Transmission Lines-2

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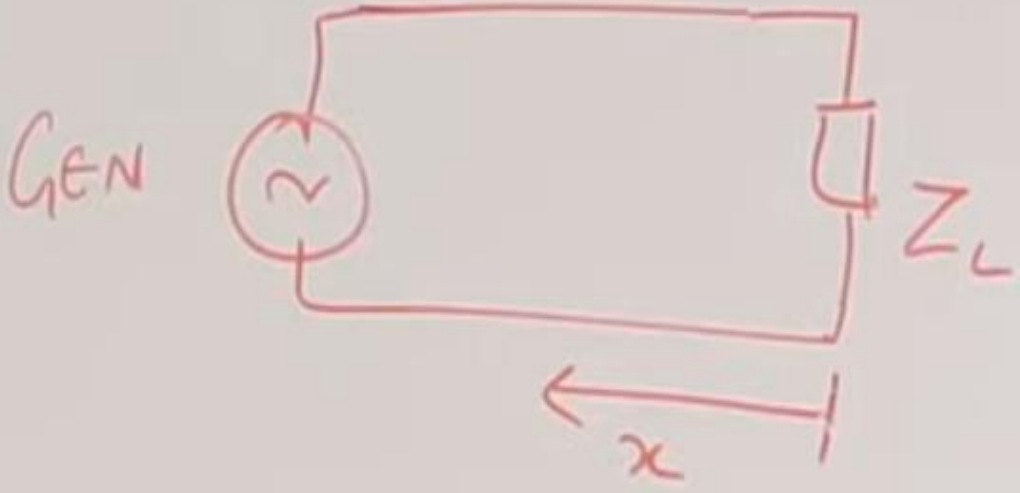
Reflection Coefficient

$$V(x,t) = V^+ e^{-\gamma x} + V^- e^{+\gamma x}$$

$$I(x,t) = I^+ e^{-\gamma x} + I^- e^{+\gamma x}$$

$$Z_0 = \frac{V^+}{I^+} \quad \Bigg| \quad -Z_0 = \frac{V^-}{I^-}$$

$$I(x,t) = \frac{V^+}{Z_0} e^{-\gamma x} - \frac{V^-}{Z_0} e^{+\gamma x}$$



REFLECTION COEFFICIENT

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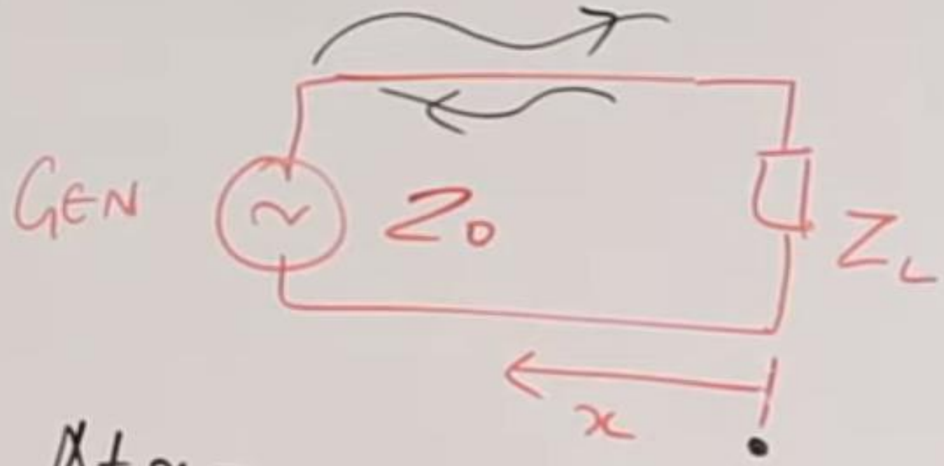


At $x=0$

$$Z_L = \frac{V}{I} = \frac{V^+ + V^-}{V^+ - V^-} \times Z_0$$

$$\Gamma = \frac{V^-}{V^+}$$

Reflection Coeff.

