$$f(x) = \begin{cases} 0 & -\pi \leq x \leq 0 \\ \pi & 0 \leq x \leq \pi \end{cases}$$

Evaluate the fourier series.

-> We have, $a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$

or $a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx\right]$

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} \pi\,dx\right]$$

$$= \frac{1}{2\pi}\left[\int_{0}^{\pi} \pi\,dx\right]$$

$$= \frac{1}{2\pi}\left[\pi \int_{0}^{\pi} 1\,dx\right]$$

$$= \frac{1}{2\pi}\left[\pi\; x\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[\pi(\pi - 0)\right]$$

$$= \frac{1}{2\pi}\left[\pi(\pi)\right]$$

$$= \frac{1}{2\cancel{\pi}} \times \pi^{\cancel{2}}$$

$$a_0 = \frac{\pi}{2}$$

Now, $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\cos nx\,dx + \int_{0}^{\pi} f(x)\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} (0)\cos nx\,dx + \int_{0}^{\pi} (\pi)\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\pi \int_{0}^{\pi} \cos nx \, dx\right]$$

$$= \frac{1}{n\pi}\left[\pi \sin nx \Big|_0^{\pi}\right]$$

$$= \frac{1}{n\pi}\left[\pi\left[\sin n\pi - \sin n0\right]\right]$$

$$= \frac{1}{n\pi}\left[\pi(0)\right]$$, we get zero for both even and odd values of n

so, $a_n = 0$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x) \sin nx \, dx + \int_{0}^{\pi} f(x) \sin nx \, dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} (0) \sin nx \, dx + \int_{0}^{\pi} \pi \sin nx \, dx\right]$$

$$= \frac{1}{\pi}\left[\pi \int_{0}^{\pi} \sin nx \, dx\right]$$

$$= \frac{1}{n\pi}\left[-\pi \cos nx \Big|_0^{\pi}\right]$$

$$= \frac{-1}{n\pi}\left[\pi\left[\cos n\pi - \cos n(0)\right]\right]$$

$$= \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{2}{n} & \text{if } n \text{ is odd} \end{cases}$$

The fourier series can be given as :-

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

Put values.

$$f(x) = \frac{\pi}{2} + 0 \cos x + 0 \cos 2x + \text{-------} +$$
$$2 \sin x + \frac{2}{3} \sin 3x + \frac{2}{5} \sin 5x + \text{---}$$

or $$f(x) = \frac{\pi}{2} + 2 \sin x + \frac{2}{3} \sin 3x + \frac{2}{5} \sin 5x + \text{.---}$$