(*) Classification of Systems:-

① Memory System & Memoryless System:-

A discrete time system is said to be memory system if its output $y[n]$ at any instant depends on its past input.

e.g. n — [N] → If it is zero then it will be M. less system and otherwise memory system.

⇒ Accumulator is a system with memory

$$y[n] = \sum_{K=0}^{n} x[K]$$

$$y[n] = \sum_{K=0}^{3} x[K]$$

$$y[n] = x[0] + x[1] + x[2] + x[3]$$

$y[0] = x[0] = 0$

$y[1] = 0 + 1 = 1$

$y[2] = 1 + 2 = 3$

$y[3] = 3 + 3 = 6$

$\Rightarrow \quad s[n] = u[n] - u[n-1]$ System with memory.

$\Rightarrow$ Storing systems are memory system i-e "$\sqcup$" or "C"

$\Rightarrow$ System involve delay is generally M. System.

(*) A discrete time system is said to be memory less system if its output $y[n]$ at any instant depends on input $x[n]$ at the same time $[n]$

e.) Identity System

$x[n]$ —[ ]— $y[n]$

Resistor is memory less system. Input is

current and output is voltage.

$$y[n] = R\,x[n]$$

⇒ Amplifier system $y[n] = a\,x[n]$

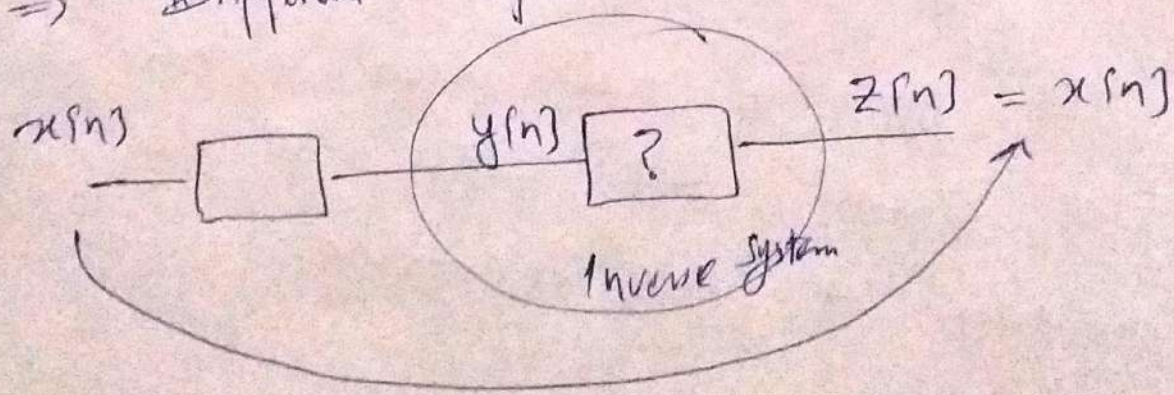
⇒ $y[n] = [2x(n) - x(n)^2]^2$

System without Memory.

