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EEEB113
CIRCUIT ANALYSIS I

Chapter 4

Circuit Theorems

Materials from Fundamentals of Electric Circuits, Alexander & Sadiku 4e, The McGraw-Hill Companies, Inc.

Circuit Theorems - Chapter 4

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4.3 Superposition
4.4 Source Transformation
4.5 Thevenin's Theorem
4.6 Norton's Theorem
4.7 Maximum Power Transfer

4.3 Superposition Theorem (1)

3

Superposition is another approach introduced to determine the value of a specific variable (voltage or current) if a circuit has two or more independent sources.

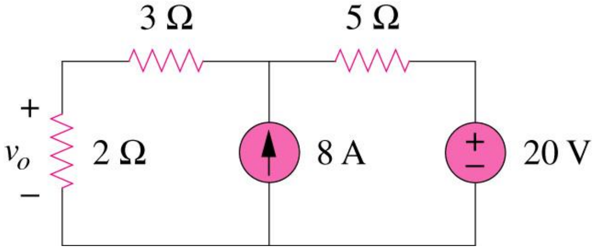
Superposition states that: the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltage across (or currents through) that element due to EACH independent source acting alone.

4.3 Superposition Theorem (2)

4

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately and then adding them up.

Example: We consider the effects of 8A and 20V one by one, then add the two effects together for final v_o.



4.3 Superposition Theorem (3)

5

Steps to apply superposition principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using **nodal** or **mesh** analysis.

2. Repeat step 1 for each of the other independent sources.

3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

4.3 Superposition Theorem (4)

6

Two things - Keep in mind:

1. When we say turn off all other independent sources:

   - Independent voltage sources are replaced by 0 V (short-circuit) and

   - Independent current sources are replaced by 0 A (open-circuit).

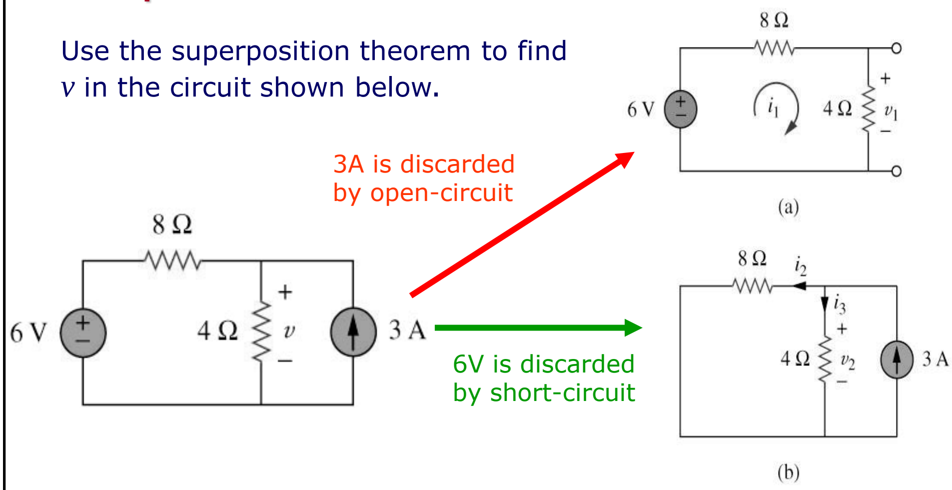
2. Dependent sources are left intact because they are controlled by circuit variables.

4.3 Superposition Theorem (5)

7

Example 1

Use the superposition theorem to find *v* in the circuit shown below.

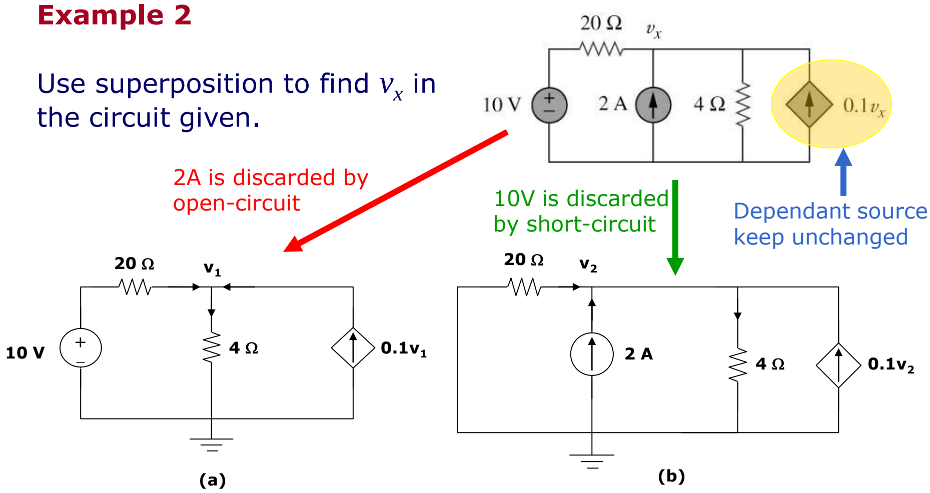


4.3 Superposition Theorem (6)

8

Example 2

Use superposition to find v_x in the circuit given.



4.3 Superposition Theorem (7)

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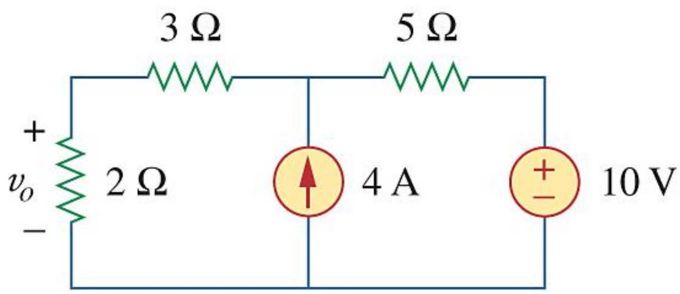
P.P.4.3

Use the superposition theorem to find v_0 in the circuit shown below.

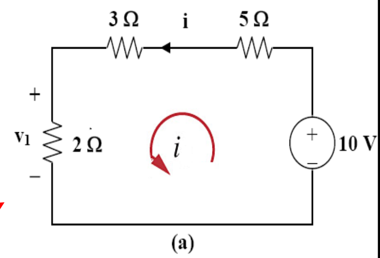


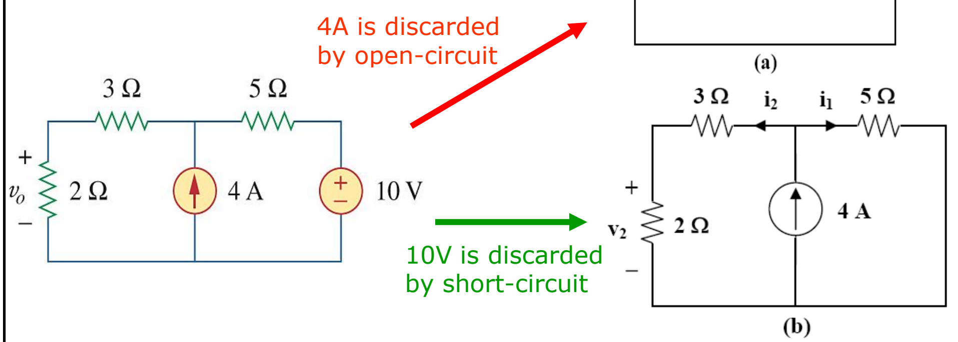
4.3 Superposition Theorem (8)

10

P.P.4.3

Use the superposition theorem to find v_0 in the circuit shown below.





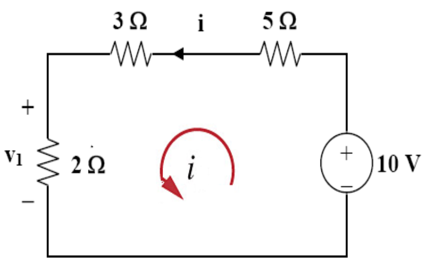
4.3 Superposition Theorem (9)

11

Soln. P.P.4.3

Let $v_0 = v_1 + v_2$,

where v_1 and v_2 are contributions to the 10V and 4A sources respectively.



