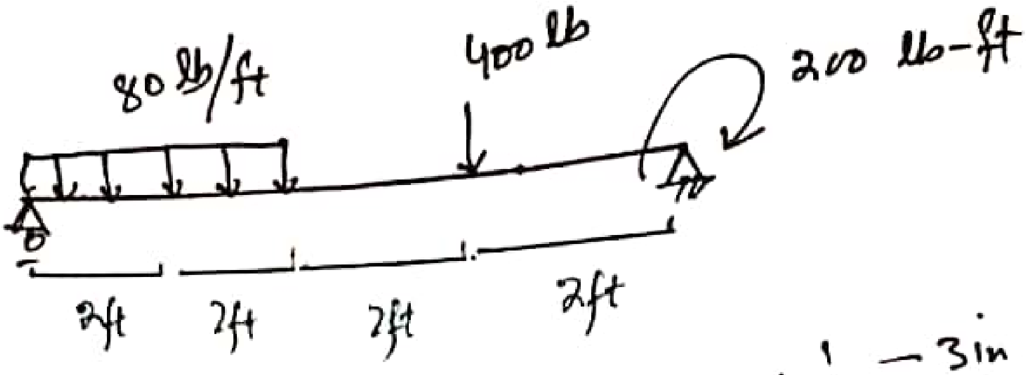
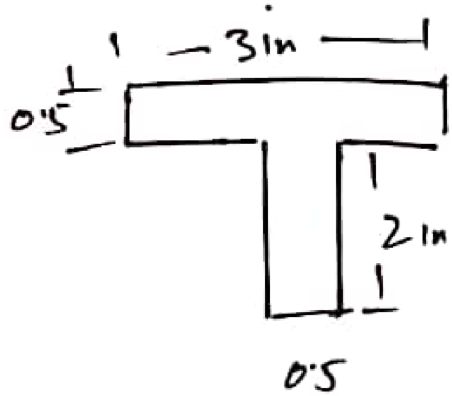
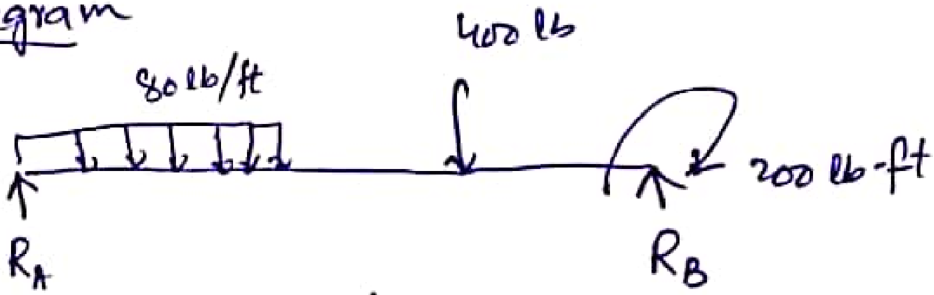




