Control Technology
Lecture 2

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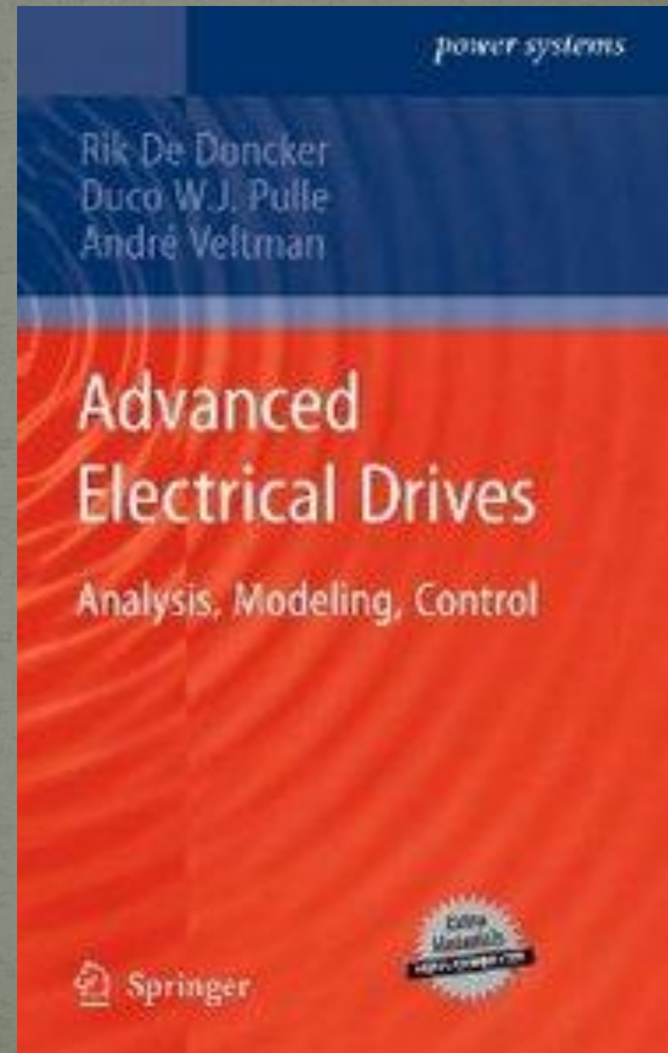
Outlines