$$Z_L = \frac{V^+\left\{1 + \frac{V^-}{V^+}\right\}}{V^+\left\{1 - \frac{V^-}{V^+}\right\}} \times Z_0$$

$$Z_L = \frac{1+\Gamma}{1-\Gamma} Z_0$$

$$Z_L = \frac{1+\Gamma}{1-\Gamma} \times Z_0$$

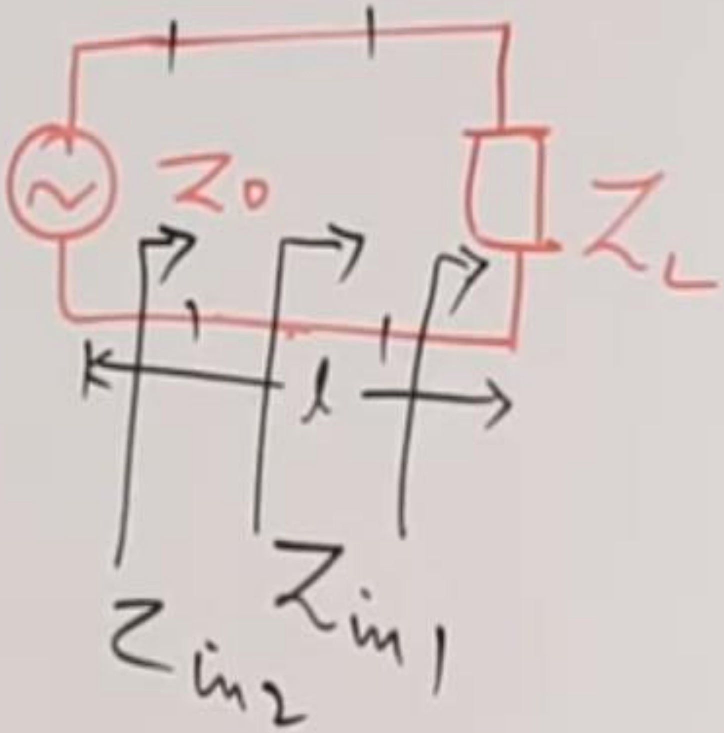
$$\boxed{\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$$\tau = \frac{2Z_L}{Z_L + Z_0}$$

INPUT IMPEDANCE OF TRANSMISSION LINE

① LOSSLESS MEDIUM.

$$Z_{in} = Z_0 \left[\frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l} \right]$$

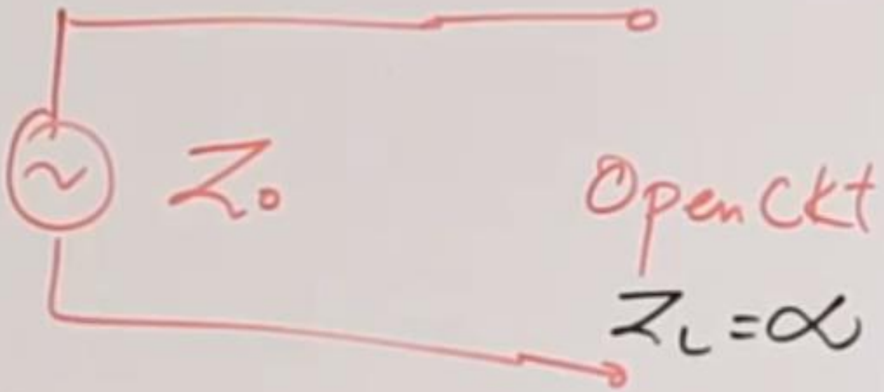


Input Impedance Of Open Ckt

$$Z_{in} = Z_0 \left\{ \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right\}$$

$$Z_{in} = Z_0 \times \cancel{Z_L} \left\{ \frac{1 + j\cancelto{0}{\frac{Z_0}{Z_L}} \tan\beta l}{\cancel{Z_L}\left\{ \cancelto{0}{\frac{Z_0}{Z_L}} + j\tan\beta l \right\}} \right\}$$

