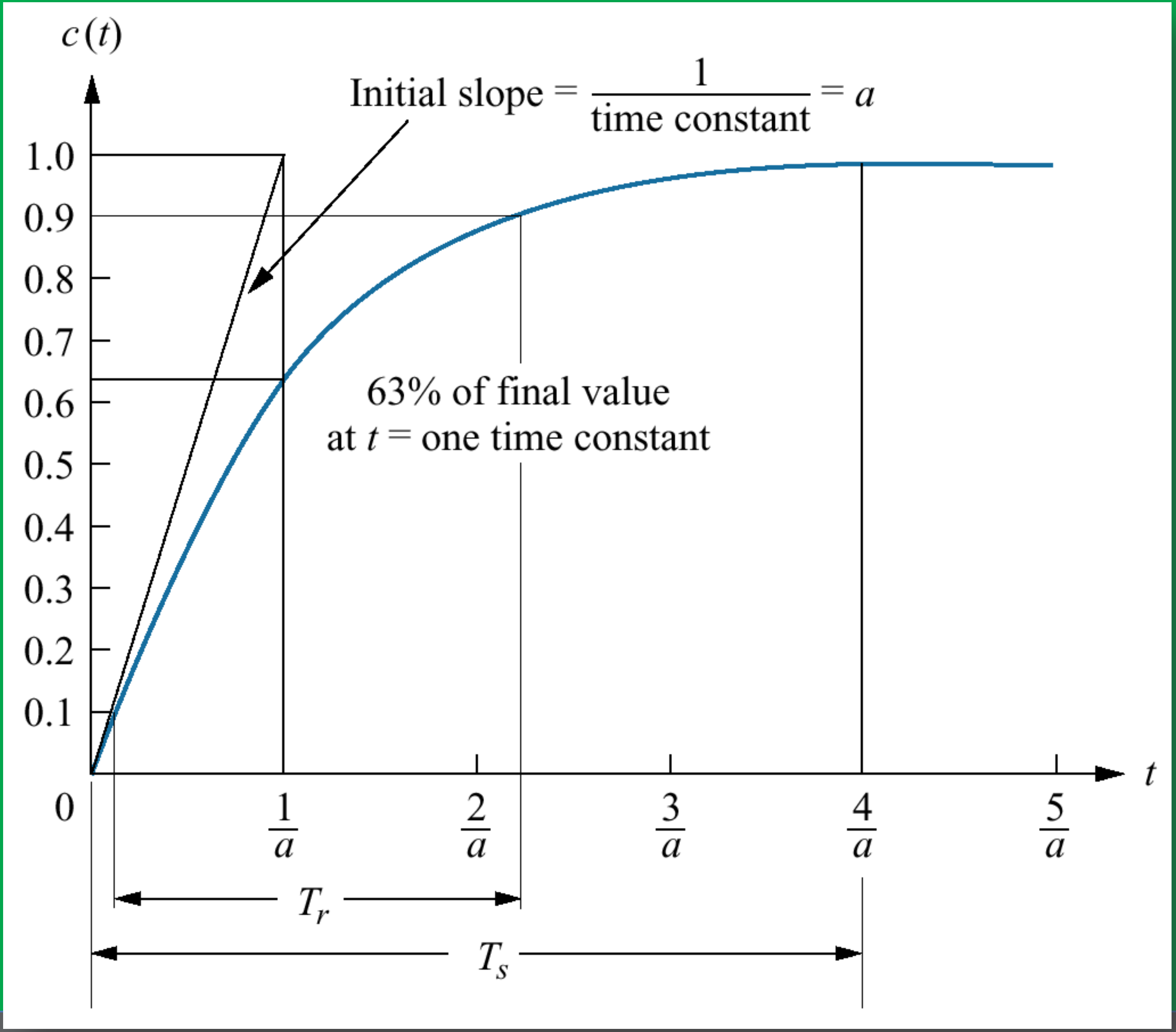
# Steady State Error




- Steady State Error ($e_{ss}$): The difference b/w the steady state output value and the reference input value at steady state
- Reflects the accuracy of the system

# Transient Performance



# Transient Performance

- Time Constant (τ)
  - The time for the waveform to go 63% of its final value

$$\tau = \frac{1}{a}$$

- Rise Time ($t_r$)
  - The time for the waveform to go from 0.1 to 0.9 of its final value

$$t_r = \frac{2.2}{a}$$

- Settling Time ($t_s$)
  - The time for the response to reach and stay within 2% of its final value

$$t_s = \frac{4}{a}$$

# Solved Example

- For a system with the transfer function shown below, find the relevant response specifications

$$G(s) = \frac{50}{s+50}$$

i. Time constant (τ)
ii. Settling time ($t_s$)
iii. Rise time ($t_r$)

