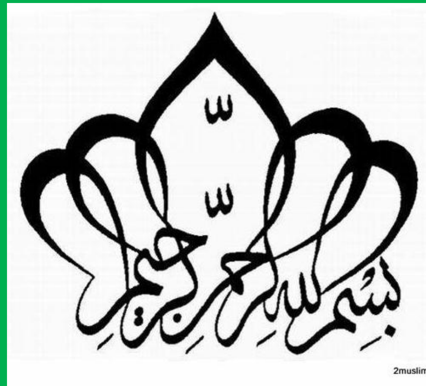

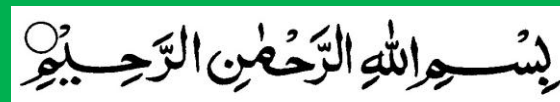
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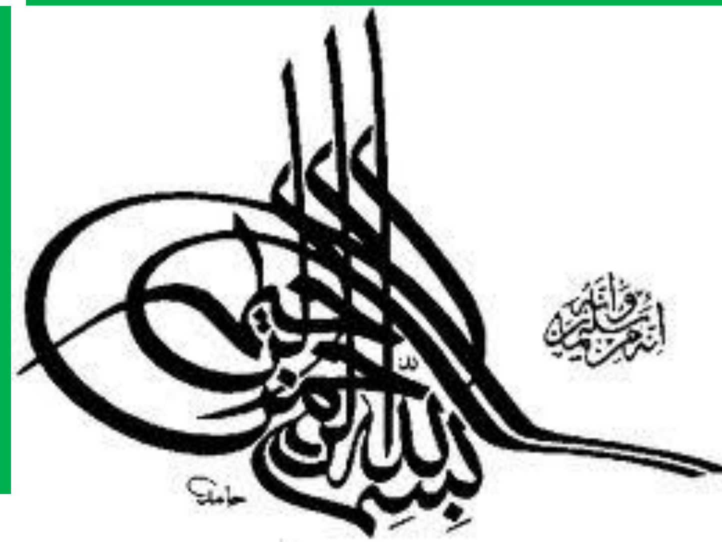


Control Technology

Engr. Muhammad Aamir Aman
Lecturer
Department of Electrical Engineering

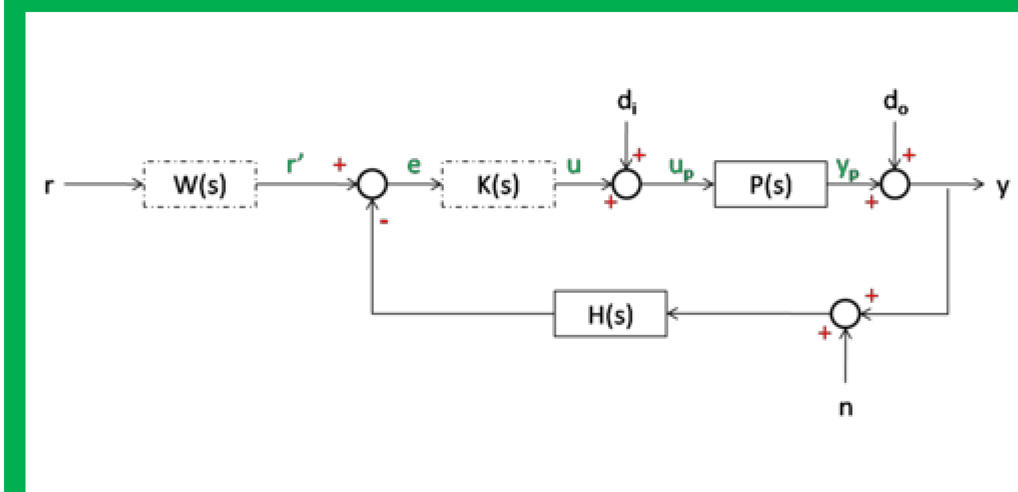
Outlines

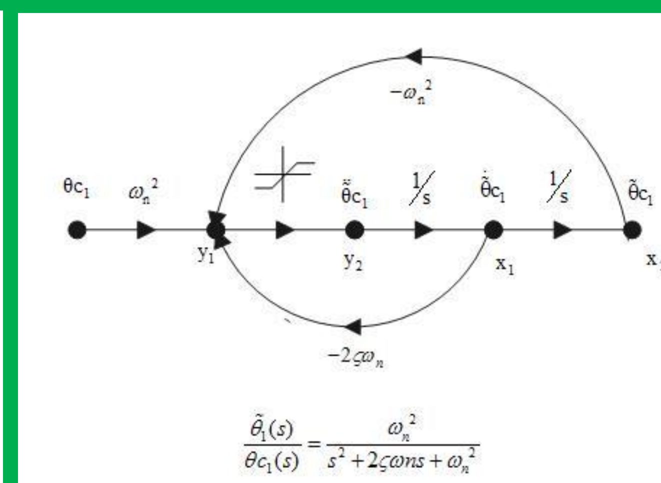
- Block Diagram Reduction
- Signal Flow Graph

Representation of Control Systems

- Three basic representations of physical components and systems
  - Differential equations and other mathematical relations (Difference eq., Laplace, Z)
  - Block diagrams
  - Signal flow graphs

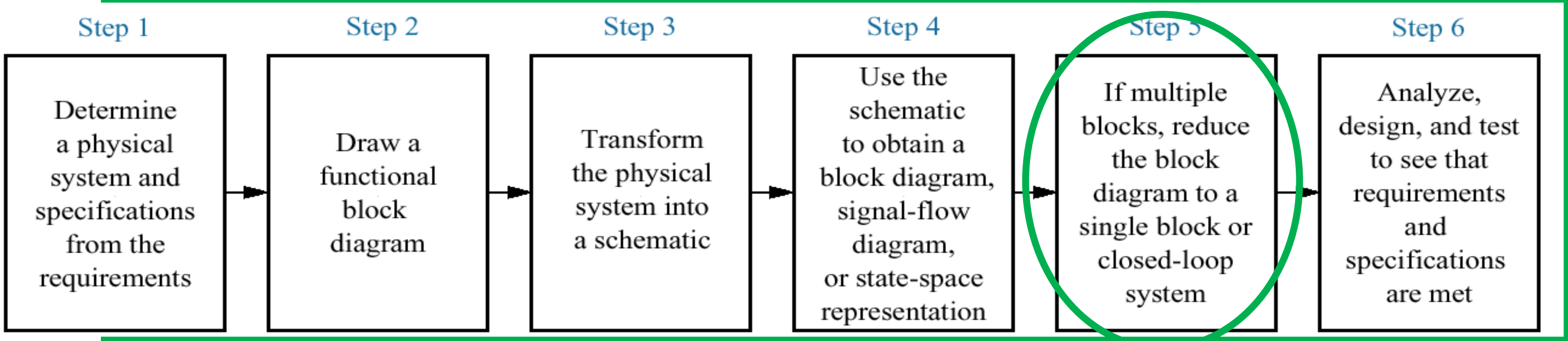
$$a_n \cdot \frac{d^n}{dt^n} y + a_{n-1} \cdot \frac{d^{n-1}}{dt^{n-1}} y + \dots + a_1 \cdot \frac{d}{dt} y + a_0 \cdot y = u(t)$$