(a)

Apply Ohm's Law

To get v_1, consider the curcuit in Fig. (a).

$$(2 + 3 + 5)i = 10$$

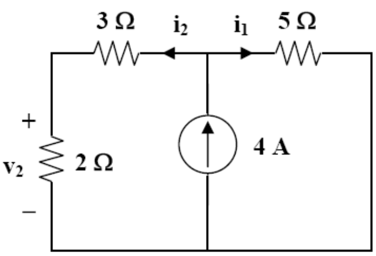
$$\longrightarrow \quad i = 10/(10) = 1A$$

$$v_1 = 2i = 2V$$

4.3 Superposition Theorem (10)

12

cont. Soln. P.P.4.3



(b)

To get v_2, consider the circuit in Fig. (b).

$$i_1 = i_2 = 2A, \; v_2 = 2i_2 = 4V$$

Thus,

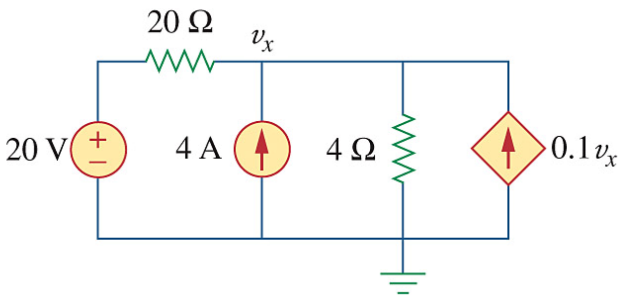
$$v = v_1 + v_2 = 2+4 = \underline{\mathbf{6\ V}}$$

4.3 Superposition Theorem (11)

13

P.P.4.4

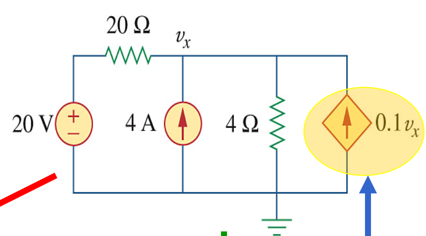
Use superposition to find v_x in the circuit given.

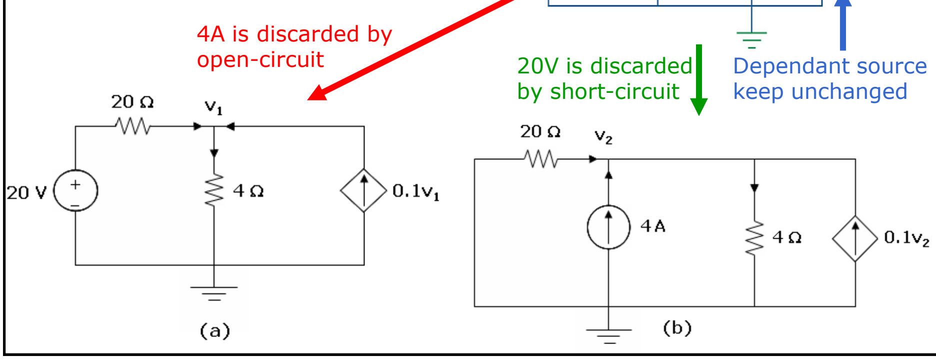


4.3 Superposition Theorem (12)

14

Soln. P.P.4.4



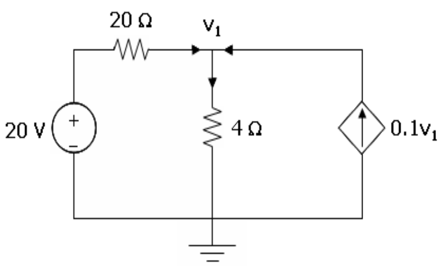


4.3 Superposition Theorem (13)

15

cont. Soln. P.P.4.4

Let $v_x = v_1 + v_2$,
where v_1 and v_2 are due to the 20V and 4A sources respectively.



(a)

Apply KCL

To obtain v_1, consider Fig. (a).

$$\frac{20 - v_1}{20} + 0.1v_1 = \frac{v_1 - 0}{4}$$

$$\longrightarrow \quad v_1 = 5\ \text{V}$$

4.3 Superposition Theorem (14)

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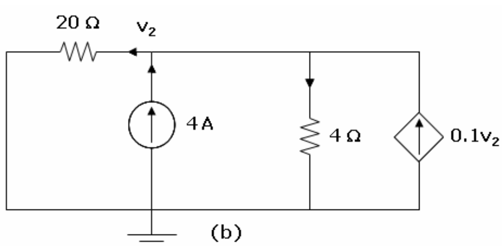
cont. Soln. P.P.4.4

Apply KCL

For v_2, consider Fig. (b).

$$4 + 0.1v_2 = \frac{v_2 - 0}{20} + \frac{v_2 - 0}{4}$$

$$\longrightarrow \quad v_2 = 20$$



(b)

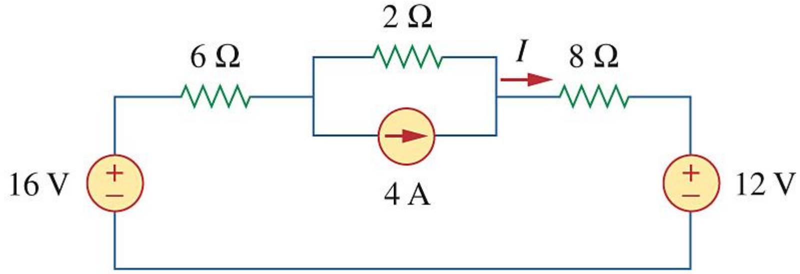
Thus,
$$v_x = v_1 + v_2 = \underline{\mathbf{25\ V}}$$

4.3 Superposition Theorem (15)

17

P.P.4.5

Use the superposition principle to find I in the circuit shown below.



4.3 Superposition Theorem (16)

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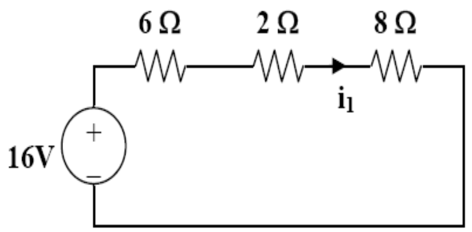
Soln. P.P.4.5

Let $i = i_1 + i_2 + i_3$

where i_1, i_2, and i_3 are contributions due to 16V, 4A, 12V sources respectively.



(a)

Apply Ohm's Law

For i_1, consider Fig. (a),

$$i_1 = \frac{16}{6+2+8} = 1\text{A}$$

4.3 Superposition Theorem (17)

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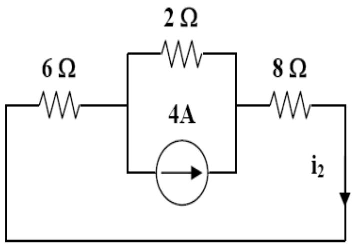
cont. Soln. P.P.4.5

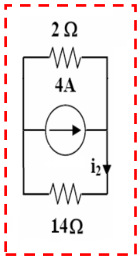


(b)

Apply Current Division

For i_2, consider Fig. (b).

By current division,

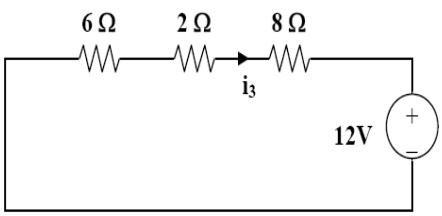


$$i_2 = \frac{2}{2+14}(4) = 0.5$$

4.3 Superposition Theorem (18)

20

cont. Soln. P.P.4.5



(c)

Apply Ohm's Law

For i_3, consider Fig. (c),

$$i_3 = \frac{-12}{16} = -0.75\text{A}$$

Thus,

$$i = i_1 + i_2 + i_3$$
$$= 1 + 0.5 - 0.75$$
$$= \underline{\mathbf{750mA}}$$

4.4 Source Transformation (1)

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- Another tool to simplify circuits.
- Use the concept of equivalent circuit where v-i characteristics are identical with the original circuit.