Solve

1 First Find The Reaction Shear force & Bending moment Diagram.

Free body Diagram



$\Sigma F_y = 0$ ↑+ upward is positive

$$R_A + R_B - 80 \times 4 - 400 = 0$$

$$R_A + R_B = 720 \text{ lb}$$

$\Sigma M_A = 0$ ↺+ Anti clockwise is positive

$$R_B(8) - 200 - 400(6) - 80 \times 4(2) = 0$$

$$R_B(8) = 3240$$

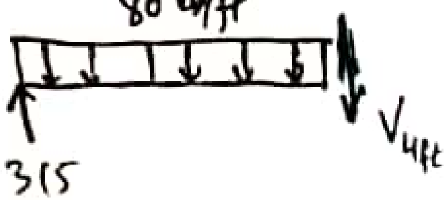
$$\boxed{R_B = 405 \text{ lb}}$$

Now

$$R_A = 720 - 405$$

$$\boxed{R_A = 315}$$

Now Shear force at change point of Beam



Shear force at <u>4ft</u> from left support

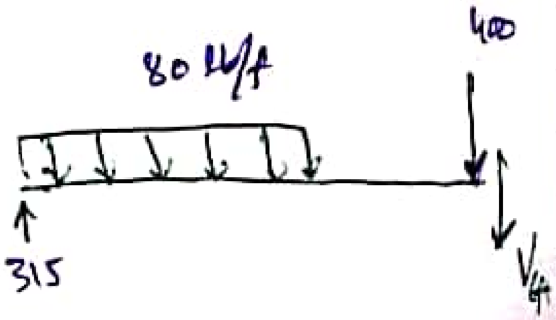
$\Sigma F_y = 0 \uparrow +$

$$-V_{4ft} + 315 - 80 \times 4 = 0$$

$$V_{4ft} = -5 \text{ lb}$$

Shear force at <u>6ft</u>

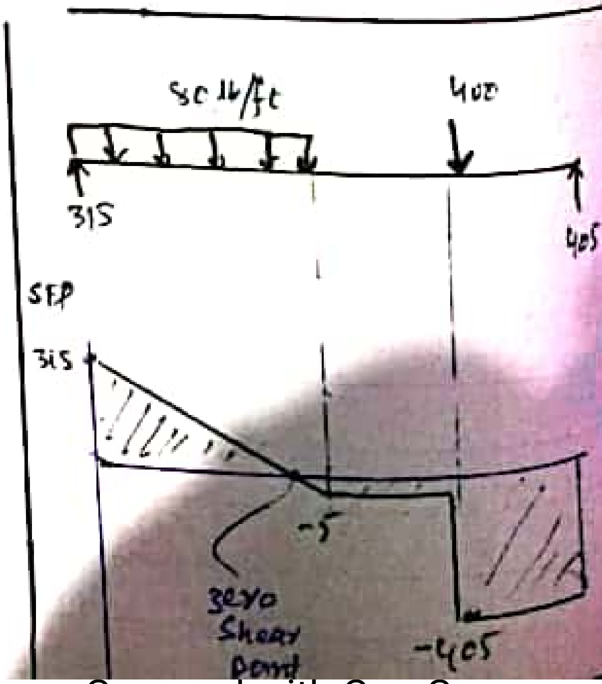