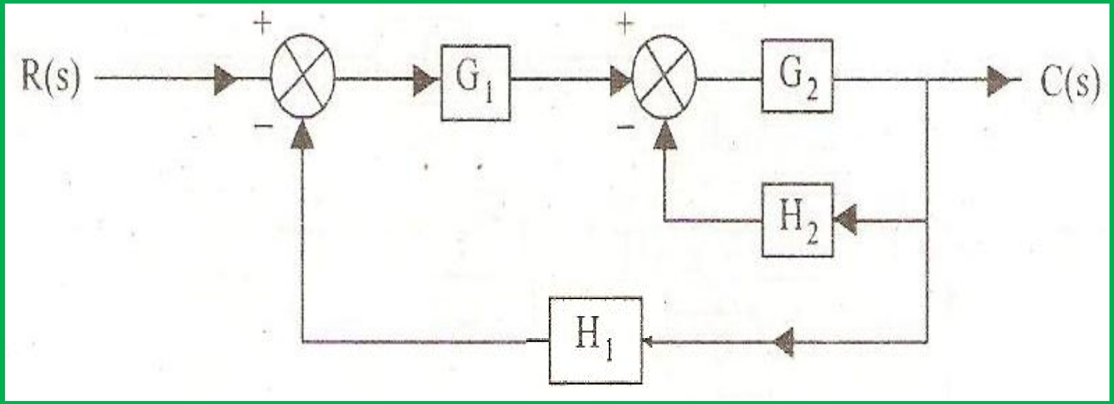




Block Diagram Reduction

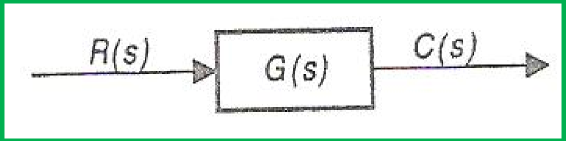
Process of Modeling





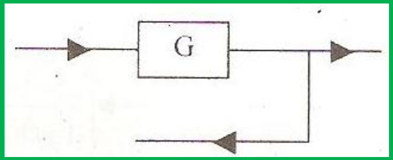
Rules (Preleminaries) for BD Reduction

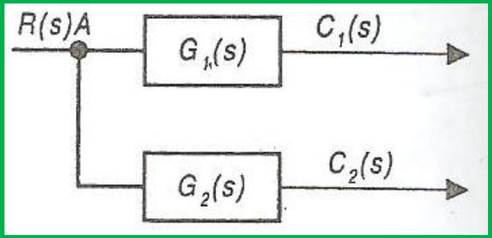
- Transfer Function



$$G(s) = \frac{C(s)}{R(s)}$$
$$C(s) = R(s) \cdot G(s)$$

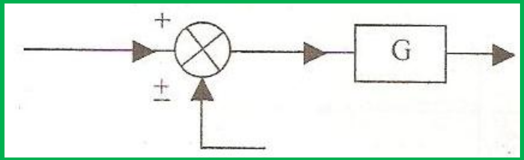
- Pick Off Point

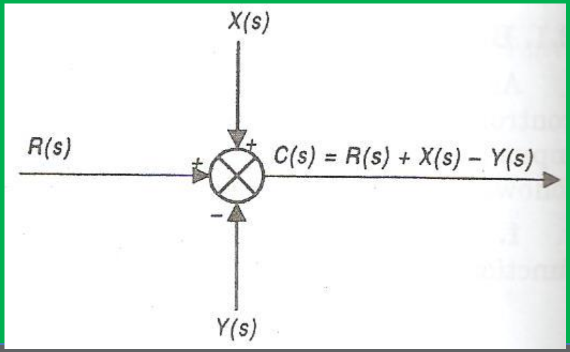




$$\frac{C_1(s)}{R(s)} = G_1(s)$$

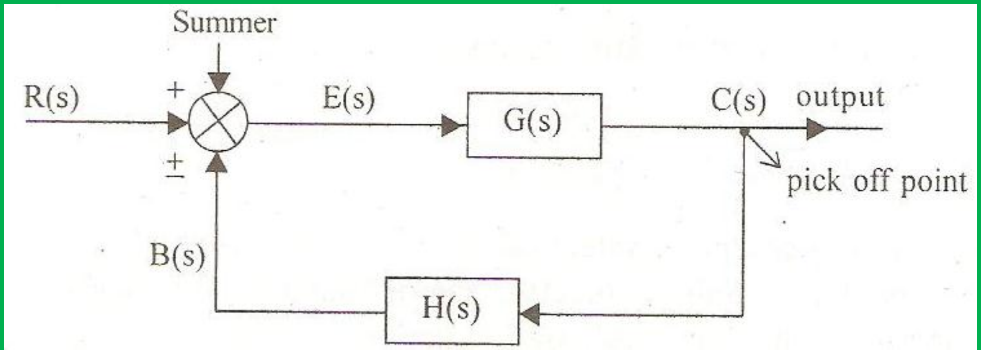
- Summing Point





Rules

➢ Simple F/Back System (Nomenclature)

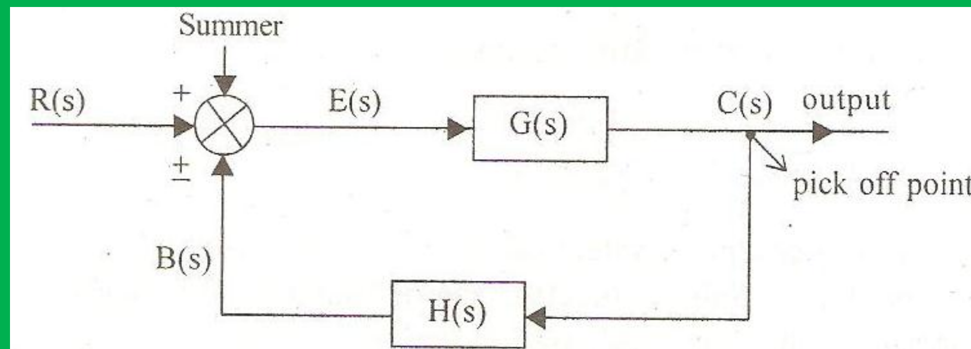


Nomenclature:

