⇒ The RADAR Equation :-

Radar equation accomplishes the following:

✓ Assists in the design of radar systems to meet the detection specifications set by the users.

✓ Establishes the relationship between the signal power received and the radar and target parameters.

✓ Describes the power received from interfering sources, including thermal noise, clutter, jamming and EMI.

✓ Provides means of predicting signal to interference ratios, and for predicting the maximum range at which targets of a given RCS (Radar cross section) will produce a specified signal-to-interference ratio.

↳ From Exp 1-11. we will discuss various parameters of Radar Equation one by one :

<u>Exp 1-11:-</u> Radar's specification:

Transmit Power: 3,000,000 Watts (3 MW)

Antenna Gain : 4,500

Antenna effective aperture : $20 m^2$

Transmit frequency: 1.27 GHz (in the L-Band)

Transmit pulse width : $2.5 \mu s$

Pulse repetition freq: 350 pps

Receiver noise factor: 2.5

The target which this radar will attempt to detect has the following specifications:

Radar cross-section : $10 m^2$

Range from radar: 300 nmi

* Transmit: A transmitter generates RF energy, in this case a pulse of width 2.5 μs.
  Exp 11.1a: In this radar would emit a pulse of about one to five μs width (1—5 μs), and would five 350 times per seconds. During the time the transmit pulse is ON, it power is 3MW

* Antenna Gain: The radio energy from the transmitter is concentrated in a prefered direction by the antenna. The degree of concentration is the gain of the antenna.

$$\boxed{G \simeq \frac{\text{Solid angle in a sphere}}{\text{Solid angle in the antenna's beam}}}$$

For large microwave antennas the directivity and gain are same.

- The power effectively radiated by transmitter/antenna combinations in direction of main beam is called Effective Radiated Power ERP.

$$\boxed{ERP = P_T G_T}$$

ERP is radar effective radiated power (watts).
P_T is the transmit power deliver to antenna. (watts).
G_T is the gain of radar's transmit antenna.

Exp 1.11b: The ERP of the example radar is the transmitter power of 3,000,000W times the antenna gain of 4,500 or 13,500,000,000 W

* Forward Propagation / Forward Power density: The power density of the forward signal is the power per unit area of beam crossection (w/m^2). It equals the power transmitted divided by the area of bean at target's range. i.e.

$$\boxed{P/A_F = \frac{P_T}{4\pi R_T^2/G_T}}$$

P/A_F = the forward power density at the target range. w/m^2

R_T = range from radar transmitter to target (meters).

$4\pi R_T^2$ = The surface area of a sphere of radius R_T

Exp-1.11c :- 300nmi equals 555,600 m ($\because$ 1nmi = 1852m). A sphere of radius 555,600 m has surface area of :

$$A = 4\pi r^2 = 4(3.14)(555600)^2$$

surface area of sphere = 3.879×10^{12} m²

The radar cross section is 10m².

The antenna beam illuminates $\frac{1}{4500}$ of that area

means that illuminated area $= \frac{1}{4500} \times 3.879\times10^{12}$

$= 862{,}000{,}000$ m².

The total transmitted power is 3,000,000W.

So the power per unit area in the beam at the target's range is :

$$= \frac{3{,}000{,}000}{862\,000\,000} = 0.00348 \text{ W/m}^2.$$

OR: we use directly formula of forward power density:

$$P/A_F = \frac{P_T}{4\pi R_T^2 / G_T}$$

$$= \frac{3{,}000{,}000 \times 4500}{(4)(3.14)(555{,}600)^2} = 0.00348 \text{ W/m}^2$$

* <u>Target Reflection / Power Reflected</u> :- The portion of the reflected energy that propagates in the direction of radar's receiving antenna is called Backscatter, It is the only reflected energy which matters to radar.

The power reflected from target is proportional to the illumination power density and to the reflection characteristic of target (radar cross-section).

$$P_{Tgt} = \frac{P}{A_F}\sigma$$

$$\boxed{P_{Tgt} = \frac{P_T G_T}{4\pi R_T^2}\sigma}$$

P_{Tgt} = the effective power reflected by the target in the direction of radar (watts)

σ = the target's radar cross section (m²).

Exp 1-11 d :- The illumination power density at the target is $0.00348\,W/m^2$

The target has a RCS $= 10m^2$.

So the capture echo is: 0.00348×10

$= 0.0348\ W$ radiated by target.

* Backscatter Propagation: is given by:

$$\boxed{P/A_B = \frac{P_T G_T \sigma}{4\pi R_T^2} \; \frac{1}{4\pi R_R^2}}$$

$$\frac{P_T G_T \sigma}{4\pi R_T^2} = P_{Tgt}$$

backscatter

P/A_B ; is the backscatter power density at the radar's receiving antenna (watts/meter^2)

R_R = the range from the target to the radar's receive antenna (m).