$\Sigma F_y = 0$



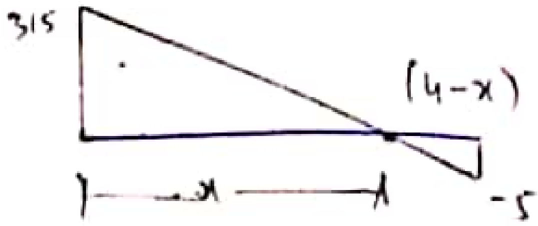
$$315 - 80 \times 4 - 400 - V_{6ft} =$$

$$-V_{6ft} = 405$$

$$\boxed{V_{6ft} = -405 \text{ lb}}$$



Moments at Change Points



Find zero Shear point

$$\frac{315}{x} = \frac{5}{(4-x)}$$

$$315(4-x) = 5(x)$$

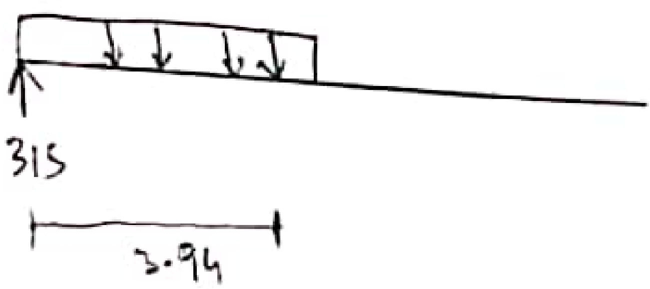
$$1260 - 315x = 5x$$

$$x = 3.93 \text{ ft}$$

As we know that Moment is max where Shear force is zero

Take Section at 3.93 ft. from left Support & Find Moment

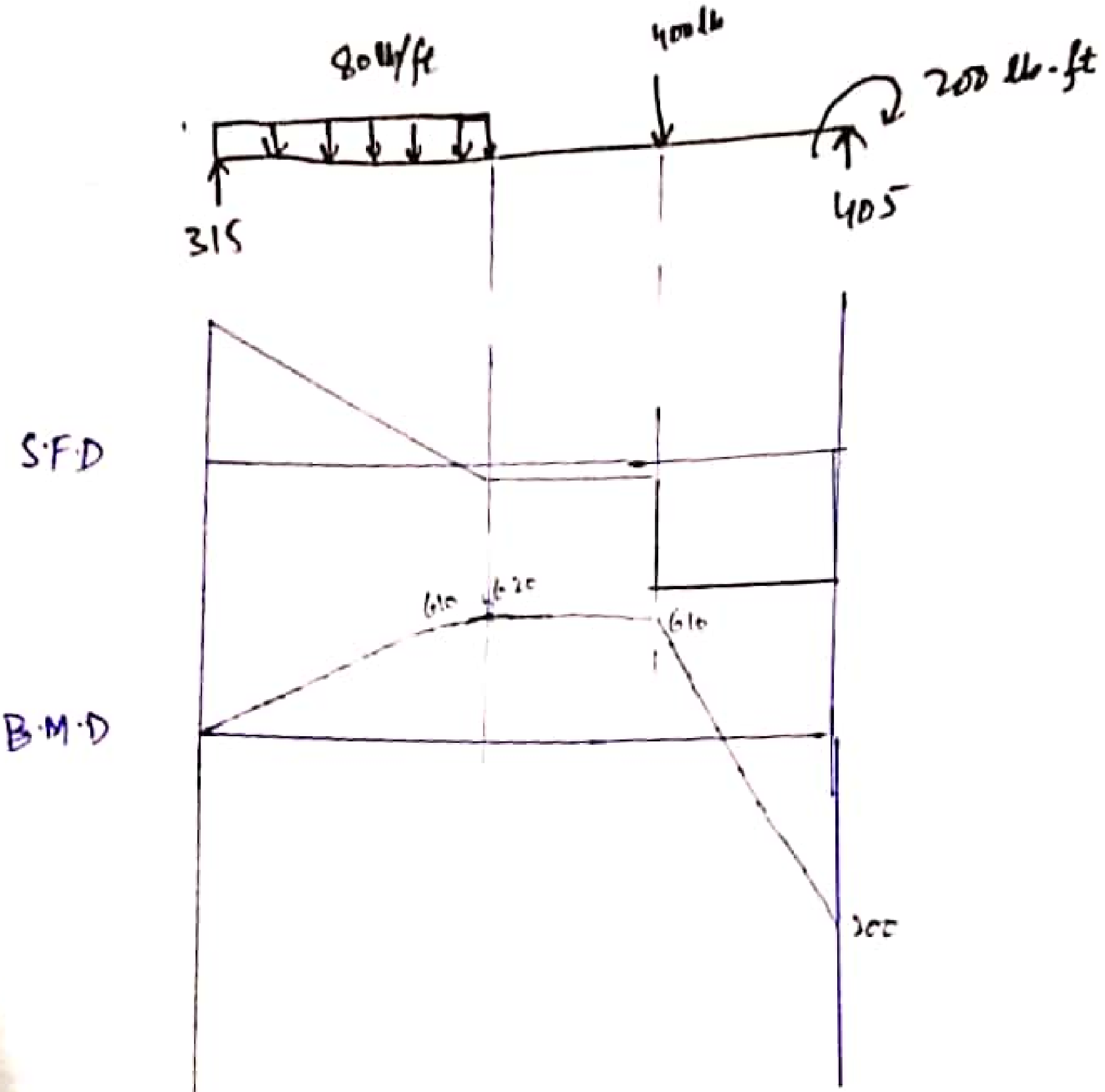
$$\Sigma M_{3.94} = 0 \circlearrowleft +$$



$$M_{3.94} - 315(3.94) + 80(4)\left(\frac{3.94}{2}\right) = 0$$

Max Moment $= M_{3.94} = 610.7$

$M_{4ft} = 620$ Moment at 4 ft from left Support.



