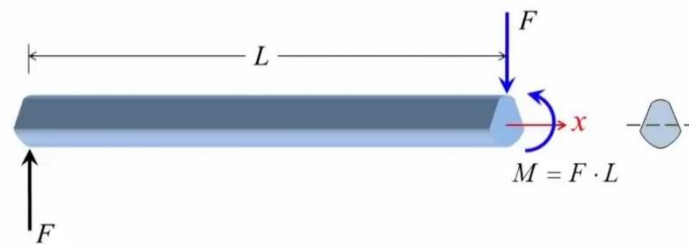
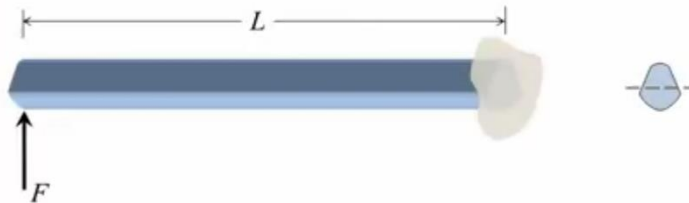
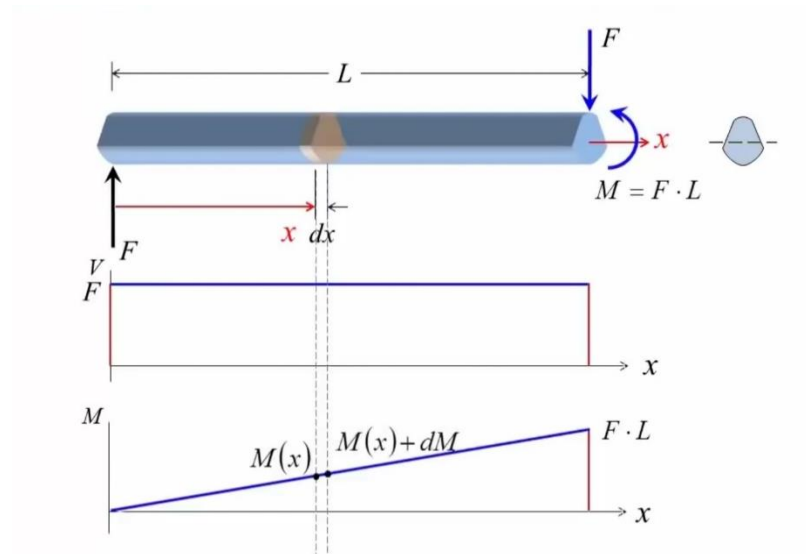
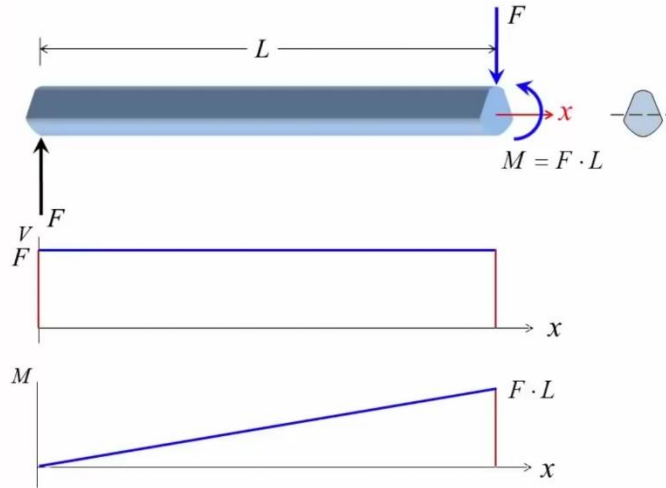