Source transformation is: the process of replacing a voltage source v_S in series with a resistor R by a current source i_S in parallel with a resistor R, or vice versa.

$$v_s = i_s R \quad \leftrightarrow \quad i_s = \frac{v_s}{R}$$

4.4 Source Transformation (2)

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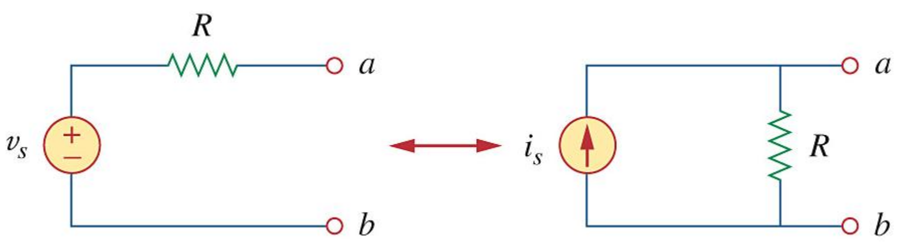

(a) Independent source transform

4.4 Source Transformation (3)

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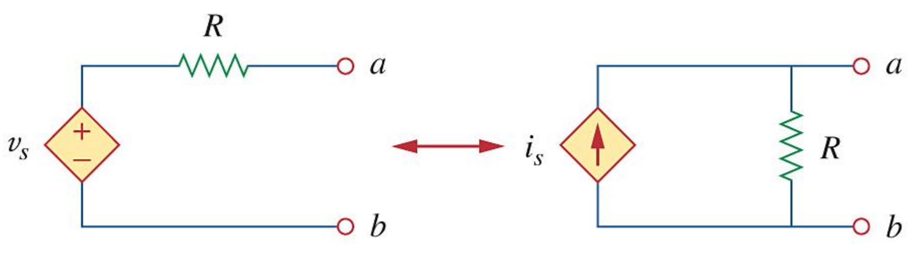


(b) Dependent source transform

4.4 Source Transformation (4)

24

Two things - Keep in mind:

1. Arrow of current source is directed toward positive terminal of voltage source.

2. Not possible when:

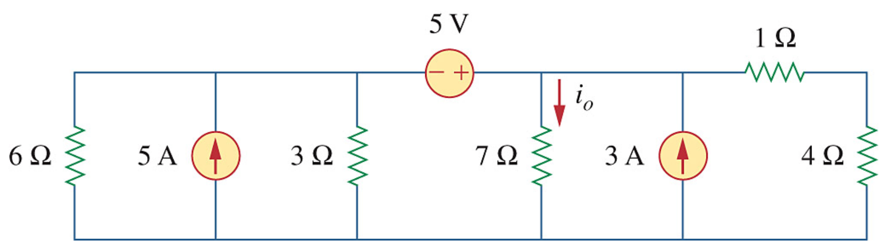
   - ➢ R = 0 for voltage source
   - ➢ R = ∞ for current source

4.4 Source Transformation (5)

25

P.P.4.6

Find i_o in the circuit shown below using source transformation.



4.4 Source Transformation (6)

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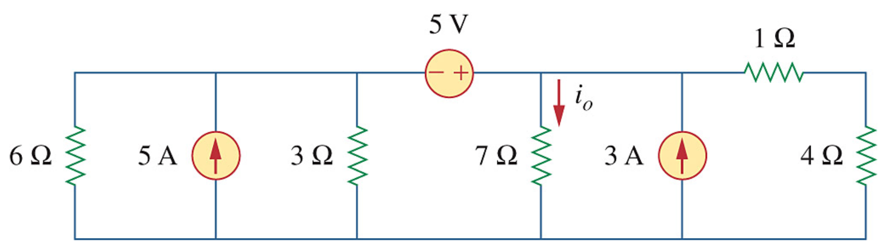
Soln. P.P.4.6



Combining the 6-Ω and 3-Ω resistors in parallel gives (6x3)/(6+3)=2Ω.

Adding the 1-Ω and 4-Ω resistors in series gives 1 + 4 = 5Ω.

Transforming the left current source in parallel with the 2-Ω resistor gives the equivalent circuit as shown in Fig. (a).

4.4 Source Transformation (7)

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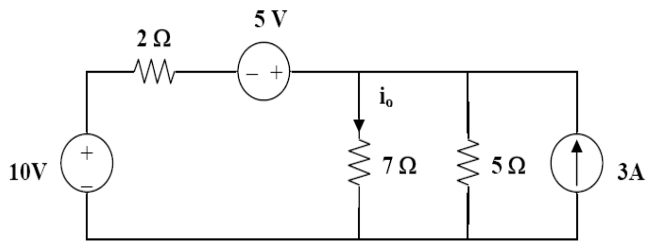
cont. Soln. P.P.4.6



(a)

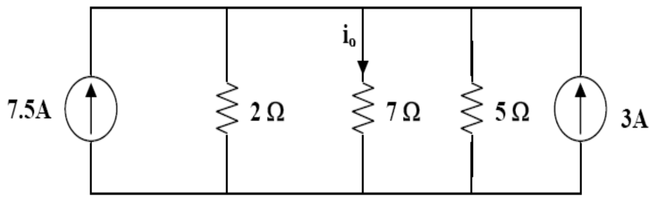
Adding the 10-V and 5-V voltage sources gives a 15-V voltage source.

Transforming the 15-V voltage source in series with the 2-Ω resistor gives the equivalent circuit in Fig. (b).

4.4 Source Transformation (8)

28

cont. Soln. P.P.4.6



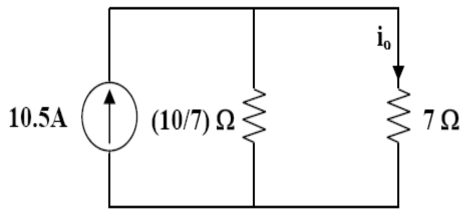
(b)

Combining the two current sources and the 2-Ω and 5-Ω resistors leads to the circuit in Fig. (c).

4.4 Source Transformation (9)

29

cont. Soln. P.P.4.6



(c)

Using current division

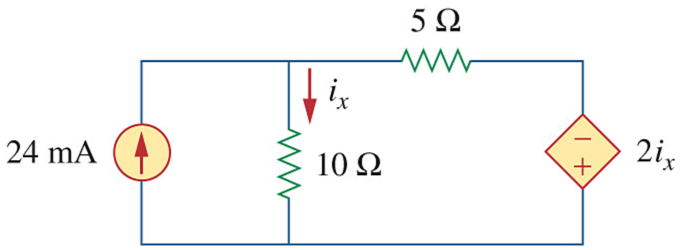
$$i_o = \frac{\frac{10}{7}}{\frac{10}{7}+7}(10.5) \;=\; \underline{\mathbf{1.78\ A}}$$

4.4 Source Transformation (10)

30

P.P.4.7

Use source transformation to find i_x in the circuit shown below.



4.4 Source Transformation (11)

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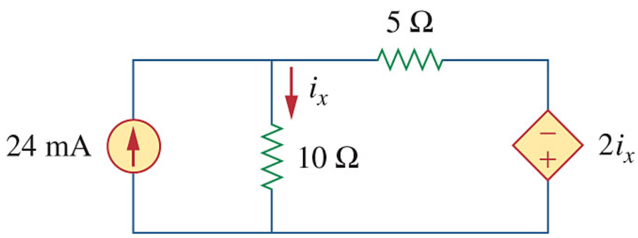
Soln. P.P.4.7



Transform the dependent voltage source as shown in Fig. (a).

4.4 Source Transformation (12)

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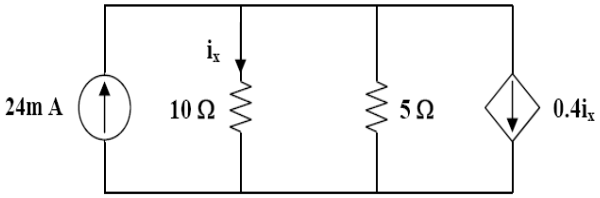
cont. Soln. P.P.4.7



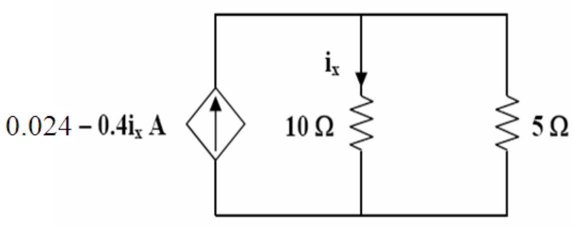
(a)

Combine the two current sources in Fig. (a) to obtain Fig. (b).

4.4 Source Transformation (13)

33

cont. Soln. P.P.4.7



(b)

By the current division principle,

$$i_x = \frac{5}{15}(0.024 - 0.4i_x) \longrightarrow i_x = \underline{7.059 \text{ mA}}$$

4.5 Thevenin's Theorem (1)

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- In practice the load usually varies, while the source is fixed - e.g. fixed household outlet terminal and different electrical appliances which constitute variable loads.
- Each time the load is changed, the entire circuit has to be analysed all over again.
- To avoid this problem, Thevenin's theorem provides a technique by which the fixed part of the circuit is replaced with equivalent circuit.

4.5 Thevenin's Theorem (2)

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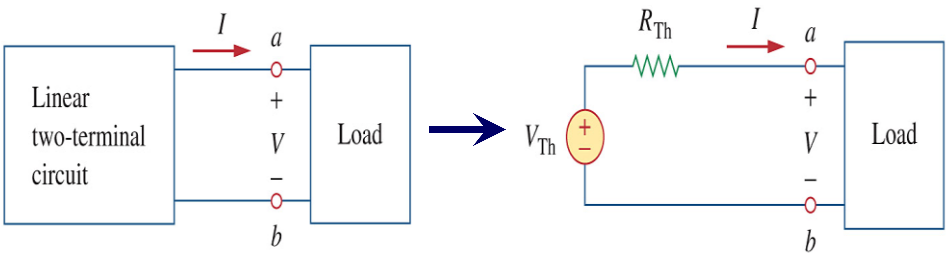
Thevenin's theorem states that: a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with resistor R_{Th}.