Shear Stress

As Per The Question the maximum Shear Stress $\tau = \frac{VQ}{It}$ accurs where the maximum Shear force lies. In above diagram max Shear force is 405 lb

Note: it is also possible to Find the Shear Stress at any other Section along The length of Beam i.e 4 ft from left Support etc. in That case V = Shear force will be other than 405 lb

Now to find the Shear Stress $\tau = \frac{VQ}{Ib}$ it is necessary to find 'I' Moment of Inertia of the Given Cross-section. In this case the Section is T Section

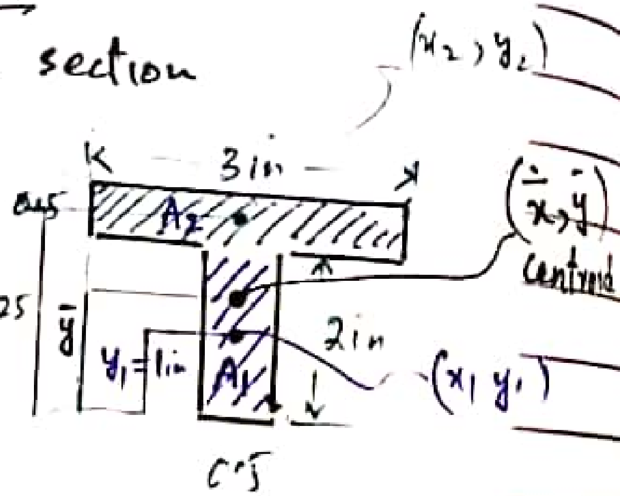
Note

If the Section of Beam given in problem is other than T section the moment of Inertia will be change

Moment of Inertia of T section

To Find the M. Inertia First Find Geometric Centre whose coordinat is $(\bar{x}, \bar{y})$

$\bar{x}$ is zero becouse Section is Symmetrical about y axis



$A_1 = 2 \times 0.5$
$= 1 in^2$

$A_2 = 3 \times 0.5$
$1.5 in^2$

$$\bar{y} = \frac{Y_1 A_1 + Y_2 A_2}{A_1 + A_2} = \frac{1(1) + (2.25)(1.5)}{1 + 1.5}$$

$$\boxed{\therefore \bar{y} = 1.75}$$

Now Moment of Inertia of the whole Section about x-axis (reference axis) is

$$I_x = I_{1c} + I_{2c}$$

where

$$I_{1c} = I_1 + A_1 d_1^2$$

$$I_1 = \frac{bh^3}{12} = \frac{(0.5)(2)^3}{12} = 0.33$$

$$d_1 = \bar{y} - y_1 = 1.75 - 1$$

$$d_1 = 0.75$$

Note: d_1 is Distance or length between geometric centre of whole Section and centre of A_1

$$I_{1c} = 0.33 + (1)(0.75)^2$$

$$= 0.8925 \text{ in}^4$$

&

$$I_{2c} = I_2 + A_2 d_2^2$$

$$I_2 = \frac{bh^3}{12} = \frac{3(0.5)^3}{12} = 0.0312$$

$$d_2 = y_2 - \bar{y} = 2.25 - 1.75$$

$$d_2 = 0.5$$

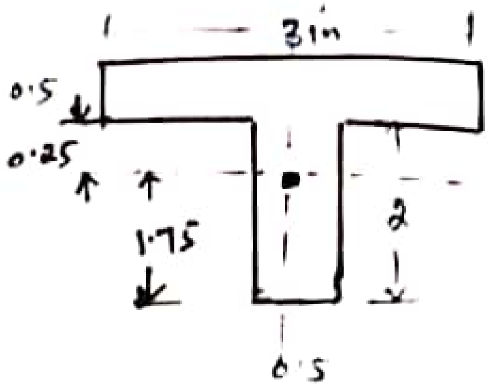
Note: d_2 is Distance or length b/w geometric Centre of whole Section & Centre of A_2

$$I_{2c} = 0.0312 + (1.5)(0.5)^2$$

$$I_{2c} = 0.4062$$

Now

$$\boxed{I_x = 0.8925 + 0.4062 = 1.3 \text{ in}^4}$$



As we evaluate the Shear force, Bending Moment Diagram and Moment of Inertia of Section it is possible to calculate the Shear Stress & Flexural Stress at any point in the Beam.

