Transmission Line-1

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Transmission Lines

Communication Link for EM waves from one point to another

- Transmission Lines
  (VHF & UHF Range)
  - Two or More conductors
    - Two wire lines
      (Balanced)
      VHF Range
    - Coaxial Lines
      (unbalanced)
      UHF Range
- Waveguides
  (Microwave Range)
  - Single conductor
    - Rectangular
      Waveguides
      (Most common)
    - Circular
      Waveguides

Following are the difference between waveguide and transmission line

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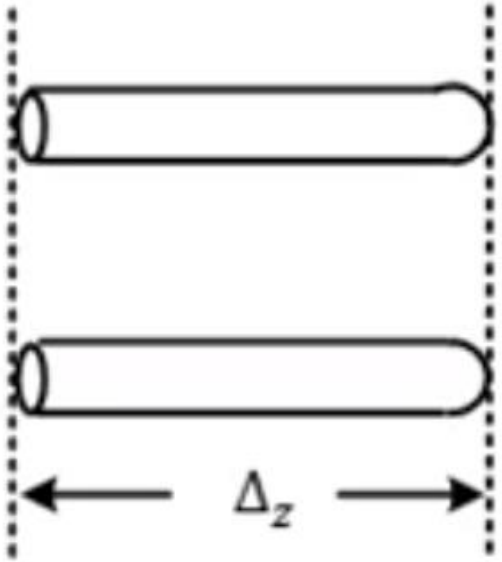
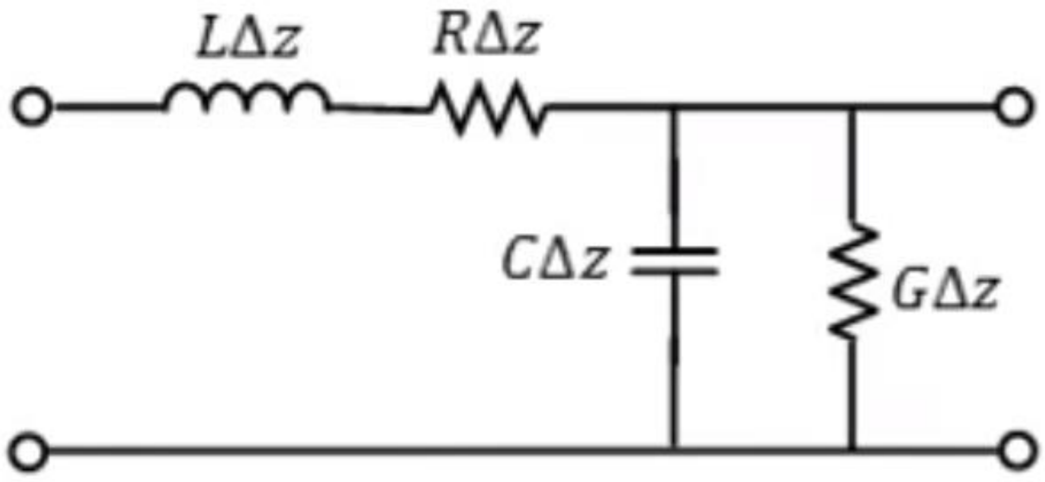
Waveguide	Transmission Line
The waveguide is a hollow metallic structure through which electric and magnetic fields are transmitted.	The transmission line is a conductor which is used to carry electrical signal over a long range.
It has simple to manufactured.	It has complex to manufactured.
In waveguide the power handling is high as compared to transmission line.	In transmission line the power handling is low as compared to waveguide.
The Operating modes are TE or TM mode.	The operating mode are TEM or quasi TEM mode.
In waveguide high power is transmitted.	In transmission line low power is transmitted.
In waveguide the electromagnetic signal is transmitted.	In transmission line the electrical signal is transmitted.
The operating frequency is 3 GHZ to 100 GHZ in waveguide.	The operating frequency is up to 18 GHz.