4.5 Thevenin's Theorem (3)

36

What is…?

V_{Th} = open-circuit voltage at the terminals.

R_{Th} = input or equivalent resistance at the terminals when the independent sources are turned off. i.e.

- ➢ voltage sources = 0V (short-circuit)
- ➢ current sources = 0 A (open-circuit)

4.5 Thevenin's Theorem (4)

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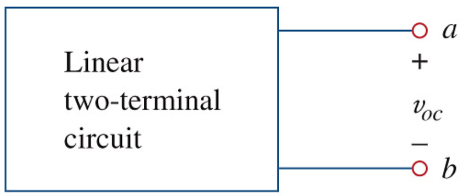
How to find… V_{Th}



$$V_{Th} = v_{oc}$$

Find the voltage across point '*a*' and '*b*' using any method in previous chapters. *(by taking out the load from the circuit.)*

4.5 Thevenin's Theorem (5)

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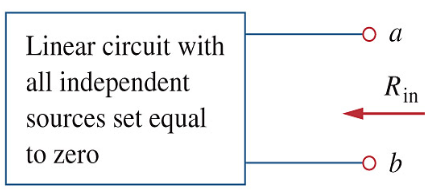
How to find… R_{Th}

Case 1: No dependent sources in the circuit.



$$R_{Th} = R_{in}$$

Turn off all independent sources.

Find R_{Th} by finding the equivalent resistance at point '*a*' and '*b*'.

4.5 Thevenin's Theorem (6)

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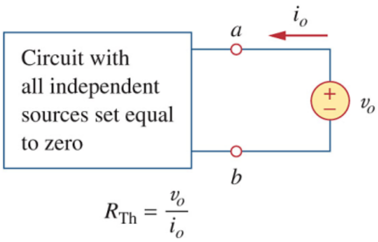
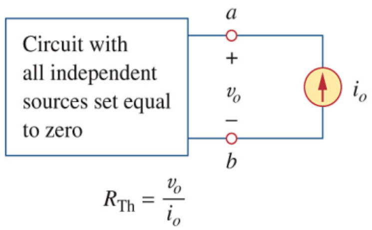
Case 2: Circuit has dependent sources. *(cannot turn off)*





Turn off all independent sources.

Leave dependent sources intact.

Apply voltage source v_0 across '*a*' and '*b*' then find $R_{Th}= v_o/i_o$. OR apply current source i_0 and find $R_{Th}= v_o/i_o$.

4.5 Thevenin's Theorem (7)

40

Two things to keep in mind - for Case 2

1. Any value can be assumed for v_0 and i_0.
   (usually assume v_0=1V and i_0=1A)

2. If R_{Th}<0, imply circuit is supplying power - possible in circuit with dependent sources.

4.5 Thevenin's Theorem (8)

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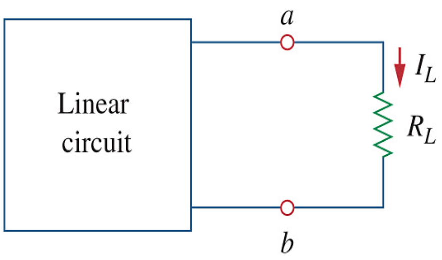
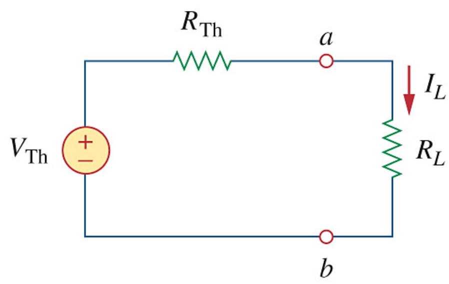
Consider linear circuit terminated by load R_L.





Current I_L through the load and voltage V_L across the load is given by:

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} \qquad V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

4.5 Thevenin's Theorem (11)

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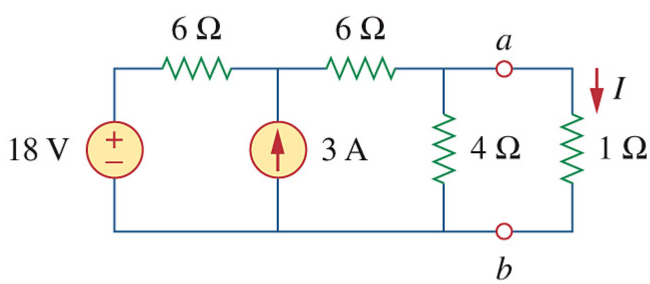
P.P.4.8

Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit shown below. Hence find i.

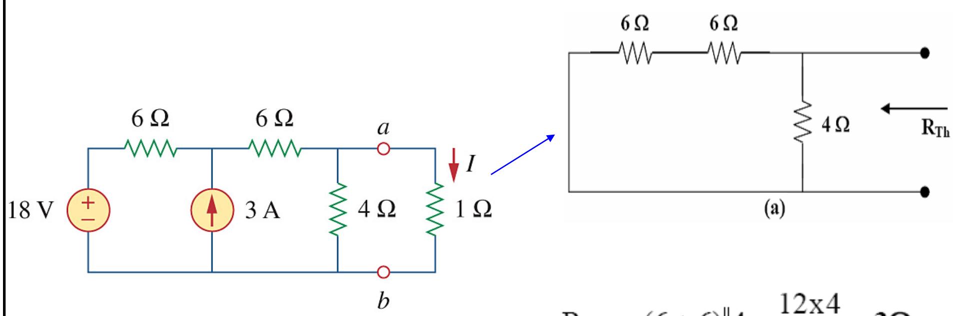


4.5 Thevenin's Theorem (12)

43

Soln.P.P.4.8

To find R_{Th}, consider the circuit in Fig. (a).



$$R_{Th} = (6+6)\|4 = \frac{12x4}{18} = \underline{\mathbf{3\Omega}}$$

4.5 Thevenin's Theorem (13)

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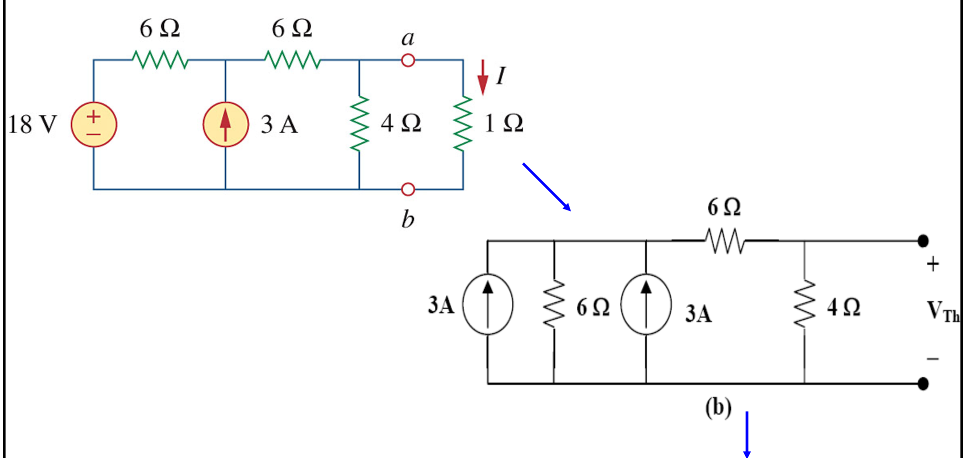
cont. Soln.P.P.4.8

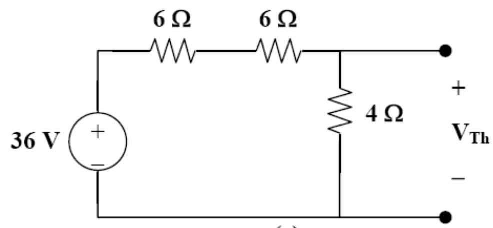
To find V_{Th}, do source transformation, as shown in Fig. (b) and (c).



4.5 Thevenin's Theorem (14)

45

cont. Soln.P.P.4.8



(c)

Using voltage division in Fig. (c),

$$V_{Th} = \frac{4}{4+12}(36) = \underline{9\text{ V}}$$

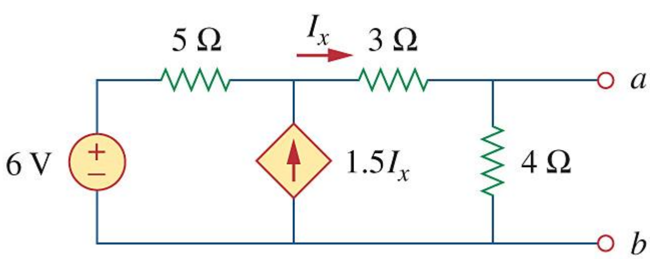
Calculate i, $i = \frac{V_{Th}}{R_{Th}+1} = \frac{9}{3+1} = \underline{2.25\text{ A}}$

4.5 Thevenin's Theorem (15)

46

P.P.4.9

Find the Thevenin equivalent circuit of the circuit shown below to the left of the terminals.