For Shear Stress $\tau = \frac{VQ}{Ib}$

$V_{max} = 405$ lb

$I = 1.3 \text{ in}^4$

Shear Stress along the Depth of Section $Q = \bar{y}A$

Cas-1 τ at Top fibre

$$\tau_{Top\ fibre} = \frac{VQ}{Ib} = \frac{(405)[0]}{(1.3)(3)}$$

$$= 0$$

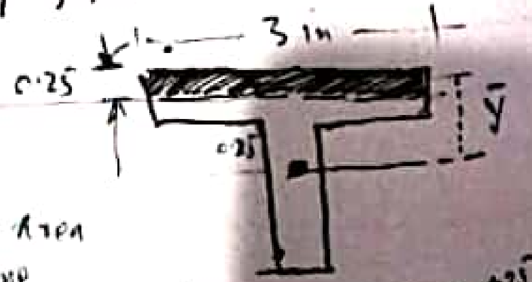
Note: As there is no Area of Section above Top fibre the value of $Q = \bar{y}A$ is zero. i.e $\boxed{A = 0}$

Cas-2 τ at 0.25 in below the Top fibre

$$\tau = \frac{405\,[(0.625)\underbrace{(0.25 \times 3)}_{A}]}{(1.3)(3)}$$

A is Shaded Area above the line where Stress is to be find

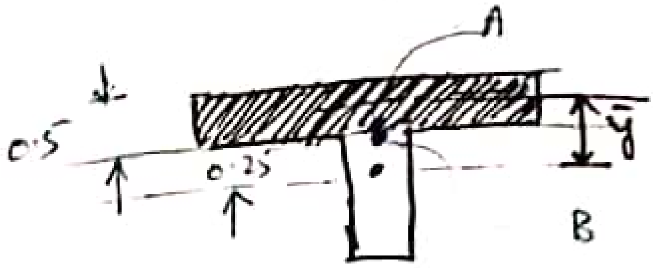
$$\tau = 48.67 \text{ psi}$$



$\bar{y} = 0.25 + 0.25 + \frac{0.25}{2}$

$\bar{y} = 0.625$ in

Cas-3 Find Stress at 0.5 in below the top fibre
Two Cases A, & B



$$\tau_A = \frac{VQ}{Ib}$$

$$= \frac{(405)(0.5 \times 1.5)}{1.3 \times 3}$$

$$\boxed{\tau_A = 77.9 \text{ psi}}$$

$A = 0.5 \times 3 = 1.5$

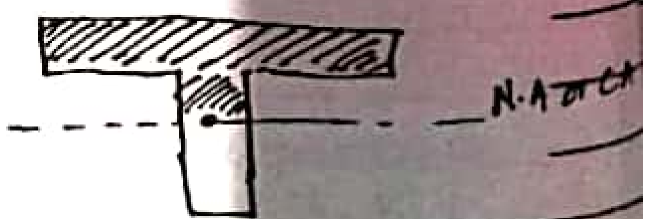
$\bar{Y} = 0.25 + 0.5/2 = 0.5$

$$\tau_B = \frac{405(0.5 \times 1.5)}{1.3 \times 0.5}$$

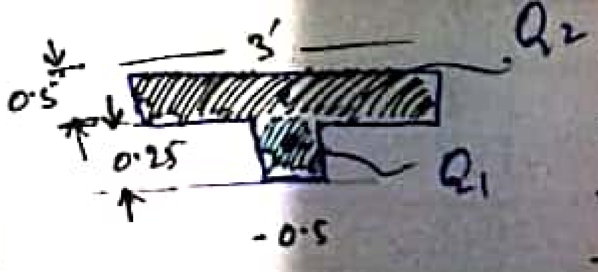
$$\tau_B = 467.30$$

Note
In above diagram
The point A lies in
Flange Section with
Thickness $b = 3$ in
&
Point B lies in
Web Section where
Thickness is $b = 0.5$

Case-4 Find Stress at the Centroidal axis
In this Case their are
Two ways to Solve the
Shear Stress at Centroidal axis