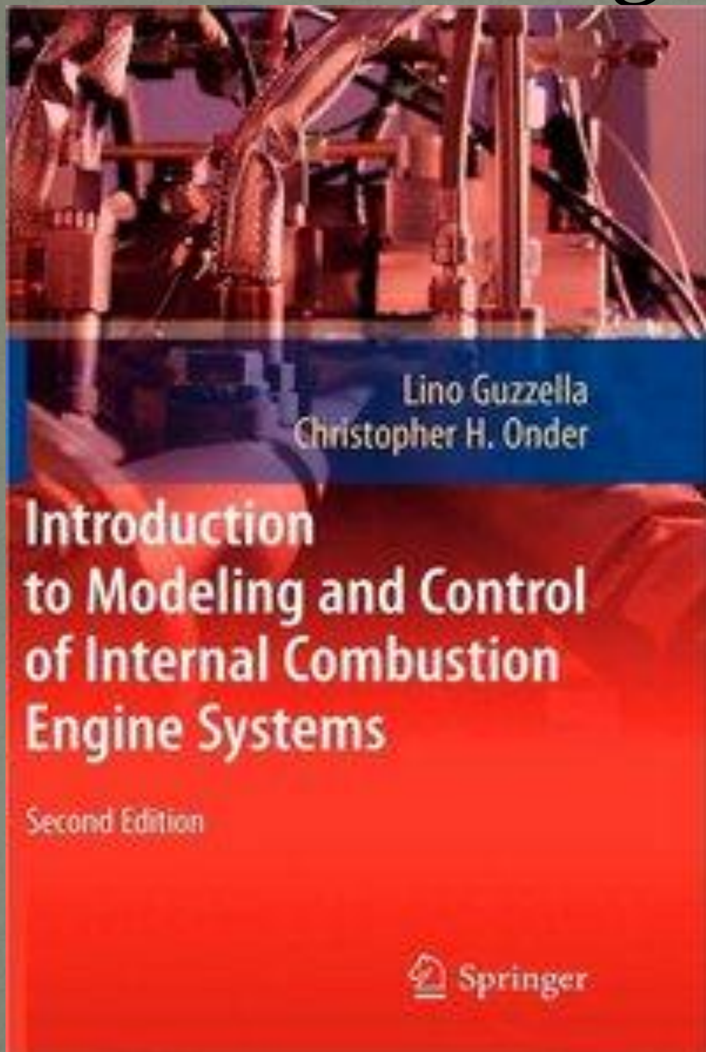
- Modeling of C/Systems

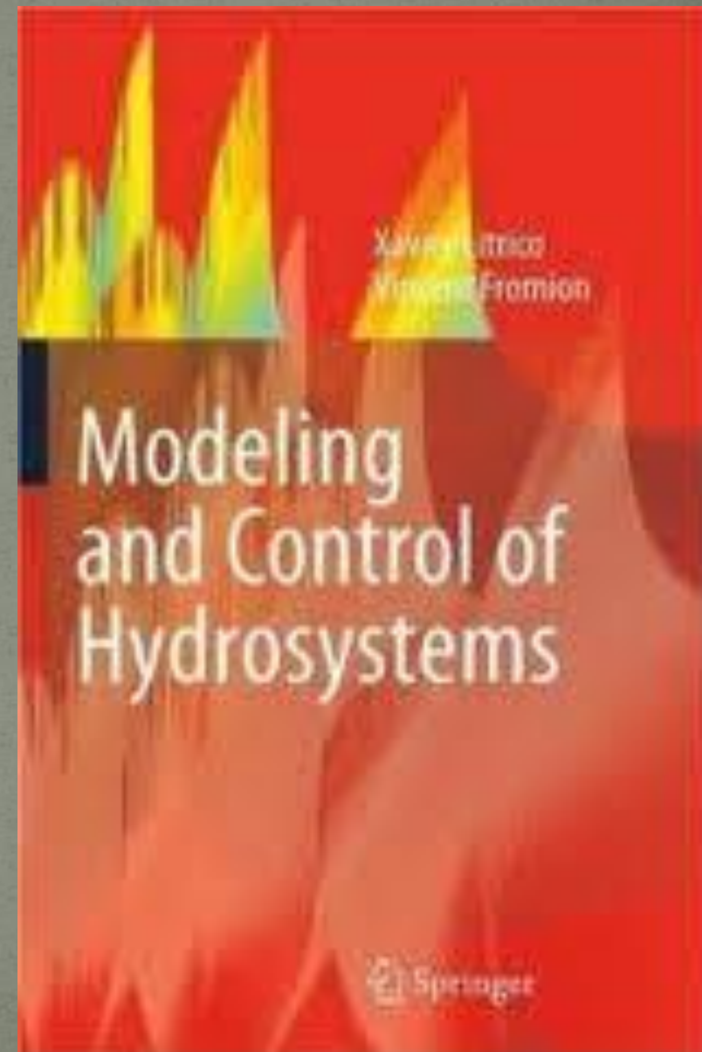
MODELING

Modeling- Examples



Modeling- Examples





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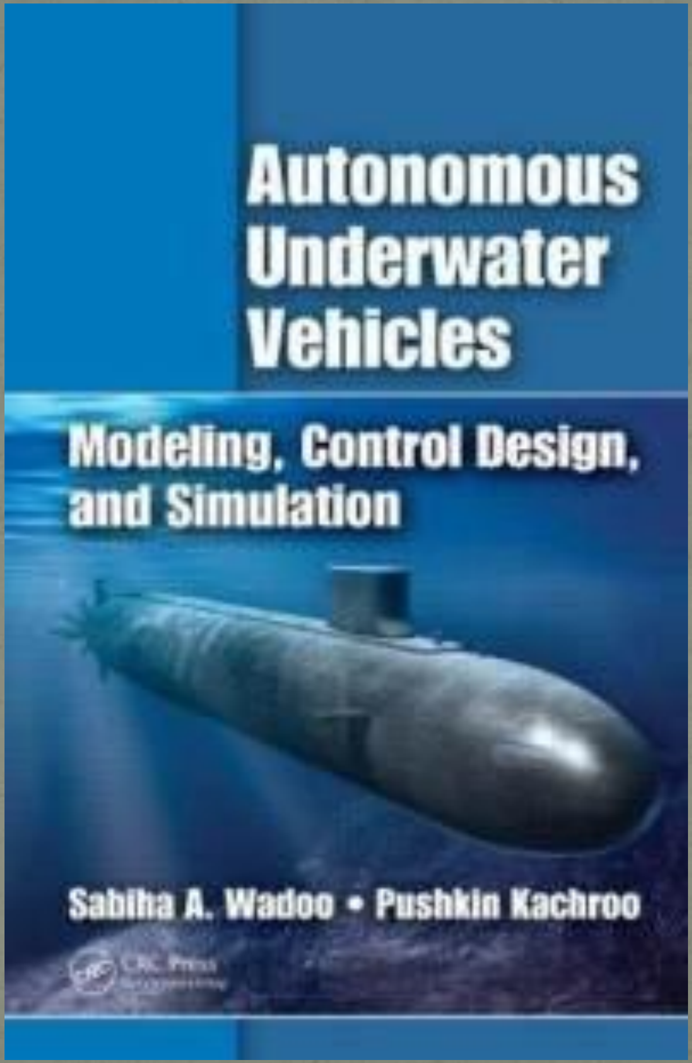