- G(s) = C(s)/E(s) is Forward path transfer function
- R(s) is Reference input or desired output
- C(s) is Output or controlled variable
- B(s) is Feedback signal
- E(s) is Error signal

Rules

- Computation of overall T/Function



$$C(s) = G(s)\ E(s)$$

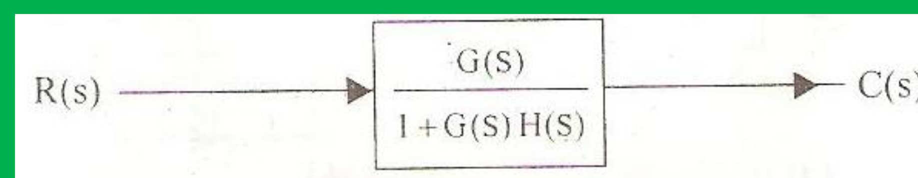
$$E(s) = R(s) - B(s)$$

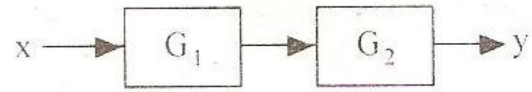
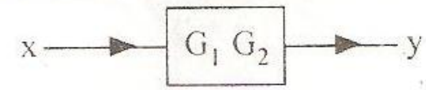
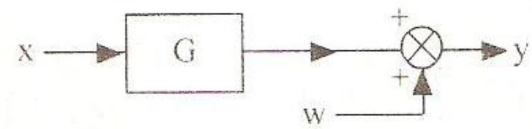
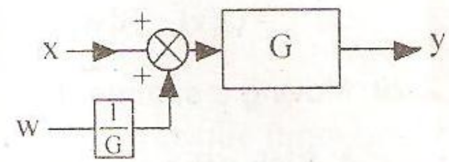
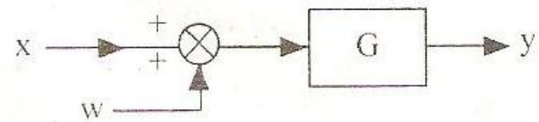
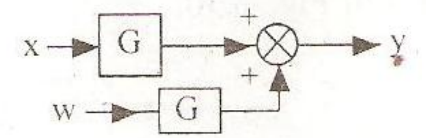
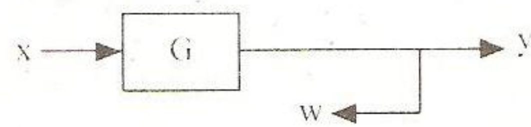
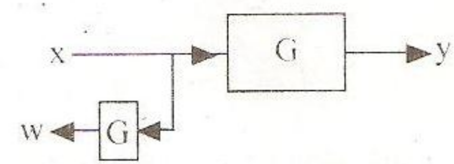
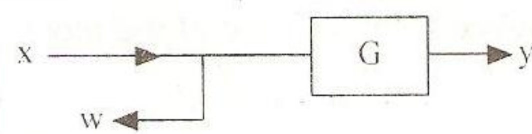
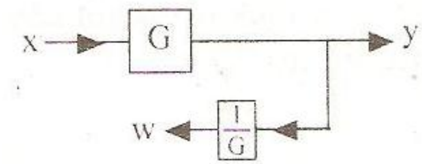
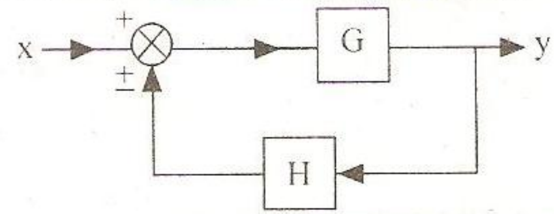
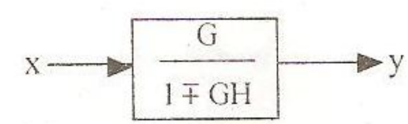
$$= R(s) - H(s)\ C(s)$$

$$C(s) = G(s)\ [R(s) - H(s)\ C(s)]$$

$$C(s)\ [1 + G(s)\ H(s)] = G(s)\ R(s)$$

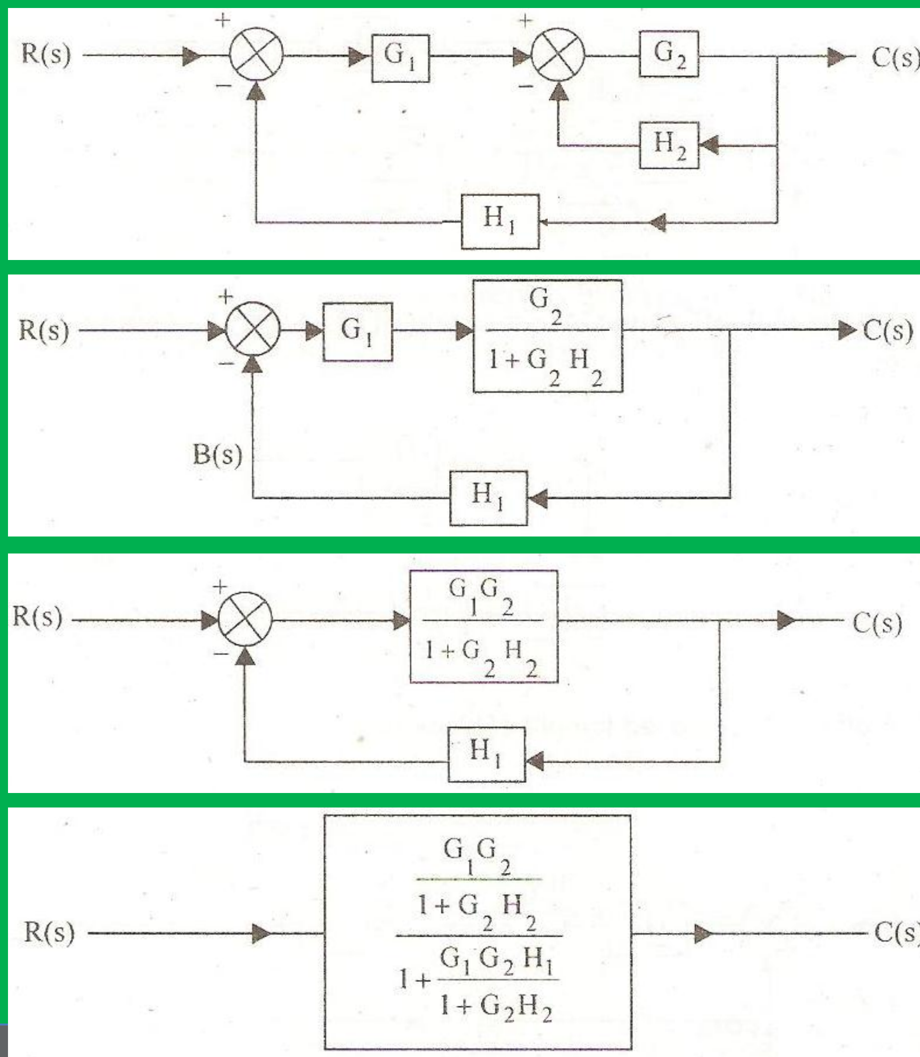
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\ H(s)}$$