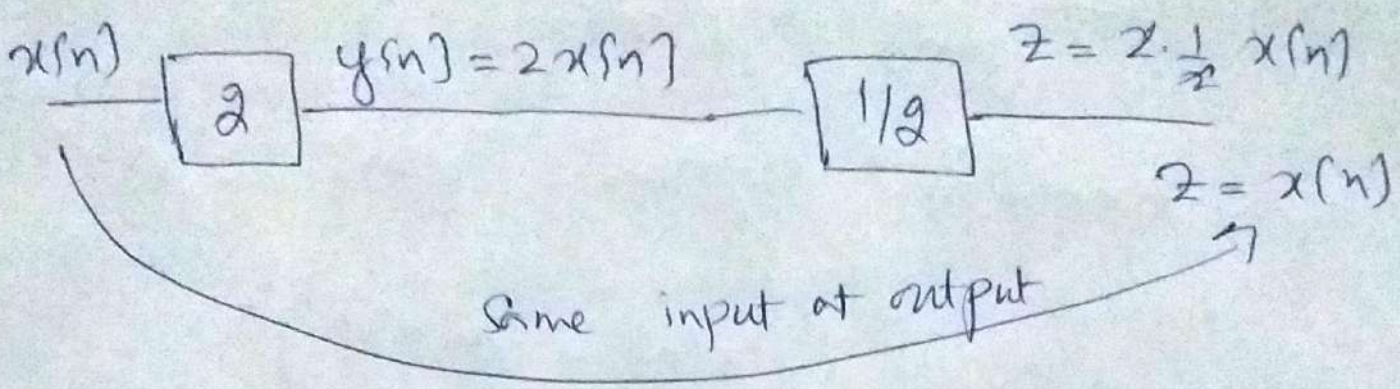
(#) Invertable & Non-Invertable :-

⇒ If the original input can be recieved from the output. Just by absorbing the output, Input can be determine.

⇒ Different inputs leads different outputs.



(#) $y[n] = 2x[n]$



⇒ This recieving side of the system is used to reduce/overcome distortion (Noise) from the input.

(*) If system is not designed like that then the system is called Non-Invertable System.

e.g. $y[n] = x^2(t)$

It gives ± two values so here we can not decide that either it is Invertable or Non Invertable.

(4) Causal & Non-Causal System:-

⇒ If a system depends on present & past a system is causal or Non-Anticepative.

⇒ If the independent variable is time the system is causal and If the independent variable is space then called Non-Causal system.

⇒ Backward Difference Equation (BDE)

$y[n] = x[n] - x[n-1]$ ⇒ Causal.

$y[n] = a\,x[n]$ Amplifier is causal system.

⇒ Amplifier depends on present value.

⇒ Accumulator is a causal system because it depends on present & Past value.

⇒ Forward Difference Equation (FDE):-

Non causal system

$y[n] = x[n+1] - x[n]$

⇒ Generally Memoryless System are causal System.

$$\Rightarrow \quad y[n] = \frac{1}{2M+1} \sum_{K=-1}^{1} x[n-K]$$

⇒ If a system is Anticepative then it is Non-Causal. (Means depends on future)

⇒ Let $M=1$

$$= \frac{1}{2(1)+1} \{ x[n+1] + x[n] + x[n-1] \}$$

$$= \frac{1}{3} \{ x[n+1] + x[n] + x[n-1] \}$$

This is Non-causal System because it depends on future.

⊛ Fourier Series:-

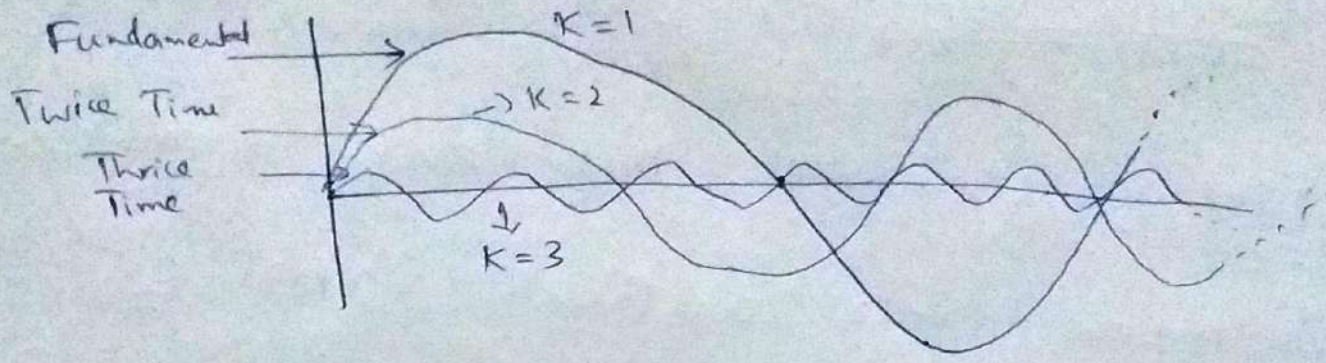
Fourier generalize that every signal is decompose into another signal e.g. Sinusoidal signal into Harmonics signal.