1st Take area above the Centroid
and find Q



$$Q = Q_1 + Q_2$$

$$Q_1 = \bar{Y}_1 A_1 = 0.25/2\,(0.25 \times 0.5)$$
$$= 0.0156$$

$Q_2 = [0.5 \times 1.5] =$

$Q = Q_1 + Q_2$

$Q = 0.0156 + 0.75 = 0.765$

Now

$$\tau_{max} = \frac{405 \times 0.765}{1.3 \times 0.5} = 477 \text{ psi}$$

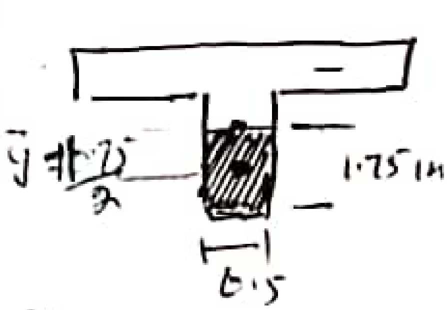
2nd Method

Taking area below the Centroid

$Q = \bar{y}A$

$\boxed{Q = 0.765}$

$\boxed{A = 1.75 \times 0.5 \quad A = 0.875}$



$$\tau_{max} = \frac{405 \times 0.765}{1.3 \times 0.5}$$

$\bar{y} = \frac{1.75}{2}$

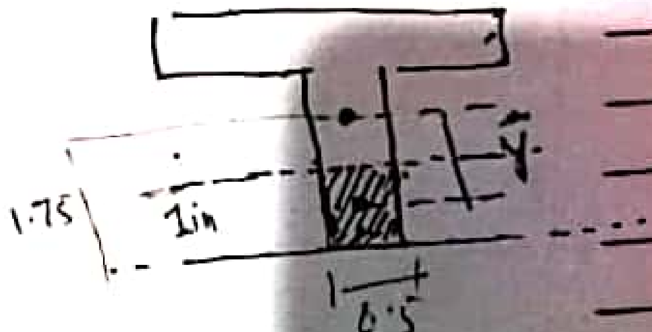
$\bar{y} = 0.875$

$$\boxed{\tau_{max} = 477 \text{ psi}}$$

Cas-5 Find Shear Stress In above the Bottom line of Bottom fibre

$$\tau = \frac{VQ}{Ib} = \frac{405\,(1.25 \times 0.5)}{1.3 \times 0.5}$$

$\tau = 390$ psi



$A = 1 \times 0.5 = 0.5 \text{ in}^2$

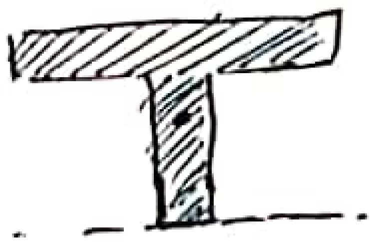
$\bar{Y} = 1.75 - \frac{1}{2} = 1.25$

Case - 6 Find Stress at Bottom fibre.