Rule no.	Given system	Equivalent system
1. Blocks in cascade		
2. Moving summing point to left		
3. Moving summing point to right		
4. Moving pick off point to left		
5. Moving pick off point to right		
6. Absorbing a loop		

Solved Example 1

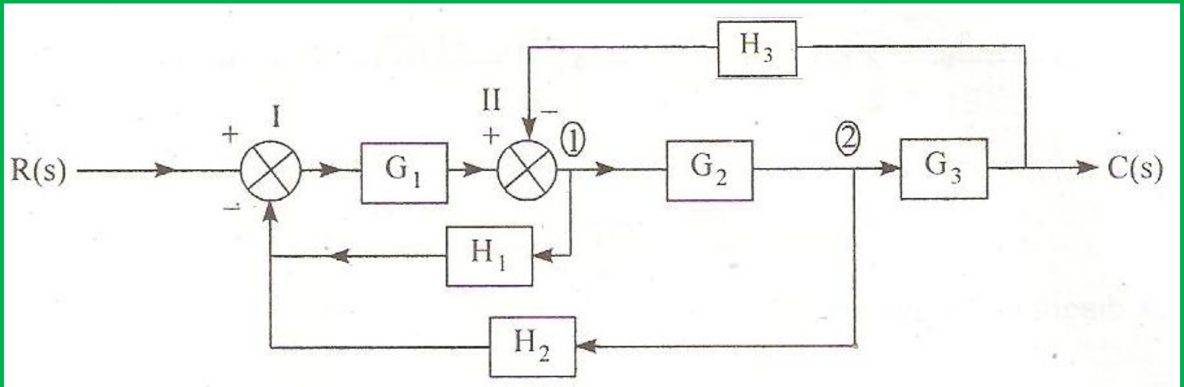
- Find the overall T/Function



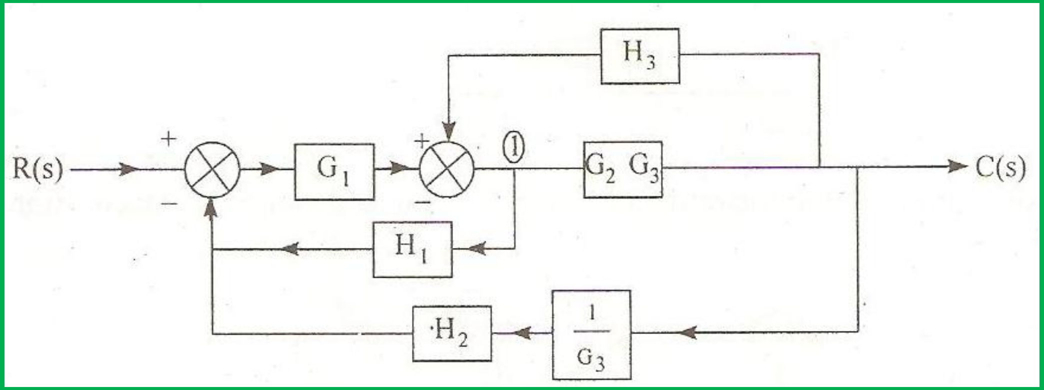
$$\frac{C(s)}{R(s)} = \frac{G_1G_2}{1+G_2H_2+G_1G_2H_1}$$

Solved Example 2

- Find the overall T/Function

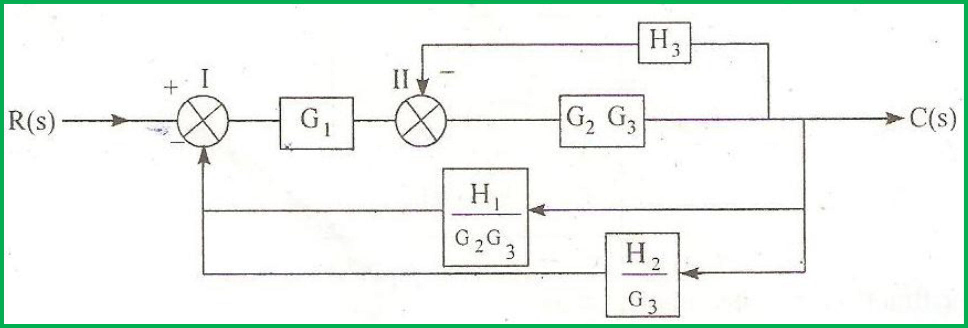


- Moving the pick off point (2) to the right of block G_3 and combining blocks G_2 and G_3 in cascade