Fourier Series is used to look the inside picture that what/which thing is present in it.

Every signal is decompose into elementary signal i-e Sinusoidal etc.

⇒ Fourier Series allows you any periodic wave form in time to be decompose into some of Harmonic waves related Sine and Cosine wave form.

Multiple of Frequencies is Harmonics.



Normal Form:-

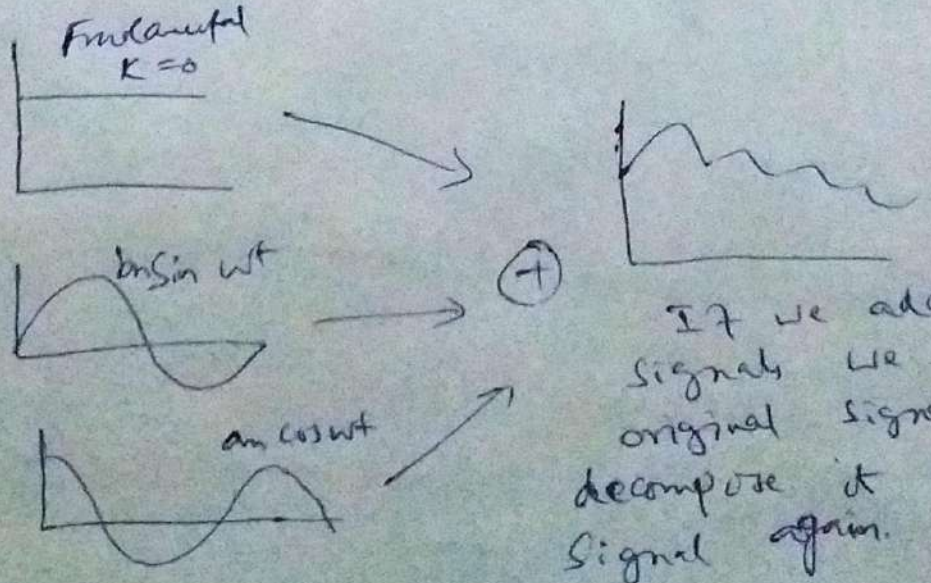
$$① \quad y(t) = a_0 + \sum_{n=-\infty}^{\infty} (a_n \cos \omega t + b_n \sin \omega t)$$

② Cos only form:-

$$y(t) = C_0 + \sum C_n \cos(\omega t + \phi)$$

③ Expnential Form:-

$$X(e^{j\omega}) = \sum x[n] e^{-j\omega n}$$



If we add all these signals we can get original signal & if we decompose it we get back signal again.

④ Discrete Fourier Series:-

$$x[n] = \sum_{K=0}^{N_0-1} C_K e^{-j\left(\frac{2\pi}{N_0}\right)Kn}$$

$N_0 - 1$ → Fundamental Period

Harmonics

$x[n]$ → DTS

C_K → Fourier Coefficient / Spectral Coefficient.

Time Domain ⟷ Frequency Domain

$$C_K = \frac{1}{N_0} \sum_{n=0}^{N_0-1} x[n] e^{-j(2\pi/N_0)Kn}$$

C_K → Fourier Coefficient

Time Domain → Frequency Domain | Analysis Equation & Synthesis Equation.