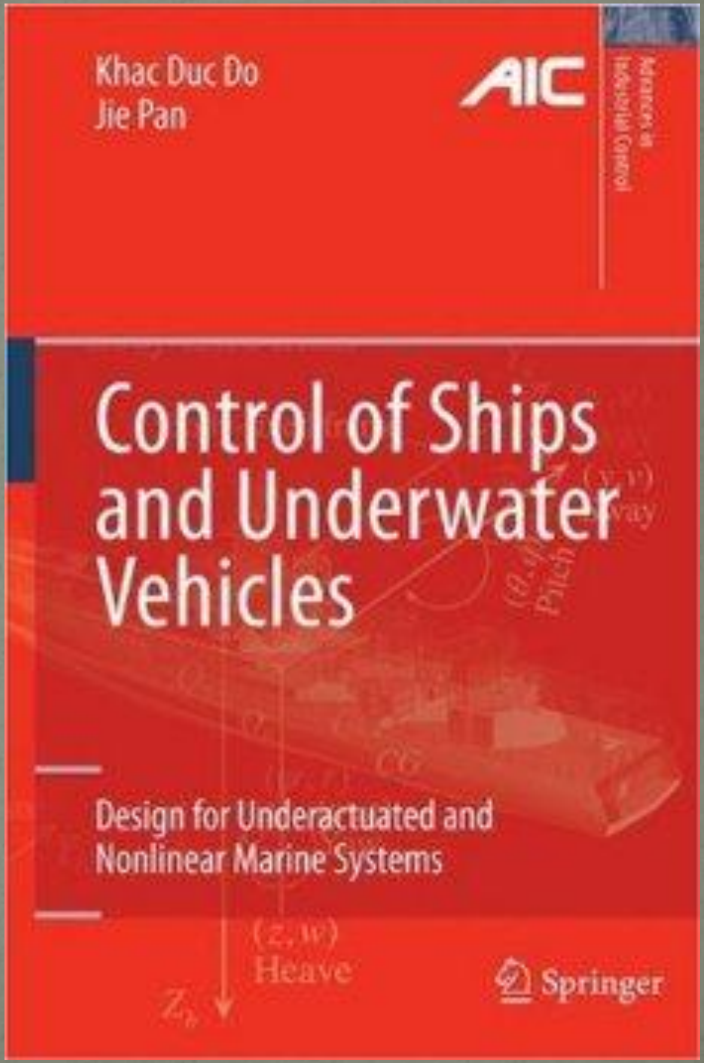

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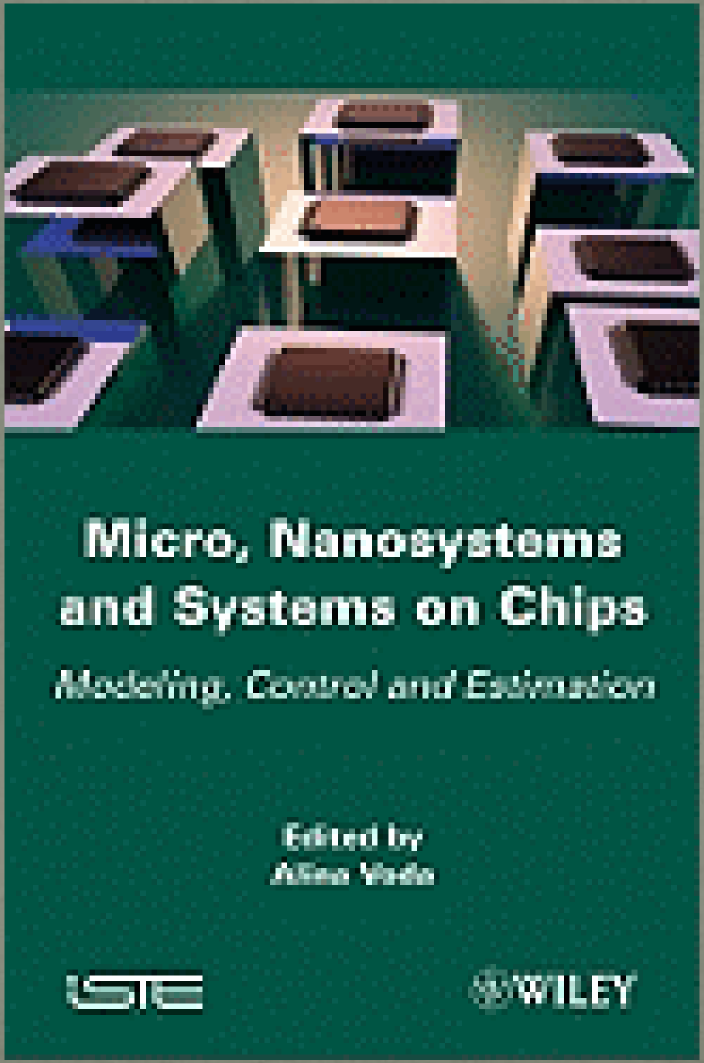
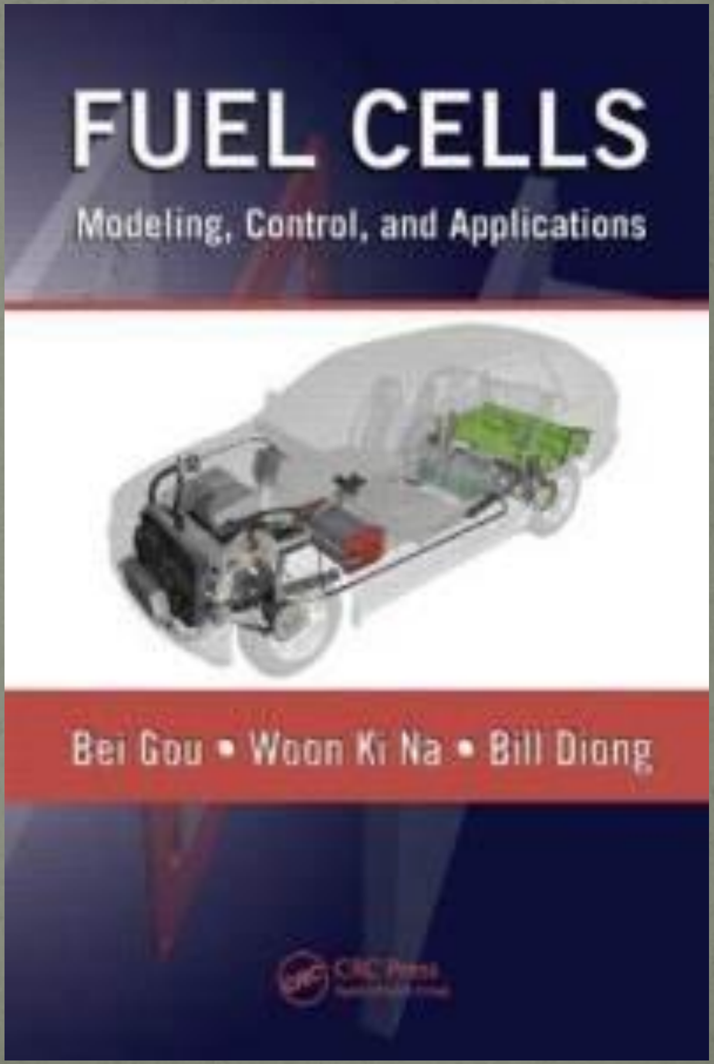
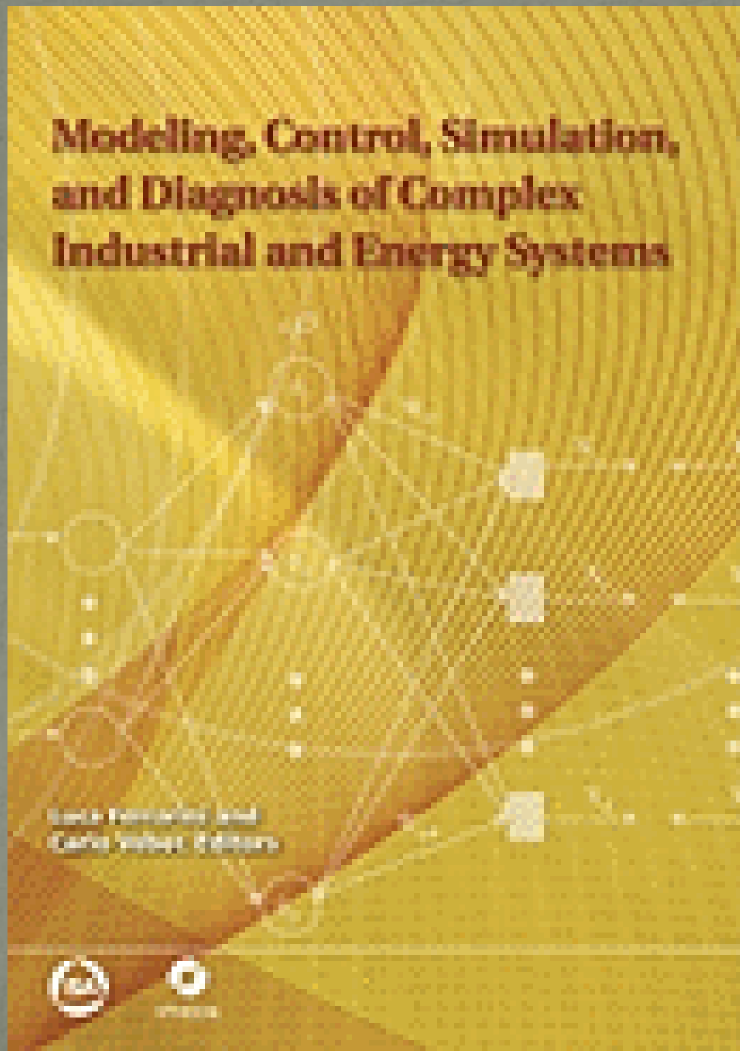
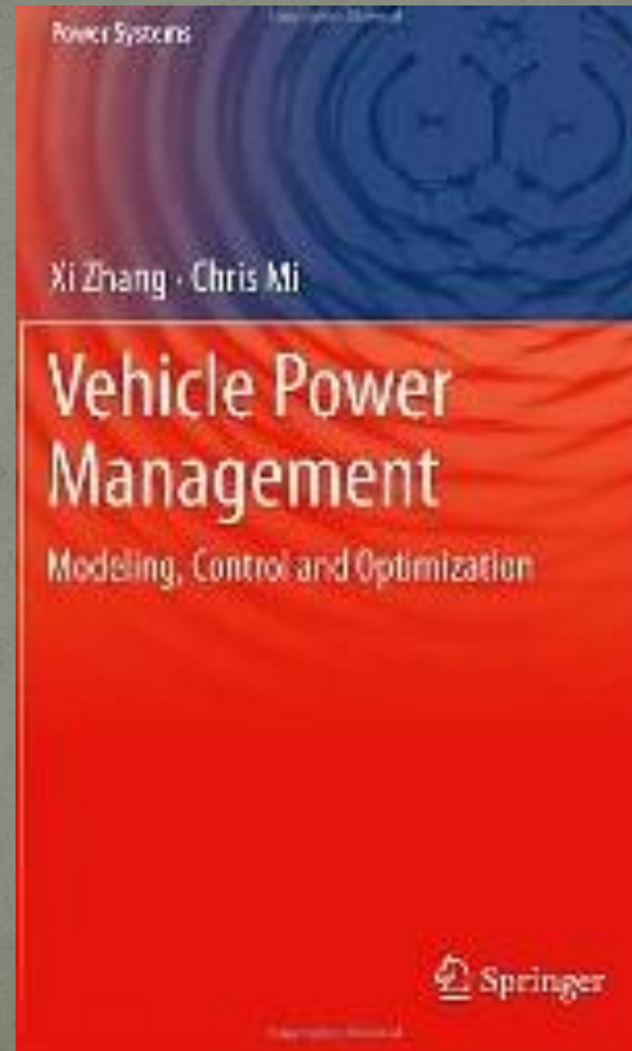




