

Q If given $X(z) = \frac{z}{3z^2 - 4z + 1}$

Give inverse z -transform.

Soln $X(z) = \frac{z}{3z^2 - 4z + 1}$

$$X(z) = \frac{z}{3z(z-1) - 1(z-1)}$$

$$X(z) = \frac{z}{(3z-1)(z-1)}$$

$$\text{or } \frac{X(z)}{z} = \frac{1}{(3z-1)(z-1)}$$

By partial fraction method

$$\frac{1}{(3z-1)(z-1)} = \frac{A}{(3z-1)} + \frac{B}{(z-1)} \rightarrow (i)$$

Multiply both sides of (i) by $(3z-1)(z-1)$

We get,

$$1 = A(z-1) + B(3z-1) \rightarrow (ii)$$

Put $z=1$ in (ii)

$$1 = A(1-1) + B[3(1)-1]$$

$$1 = A(0) + B(3-1)$$

$$1 = B(2)$$

$$\text{or } B = \frac{1}{2}$$

$$\text{Put } z = \frac{1}{3} \text{ in (ii)}$$

$$1 = A\left(\frac{1}{3} - 1\right) + B\left(3 \times \frac{1}{3} - 1\right)$$

$$1 = A\left(\frac{2-3}{3}\right) + B[1-1]$$

$$1 = A\left(-\frac{1}{3}\right) + B(0)$$

$$1 = A\left(-\frac{1}{3}\right)$$

$$\text{or } A = -\frac{3}{2}$$

Put values of A & B in (ii)

$$\frac{1}{(3z-1)(z-1)} = \frac{(-3/2)}{(3z-1)} + \frac{1/2}{(z-1)}$$

$$\text{or } \frac{X(z)}{z} = \frac{(-3/2)}{(3z-1)} + \frac{(1/2)}{(z-1)}$$

$$\text{Also, } X(z) = \left(\frac{-3}{2}\right) \frac{z}{(3z-1)} + \left(\frac{1}{2}\right) \frac{z}{(z-1)}$$

$$X(z) = -\frac{3}{2} \frac{z}{3(z-1/3)} + \left(\frac{1}{2}\right) \frac{z}{(z-1)}$$

Apply inverse z-transform

$$\text{We get, } \frac{z}{(z-1/3)} = \left(\frac{1}{3}\right)^k \quad \& \quad \frac{z}{(z-1)} = [1]$$

$$x[n] = -\frac{3}{2} \left(\frac{1}{3}\right)^k + \frac{1}{2} [1]$$

$$\text{or } x[n] = -\frac{1}{2} \left[\frac{1}{3}\right]^k + \frac{1}{2} [1]$$