4.5 Thevenin's Theorem (16)

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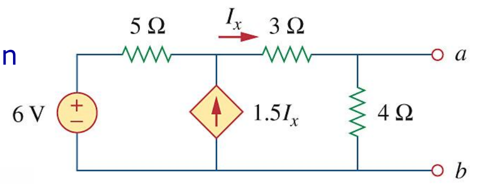
Soln.P.P.4.9

To find V_{Th} consider the circuit in Fig. (a).



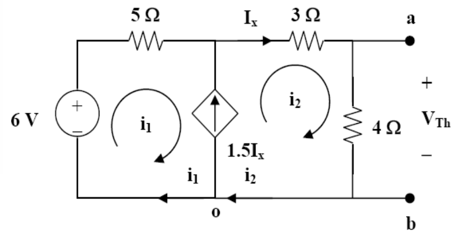
$I_x = i_2$

$i_2 - i_1 = 1.5I_x = 1.5i_2 \longrightarrow i_2 = -2i_1 \quad (1)$

For the supermesh, $-6 + 5i_1 + 7i_2 = 0 \quad (2)$

From (1) and (2), $i_2 = 4/(3)$A

$V_{Th} = 4i_2 =$ **5.333V**



(a)

4.5 Thevenin's Theorem (17)

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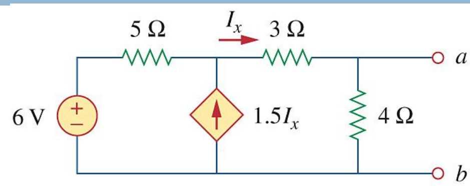
cont. Soln.P.P.4.9

To find R_{Th} consider the circuit in Fig. (b).

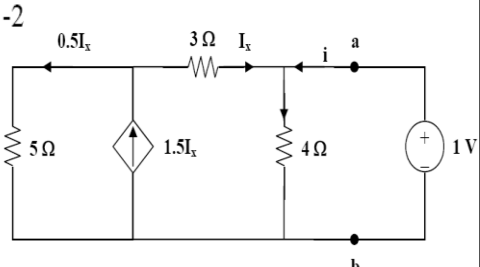


Applying KVL around the outer loop,

$$5(0.5I_x) - 1 - 3I_x = 0 \longrightarrow I_x = -2$$

$$i = \frac{1}{4} - I_x = 2.25$$

$$R_{Th} = \frac{1}{i} = \frac{1}{2.25} = \underline{\mathbf{444.4\ m\Omega}}$$



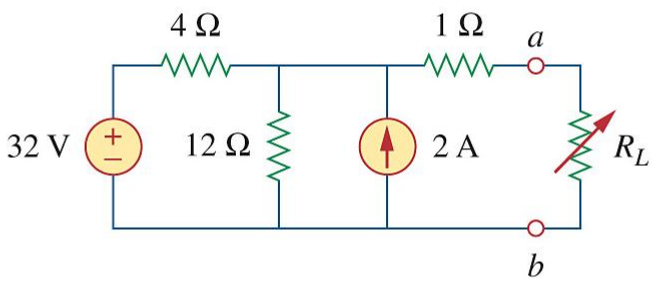
(b)

4.5 Thevenin's Theorem (i)

49

e.g.4.8

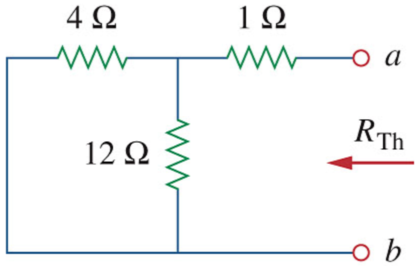
Find the Thevenin equivalent circuit of the circuit shown below, to the left of the terminals *a*-*b*. Then find the current through R_L = 6, 16 and 36 ohms.



4.5 Thevenin's Theorem (i)

50

e.g.4.8 Solve R_{Th}

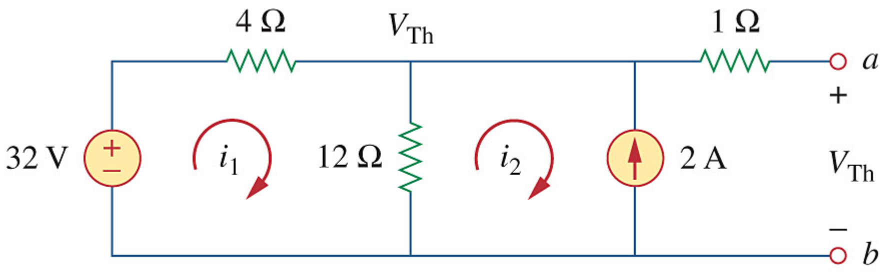


(a)

4.5 Thevenin's Theorem (i)

51

e.g.4.8 Solve V_{Th}



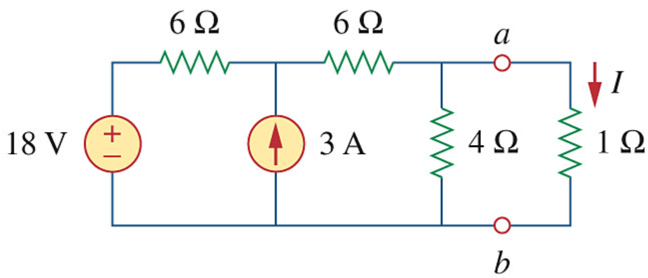
(b)

4.5 Thevenin's Theorem (ii)

52

P.P.4.8

Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit shown below. Hence find *i*.

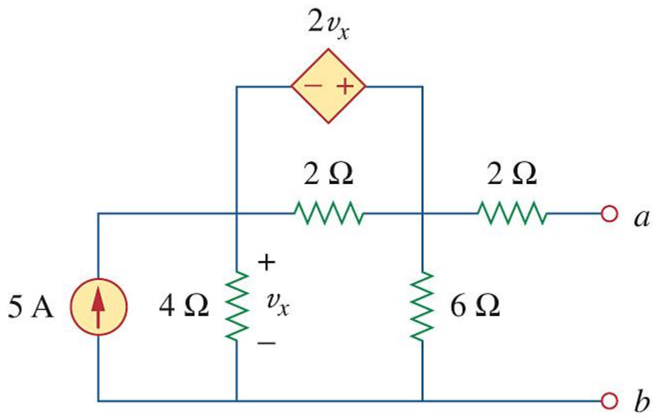


4.5 Thevenin's Theorem (iii)

53

e.g.4.9

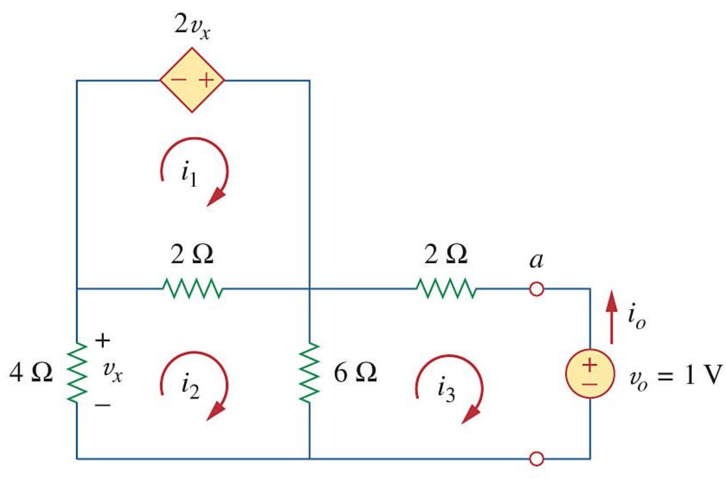
Find the Thevenin equivalent circuit of the circuit shown below at terminals *a*-*b*.