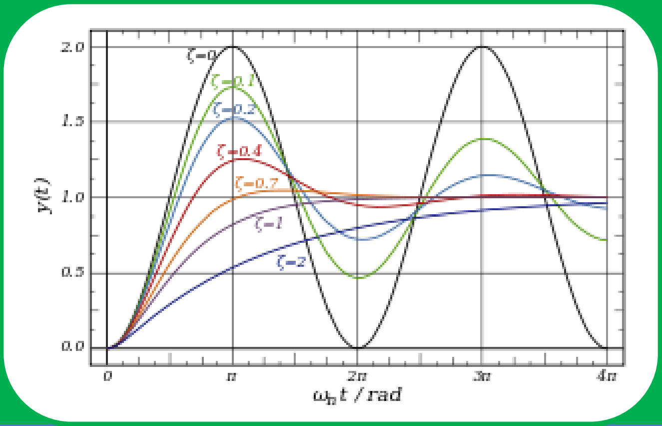
# Time Response of 2nd Order System

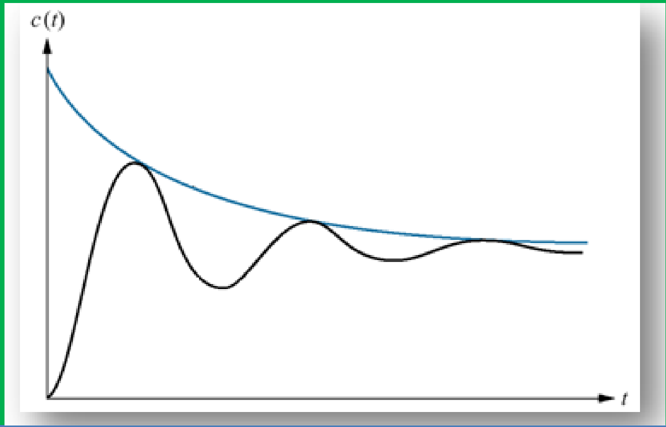
# Preliminaries - Terminologies

- Natural frequency ($\omega_n$)
  - Freq. of oscillation of the system without damping

- Damping ratio
  - Quantity that compares the exponential decay frequency of the envelope to the natural frequency

$$\varsigma = \frac{\text{Exponential decay frequency}}{\text{Natural frequency (rad/s)}}$$



# Response of 2nd Order System

- The model of a second-order system

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

Where,

K : Gain

ζ : Damping ratio

$\omega_n$ : Undamped natural frequency

PROOF: See any Control Book

# Response of 2nd Order System

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

- Roots of characteristic equation (Poles)

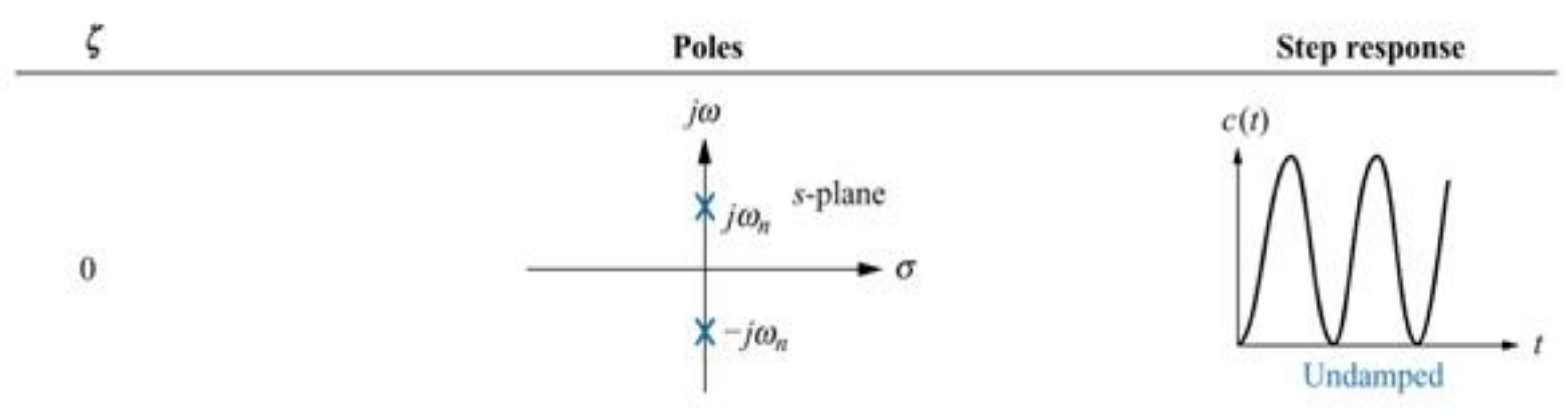
$$s^2 + 2\varsigma\omega_n s + \omega_n^2 = 0$$