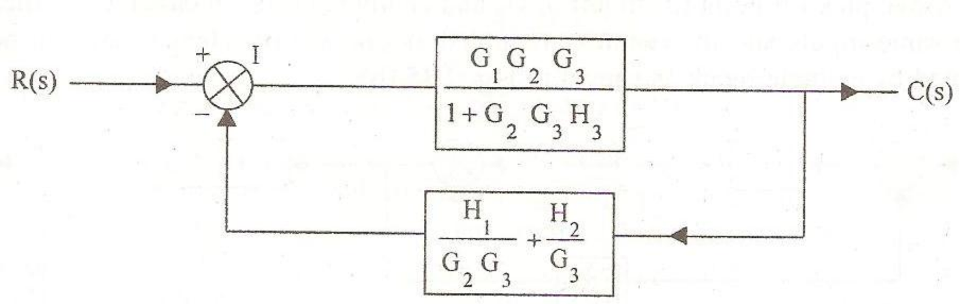


Solved Example 2

- Moving the pick off point(1) to the right block G_2G_3

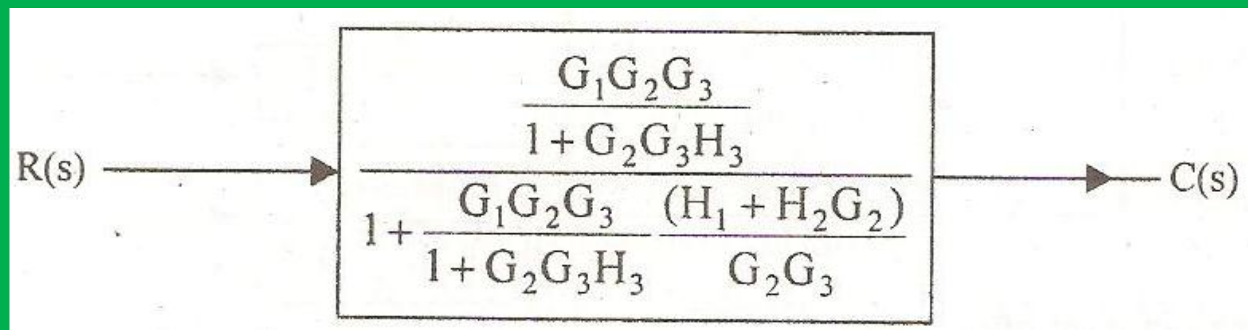


- Absorbing the loop with G_2G_3 and H_3
- Moreover, the two feedback path blocks have the same inputs and both are substratced from R(s) at the summer, so they can be added and represented by a single block



Solved Example 2

- Finally the closed loop is absorbed and the simplified block is



- The transfer function of the system is

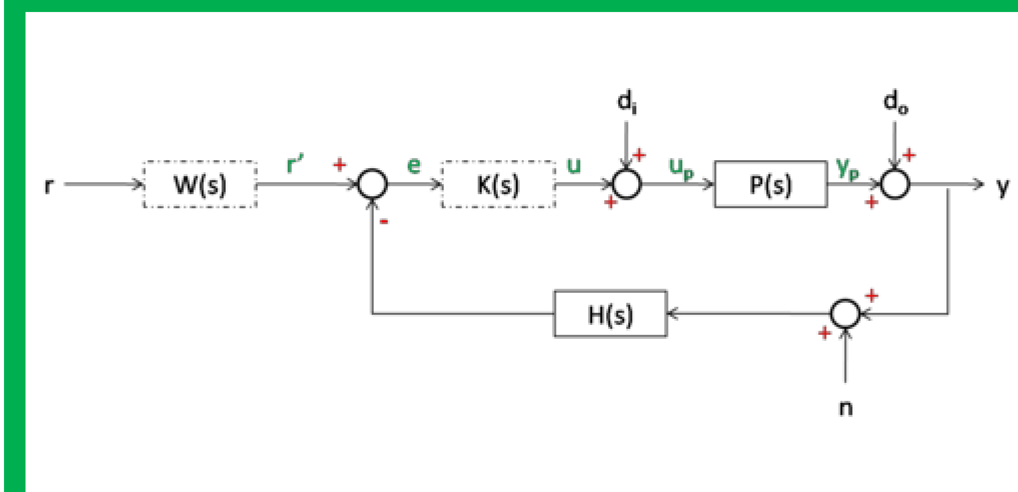
$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 + G_2 G_3 H_3 + G_1 H_1 + G_1 G_2 H_2}$$

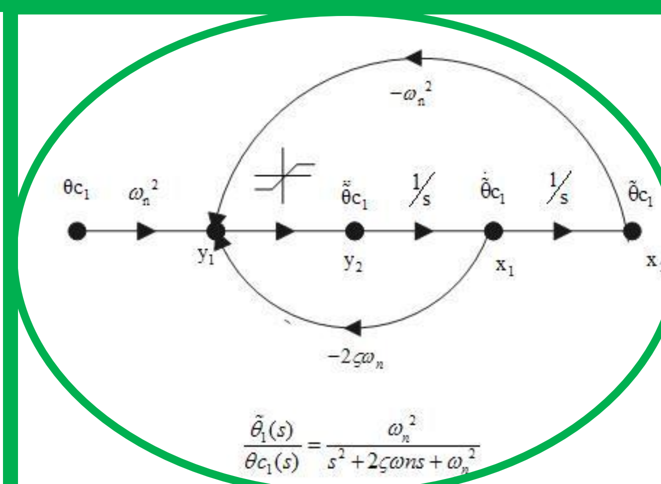
SIGNAL FLOW GRAPH

Modeling of Control Systems

- Three basic representations of physical components and systems
  - Differential equations and other mathematical relations (Difference eq., Laplace, Z)
  - Block diagrams
  - Signal flow graphs

$$a_n \cdot \frac{d^n}{dt^n} y + a_{n-1} \cdot \frac{d^{n-1}}{dt^{n-1}} y + \dots + a_1 \cdot \frac{d}{dt} y + a_0 \cdot y = u(t)$$





Signal Flow Graphs

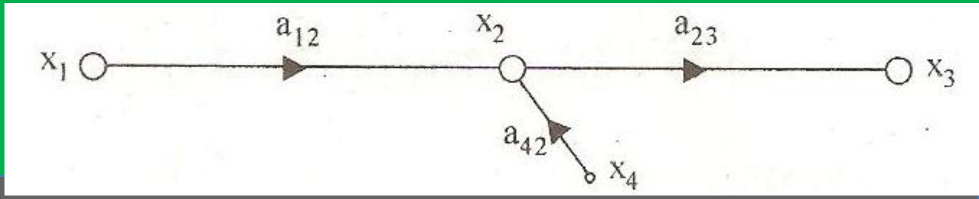
- Motivation & Intro:
  - Block diagram is a simple representation of describing a system but it is cumbersome to use block diagram algebra and obtain its overall transfer function
  - A signal flow diagram describes how a signal gets modified as it travels from I/p to O/p
  - The overall transfer function can be obtained very easily using Mason‘s gain formula

Signal Flow Graphs: Terminologies

- Signal Flow Graph:
  - It is graphical representation of the relationships between the variables of a system
- Node:
  - Every variable in a system is represented by a node
  - The value of the variable is equal to the sum of the signals coming towards the node
  - Its value is unaffected by the signals which are going away from the node
  - Example:

    $x_2 = a_{12}x_1 + a_{42}x_4$

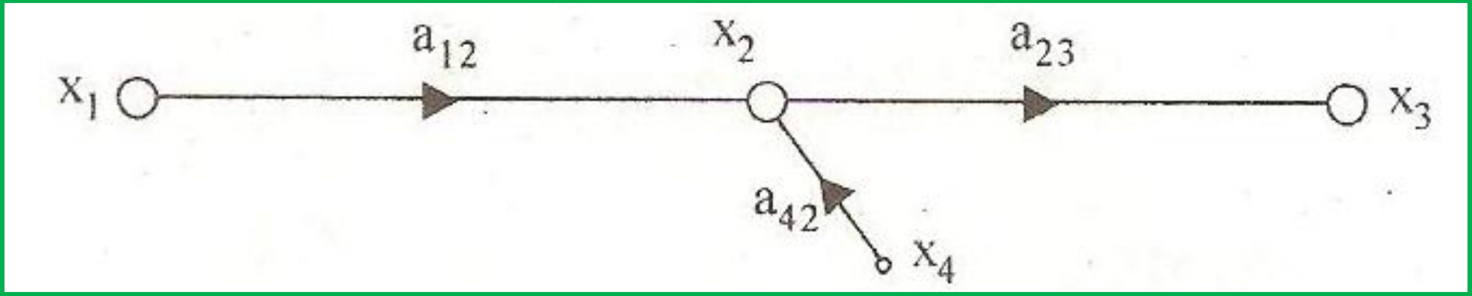
    $x_3 = a_{23}x_2$



Signal Flow Graphs: Terminologies

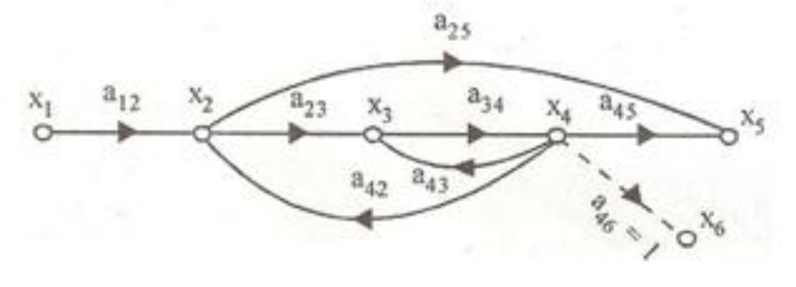
- Branch:
  - A signal travels along a branch from one node to another node in the direction indicated on the branch
  - Every branch is associated with a gain constant or transmittance (Example)

$$x_2 = a_{12}x_1 + a_{42}x_4$$ (Circles show nodes)



Signal Flow Graphs: Terminologies

- Input node (Source node):
  - It is a node at which only outgoing signals are present
  - Example: Node x_1



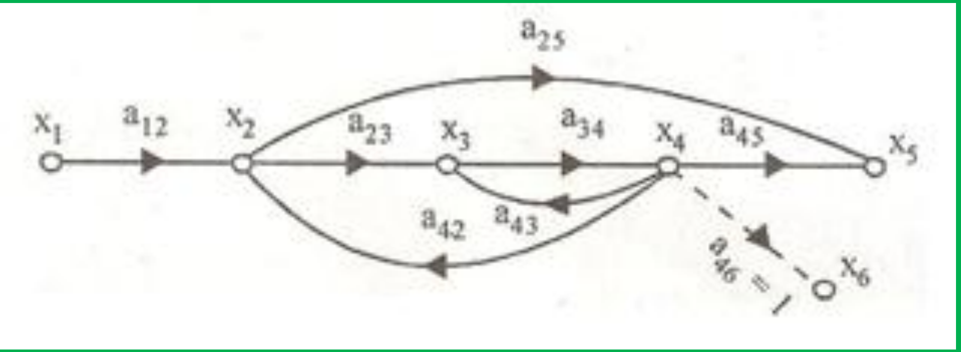
- Output node:
  - It is a node at which only incoming branches are present
  - Example: Node x_5 is an O/p node

Signal Flow Graphs: Terminologies

- Path:
  - It is traversal from one node to another node through the branches such that no node is traversed twice
- Forward Path:
  - It is a path from input node to output node
  - Examples:

    i) $x_1 - x_2 - x_3 - x_4 - x_5$

    ii) $x_1 - x_2 - x_5$



Signal Flow Graphs: Terminologies

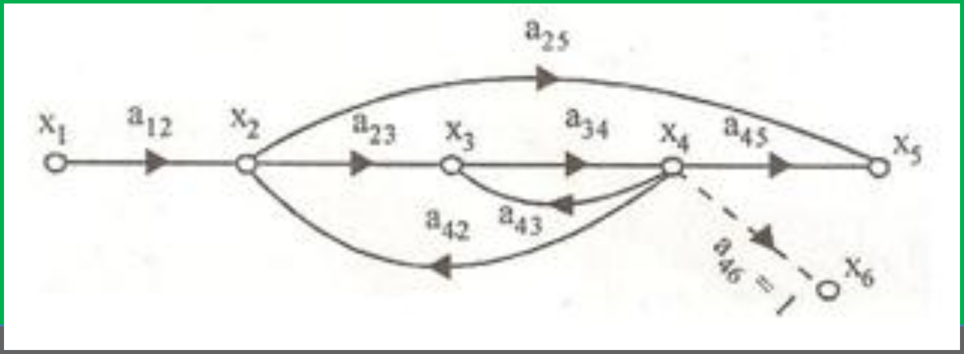
- **Loop:**
  - It is a path starting and ending on the same node
  - Examples:

    i) $x_3 - x_4 - x_3$

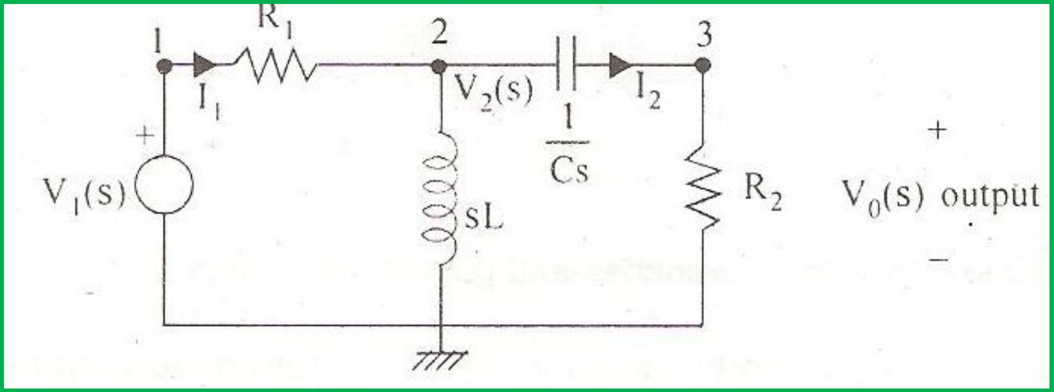
    ii) $x_2 - x_3 - x_4 - x_2$

- **Non-touching Loops:**
  - Loops which have no common node

- **Forward Path Gain:**
  - Gain product of the branches in the forward path



Solved Example: Construction of SFG

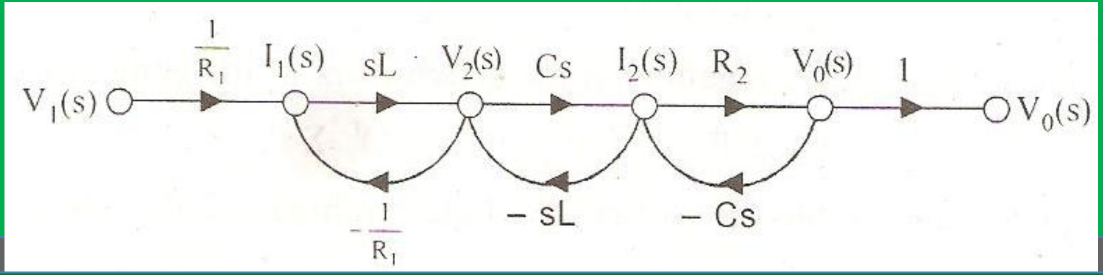


$$I_1(s) = \frac{V_1(s) - V_2(s)}{R_1}$$

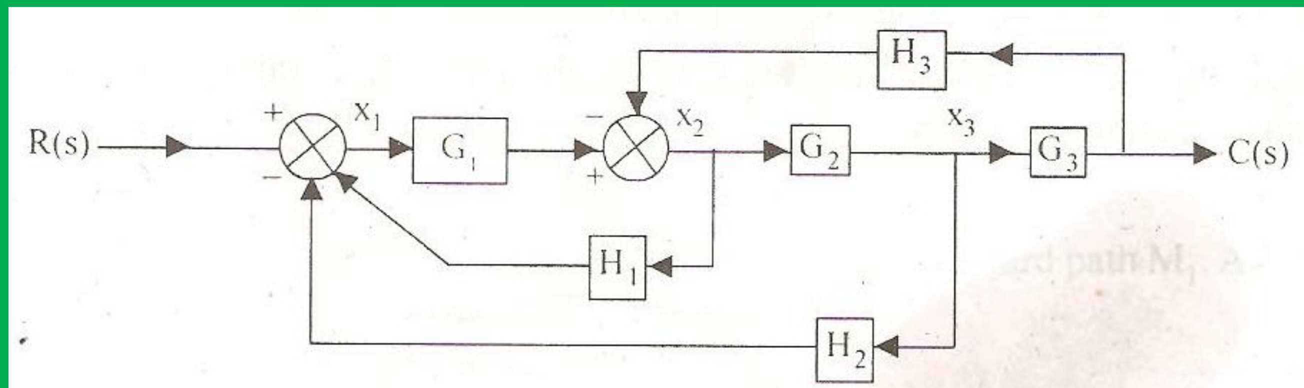
$$V_2(s) = [I_1(s) - I_2(s)]\, sL$$

$$I_2(s) = [V_2(s) - V_0(s)]Cs$$

$$V_0(s) = I_2(s)\, R_2$$



Solved Example 2: Construction of SFG

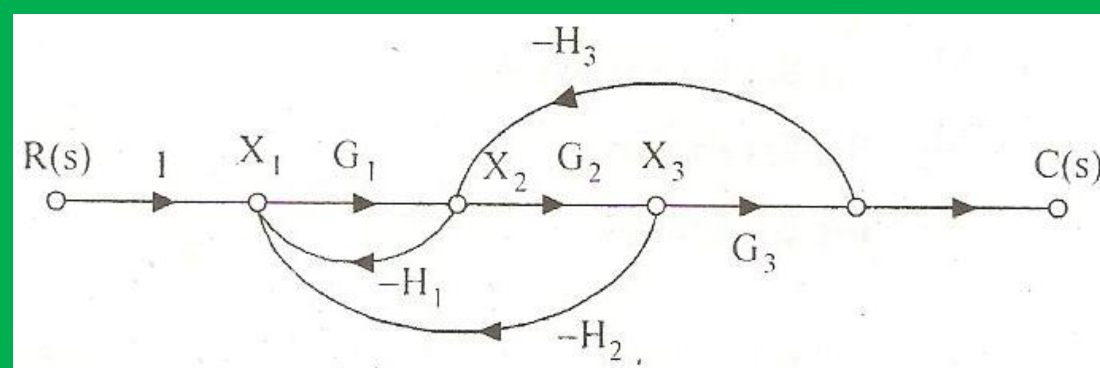


$$X_1(s) = R_1(s) - H_1(s)\ X_2(s) - H_2(s)\ X_3(s)$$

$$X_2(s) = G_1(s)\ X_1(s) - H_3(S)\ C(s)$$

$$X_3(s) = G_2(s)\ X_2(s)$$

$$C(s) = G_3(s)\ X_3(s)$$



Mason Formula

- Motivation & Intro:
  - Mason’s Gain Rule is a technique for finding an overall transfer function
  - The purpose of using Mason’s is the same as that of Block reduction
  - Mason’s method was particularly helpful before the advent of modern computers, and tools such as MATLAB which can also be used to find the overall transfer function of a complex system

Mason Formula: Algorithm

- For a Signal Flow Graph:
  - The Transfer function (T) is given by

$$T = \frac{\sum_k M_k \Delta_k}{\Delta}$$

Where

M_k is the k^{th} forward path gain

Δ is the determinant of the graph, given by

$\Delta = 1$ – (sum of gains of individual loops)

+ sum of gain products of possible combinations of 2 non touching loops)

- (sum of gain products of all possible combinations of 3 nontouching loops)

+ .....

Δ_k is the value of Δ for that part of the graph which is non touching with k^{th} forward path