Transmission Line Analysis:

Analysis by representing small section by equivalent lumped circuit



Line Section



Equivalent Circuit

R, L, G, C are primary line constants

■ Equivalent circuit

Since the voltage and current of a transmission line vary with position z (and time t), we have to characterize it by a "distributed" circuit model. Consider an infinitesimal line of length Δz, the currents set up magnetic field between the conductors (by Ampere's law), causing magnetic flux. When currents are time-varying, so is the magnetic flux, and a voltage variation "along" the conductor (electromotive force) is induced (by Faraday's law) in an attempt to drive the currents oppositely (by Lenz's law). This behavior can be modeled by a **series inductor** $\left(v = L \cdot \frac{d}{dt} i\right)$. Meanwhile, two separated conductors form a capacitor. Since the upper and lower conductors of adjacent infinitesimal lines are connected respectively, the capacitive behavior of an infinitesimal line can be modeled by a **shunt capacitor**. In the presence of imperfect conducting and imperfect insulating materials, voltage drop along the conducting line and leakage current between them exist, which can be modeled by a series resistor and a shunt conductor, respectively.

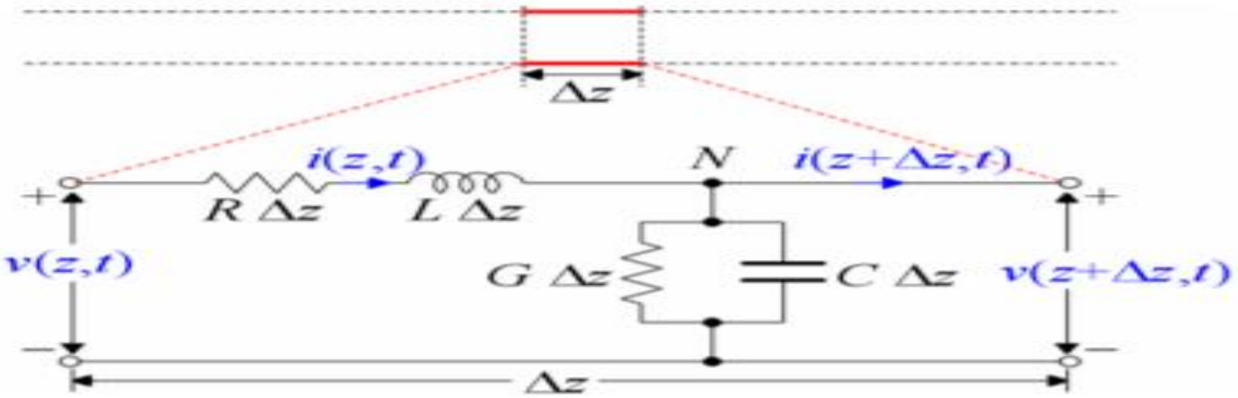


Fig. 2-5. Equivalent circuit of a real transmission line.

CHARACTERISTIC IMPEDANCE

$$\frac{dv}{dx} = -(R+j\omega L)I$$

(Transmission line equation)

$$\frac{d}{dx}\left(V^{+}e^{-\gamma x}+V^{-}e^{+\gamma x}\right) = -(R+j\omega L)\{I^{+}e^{-\gamma x}+I^{-}e^{+\gamma x}\}$$

$$V^{+}e^{-\gamma x}\times -\gamma + V^{-}e^{\gamma x}\times \gamma = -(R+j\omega L)\{I^{+}e^{-\gamma x}+I^{-}e^{+\gamma x}\}$$

Part of eq travelling forward

$$V^{+}e^{-\gamma x}\times -\gamma = -(R+j\omega L)I^{+}e^{-\gamma x}$$

$$\frac{V^{+}}{I^{+}} = \frac{R+j\omega L}{\gamma}$$

$$\left\{\begin{array}{l}\gamma^{2} = (R+j\omega L)(G+j\omega C)\\ \gamma = \sqrt{(R+j\omega L).(G+j\omega C)}\end{array}\right.$$

$$Z_0 = \frac{V^{+}}{I^{+}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

CHARACTERISTIC IMPEDANCE

$$\frac{dv}{dx} = -(R+j\omega L)I$$

$$\frac{d}{dx}\left(V^+e^{-\gamma x}+V^-e^{+\gamma x}\right) = -(R+j\omega L)\left\{I^+e^{-\gamma x}+I^-e^{+\gamma x}\right\}$$

$$V^+e^{-\gamma x}\times -\gamma + V^-e^{\gamma x}\times\gamma = -(R+j\omega L)\left\{I^+e^{-\gamma x}+I^-e^{+\gamma x}\right\}$$