$$s_{1,2} = -\varsigma\omega_n \pm \omega_n\sqrt{\varsigma^2 - 1}$$

- The response depends on ζ and $\omega_n$

# Response of 2nd Order System

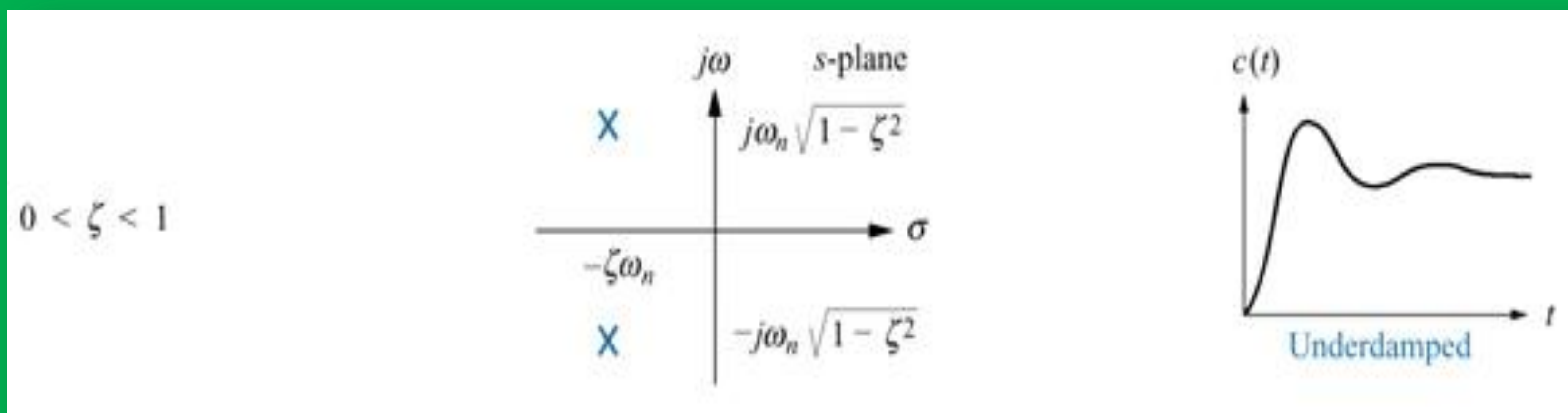
| Unit Step Response | | |
|---|---|---|
| If | Roots | Response |
| ζ=0 | two complex conjugate | Undamped |




$$s_{1,2} = -\varsigma\omega_n \pm \omega_n\sqrt{\varsigma^2 - 1}$$

# Response of 2nd Order System

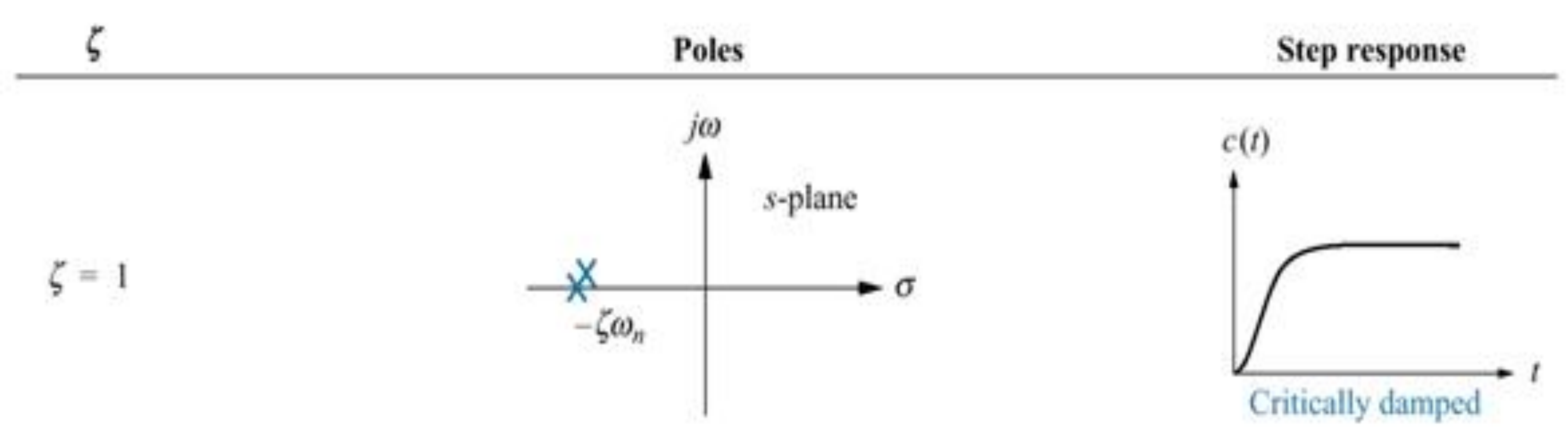
| Unit Step Response | | |
|---|---|---|
| If | Roots | Response |
| 0<ζ<1 | two –ive real-part | Underdamped |




