Recursive Definition

Recursive Definition (RD):

An RD is a three step process to validate the instances of any given language. In which

1. First we specify some basic objects in the set.
2. Second, we give Rules for constructing more objects.
3. Third, we declare that only those objects are valid that are in accordance with Rule-1 and rule-2.

A Recursive Definition is called Recursive because one of the rules is called reclusively by itself again and again.

Example-1: RD for Even Numbers.

Rule-1: 2 is in Even (Hint: Basic object of Even Numbers is 2)
Rule-2: If x is in Even, then so is x+2 (Hint: Rule for constructing more objects in Even Numbers)

Let's try to understand it by proving that 12 is in Even or not.

Proof:

By Rule-1: 2 is in Even
By Rule-2: 2+2 = 4 → 4 is in Even
By Rule-2: 4+2 = 5 → 6 is in Even
By Rule-2: 6+2 = 8 → 8 is in Even
By Rule-2: 8+2 = 10 → 10 is in Even
By Rule-2: 10+2 = 12 → 12 is in even
Hence Proved

Example-2: RD for Polynomials.

Rule-1: Any number is in Polynomial
Rule-2: The variable x is in Polynomial
Rule-3: If p and q are in Polynomials then so are
$p + q$
$p – q$
pq OR p*q
p / q
p^q
(p)

Show that $3x^2 + 7x - 9$ is in Polynomial using above RD.

Proof:

By Rule-1:	3 is in Polynomial		
By Rule-2:	x is in Polynomial		
By Rule-3:	3*x is in Polynomial		
By Rule-3:	$3x^2$ is in Polynomial	→	$3x^2$ is in Polynomial
By Rule-1:	7 is in Polynomial		
By Rule-3:	$3x^2+7$ is in in Polynomial	→	$3x^2+7$ is in Polynomial
By Rule-1:	9 is in Polynomial		
By Rule-3:	$3x^2+7-9$ is in Polynomial	→	$3x^2+7-9$ is in Polynomial

Hence Proved

Exapmle-3: RD for L = x^+ = {x xx xxx xxxx xxxxx ……}

Rule-1: x is in L
Rule-2: If w is any word in L, then xw will also be in L

Example-4: RD for L = x* = {^ x xx xxx xxxx …..}

Rule-1: ^ is in L
Rule-2: If w is in L, then xw is also in L

Example-5: RD for Kleene Closure

Rule-1: If S is a language, then all the words of S are in S*.
Rule-2: ^ is in S*.
Rule-3: If x and y are in S*, then so is their concatenation xy.

Example-6: RD for Arithmetic Expression (AE)

Rule-1: Any number (+ve, -ve or 0) is in AE
Reule-2: If x is in AE, then so are
   i. (x)
   ii. –x
Rule-3: If x and y are in AE, then so are
   i. x+y
   ii. x-y
   iii. x*y
   iv. x/y
   v. x^y

Theorem: An AE cannot contain the character $.

Proof:

By Rule-1: $ is not part of any number, so it cannot be included in an AE.
By Rule-2: As x does not contain $, so as (x) or (-x) cannot contain $.
By Rule-3: As neither x nor y can contain $, so any of the expressions defined by Rule-3 can also not contain $.

Therefore: the character $ can never get into an AE.

You may watch the following video to make your concepts more clear.

https://www.youtube.com/watch?v=2-3kzQU_pfM