SHEAR STRESS EQUATION DERIVATION

Shear stress in a straight member

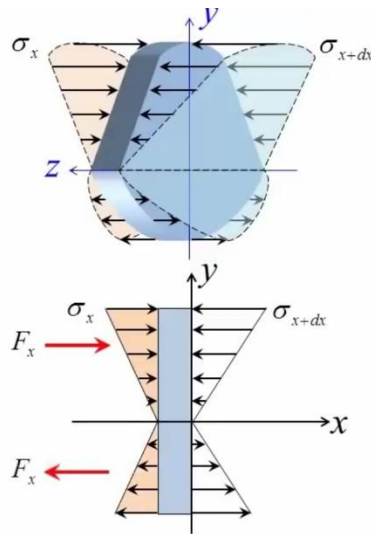
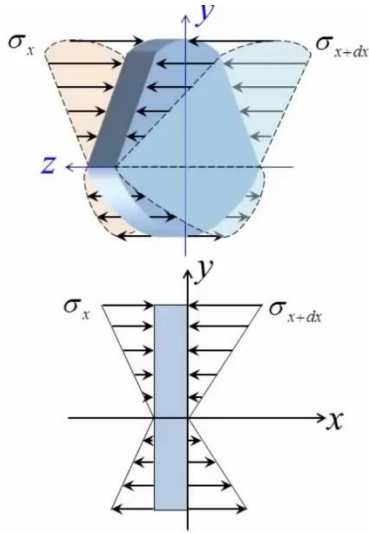