Modeling- Examples





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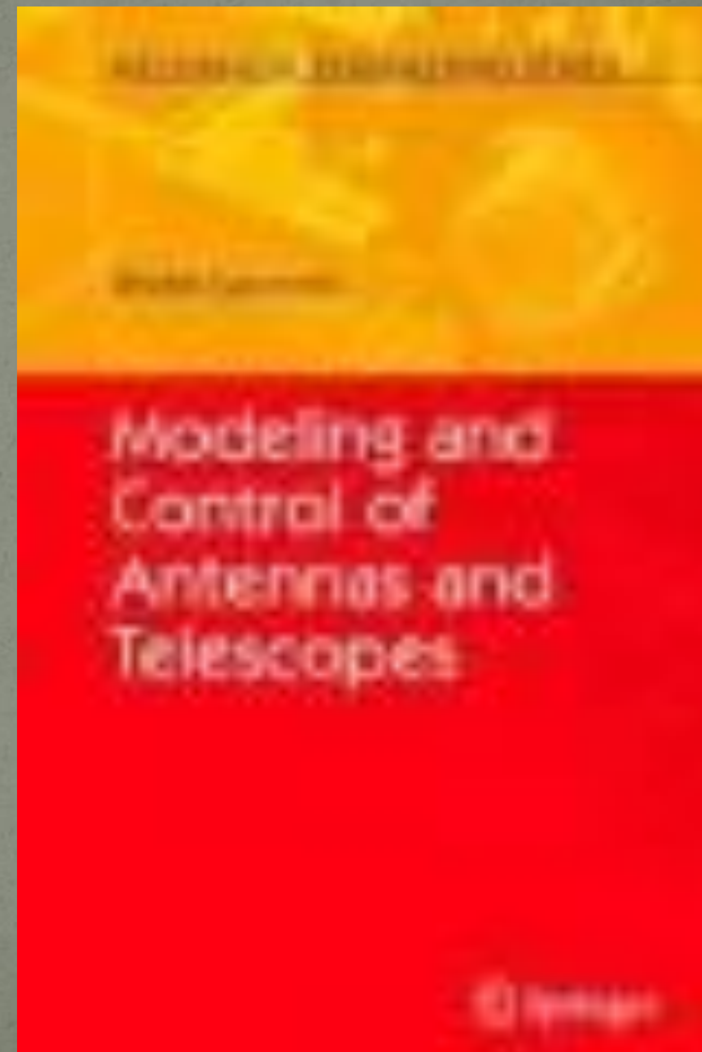
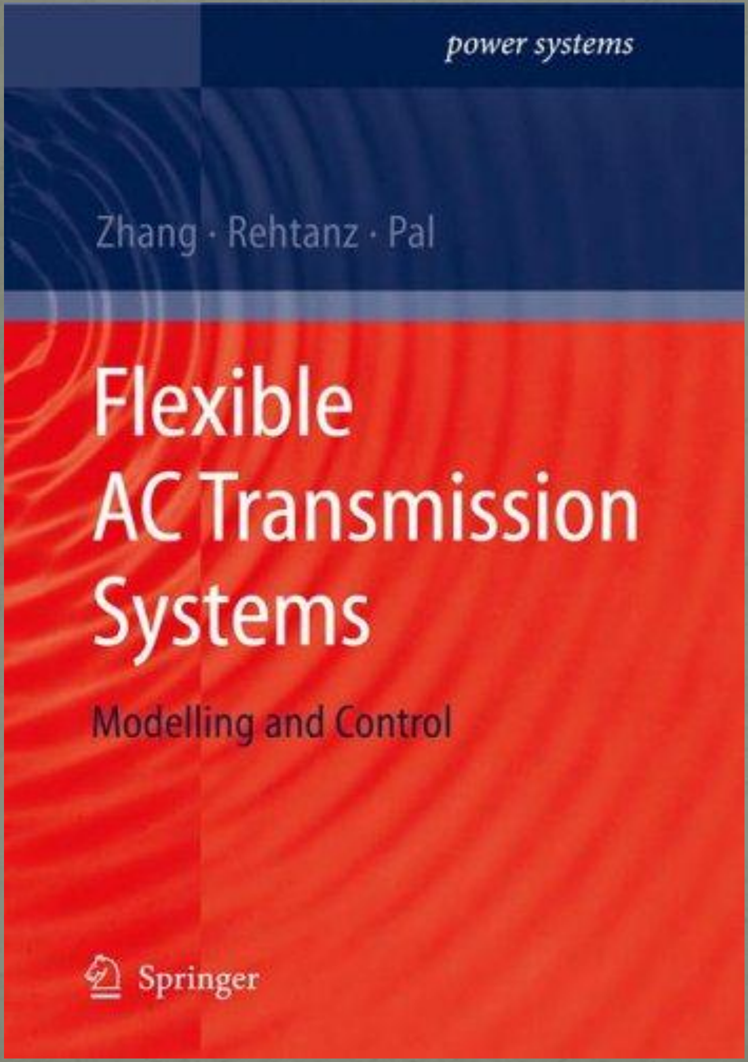
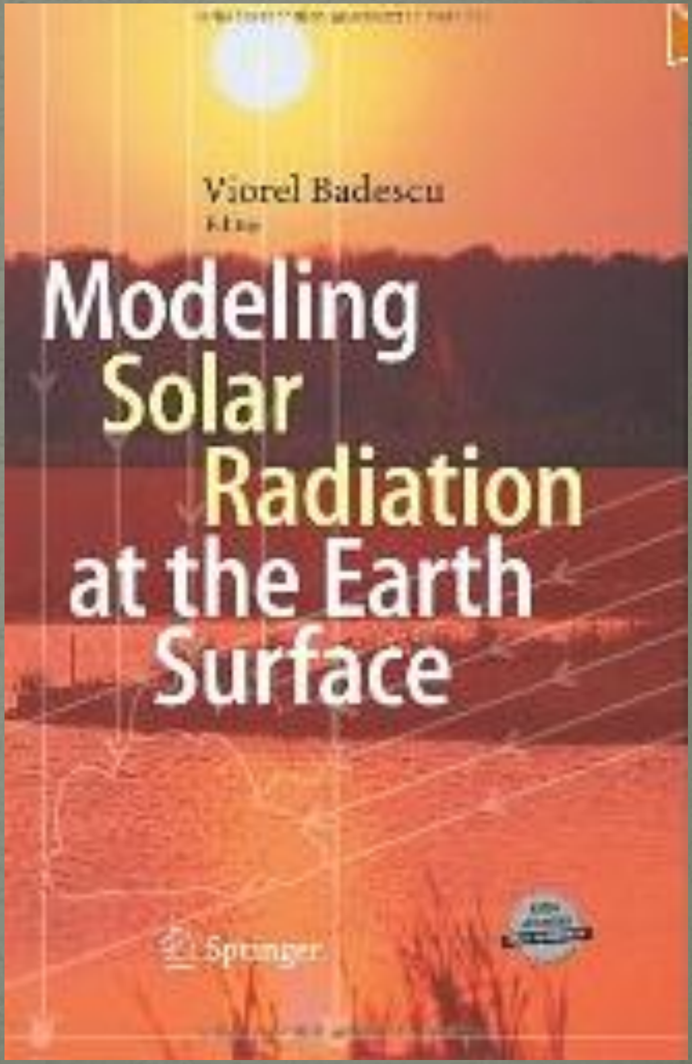
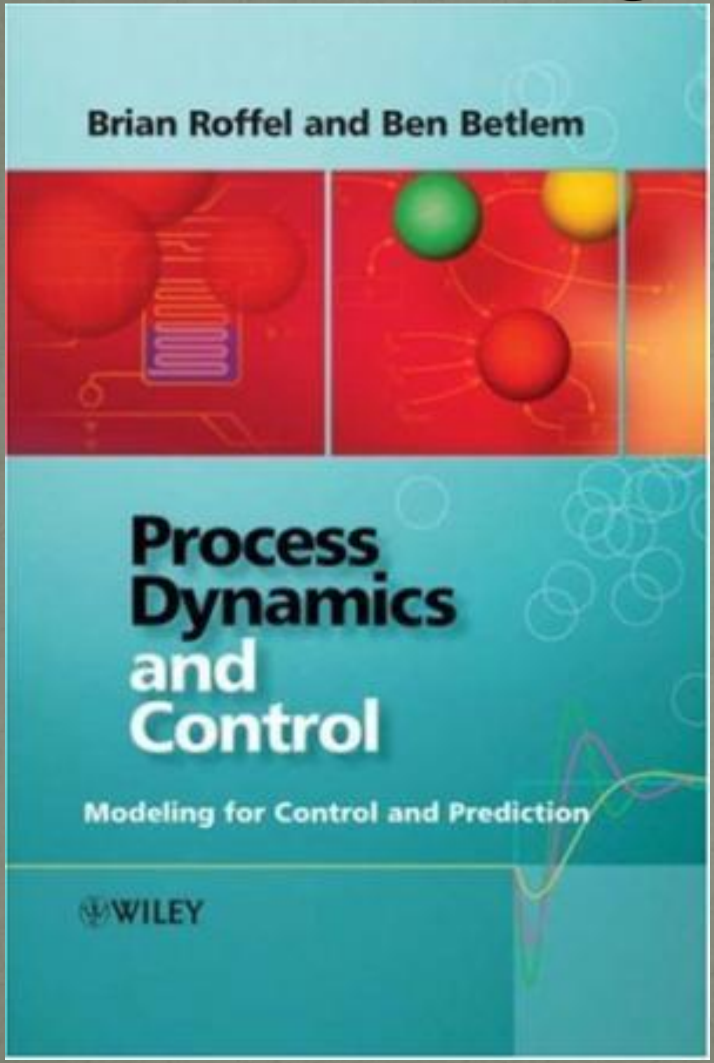