Exp 1-11e: The target now transmit with power $= 0.0348\ W$. The range from target to radar $= 555600m$. Find ~~backsk~~ backscatter propagation / power density?

$$P/A_B = P_{Tgt} \frac{1}{4\pi R_R^2} = 0.0348 \times \frac{1}{4\pi (555600m)^2}$$

$$P/A_B = 8.97 \times 10^{-15}\ w/m^2$$

* Affective Area: Echo signal and Interference capture :- The echo power which propagates back to the radar is captured by the Effective area of the receiving antenna. "The effective area of a typical radar antenna is about one-half of it's actual area." So the echo power received from target is:

$$\boxed{P_R = P/A_B\ A_E}$$

P_R is echo power received from target (watts)

A_E effective capture area of receiving antenna (sq. meter)

Exp 1-11f: Effective area of radar $= 20m^2$

Power density of echo $= 8.97 \times 10^{-15}\ w/m^2$

So, power received / capture from target is:

$$P_R = 8.97 \times 10^{-15} \times 20 = 1.79 \times 10^{-13}\ w.$$

We can also write echo power as:

$$\boxed{P_R = \frac{P_T G_T \sigma A_E}{(4\pi)^2 R^4}}$$

$$\because P/A_{e} = \frac{P_T G_T}{(4\pi)^2 R_T^2 R_R^2}$$

Above is simplified version of Radar Equation, but it ignores all losses (losses in radar system & losses in the propagation path).

$$P_R = \frac{K_R \sigma}{R^4 L_A}$$

rearranging and including losses, RCS of target and range.

where, $K_R = \dfrac{P_T G_T A_E}{(4\pi)^2 L_S}$

L_S losses in radar system.
L_A " " propagation path.
K_R represent Radar's parameter.

can be further solved as:

$$K_R = \frac{P_T G_T^2 \lambda^2}{(4\pi)^3 L_S} \qquad \because \quad G = \frac{4\pi}{\lambda^2} A_E \Rightarrow A_E = \frac{G\lambda^2}{4\pi}$$

Exp#1.g: Find K_R for radar used in this example: Find signal echo power received from a $0.1\,m^2$ target at 95nmi. Ignore losses (L_S and L_A are unity).

We know that $K_R = \dfrac{P_T G_T A_E}{(4\pi)^2 L_S}$

$$K_R = \frac{(3{,}000{,}000)(4500)(20)}{(4)^2(\pi)^2(1)} = 1.71 \times 10^9 \ \mathrm{wm}^2.$$

Find $P_R = ?$

$$P_R = \frac{K_R \sigma}{R^4 L_A}$$

RCS = $0.1\,m^2$
R = 95nmi = 175,940m

$$= \frac{1.71\times 10^9 \times 0.1}{(175940)^2 (1)}$$

$$P_R = 1.78 \times 10^{-13}\ \mathrm{W}$$

* **Thermal Noise at Receiver:-** Thermal noise is mainly generated at radar itself. The purpose of receiver's match filter is to reduce effect of thermal noise and noise jamming effectively. The thermal noise power, at the receiver's input is:

$$\boxed{P_N = K T_0 B F}$$

P_N is noise power at the input to the receiver.

K is Boltzman's constant (1.38×10^{-23} J/K)

$T_0 = 290°K$

B is noise bandwidth of system.

F is noise factor (if in dB, then it is noise figure).

Exp 1-11h: The noise factor is 2.5 at 290°K. Find the noise power:

We know that noise bandwidth is the reciprocal of processed pulse width, so.

$$B = \frac{1}{2.5\,\mu s} = 400000 \text{ Hz}.$$

$$F = 2.5$$

$$K = 1.38 \times 10^{-23} \text{ J/K}$$

$$T_0 = 290°K$$

So $P_N = K T_0 B F$

$$= (1.38 \times 10^{-23})(290°)(400{,}000)(2.5)$$

$$P_N = 4.0 \times 10^{-15} \text{ W}$$

* <u>Signal to Interference Ratio</u> :- determines whether or not sufficient signal is present to detect the target. It is the ratio of signal echo power to the interfering power. and given as:

$$\boxed{S/N = \frac{P_R}{P_N}}$$

S/N is signal power to noise ratio.
P_R is signal echo power at the input to the receiver
P_N the noise power at the input to the receiver.

<u>Exp 1-11</u>: Find S/N in the example?

we know: $S/N = \frac{P_R}{P_N}$

we have already found: $P_R = 1.79 \times 10^{-13}$ W

$P_N = 4.0 \times 10^{-15}$ W

So $S/N = \frac{1.79 \times 10^{-13}}{4.0 \times 10^{-15}}$

$\frac{S}{N} = 44.75$

or in dB

$\frac{S}{N}_{dB} = 10 \log 44.75$

$\frac{S}{N}_{dB} = 16.5$ dB.

* <u>Radar Eq</u>:

$$\frac{S}{N} = \frac{P_R}{P_N}$$

$$\boxed{\frac{S}{N} = \frac{P_T G_T^2 \lambda^2 \sigma}{R^4 (4\pi)^3 L_a L_s K T_o B F}}$$