SHEAR STRESS EQUATION DERIVATION

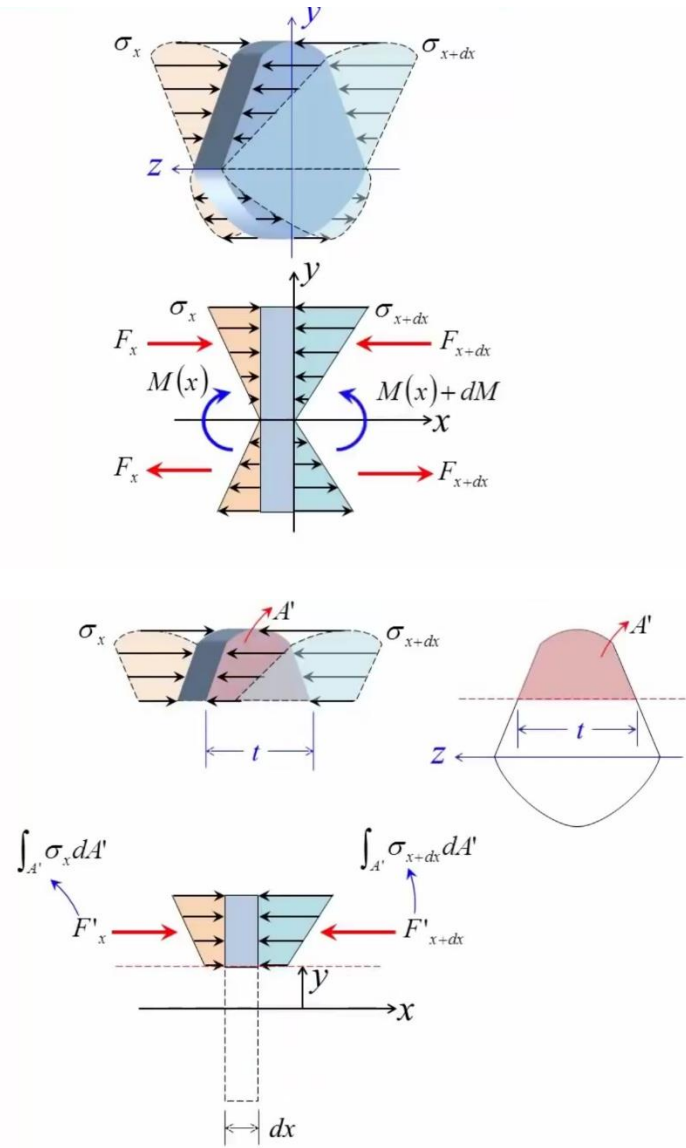


Flexural stress = MY/I

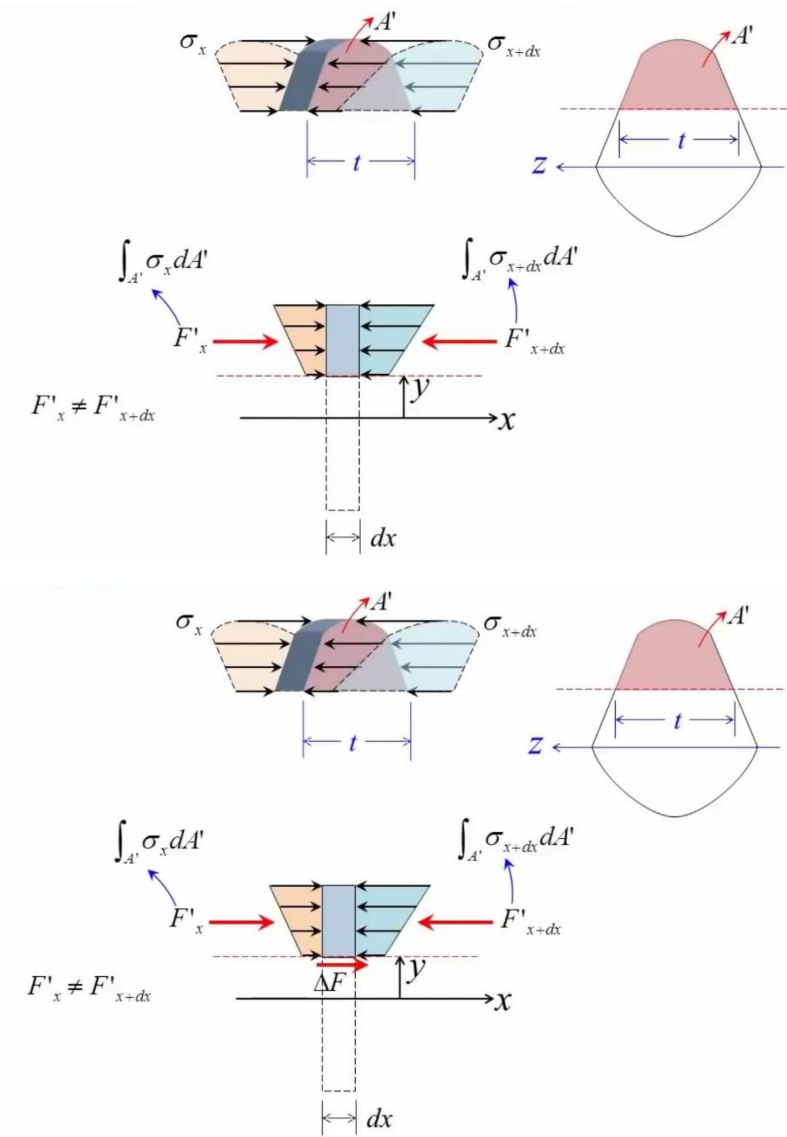
SHEAR STRESS EQUATION DERIVATION



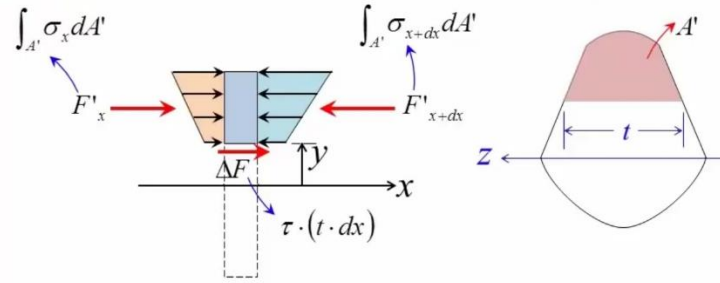
SHEAR STRESS EQUATION DERIVATION



SHEAR STRESS EQUATION DERIVATION



SHEAR STRESS EQUATION DERIVATION

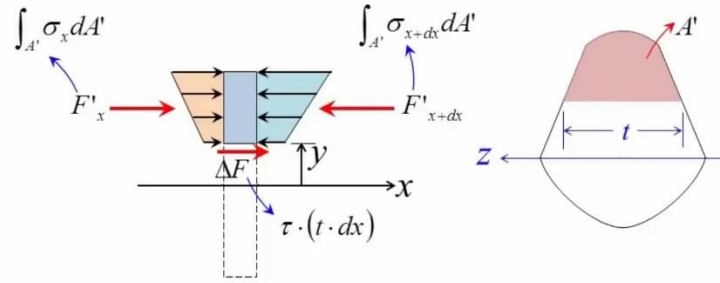


$$\sum F_x = 0 = \int_{A'} \sigma_x dA' + \tau \cdot t dx - \int_{A'} \sigma_{x+dx} dA' \quad \sigma = \frac{My}{I}$$

$$\int_{A'} \frac{My}{I} dA' + \tau \cdot t dx - \int_{A'} \frac{(M + dM)y}{I} dA' = 0$$