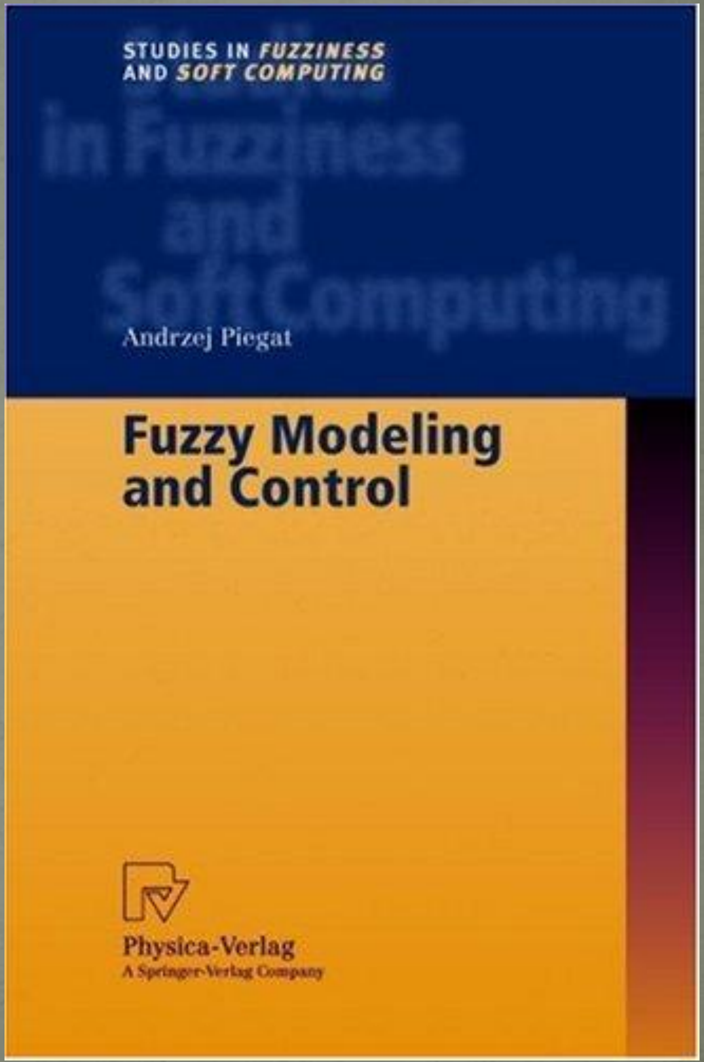
Modeling- Examples





Modeling- Examples





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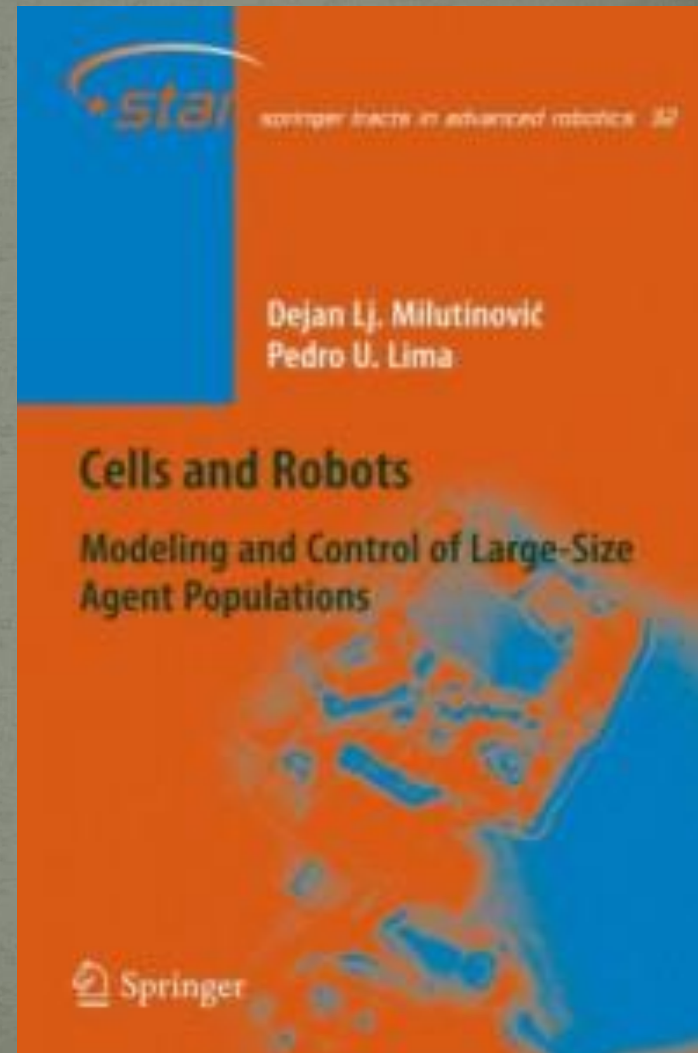
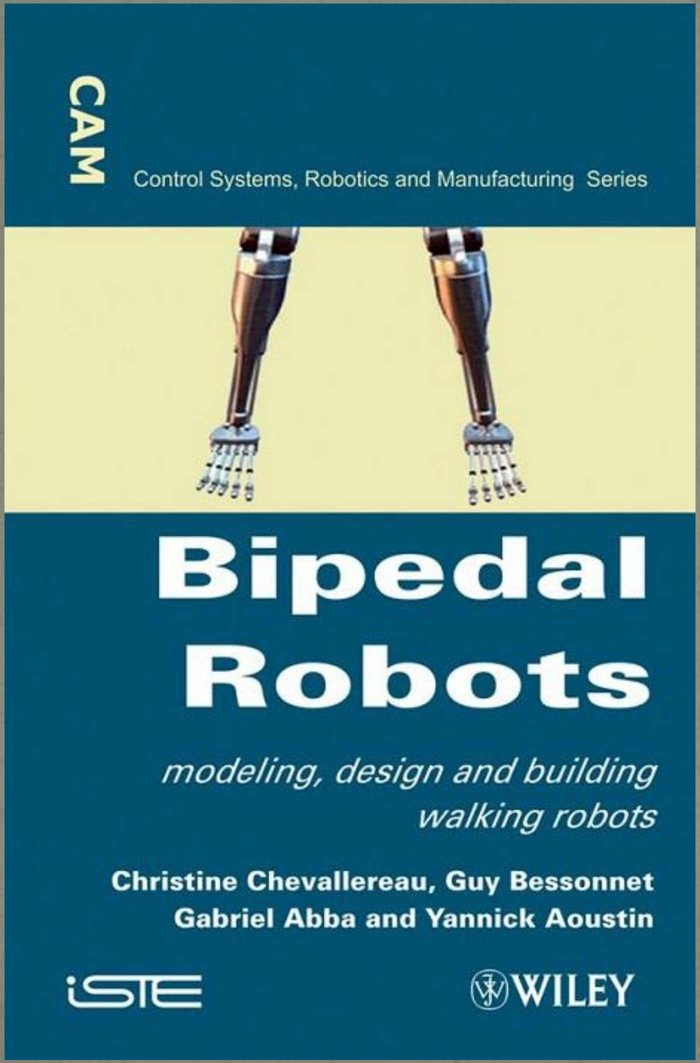
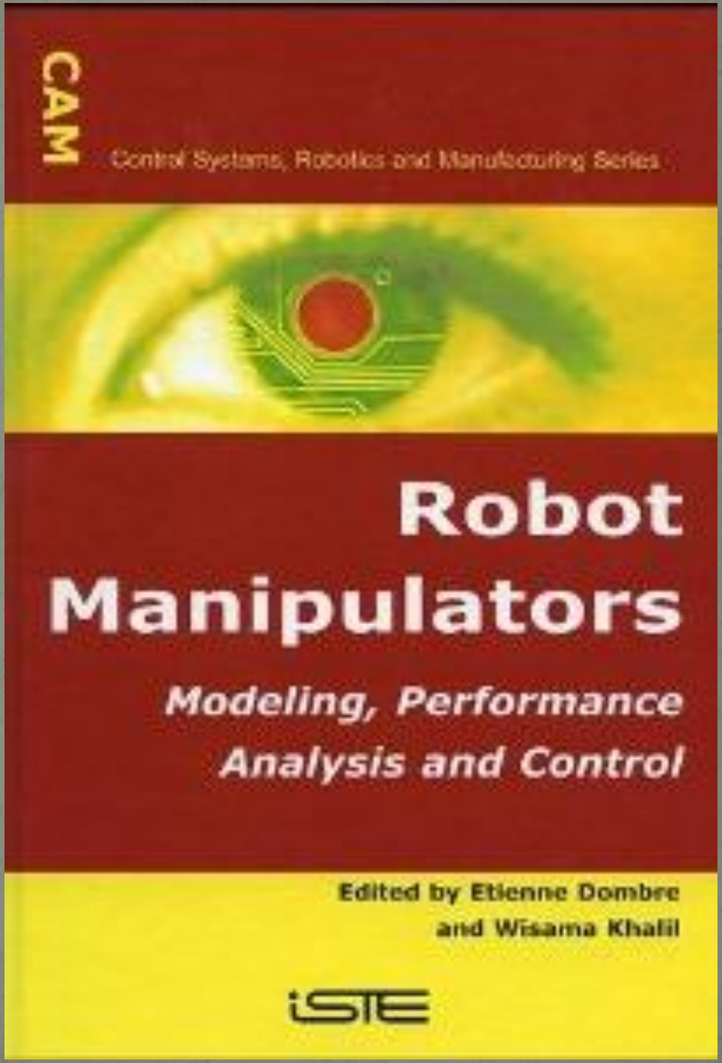