Part of eq travelling Backward

$$V^-e^{\gamma x}\gamma = -(R+j\omega L)I^-e^{+\gamma x}$$

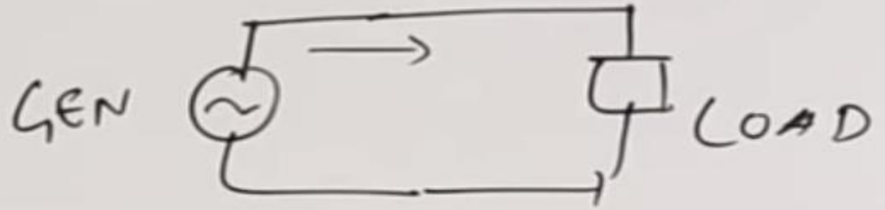
$$\frac{V^-}{I^-} = -\frac{(R+j\omega L)}{\gamma}$$

$$Z0 = \frac{V^-}{I^-} = -\sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\left\{\begin{array}{l}\gamma^2 = (R+j\omega L)(G+j\omega C)\\ \gamma = \sqrt{(R+j\omega L).(G+j\omega C)}\end{array}\right.$$

Different Type Of Transmission Line

(1) Lossless Tx Line



$$R = 0 \,,\; G = 0$$

$$\gamma = \sqrt{(R + j\omega L).(G + j\omega C)}$$

$$\gamma = \sqrt{j^2 \omega^2 LC}$$

$$\gamma = j\omega\sqrt{LC}$$

$$\alpha + j\beta = j\omega\sqrt{LC}$$

$$\alpha = 0$$

$$\beta = \omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_0 = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{L/C}$$

$$\beta = \omega\sqrt{LC}$$

$$\frac{2\pi}{\lambda} = 2\pi f\sqrt{LC}$$

$$f\lambda = \frac{1}{\sqrt{LC}}$$

$$V_p = \frac{1}{\sqrt{LC}}$$

$$\left.\begin{matrix} f = \frac{c}{\lambda} \\ f\lambda = c \end{matrix}\right\}$$

Propagation constant
Attenuation constant
Phase constant
Characteristic Impedance

DIFFERENT TYPE OF TRANSMISSION LINE

2) Low Loss Tx line

$R << \omega L, \; G << \omega C$

$$\gamma = \sqrt{(R+j\omega L).(G+j\omega C)}$$

$$\gamma = \sqrt{j\omega L\left(1+\frac{R}{j\omega L}\right) j\omega C\left(1+\frac{G}{j\omega C}\right)}$$

$$\sqrt{j\omega L\left(1-j\frac{R}{\omega L}\right) j\omega C\left(1-j\frac{G}{\omega C}\right)}$$

$$= \left\{j\omega L \cdot j\omega C\left(1-j\frac{R}{\omega L}\right)\left(1-j\frac{G}{\omega C}\right)\right\}^{1/2}$$

$$= j\omega\sqrt{LC}\left\{\left(1-j\frac{R}{\omega L}\right)\cdot\left(1-j\frac{G}{\omega C}\right)\right\}^{1/2}$$

$$= j\omega\sqrt{LC}\left\{1-j\frac{R}{\omega L}-j\frac{G}{\omega C}\right\}^{1/2}$$

$$= j\omega\sqrt{LC}\left\{1-\frac{jR}{2\omega L}-j\frac{G}{2\omega C}\right\}$$

$$= j\omega\sqrt{LC}+\frac{R}{2}\sqrt{\frac{C}{L}}+\frac{G}{2}\sqrt{\frac{L}{C}}$$

$$\alpha + j\beta = \underbrace{j\omega\sqrt{LC}}_{\beta}+\underbrace{\frac{R}{2}\sqrt{\frac{C}{L}}+\frac{G}{2}\sqrt{\frac{L}{C}}}_{\alpha}$$

DIFFERENT TYPE OF TRANSMISSION LINE

② Low Loss Tx Line

$R << \omega L,\ G << \omega C$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_0 = \sqrt{\frac{j\omega L\left(1 - j\frac{R}{\omega L}\right)}{j\omega C\left(1 - j\frac{G}{\omega C}\right)}}$$

$$Z_0 = \sqrt{\frac{L}{C}}\left\{1 - j\frac{R}{2\omega L} + j\frac{G}{2\omega C}\right\}$$

$$= j\omega\sqrt{LC}\left\{1 - j\frac{R}{\omega L} - j\frac{G}{\omega C}\right\}^{1/2}$$

$$= j\omega\sqrt{LC}\left\{1 - \frac{jR}{2\omega L} - j\frac{G}{2\omega C}\right\}$$

$$= j\omega\sqrt{LC} + \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}$$

$$\alpha + j\beta = \underbrace{j\omega\sqrt{LC}}_{\beta} + \underbrace{\frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}}_{\alpha}$$