$$\tau = \frac{VQ}{Ib}$$

$$= \frac{(405)(0)}{1.3 \times 0.5}$$

$$\boxed{\tau = 0}$$



$Q = \bar{y}A$

Area is Maximum

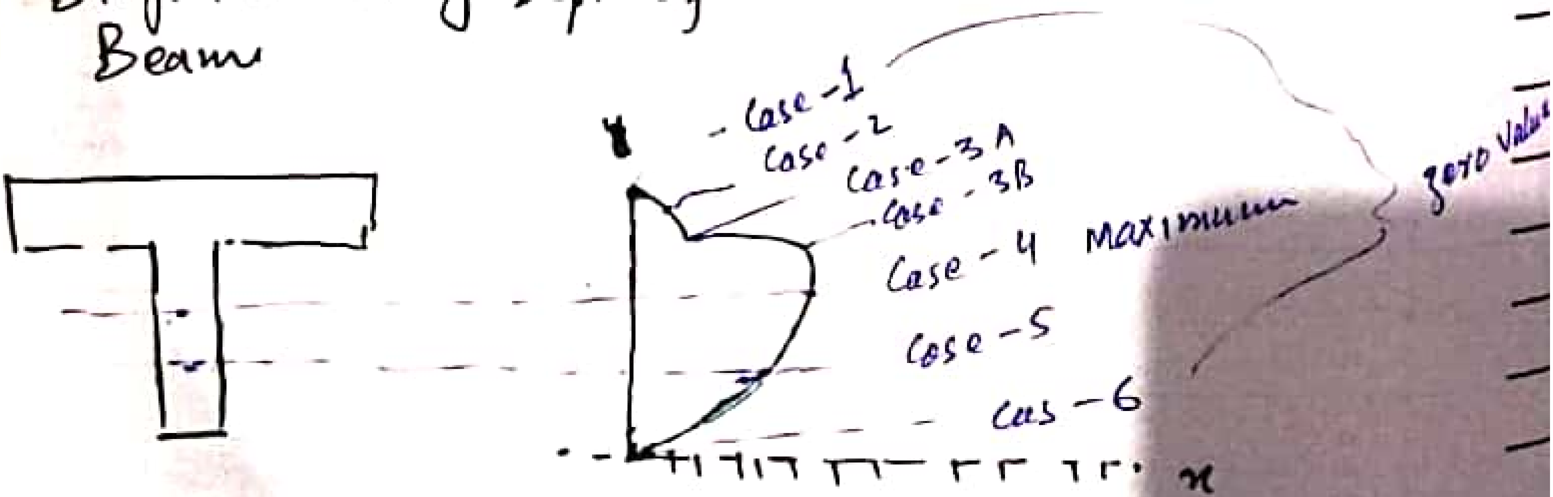
$A = 3\times0.5 + 2\times0.5$

But $\bar{y} = 0$ Because the centre of the Shaded area Coincide with the geometric centroide

$\bar{y} - \bar{Y} = 0$

$Q = 0$

Shear Stress Variation Diagram along Depth of Beam



Case – 7

Find Maximum Shear Stress at a distance of 4 ft. from left Support of Beam along its length

As we know from Shear force Diagram The Shear force at 4ft from left support is $V = 5 lb$ (See on page 2 of This notes)

$$\tau_{max} = \frac{VQ}{Ib} = \frac{5 \times 0.765}{1.3 \times 0.5}$$

Note
$Q = 0.765$
(See Case-4 of This notes page #9)

$$\tau_{max} = 5.88 \text{ psi}$$

Case – 8

Find Shear Stress at a distance of 4ft from left Support and 0.25 in below The Top fibre

$$\tau = \frac{VQ}{Ib}$$

for This case
$V = 5 lb$ & (see shear force Diagram)
$Q = 0.468$ (See case-2 on page

$$\tau = \frac{5 \times 0.468}{1.3 \times 3}$$

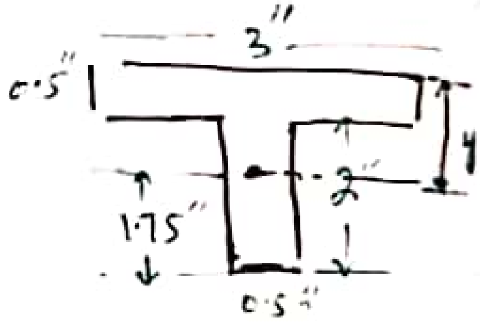
$$\tau = 0.6 \text{ psi}$$

Flexural Stress Analysis

Although it is possible to find flexural Stress anywhere along the length of Beam. But for Structural Design the maximum moment is to be considered