Modeling- Examples

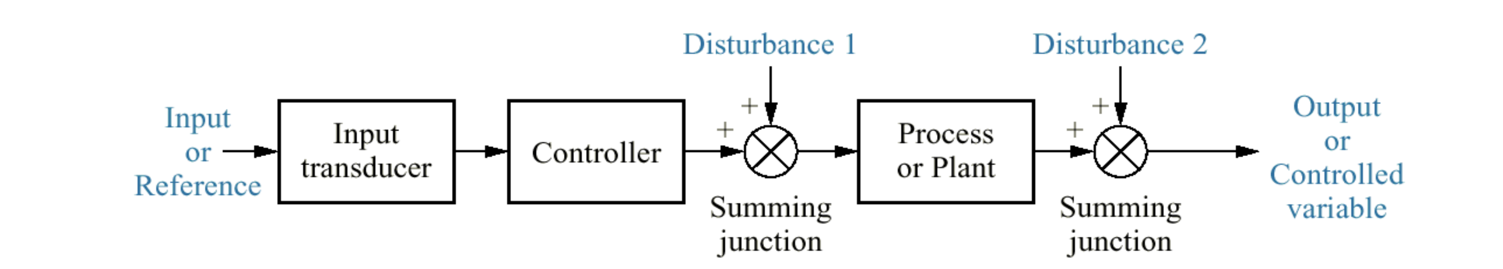


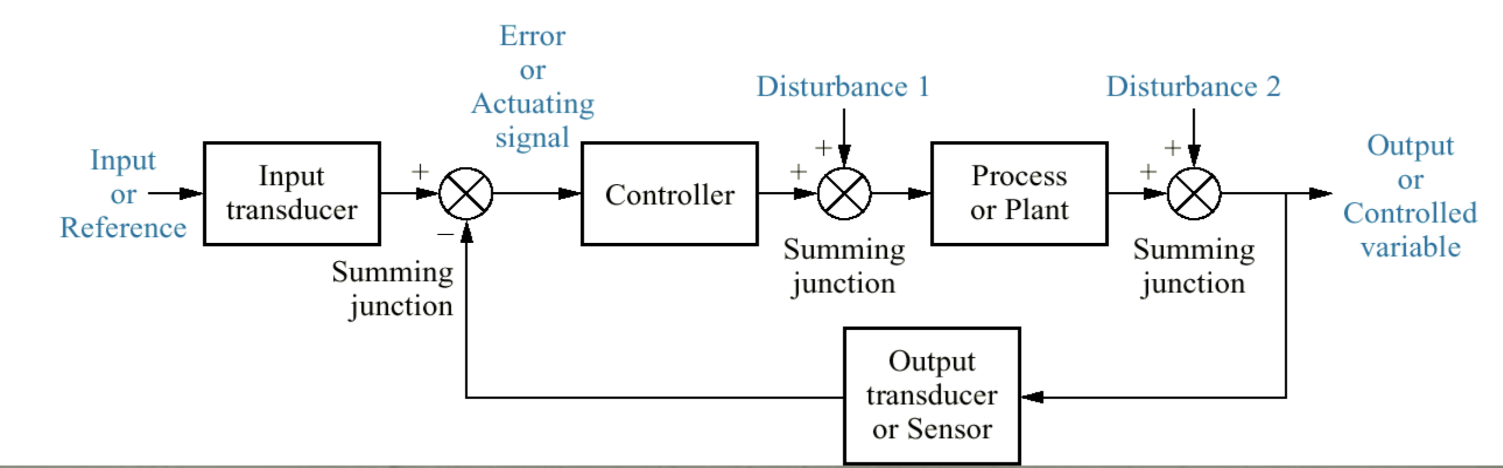


Control Systems to be Modeled

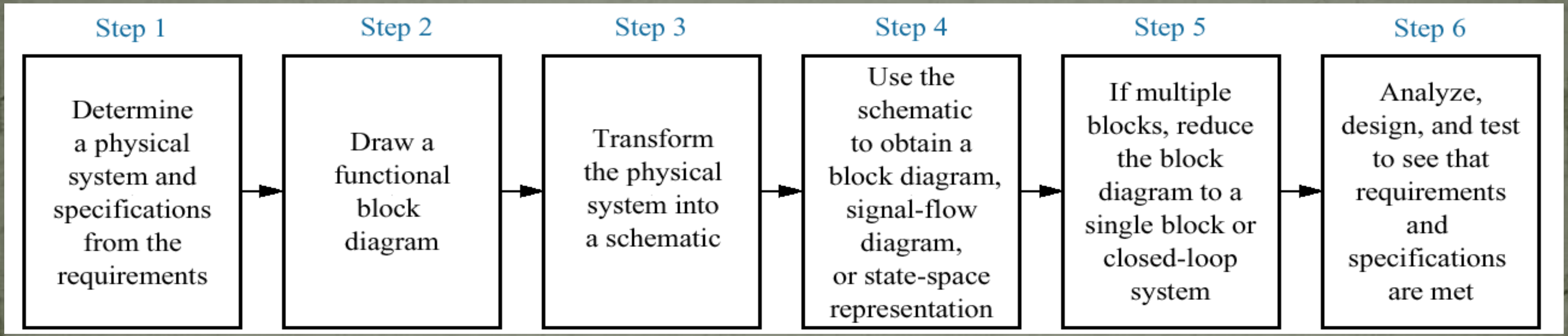
- Modeling of THREE types of control systems will be studied in this course:
  - Electrical/Electronic Systems
  - Mechanical Systems
  - Biological system

Typical Control Systems Block Diagrams

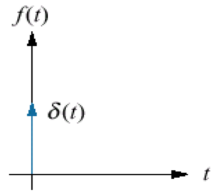
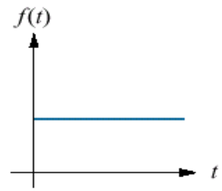
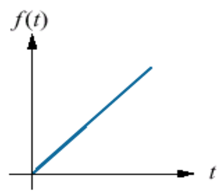
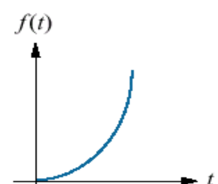
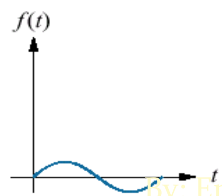


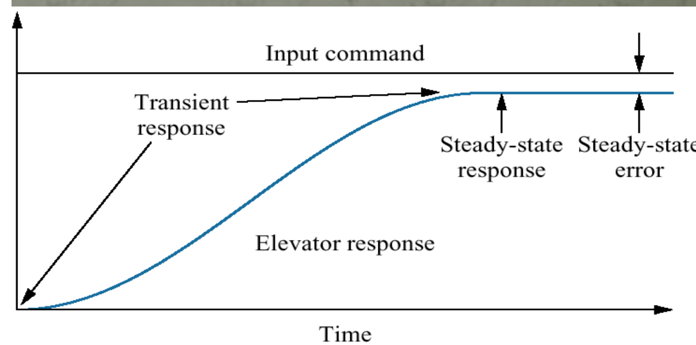


Process of Modeling



Typical Test Inputs for Control Systems Analysis

Input	Function	Description	Sketch	Use
Impulse	$\delta(t)$	$\delta(t) = \infty$ for $0- < t < 0+$ $= 0$ elsewhere $\int_{0-}^{0+} \delta(t)\,dt = 1$		Transient response Modeling
Step	$u(t)$	$u(t) = 1$ for $t > 0$ $= 0$ for $t < 0$		Transient response Steady-state error
Ramp	$tu(t)$	$tu(t) = t$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error
Parabola	$\frac{1}{2}t^2u(t)$	$\frac{1}{2}t^2u(t) = \frac{1}{2}t^2$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error
Sinusoid	$\sin \omega t$			Transient response Modeling Steady-state error



ELECT. SYSTEMS

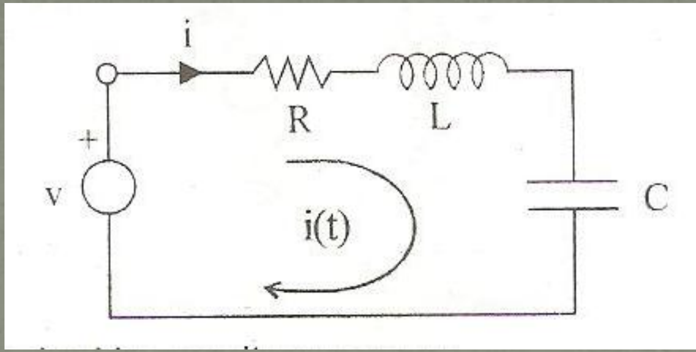
Modeling Elements & Laws

- Modeling means?
  - To describe the system (writing Transfer function)
- Elements
  - Resistor
  - Inductor
  - Capacitor
- Laws
  - KVL
  - KCL
  - Voltage Division Rule