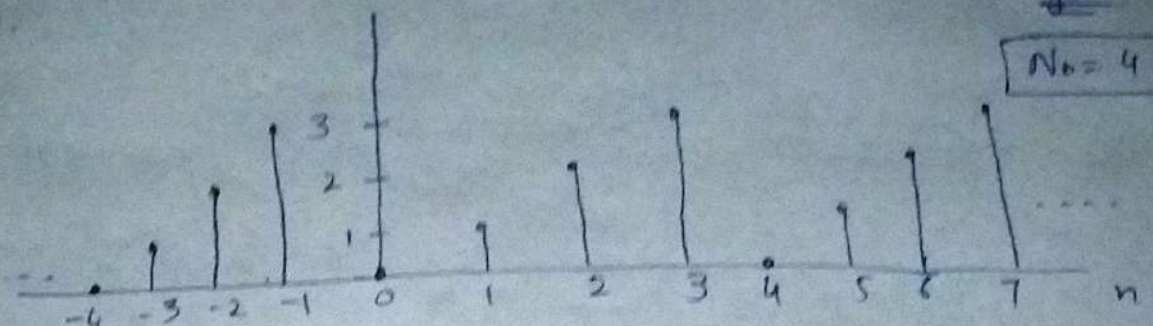
Two Important Properties:-

① $C_{K+N_0} = C_K$

② $C_{-K} = C_{N_0-K} = C_K^*$ → conjugate.

Exple:

$N_0 = 4$



$$x[n] = \{\underset{\uparrow}{0}, 1, 2, 3\}$$

$$C_k = \frac{1}{N_0} \sum_{n=0}^{N_0-1} x[n]\, e^{-j\left(\frac{2\pi}{N_0}\right)kn}$$

$\therefore e^{i\theta} = \cos\theta + j\sin\theta$ (Euler form)

So $e^{-j\left(\frac{\pi}{2}k\right)} = \cos(\pi/2) - j\sin(\pi/2)$

or $e^{j(\pi/2)} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2}$

$= \boxed{-j}$

$$C_k = \frac{1}{4} \sum_{n=0}^{4-1} x[n]\,(-j)^{1kn}$$

$$C_k = \frac{1}{4} \sum_{n=0}^{3} x[n]\,(-j)^{kn}$$

at $k=0$; $\quad C_0 = \frac{1}{4} \sum_{n=0}^{3} x[n]\,(1)$

$$C_0 = \frac{1}{4}\left[x[0] + x[1] + x[2] + x[3]\right]$$

$$C_0 = \frac{1}{4}[0+1+2+3] = \frac{6}{4} = \boxed{3/2}$$

$\boxed{C_0 = 3/2} \Rightarrow$ DC component

Now at $k=1$

$$C_1 = \frac{1}{4}\sum_{n=0}^{3} x[n](-j)^n$$

$$C_1 = \frac{1}{4}\{(-j)^0 x[0] + (-j)^1 x[1] + (-j)^2 x[2] + (-j)^3 x[3]\}$$

$$C_1 = \frac{1}{4}\{0 + (-j)(1) + j^2(2) + (-j^3)(3)\}$$

$$C_1 = \frac{1}{4}\{-j - 2 + 3j\}$$

$$C_1 = \frac{1}{4}\{-2 + 2j\}$$

$$C_1 = -\frac{1}{2} + j/2$$

Also solve for C_2 & C_3 we find coefficient upto time period range. Here time period is 4.