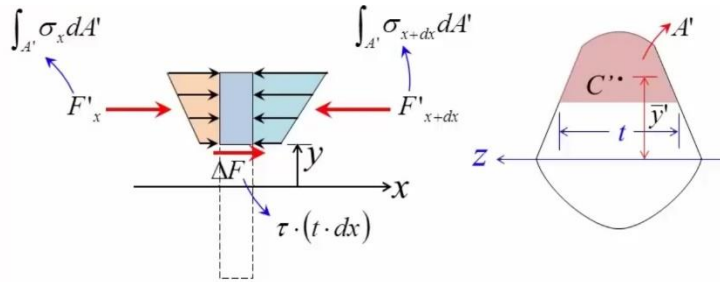
$$\tau \cdot t dx - \frac{dM}{I} \int_{A'} y dA' = 0 \quad \therefore \tau = \frac{1}{It} \frac{dM}{dx} \int_{A'} y dA'$$

SHEAR STRESS EQUATION DERIVATION



$$\tau = \frac{1}{It} \frac{dM}{dx} \int_{A'} y dA'$$

$$\frac{dM}{dx} = V$$



The shear formula

$$\tau = \frac{1}{It} \frac{dM}{dx} \int_{A'} y dA'$$

$$\frac{dM}{dx} = V$$

$$\bar{y}' = \frac{\int_{A'} y dA'}{A'}$$

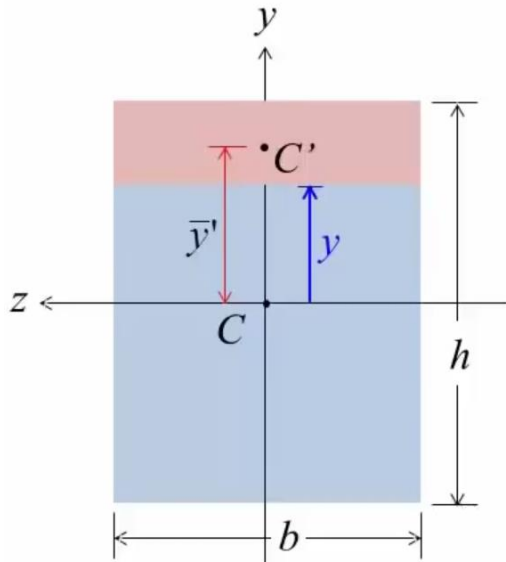
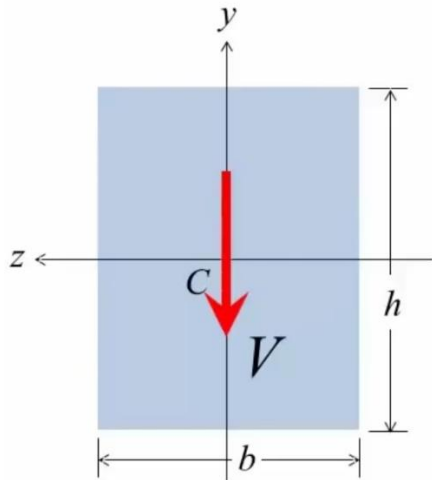
$$\int_{A'} y dA' = \bar{y}' A' = Q$$