Modeling Elements

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Example 1: Series RLC Circuit

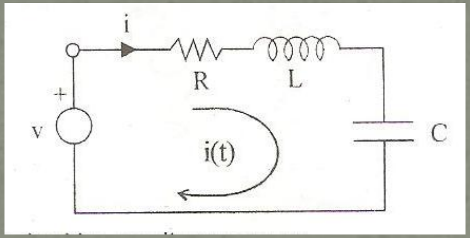


$$R\,i(t) + L\frac{d\,i(t)}{dt} + \frac{1}{C}\int i(t)\,dt = v$$

$$R\,I(s) + Ls\,I(s) + \frac{I(s)}{Cs} = V(s)$$

$$\frac{I(s)}{V(s)} = \frac{1}{Ls + R + \frac{1}{Cs}} = \frac{Cs}{LCs^2 + RCs + 1}$$

Example 1: Series RLC Circuit



$$R\,i(t) + L\frac{d\,i(t)}{dt} + \frac{1}{C}\int i(t)\,dt = v$$

$\int$ i(t) d(t) by q(t)

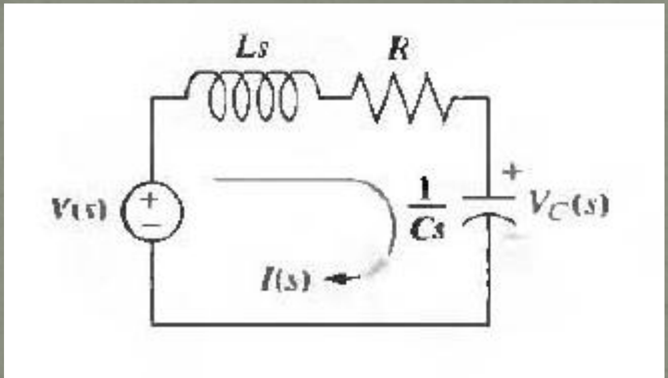
$$L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{q(t)}{C} = v$$

$$Ls^2\,Q(s) + Rs\,Q(s) + \frac{1}{C}Q(s) = V(s)$$

$$\frac{Q(s)}{V(s)} = \frac{1}{Ls^2 + Rs + \frac{1}{C}} = \frac{C}{LCs^2 + RCs + 1}$$

Method 2: Transform Method

Make 'Impedance' equivalent circuit



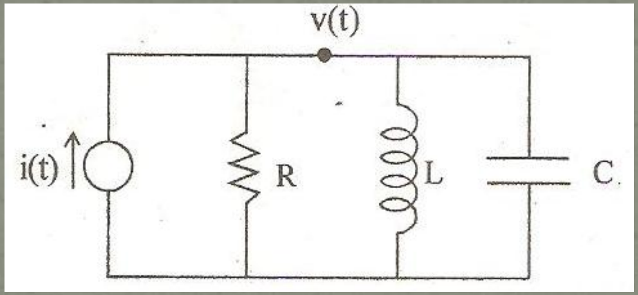
Apply KVL/KCL

$$\left(Ls + R + \frac{1}{Cs}\right)I(s) = V(s)$$

Rearrange in terms of Transfer Function

$$\frac{I(s)}{V(s)} = \frac{1}{Ls + R + \frac{1}{Cs}}$$

Example 2: Parallel RC Circuit



$$\frac{v(t)}{R} + \frac{C\,dv(t)}{dt} + \frac{1}{L}\int v\,dt = i(t)$$

$$GV(s) + CsV(s) + \frac{1}{Ls}V(s) = I(s)$$

$$\frac{V(s)}{I(s)} = \frac{1}{Cs + G + \frac{1}{Ls}} = \frac{Ls}{LCs^2 + LGs + 1}$$

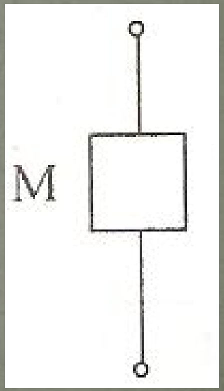
Mechanical Systems

Modeling Elements (Translational Sys.)

- Elements:
  - Mass
  - Spring
  - Damper

- Mass:
  - An element which resists the motion due to inertia

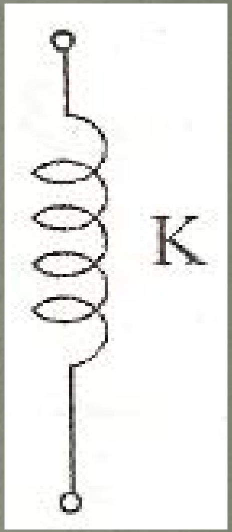
$$f_M = Ma = M.\frac{dv}{dt} = M\frac{d^2x}{dt^2}$$



Modeling Elements (Translational Sys.)

- Spring:
  - An element which opposes motion in the spring (if you compress/decompress the spring, it resists)

$$f_K = K\,x$$

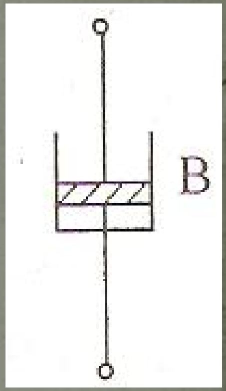


Where K = Spring constant or
Stiffness of the spring

Modeling Elements (Translational Sys.)

- Damper:
  - An element which opposes motion due to friction. If the friction is viscous friction, then the frictional force is proportional to velocity

$$f_B = Bv = B\frac{dx}{dt}$$



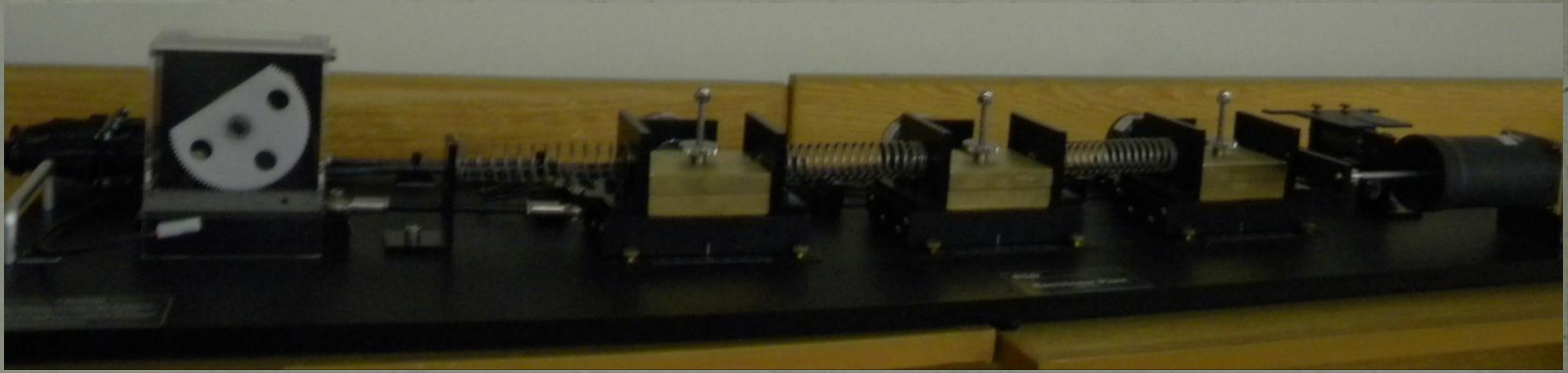
Where B = Damping coefficient

Modeling Elements (Translational Sys.)

- Damping:
  - Although friction opposes motion, sometimes it is introduced intentionally
  - Examples

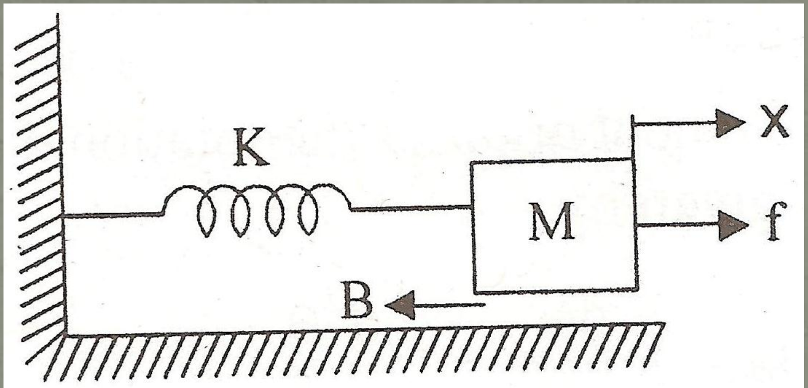
    - Car shocks
    - Tower buildings
    - Robots

Modeling Elements (Translational Sys.)