LOW-LOSS PROPAGATION | LOSSES ARE SMALL

Requirement : $\boxed{R \ll \omega L \quad \& \quad G \ll \omega C}$

so, $\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{j\omega L\left(1+\frac{R}{j\omega L}\right)j\omega C\left(1+\frac{G}{j\omega C}\right)}$

$$\gamma = j\omega\sqrt{LC}\sqrt{1+\frac{R}{j\omega L}}\sqrt{1+\frac{G}{j\omega C}}$$

Using Binomial series, upto first three terms for Low-Loss approximation.

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8}$$

$$\gamma = j\omega\sqrt{LC}\left[\left(1+\frac{R}{j2\omega L} - \frac{R^2}{j^2 8\omega^2 L^2}\right)\left(1+\frac{G}{j2\omega C} - \frac{G^2}{j^2 8\omega^2 C^2}\right)\right]$$

$$\rightarrow \gamma = j\omega\sqrt{LC}\left[\left(1+\frac{R}{j2\omega L}-\frac{R^2}{j^2 8\omega^2 L^2}\right)\left(1+\frac{G}{j2\omega C}-\frac{G^2}{j^2 8\omega^2 C^2}\right)\right]$$

$$= j\omega\sqrt{LC}\left[\left(1+\frac{R}{j2\omega L}+\frac{R^2}{8\omega^2 L^2}\right)\left(1+\frac{G}{j2\omega C}+\frac{G^2}{8\omega^2 C^2}\right)\right]$$

$$= j\omega\sqrt{LC}\left[1+\frac{G}{j2\omega C}+\frac{G^2}{8\omega^2 C^2}+\frac{R}{j2\omega L}+\underbrace{\frac{RG}{j^2 4\omega^2 LC}}+\frac{RG^2}{16j\omega^3 C^2 L}+\frac{R^2}{8\omega^2 L^2}\right.$$

$$\left.+\frac{R^2 G}{16j\omega^3 L^2 C}+\frac{R^2 G^2}{64\omega^4 C^2 L^2}\right]$$

$$= j\omega\sqrt{LC}\left[1 + \frac{1}{j2\omega}\left(\frac{R}{L}+\frac{G}{C}\right) + \frac{1}{8\omega^2}\left(\frac{R^2}{L^2} - \frac{2RG}{LC} + \frac{G^2}{C^2}\right)\right]$$

$\left\{ RG^2, R^2G \;\&\; R^2G^2 \right.$ negligible compared to all others.

$$\gamma = j\omega\sqrt{LC}\left[1 + \frac{1}{j2\omega}\left(\frac{R}{L}+\frac{G}{C}\right) + \frac{1}{8\omega^2}\left(\frac{R}{L} - \frac{G}{C}\right)^2\right]$$

$$\alpha + j\beta = \underbrace{\sqrt{LC}\left(\frac{1}{2}\right)\left(\frac{R}{L}+\frac{G}{C}\right)} + \underbrace{j\omega\sqrt{LC}\left[1 + \frac{1}{8\omega^2}\left(\frac{R}{L} - \frac{G}{C}\right)^2\right]}$$

Real part

Img. Part

EPOV CHANNEL @ YOUTUBE

CHARACTERISTIC IMPEDANCE for LOW-LOSS APPROXIMATION

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{j\omega L\left(1+\frac{R}{j\omega L}\right)}{j\omega C\left(1+\frac{G}{j\omega C}\right)}} = \sqrt{\frac{L}{C}}\left[\frac{1+\frac{R}{j2\omega L}+\frac{R^2}{8\omega^2L^2}}{1+\frac{G}{j2\omega C}+\frac{G^2}{8\omega^2C^2}}\right]$$

$$Z_0 = \sqrt{\frac{L}{C}}\,\frac{\left(1+\frac{R^2}{8\omega^2L^2}-j\frac{R}{2\omega L}\right)}{\left(1+\frac{G^2}{8\omega^2C^2}-j\frac{G}{2\omega C}\right)} \times \frac{\left(1+\frac{G^2}{8\omega^2C^2}+j\frac{G}{2\omega C}\right)}{\left(1+\frac{G^2}{8\omega^2C^2}+j\frac{G}{2\omega C}\right)}$$