$$C_2 = -1/2$$

$$C_3 = -\frac{1}{2} - \frac{1}{2}j$$

1st Property:-

$$C_{K+N_0} = C_K$$

$$C_{1+4} = C_1$$

$$C_5 = C_1$$

2nd Property:-

$$C_{-K} = C_{N_0-K} = C_K^*$$

$$\Rightarrow C_{4-1} = C_1^*$$

$$C_3 = C_1^*$$

$$-\frac{1}{2} - \frac{1}{2}j = -\frac{1}{2} + \frac{1}{2}j$$

conjugate of C_3

$C_0 = \frac{3}{2}$

$C_1 = -\frac{1}{2} + j/2$

$C_2 = -1/2$

$C_3 = -\frac{1}{2} - j/2$

convert it to Polar form

Rectangular form
$Z = x + jy$

Polar form
$Z = r \angle \theta$

Polar form will be:-

$$C_0 = 3/2$$

$$C_0 = 1.5$$

$$C_1 = 0.707 \angle 135^\circ$$

$$C_2 = 0.5 \angle 180^\circ$$

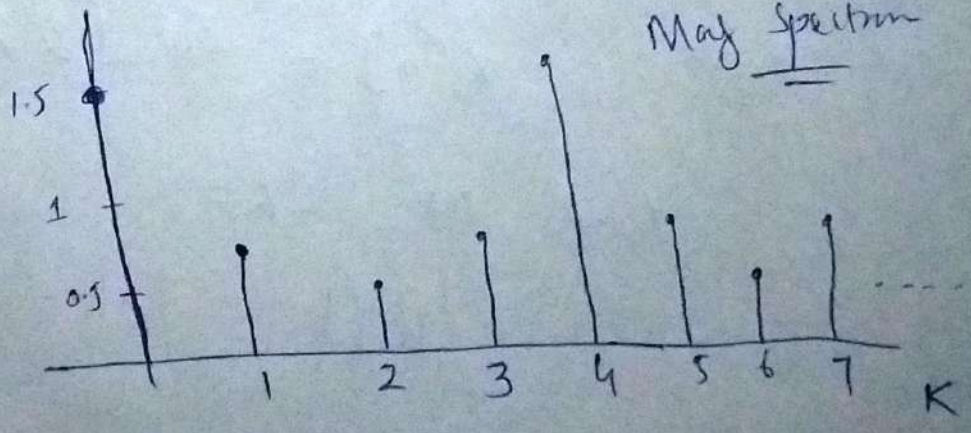
$$C_3 = 0.707 \angle -135^\circ$$

⇒ Two Important Plots:-

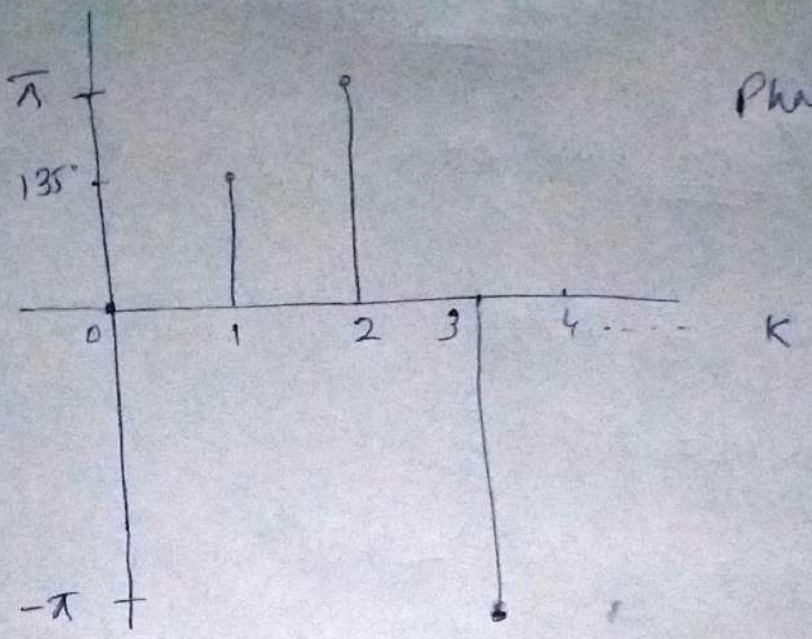
① Magnitude Spectrum

② Phase Spectrum.

① |CK|



Phase Spectrum



Exple:- $x[n] = \cos\left(\frac{\pi}{4}\right)n$

Find the Fourier Coefficient $C_K = ?$

DYS

Exple:- $x[n] = \cos\sqrt{2}\pi\, n$

Find the $C_K = ?$

DYS