4.5 Thevenin's Theorem (iii)

54

e.g.4.9 Solve R_{Th}

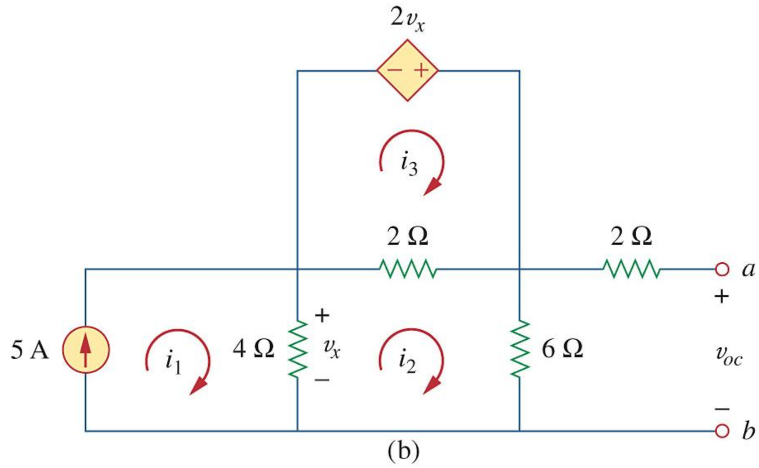


(a)

4.5 Thevenin's Theorem (iii)

55

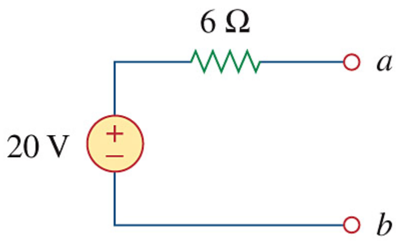
e.g.4.9 Solve V_{Th}



4.5 Thevenin's Theorem (iii)

56

e.g.4.9 Thevenin's equivalent

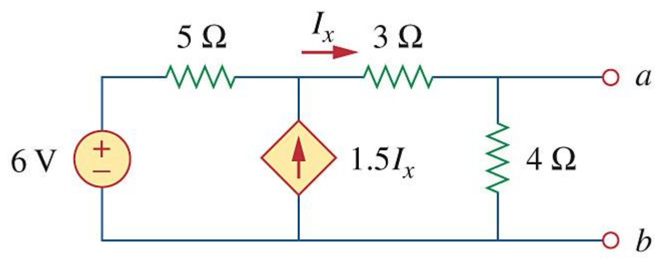


4.5 Thevenin's Theorem (iv)

57

P.P.4.9

Find the Thevenin equivalent circuit of the circuit shown below to the left of the terminals.

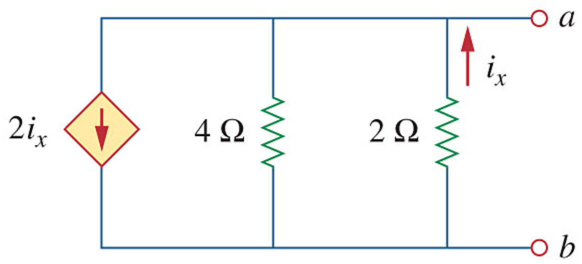


4.5 Thevenin's Theorem (v)

58

e.g.4.10

Determine the Thevenin equivalent circuit in the Figure (a) shown below at terminals *a*-*b*.

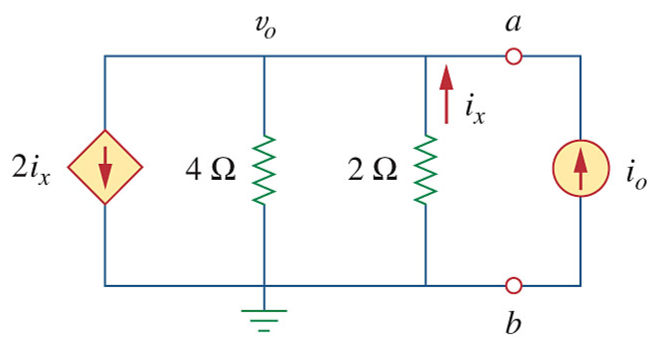


(a)

4.5 Thevenin's Theorem (v)

59

e.g.4.10 Solve R_{Th}

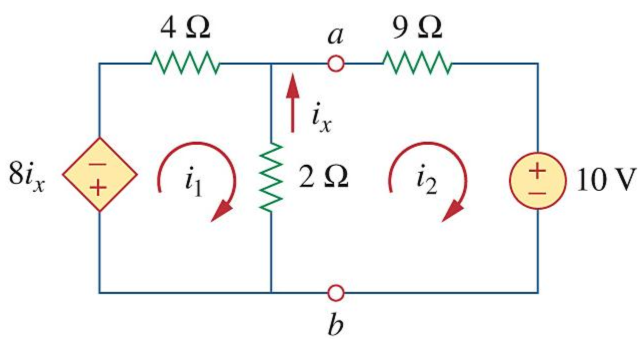


4.5 Thevenin's Theorem (v)

60

e.g.4.10 Solve V_{Th}

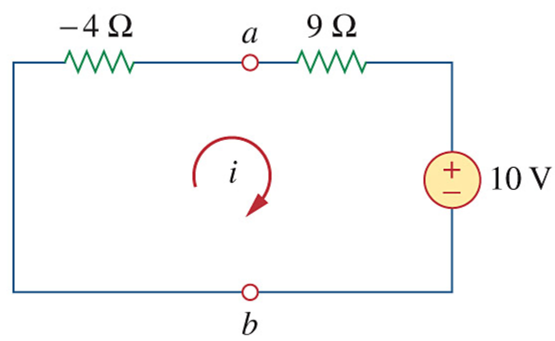
Do source transformation



4.5 Thevenin's Theorem (v)

61

e.g.4.10 Solve *i*

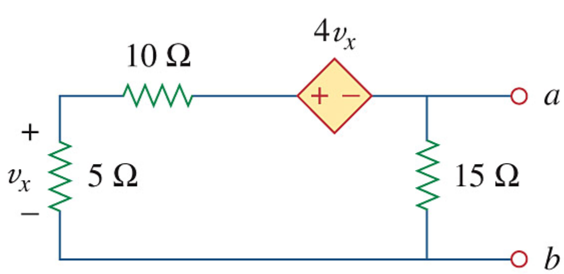


4.5 Thevenin's Theorem (v)

62

P.P.4.10 Solve *I*

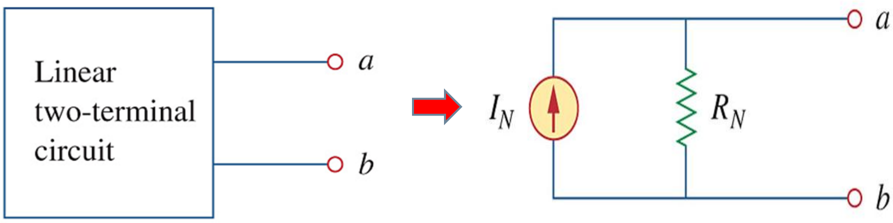
Obtain the Thevenin equivalent of the circuit given below.



4.6 Norton's Theorem (1)

63

Norton's theorem states that: a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with resistor R_N.



4.6 Norton's Theorem (2)

64

What is…?

I_N = short-circuit current through the terminals.

R_N = input or equivalent resistance at the terminals when the independent sources are turned off. i.e.

- voltage sources = 0V (short-circuit)
- current sources = 0 A (open-circuit)

4.6 Norton's Theorem (3)

65

Relation between Norton's & Thevenin's Theorem

The Thevenin's and Norton equivalent circuits are **related by a source transformation**.

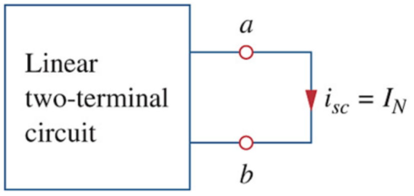
In source transformation, the resistor does not change…

Thus: $R_N = R_{Th}$

4.6 Norton's Theorem (4)

66

How to find… I_N



The short-circuit current flowing from terminal 'a' to 'b' is I_N.

Since resistors $R_N = R_{Th}$,

$$I_N = \frac{V_{Th}}{R_{Th}}$$

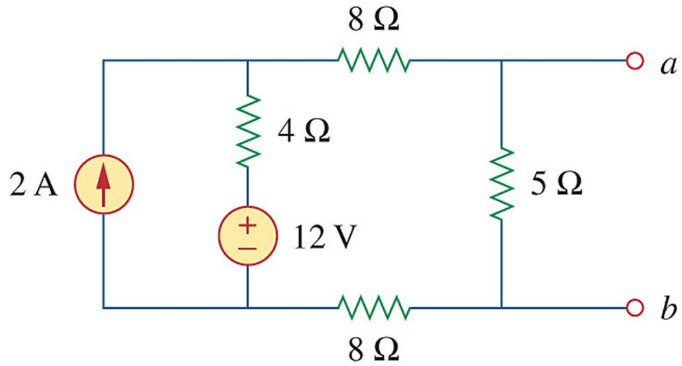
Dependent and independent sources are treated the same way as in Thevenin's Theorem.