Static moment of area **change with y**

$$\therefore \tau = \frac{VQ}{It}$$

SHEAR STRESS EQUATION DERIVATION

Example 1: For the rectangular cross-sectional area subjected to internal shear force V , determine the shear stress distribution as a function of y .



$$\tau = \frac{VQ}{It} \quad I = \frac{1}{12}bh^3$$

$$\bar{y}' = y + \frac{1}{2} \cdot \left(\frac{h}{2} - y \right) = \frac{1}{2} \left(\frac{h}{2} + y \right)$$

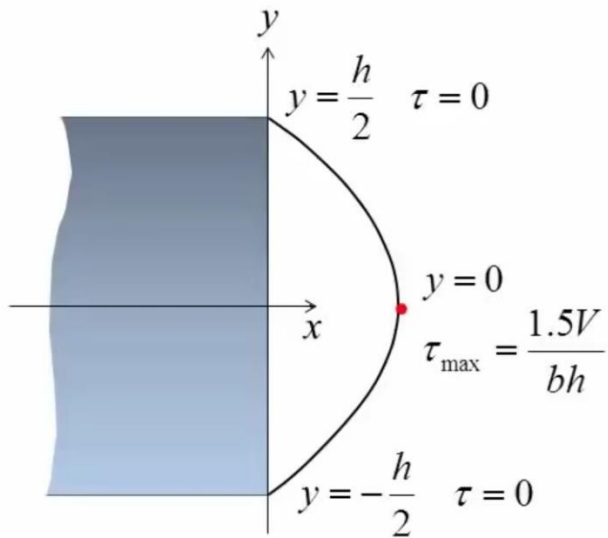
$$A' = b \cdot \left(\frac{h}{2} - y \right)$$

$$Q = \bar{y}' A' = \frac{b}{2} \left(\frac{h^2}{4} - y^2 \right) \quad t = b$$

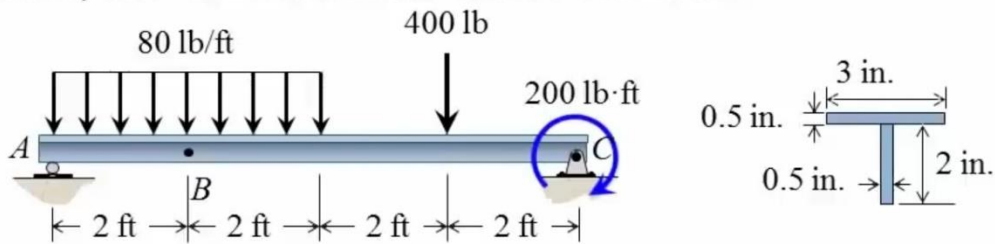
$$\therefore \tau = \frac{6V}{bh^3} \left(\frac{h^2}{4} - y^2 \right)$$