$$= \frac{Z_0}{j\tan\beta l}$$



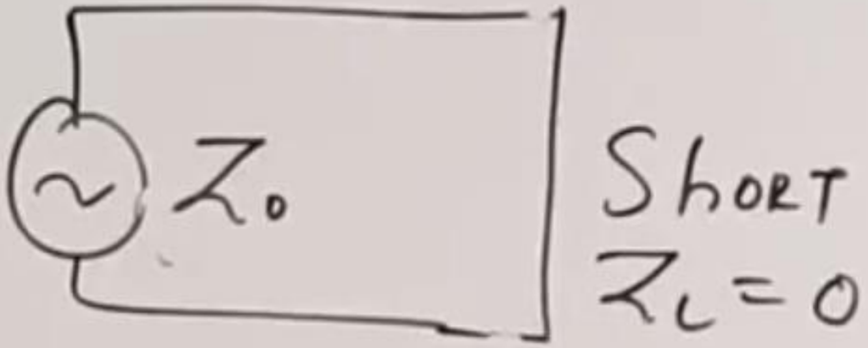
$$\boxed{Z_{in} = -jZ_0 \cot\beta l}$$

INPUT IMPEDANCE OF SHORT CKT

$$Z_{in} = Z_0 \left\{ \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right\}$$

$$Z_{in} = \cancel{Z_0} \left\{ \frac{0 + jZ_0 \tan\beta l}{\cancel{Z_0} + 0} \right\}$$

$$\boxed{Z_{in} = jZ_0 \tan\beta l}$$

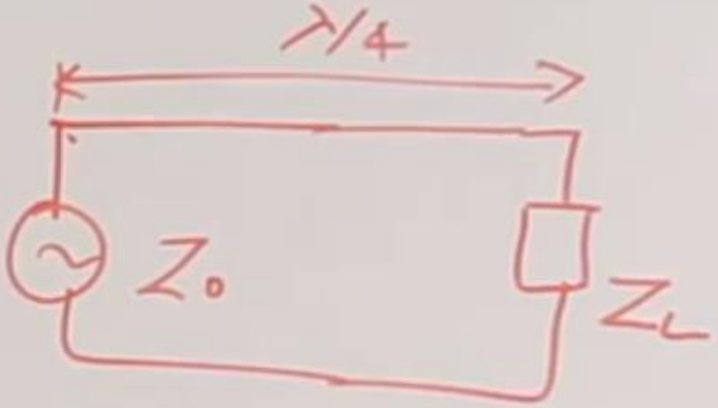


INPUT IMPEDANCE OF λ/4 LENGTH OF Tx LINE

$$Z_{in} = Z_0 \left\{ \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right\}$$

$$Z_{in} = Z_0 \frac{\cancel{\tan\beta l}\left\{ \cancel{\frac{Z_L}{\tan\beta l}}^{\to 0} + jZ_0 \right\}}{\cancel{\tan\beta l}\left\{ \cancel{\frac{Z_0}{\tan\beta l}}^{\to 0} + jZ_L \right\}}$$

$$Z_{in} = Z_0 \times \frac{jZ_0}{jZ_L} = \frac{Z_0^2}{Z_L}$$



$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

$$\boxed{Z_{in} = \frac{Z_0^2}{Z_L}}$$

POWER IN TRANSMISSION LINE

$$V(x,t) = (V^+ e^{-\gamma x} + V^- e^{+\gamma x}) e^{j\omega t}$$

FOR FORWARD PART

$$V(x,t) = V^+ e^{-\gamma x} e^{j\omega t}$$

$$\gamma = \alpha + j\beta$$

$$V(x,t) = V^+ e^{-(\alpha + j\beta)x} e^{j\omega t}$$

$$= V^+ e^{-\alpha x} e^{-j\beta x} e^{j\omega t}$$

$$= V^+ e^{-\alpha x} e^{+(\omega t - \beta x) j}$$

$$= V^+ e^{-\alpha x} \cos(\omega t - \beta x)$$

$$I(x,t) = I^+ e^{-\gamma x} + I^- e^{+\gamma x}$$

FOR FORWARD PART

$$I(x,t) = I^+ e^{-\alpha x} \cos(\omega t - \beta x)$$

$$P(x,t) = V(x,t)\, I(x,t)$$

Power In Transmission Line

$$P(x,t) = V^{+} e^{-\alpha x} \cos(\omega t - \beta x)\, I^{+} e^{-\alpha x} \cos(\omega t - \beta x + \theta)$$

$$P(x,t) = V^{+} I^{+} e^{-2\alpha x} \cos(\omega t - \beta x) \cos(\omega t - \beta x + \theta)$$

$$P_{av} = \frac{1}{T} \int_0^T P(x,t)\, dt$$

$$= \frac{1}{T} \int_0^T V^{+} I^{+} e^{-2\alpha x} \cos(\omega t - \beta x) \cos(\omega t - \beta x + \theta)\, dt$$

$$= V^{+} I^{+} e^{-2\alpha x} \cdot \frac{1}{T} \int_0^T \cos(\omega t - \beta x) \cos(\omega t - \beta x + \theta)\, dt$$