Modeling Elements (Translational Sys.)

Modeling Example (Translational Sys.)



external force = f

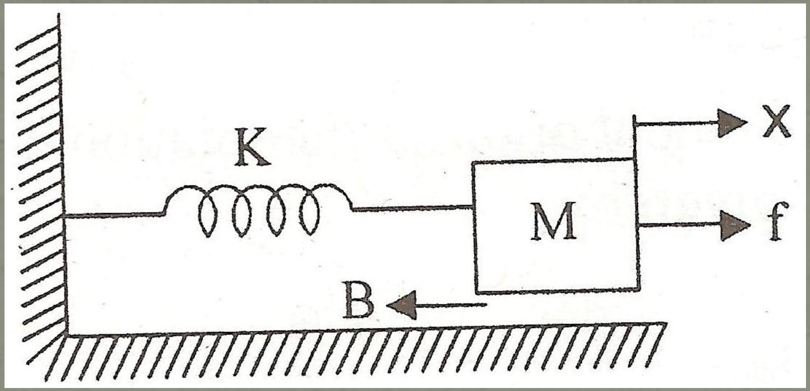
resisting forces :

Inertia force, $f_M = M \frac{d^2x}{dt^2}$

Damping force, $f_B = B \frac{dx}{dt}$

Spring force, $f_K = KX$

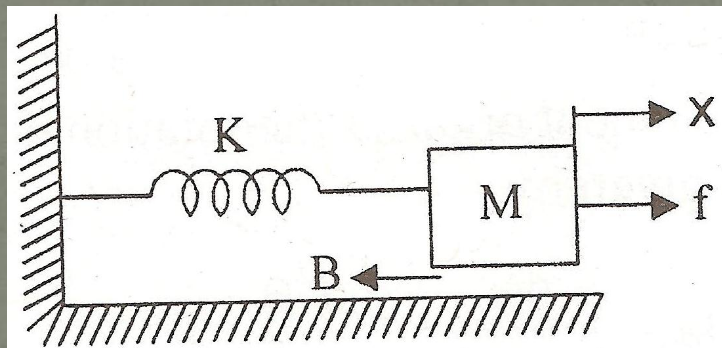
Modeling Example (Translational Sys.)

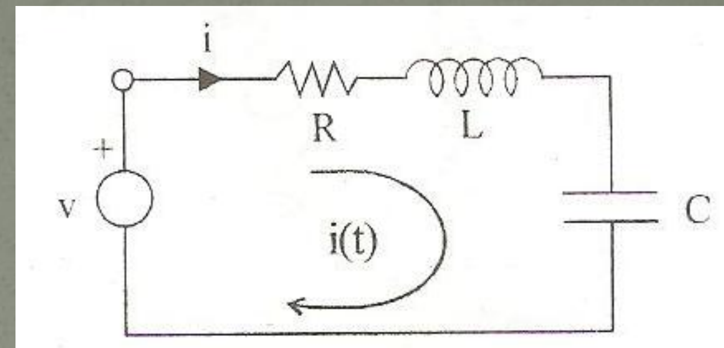


$$M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx = f$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

Modeling Example (Translational Sys.)





$$M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx = f$$

$$L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{q(t)}{C} = v$$

?

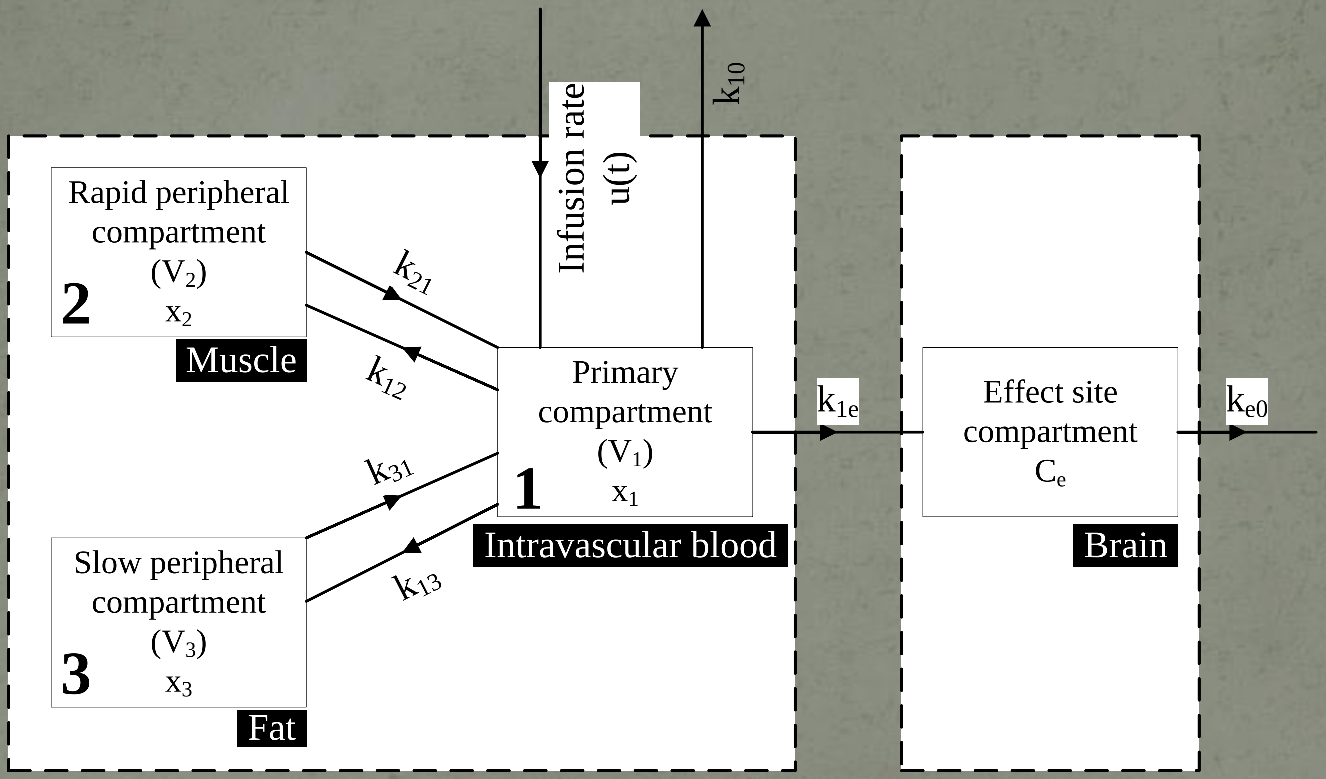
Analogy exists

Modeling Example (Translational Sys.)

Electrical system		Mechanical system			
		Translational		Rotational	
Voltage	V	Force	f	Torque	T
Current	i	Velocity	u	angular Velocity	ω
Charge	q	Displacement	x	angular displacement	θ
Inductance	L	Mass	M	Moment Inertia	J
Capacitance	C	Compliance	$\frac{1}{K}$.	Compliance	$\frac{1}{K}$
Resistance	R	Damping coefficient	B	Damping coefficient	B

Analogous quantities based on force voltage analogy

Modeling of biological system



State equations

- $\dot{x_1}(t) = -k_{10}x_1(t) - k_{12}x_1(t) - k_{13}x_1(t) + k_{21}x_2(t) + k_{31}x_3(t) + U(t)$
- $\dot{x_2}(t) = k_{12}x_1(t) - k_{21}x_2(t)$
- $\dot{x_3}(t) = k_{13}x_1(t) - k_{31}x_3(t)$
- $\dot{x}_e(t) = k_{1e}x_1(t) - k_{e0}x_e(t)$

Modeling of Other Systems

- Modeling of other systems
  - Thermal
  - Fluid
  - Pneumatic
  - Hydraulic
  - …

THANKS

By: Engr. M. Aamir Aman