$$s_{1,2} = -\varsigma\omega_n \pm \omega_n\sqrt{\varsigma^2 - 1}$$

# Response of 2nd Order System

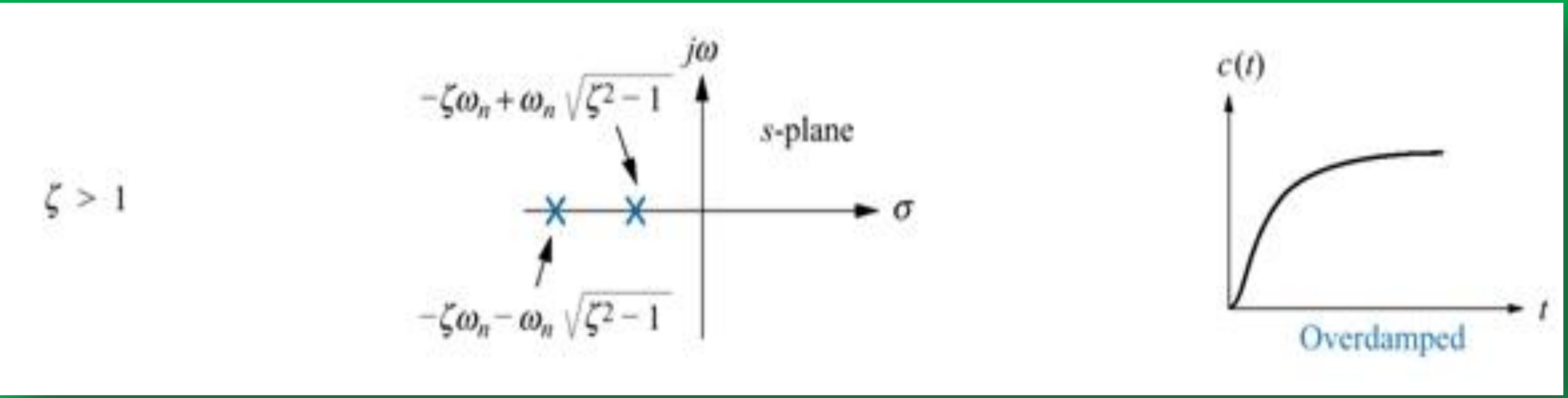
| Unit Step Response | | |
|---|---|---|
| If | Roots | Response |
| ζ=1 | two equal –ive real | Critically damped |




$$s_{1,2} = -\varsigma\omega_n \pm \omega_n\sqrt{\varsigma^2 - 1}$$

# Response of 2nd Order System

| Unit Step Response | | |
|---|---|---|
| If | Roots | Response |
| ζ>1 | two distinct –ive real | Overdamped |