Power In Transmission Line

$$P = V^+ I^+ e^{-2\alpha x} \frac{1}{T}\int_0^T \left(\frac{1}{2}\cancel{\cos(2\omega t - 2\beta x + \theta)} + \frac{1}{2}\cos(\theta)\right) dt$$

$$= V^+ I^+ e^{-2\alpha x} \cdot \frac{1}{2}\cos\theta \cdot \frac{1}{T}\int_0^T dt$$

$$P = \frac{1}{2} V^+ I^+ e^{-2\alpha x} \cos\theta$$

$$Z_0 = \frac{V^+}{I^+}$$

$$I^+ = \frac{V^+}{Z_0}$$

$$= \frac{1}{2} V^+ \cdot \frac{V^+}{Z_0} e^{-2\alpha x} \cos\theta$$

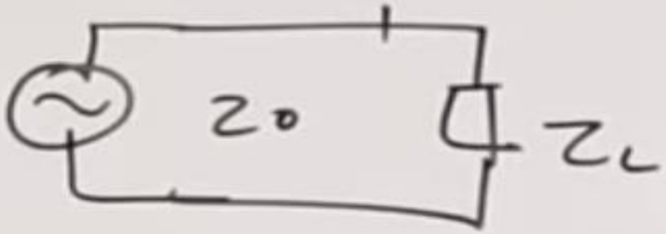
$$\boxed{= \frac{|V^+|^2}{2 Z_0} e^{-2\alpha x} \cos\theta \ W}$$

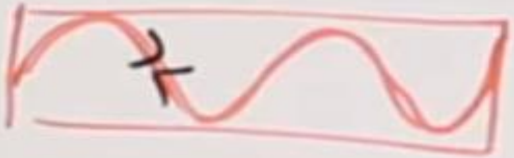
$$P = \frac{E^2}{2\eta} e^{-2\alpha z} \cos\theta$$

VSWR

Voltage Standing Wave Ratio

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$





$$V_{max} \Leftrightarrow z_{min} = \frac{-1}{2\beta}\left(\phi + (2m+1)\pi\right)$$

$$V_{min} \Leftrightarrow z_{max} = \frac{-1}{2\beta}\left(\phi + 2m\pi\right)$$

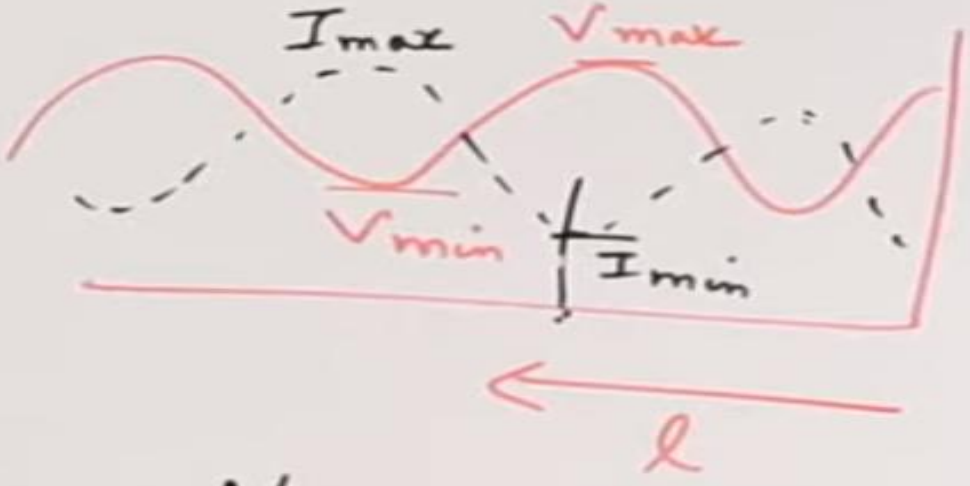
$$\left\{ m = 0, 1, 2 \ldots \right.$$

VSWR (S)