4.6 Norton's Theorem (5)

67

e.g. 4.11

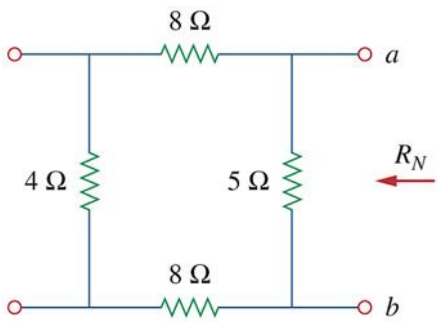
Find the Norton equivalent circuit of the circuit shown below, at terminals *a*-*b*.



4.6 Norton's Theorem (6)

68

e.g. 4.11: Solve R_N

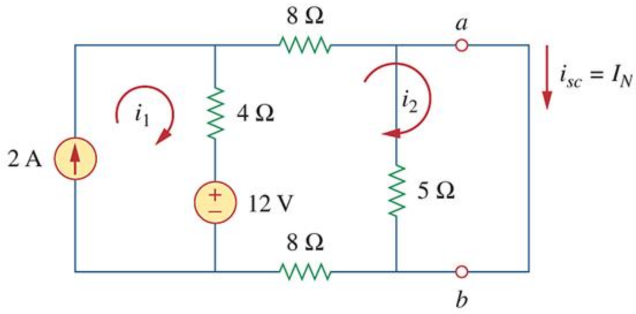


(a)

4.6 Norton's Theorem (7)

69

e.g. 4.11: Solve I_N

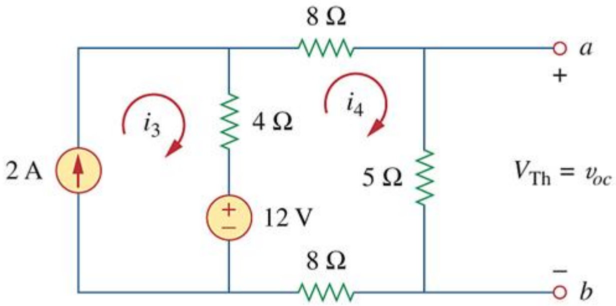


(b)

4.6 Norton's Theorem (8)

70

e.g. 4.11: Alternatively solve I_N from V_{Th}/R_{Th}

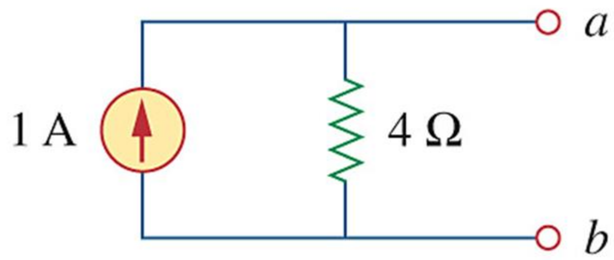


(c)

4.6 Norton's Theorem (9)

71

e.g. 4.11: Thus Norton's equivalent circuit is

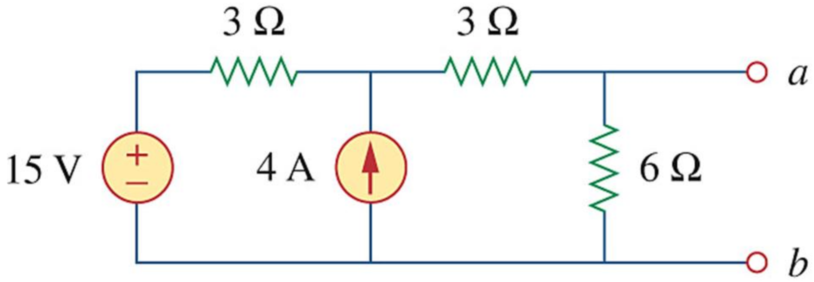


4.6 Norton's Theorem (10)

72

P.P.4.11

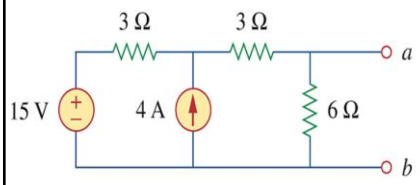
Find the Norton equivalent circuit of the circuit shown below, at terminals *a*-*b*.

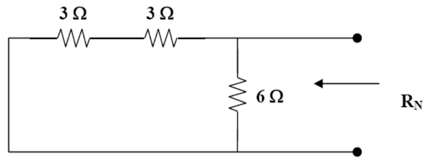


4.6 Norton's Theorem (11)

73

Soln. P.P.4.11

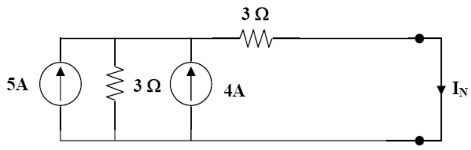




(a)

From Fig. (a), $R_N = (3+3)\|6 = \underline{\mathbf{3\ \Omega}}$

From Fig. (b), $I_N = \frac{1}{2}(5+4) = \underline{\mathbf{4.5A}}$



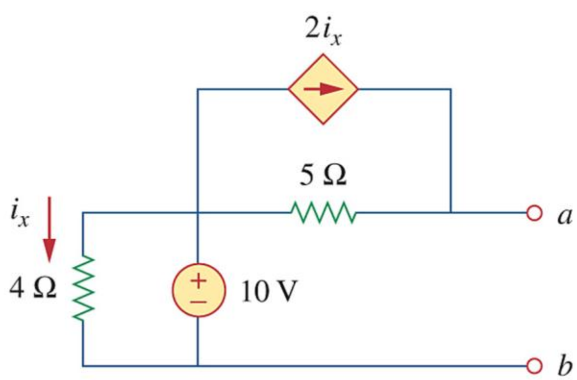
(b)

4.6 Norton's Theorem (12)

74

e.g. 4.12

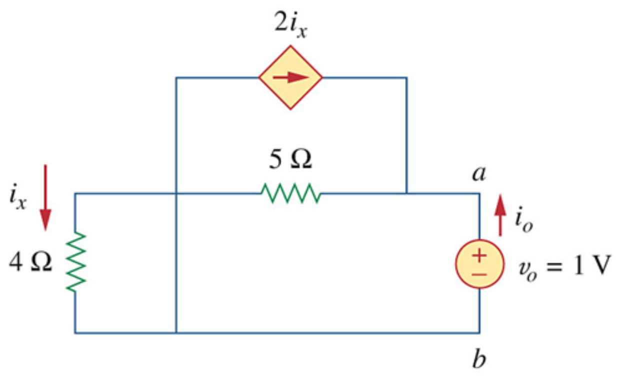
Find the Norton equivalent circuit of the circuit shown below, at terminals *a*-*b*.



4.6 Norton's Theorem (13)

75

e.g. 4.12: Solve R_N

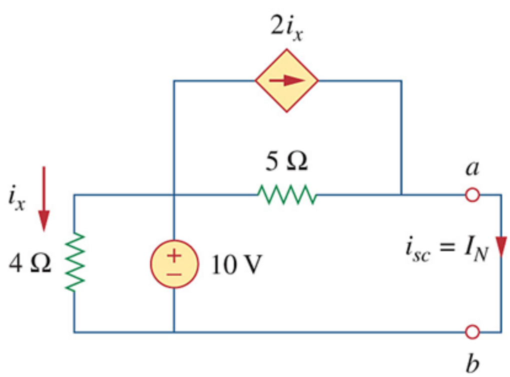


(a)

4.6 Norton's Theorem (14)

76

e.g. 4.12: Solve I_N



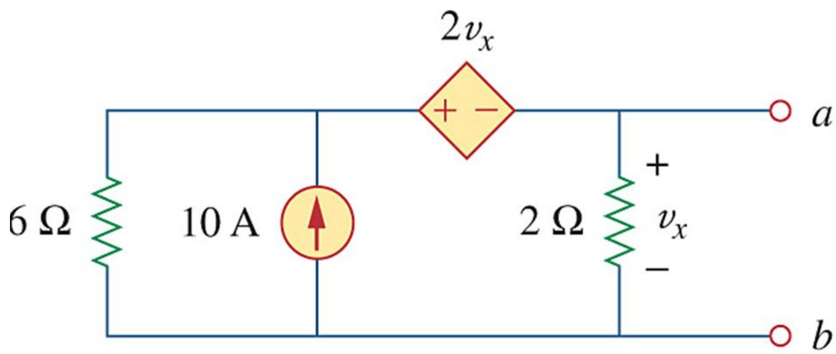
(b)

4.6 Norton's Theorem (15)

77

P.P.4.12

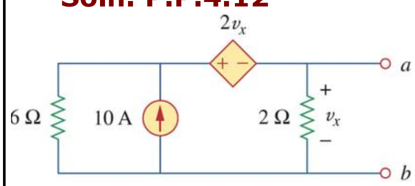
Find the Norton equivalent circuit of the circuit shown below.