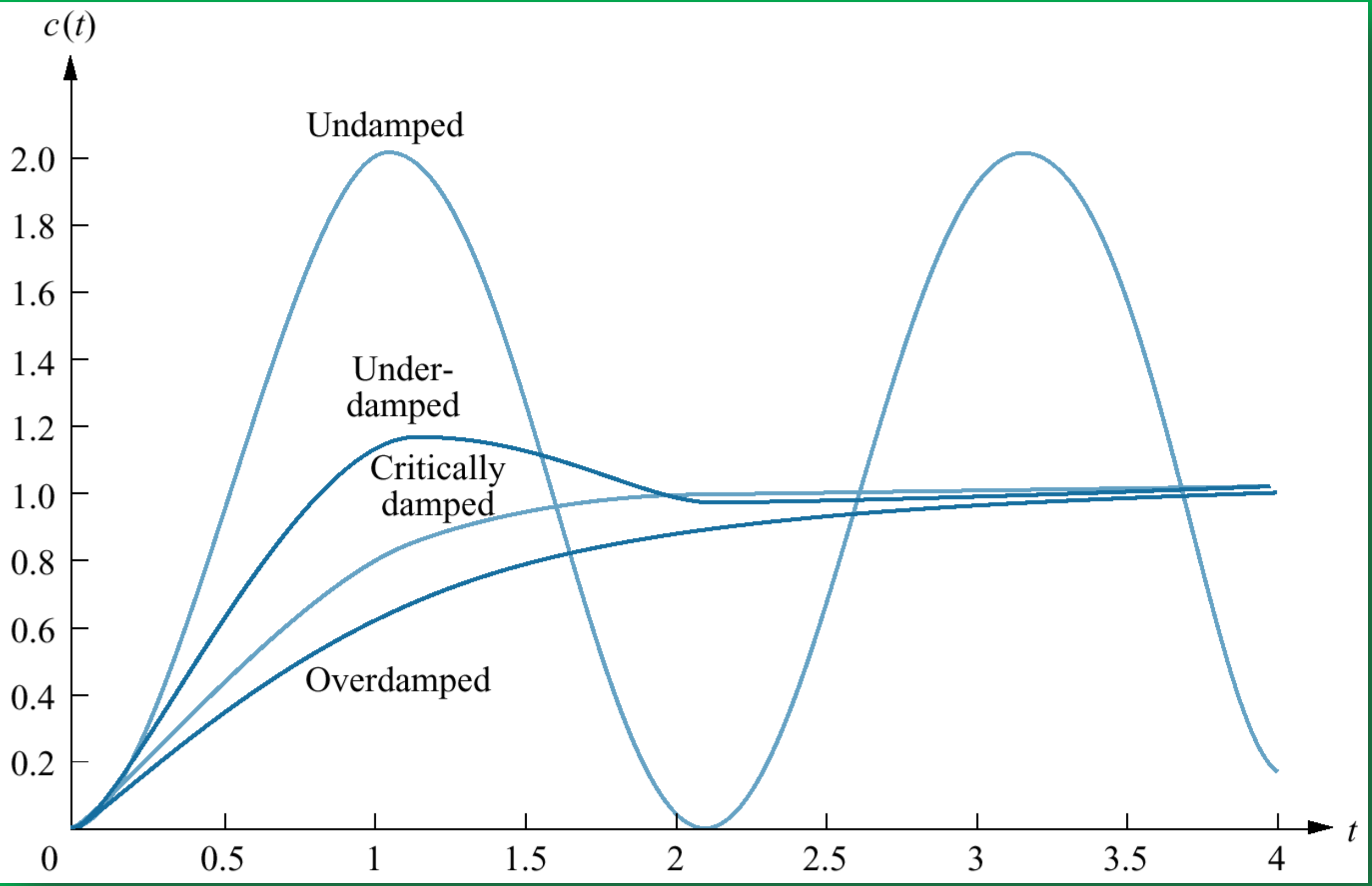



$$s_{1,2} = -\varsigma\omega_n \pm \omega_n\sqrt{\varsigma^2 - 1}$$

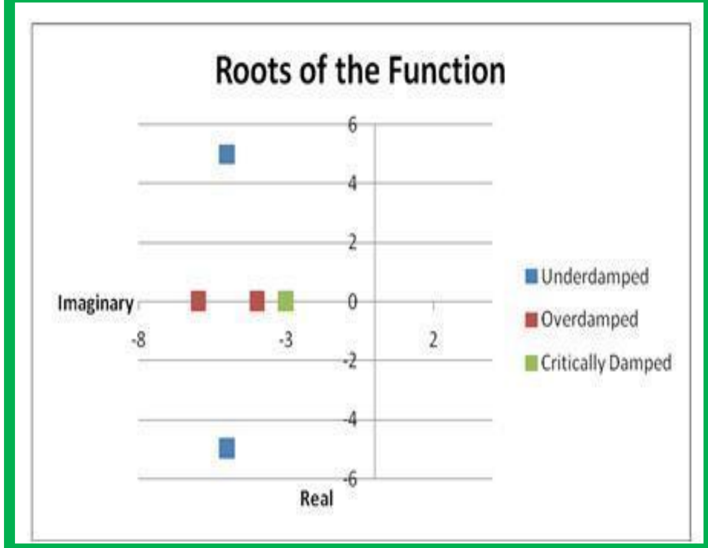
# Response of 2nd Order System

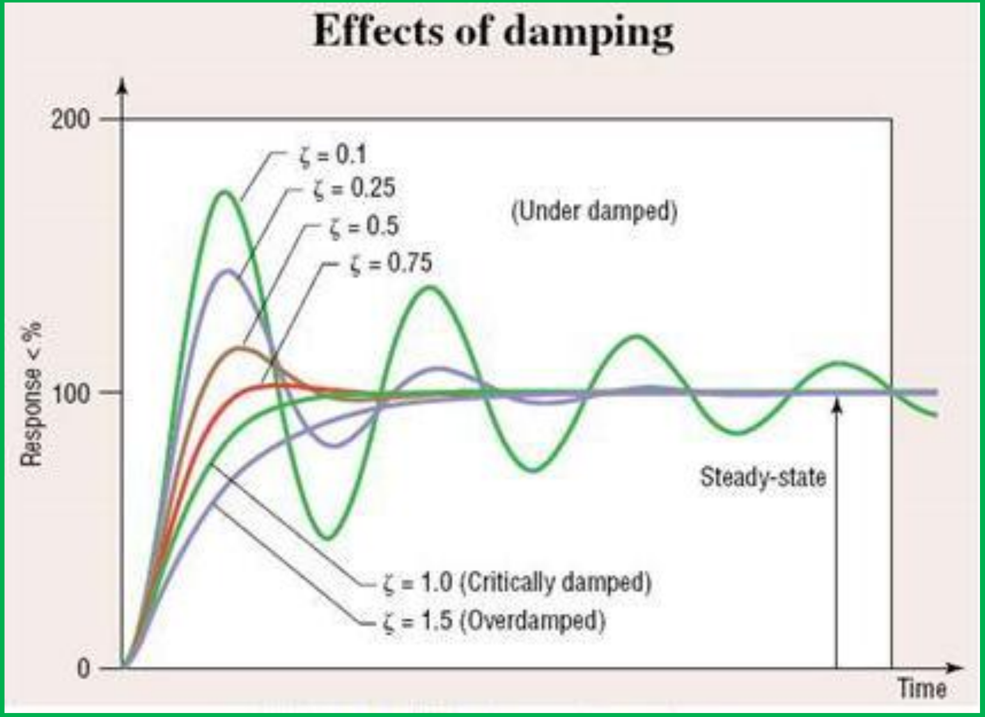
| Unit Step Response | | |
|---|---|---|
| If | Roots | Response |
| $\zeta<0$ | two +ive real-part | Unstable |
| $0<\zeta<1$ | two –ive real-part | Underdamped |
| $\zeta=1$ | two equal –ive real | Critically damped |
| $\zeta>1$ | two distinct –ive real | Overdamped |
| $\zeta=0$ | two complex conjugate | Undamped |

# Response of 2nd Order System



# Response of 2nd Order System



# Response of 2nd Order System

- Underdamped
  - Oscillatory response
  - No steady-state error

- Critically damped
  - Mono-incremental response
  - No oscillation
  - No steady-state error

- Overdamped
  - Mono-incremental response
  - Slower than critically damped
  - No oscillation
  - No steady-state error

# Transient Response Specifications

- Rise Time ($T_r$)
  - The time for the waveform to go from 0.1 to 0.9 of its final value
- Peak Time ($T_p$)
  - The time required to reach the first or maximum peak

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

- Settling Time ($T_s$)
  - The time required for the transient's damped oscillation to reach and stay within ±2% of the steady-state value

$$T_s = \frac{4}{\zeta\omega_n}$$

# Transient Response Specifications

- Percent Overshoot (%OS)
  - The amount that the waveform overshoots the steady-state, or final value at peak time, expressed as a percentage of the steady-state value

$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100\%$$

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}$$

# Effect of Poles & Zeros on System's Response

# Concept of Pole-Zero

- The concept of poles and zeros is fundamental to the analysis and design of control system
- Simplifies the evaluation of system response
- The poles of a transfer function are
  - Values of the Laplace Transform variables s, that cause the transfer function to become infinite. Decide the system behavior.
- The zeros of a transfer function are
  - Values of the Laplace Transform variable s, that cause the transfer function to become zero

# Concept of Pole-Zero

- Example

  G(s) = (s+2)/(s+5)

  Pole at s=-5

  Zero at s=-2