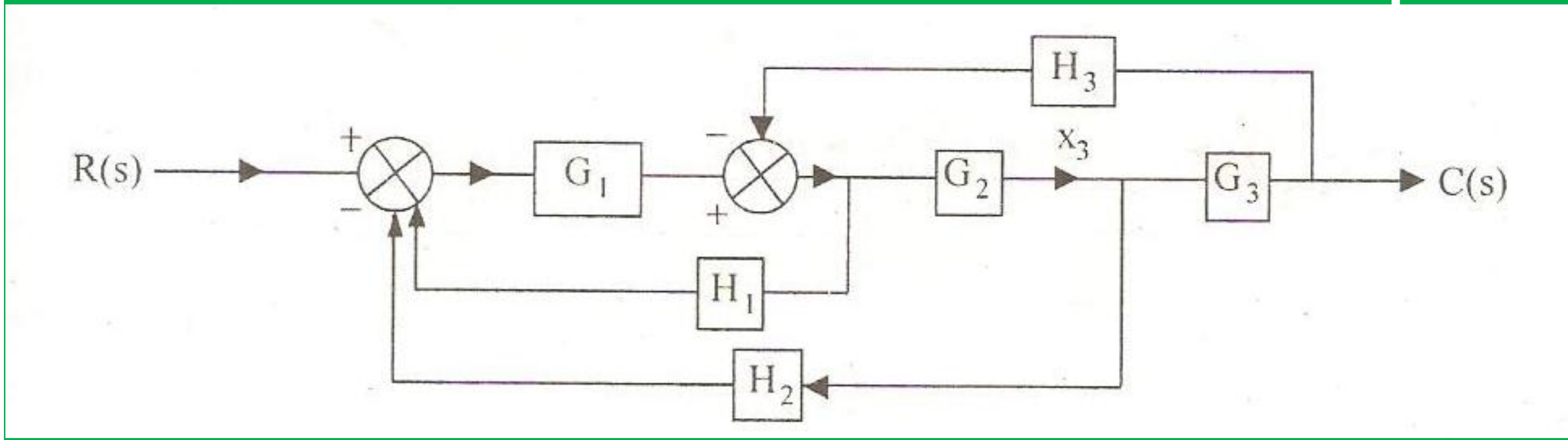
Mason Formula

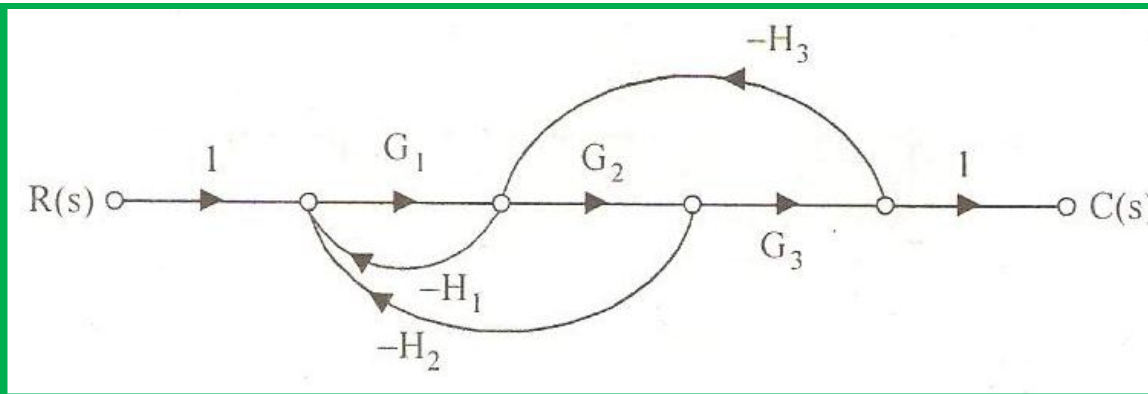
- Steps:
  - I: Analyze the SFG to see how many forward paths and how many loops are there
  - II: Write the forward paths gains (M_1, M_2,...)
  - III: Write the Loop gains (L_1, L_2,...)
  - IV: Identify non-touching loops.

    Two non-touching loops: (Calculate the product of their gains)

    Same for three non-touching loops (If any)
  - V: Calculate the determinant of the graph (Δ) and all applicable Δ_k
  - VI: Calculate the transfer function by using formula

Mason Formula: Solved Example 1





Step-I:

Forward paths = 1

Loops = 3

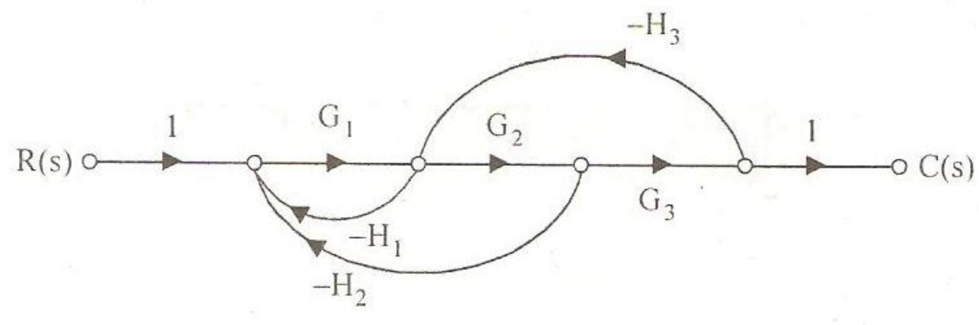
Step-II: Gain of forward path:

$$M_1 = G_1 G_2 G_3$$

Mason Formula: Solved Example 1

Step-III: Loop Gains

$$L_1 = -G_1 H_1$$
$$L_2 = -G_1 G_2 H_2$$
$$L_3 = -G_2 G_3 H_3$$



Step-IV: Identify non-touching loops

Two or more non-touching loops are not present

Step-V: Calculate the Determinant (Δ) of the Graph and Δ_1

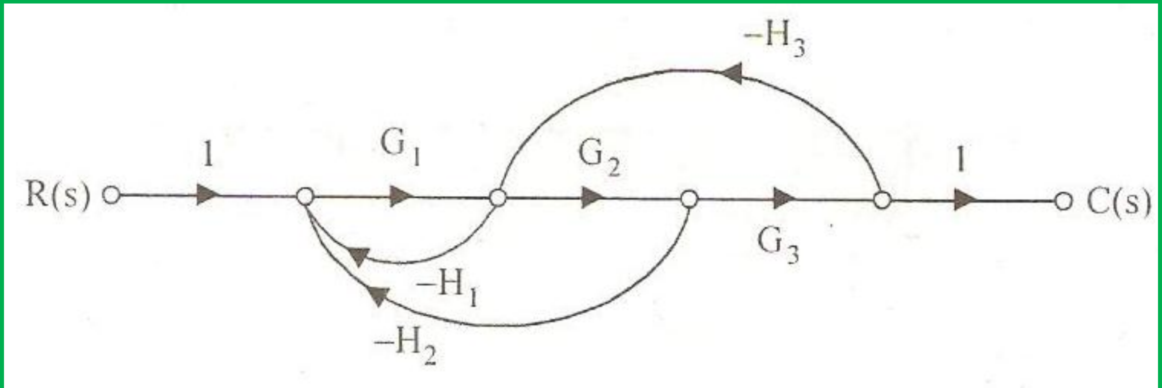
$$\Delta = 1 + G_1 H_1 + G_1 G_2 H_2 + G_2 G_3 H_3$$

Now Δ_1 is the value of Δ which is non-touching with 1st Forward path (M_1)

Means to write the expression for Δ_1,eliminate all terms in Δ which have any node in common with forward path M_1

$$\Delta_1 = 1$$

Mason Formula: Solved Example 1



Step-VI: Calculate the Transfer Function

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 + G_1 H_1 + G_1 H_2 + G_2 G_3 H_3}$$