Voltage Standing Wave Ratio

$$\text{VSWR} = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

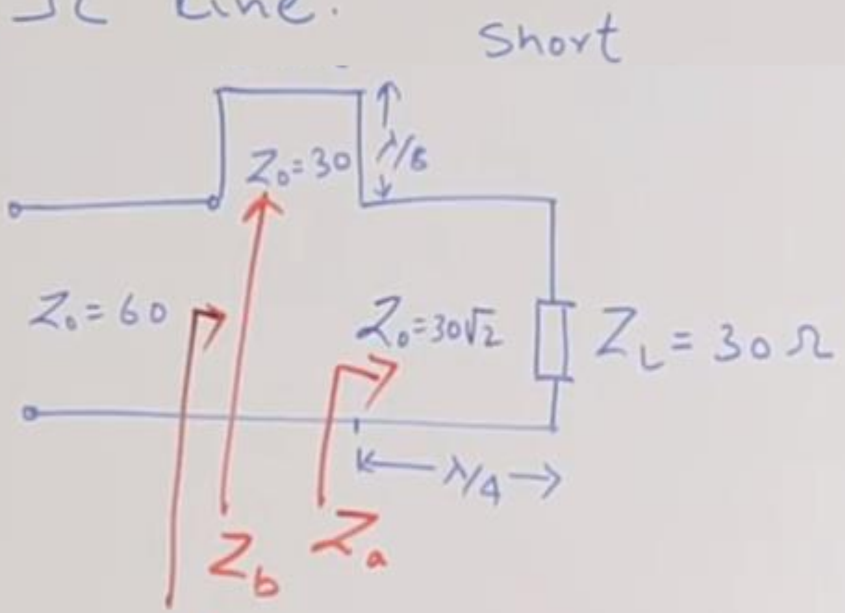
Relation b/w V & I



$$Z_{max} = \frac{V_{max}}{I_{min}} = Z_0 S$$

$$Z_{min} = \frac{V_{min}}{I_{max}} = \frac{Z_0}{S}$$

In the Circuit shown all the transmission line section are lossless.
The Voltage Standing Wave ratio (VSWR) on the 60 Ω Line.



$$Za = \frac{Z_0^2}{Z_L} = \frac{(30\sqrt{2})^2}{30} = 60\,\Omega$$

$$Zb = j Z_0 \tan\beta l = j30\tan\beta l$$

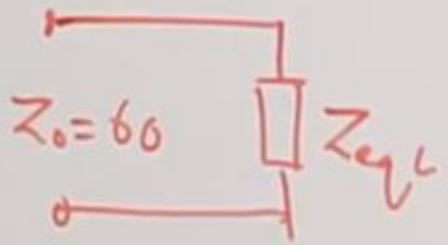
$$= j30\tan\frac{2\pi}{\lambda}\cdot\frac{\lambda}{8}$$