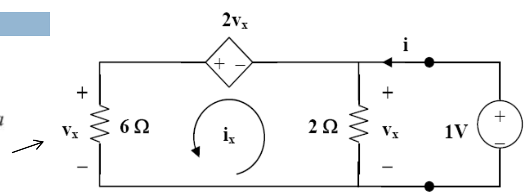


4.6 Norton's Theorem (16)

78

Soln. P.P.4.12





(a)

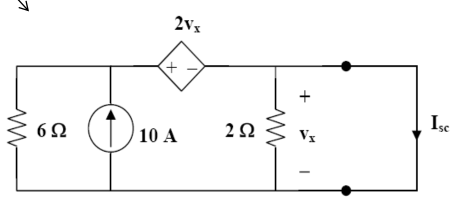
To get R_N consider the circuit in Fig. (a).

Applying KVL, $6i_x - 2v_x - 1 = 0$

But $v_x = 1$, $6i_x = 3 \longrightarrow i_x = 0.5$

$$i = i_x + \frac{v_x}{2} = 0.5 + 0.5 = 1$$

$$R_N = R_{Th} = \frac{1}{i} = \underline{\mathbf{1\Omega}}$$



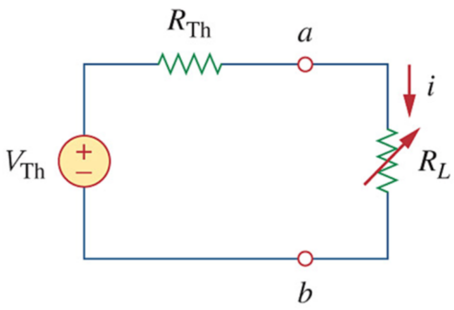
(b)

To find I_N, consider the circuit in Fig. (b). Because the 2Ω resistor is shorted, $v_x = 0$ and the dependent source is inactive. Hence, $I_N = i_{sc} = \underline{\mathbf{10A}}$.

4.7 Maximum Power Transfer (1)

-To find the maximum power that can be delivered to the load.

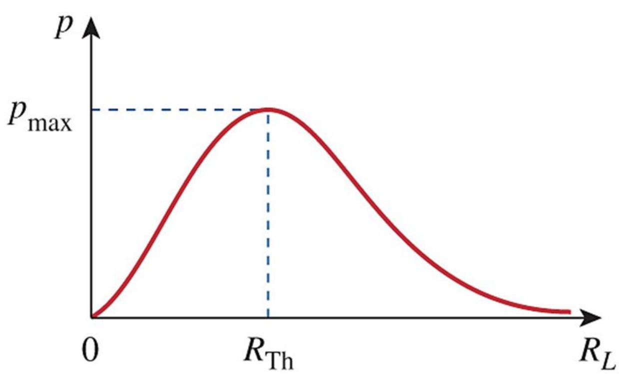
- From Thevenin's equivalent circuit,



$$P = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

4.7 Maximum Power Transfer (2)

- By varying the load resistance R_L, the power delivered will also vary - as per the graph:



Power transfer profile with different R_L

4.7 Maximum Power Transfer (3)

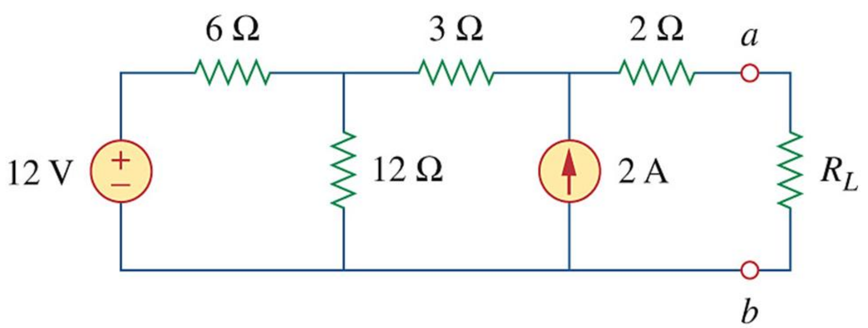
Maximum power is transferred to the load when the load resistance equals the Thevenin resistance, as seen from the load.

$$R_L = R_{TH} \quad \Rightarrow \quad p_{max} = \frac{V_{Th}^{\ 2}}{4R_{Th}}$$

4.7 Maximum Power Transfer (4)

e.g. 4.13

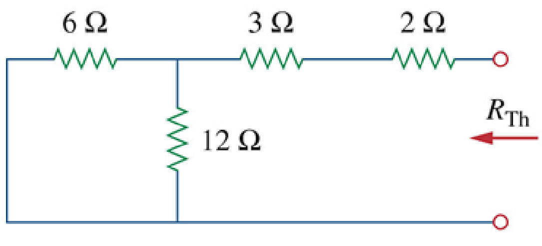
Find the value of R_L for maximum power transfer in the circuit shown below. Find the maximum power.



4.7 Maximum Power Transfer (5)

Soln. 4.13

Find R_{Th}

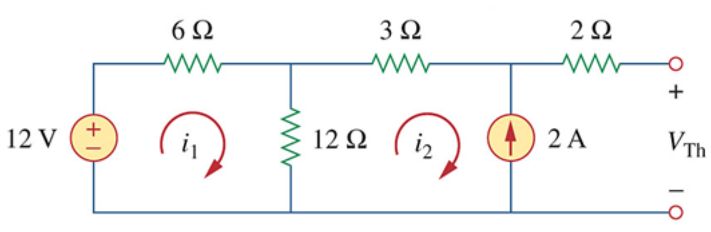


(a)

4.7 Maximum Power Transfer (6)

cont. Soln. 4.13

Find V_{Th}

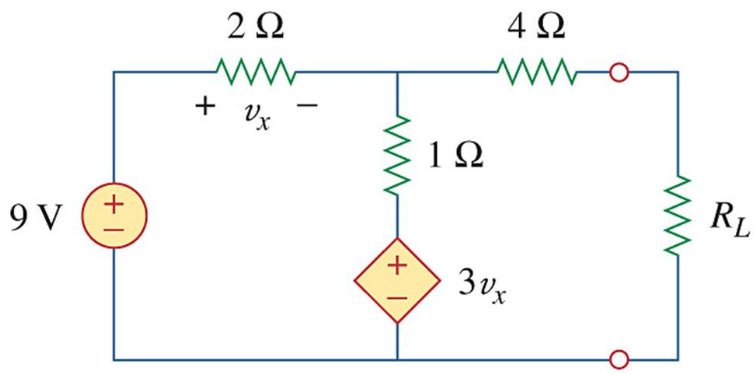


(b)

4.7 Maximum Power Transfer (7)

P.P. 4.13

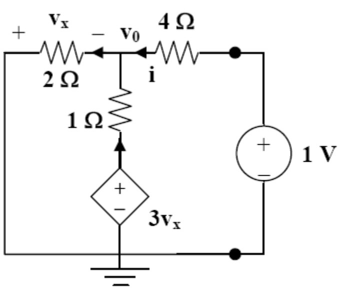
Determine the value of R_L that will draw the maximum power from the rest of the circuit shown below. Calculate the maximum power.



4.7 Maximum Power Transfer (8)

Soln. P.P. 4.13

Find R_{Th}



(a)

Applying KCL at the top node gives

$$\frac{1-v_o}{4}+\frac{3v_x-v_o}{1}=\frac{v_o}{2}$$

But $v_x = -v_o$. Hence

$$\frac{1-v_o}{4}-4v_o=\frac{v_o}{2} \longrightarrow v_o = 1/(19)$$

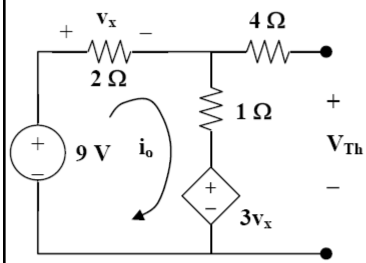
$$i=\frac{1-v_o}{4}=\frac{1-\frac{1}{19}}{4}=\frac{9}{38}$$

$R_{Th} = 1/i = 38/(9) = 4.222\Omega$

4.7 Maximum Power Transfer (9)

cont. Soln. P.P 4.13

Find V_{Th}



(b)

$$-9 + 2i_o + i_o + 3v_x = 0$$

But $v_x = 2i_o$. Hence,

$$9 = 3i_o + 6i_o = 9i_o \longrightarrow i_o = 1A$$

$$V_{Th} = 9 - 2i_o = 7V$$

$$R_L = R_{Th} = \underline{\mathbf{4.222\Omega}}$$

$$P_{max} = \frac{v_{Th}^2}{4R_L} = \frac{49}{4(4.222)} = \underline{\mathbf{2.901\ W}}$$