$$Z_0 = \sqrt{\frac{L}{C}}\,\frac{1+\frac{G^2}{8\omega^2C^2}+j\frac{G}{2\omega C}+\frac{R^2}{8\omega^2L^2}+\frac{R^2G^2}{64\omega^4L^2C^2}+j\frac{GR^2}{16\omega^3L^2C}-j\frac{R}{2\omega L}-j\frac{RG^2}{16\omega^3C^2L}+\frac{RG}{4\omega^2LC}}{\left(1+\frac{G^2}{8\omega^2C^2}\right)^2+\left(\frac{G}{2\omega C}\right)^2 \dashrightarrow \left(1+\frac{G^4}{64\omega^4C^4}+\frac{G^2}{4\omega^2C^2}+\frac{G^2}{4\omega^2C^2}\right)}$$

simplify by neglecting all terms on the order of R^2G, G^2R and higher

$$Z_0 = \sqrt{\frac{L}{C}} \; \frac{1 + \frac{G^2}{8\omega^2C^2} + j\frac{G}{2\omega C} + \frac{R^2}{8\omega^2L^2} - j\frac{R}{2\omega L} + \frac{RG}{4\omega^2LC}}{1 + \left(\frac{G^4}{64\omega^4C^4} + \frac{G^2}{2\omega^2C^2}\right)}$$

$\because \frac{1}{1+x} = 1 - x$ as $x \ll 1$

$$= \sqrt{\frac{L}{C}} \left(1 + \frac{G^2}{8\omega^2C^2} + j\frac{G}{2\omega C} + \frac{R^2}{8\omega^2L^2} - j\frac{R}{2\omega L} + \frac{RG}{4\omega^2LC}\right)\left(1 - \frac{G^4}{64\omega^4C^4} - \frac{G^2}{2\omega^2C^2}\right)$$

$$= \sqrt{\frac{L}{C}} \left(1 + \frac{G^2}{8\omega^2C^2} + \frac{R^2}{8\omega^2L^2} - j\frac{R}{2\omega L} + j\frac{G}{2\omega C} + \frac{RG}{4\omega^2LC} - \frac{G^2}{2\omega^2C^2}\right)$$

$$= \sqrt{\frac{L}{C}} \left\{ 1 + \frac{1}{2\omega^2} \left[\frac{1}{4} \frac{G^2}{C^2} + \frac{1}{4} \frac{R^2}{L^2} + \frac{RG}{2LC} - \frac{G^2}{C^2} \right] + \frac{j}{2\omega} \left(\frac{G}{C} - \frac{R}{L} \right) \right\}$$

$$= \sqrt{\frac{L}{C}} \left\{ 1 + \frac{1}{2\omega^2} \left[\frac{1}{4} \left(\frac{G^2}{C^2} + \frac{R^2}{L^2} + 2 \frac{RG}{LC} \right) - \frac{G^2}{C^2} \right] + \frac{j}{2\omega} \left(\frac{G}{C} - \frac{R}{L} \right) \right\}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}} \left\{ 1 + \frac{1}{2\omega^2} \left[\frac{1}{4} \left(\frac{R}{L} + \frac{G}{C} \right)^2 - \frac{G^2}{C^2} \right] + \frac{j}{2\omega} \left(\frac{G}{C} - \frac{R}{L} \right) \right\}}$$

Distortionless Transmission Line

$$\frac{R}{L} = \frac{G}{C}$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{R\left(1+j\omega\frac{L}{R}\right)G\left(1+j\omega\frac{C}{G}\right)}$$

$$\gamma = \sqrt{RG\left(1+j\omega\frac{C}{G}\right)^2}$$

$$\alpha + j\beta = \sqrt{RG}\left(1+j\omega C/G\right)$$

$$\alpha = \sqrt{RG}$$

$$\beta = j\sqrt{RG}\cdot\omega\frac{C}{G}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$Z_0 = \sqrt{\frac{R\left(\cancel{1+j\omega\frac{L}{R}}\right)}{G\left(\cancel{1+j\omega\frac{C}{G}}\right)}} = \sqrt{R/G}$$

$$Z_0 = \sqrt{\frac{j\omega L\left(1+\cancel{\frac{R}{j\omega L}}\right)}{j\omega C\left(1+\cancel{\frac{G}{j\omega C}}\right)}} = \sqrt{L/C}$$

$$Z_0 = \sqrt{R/G} = \sqrt{L/C}$$

https://www.youtube.com/watch?v=Cf4jJ9pLzCw