$$= j30\tan\pi/4$$

$$= +j30$$

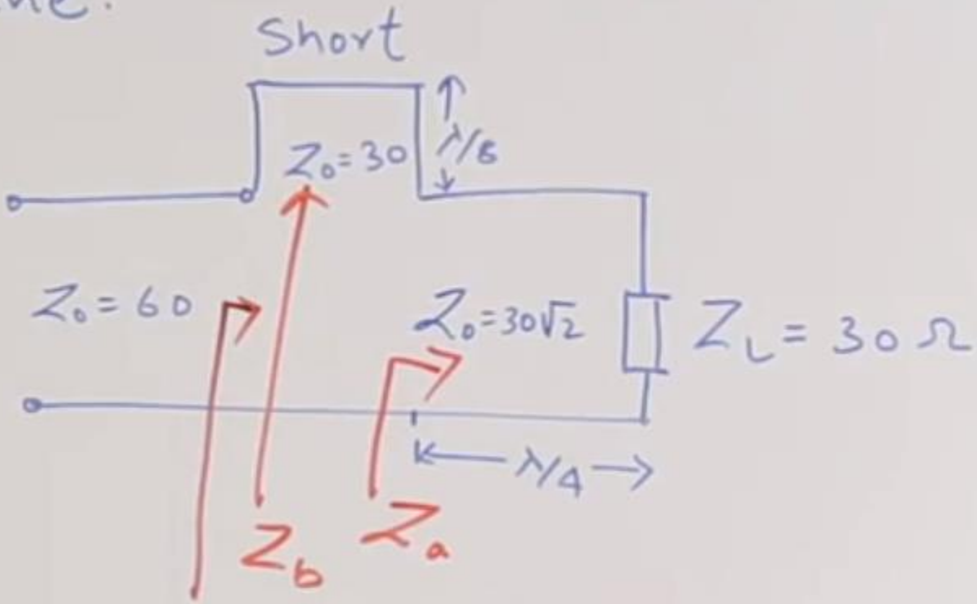
$Z_{eqL} = Z_a + Z_b$

$Z_{eqL} = 60 + 30j$



cuit shown all the transmission line section are lossless.

Standing Wave ratio (VSWR) on the 60 Ω line.



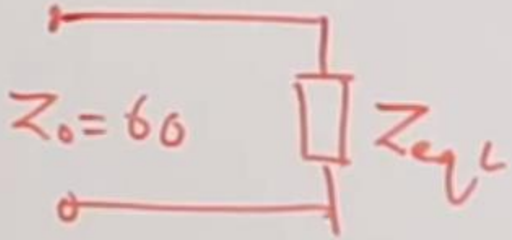
$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\Gamma = \frac{Z_{eqL}-Z_0}{Z_{eqL}+Z_0} = \frac{60+30j-60}{60+30j+60}$$

$$\Gamma = \frac{30j}{120+30j}, \quad |\Gamma| = 0.24$$

$$Z_{eqL} = Z_a + Z_b$$

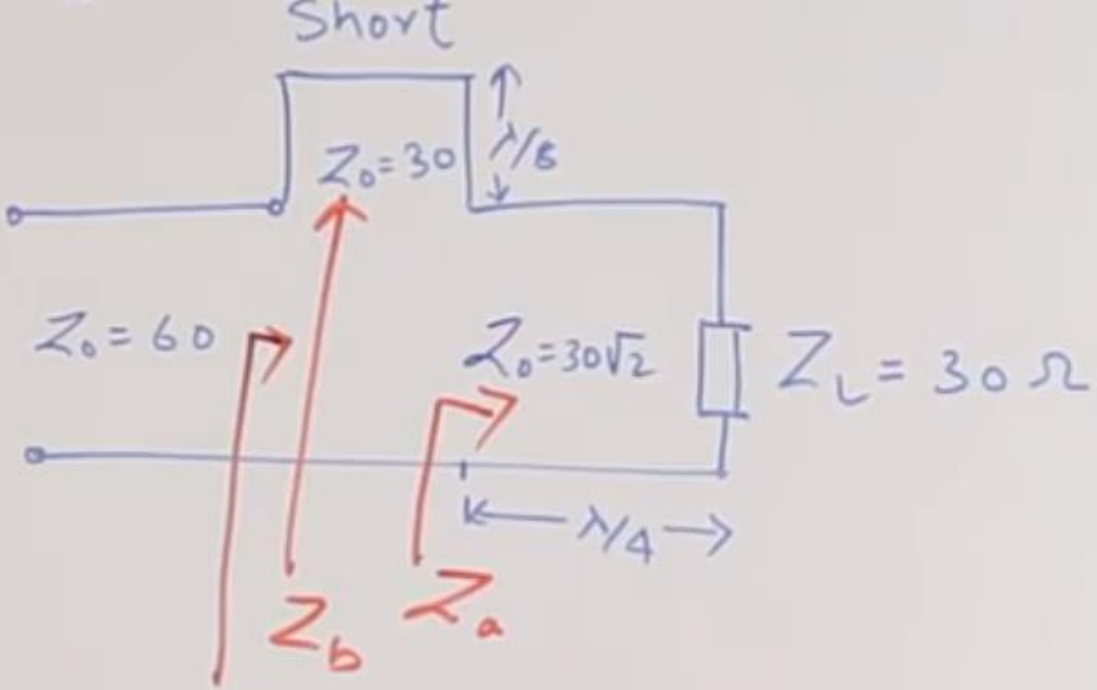
$$Z_{eqL} = 60+30j$$



$$\mathrm{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$= \frac{1+0.2425}{1-0.2425}$$

$$\boxed{\mathrm{VSWR} = 1.64}$$



$$Z_{eqL} = Z_a + Z_b$$

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