SHEAR STRESS EQUATION DERIVATION

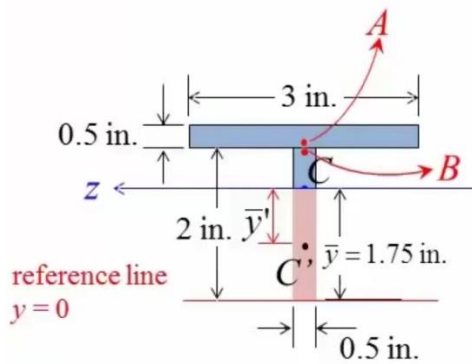
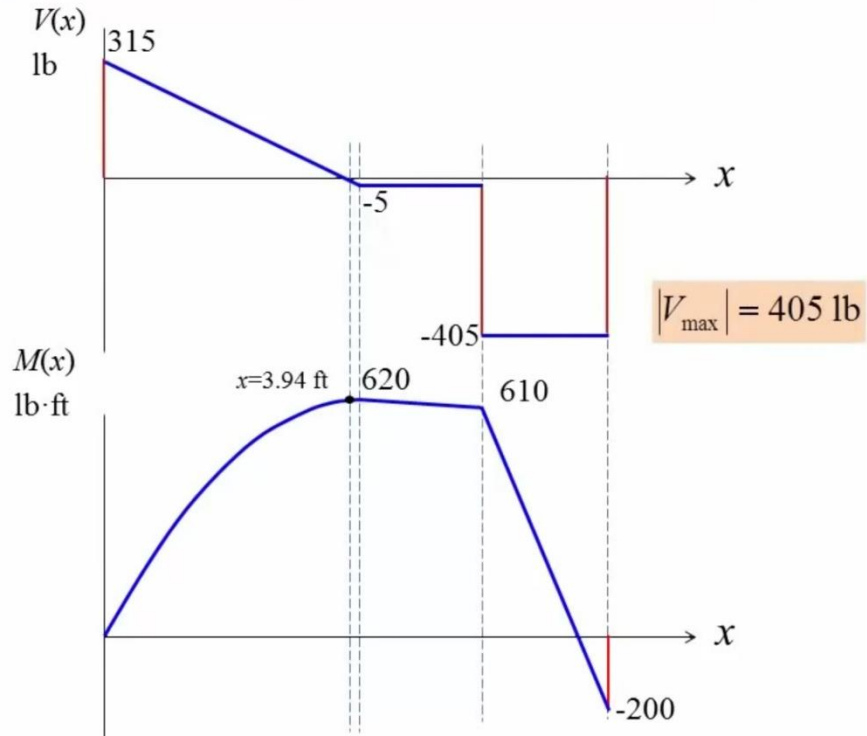
$$\tau = \frac{6V}{bh^3} \left(\frac{h^2}{4} - y^2 \right)$$



Example 2: For the over hanging beam with the shown loadings and cross section, determine the absolute maximum **shear** stress in the beam, and stretch the shear stress distribution.



SHEAR STRESS EQUATION DERIVATION



$$\tau = \frac{VQ}{It} \quad |V_{\max}| = 405 \text{ lb}$$

$$I_z = 1.30 \text{ in.}^4$$

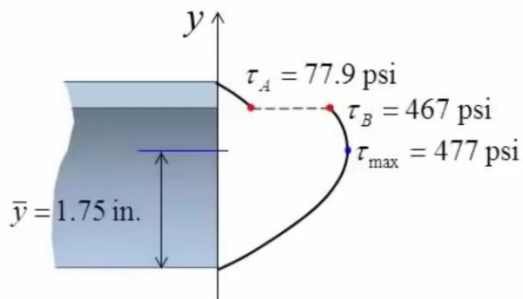
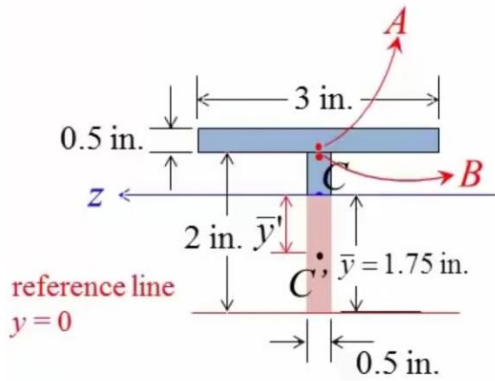
$$Q = \bar{y}' A' = \frac{1.75}{2} \cdot 1.75 \cdot 0.5 = 0.766 \text{ (in.}^3\text{)}$$

$$\tau_{\max} = \frac{405 \cdot 0.766}{1.30 \cdot 0.5} = 477 \text{ (psi)}$$

$$\tau_A = 77.9 \text{ psi}$$

$$\tau_B = 467 \text{ psi}$$

SHEAR STRESS EQUATION DERIVATION



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