The Maximum Moment in B.M.D (see on page #09) is 620 lb-ft

Flexural Stress $= \sigma = \dfrac{My}{I}$



Cas-1 Find Stress at Top fibre

$$\sigma_{Top} = \frac{(620)(0.75)}{1.3}$$

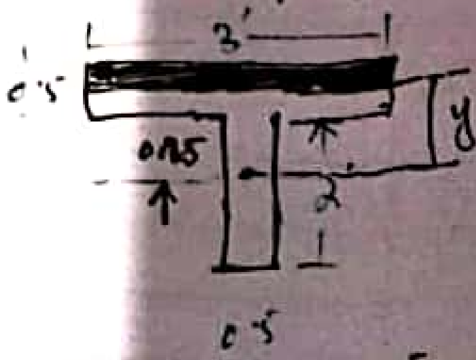
$y = 0.25 + 0.5 = 0.75$

$$\boxed{\sigma_{Top} = 357.6 \text{ psi}}$$

Cas-2 Find Stress at 0.25 in below the Top fibre

$$\sigma_{0.25} = \frac{My}{I} = \frac{620(0.5)}{1.3}$$

$$\boxed{\sigma_{0.25} = 238.4 \text{ psi}}$$

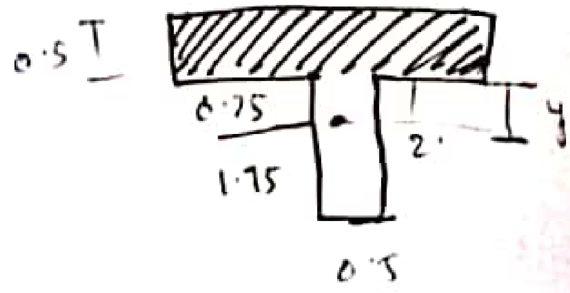


$y = 0.25 + 0.25$

Cas-3 Find Flexural Stress at 0.5in below the Top fibre

$$\sigma_{0.5} = \frac{My}{I} = \frac{620(0.25)}{1.3}$$

$$\boxed{\sigma_{0.5} = 119.2 \text{ psi}}$$



$y = 0.25$

Cas-4 Find Flexural Stress at the geometrical Centroid

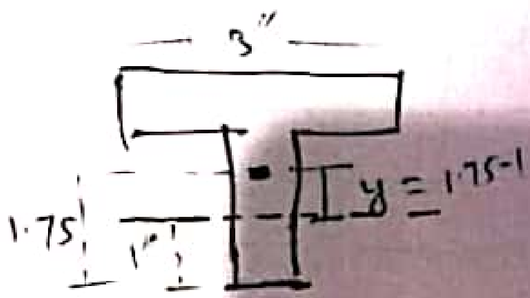
As $y = 0$

$$\sigma_{centr} = \frac{My}{I} = 0$$

Cas-5 Find Flexural Stress at 1in above the Bottom fibre

$$\sigma = \frac{My}{I} = \frac{620 \times 0.75}{1.3}$$

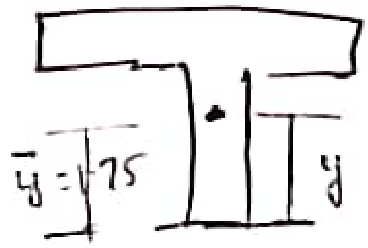
$$\boxed{\sigma = 357.69 \text{ psi}}$$



$y = 1.75 - 1$

$$\boxed{y = 0.75}$$

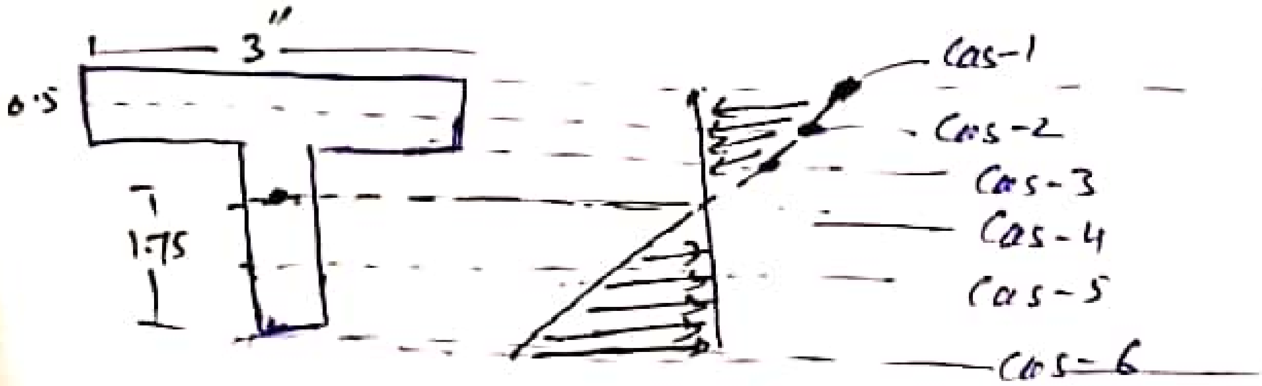
Case -6 Find Flexural Stress at the Bottom fibre



$$\sigma_{min} = \frac{My}{I}$$

$$= \frac{620 \times (-1.75)}{1.3}$$

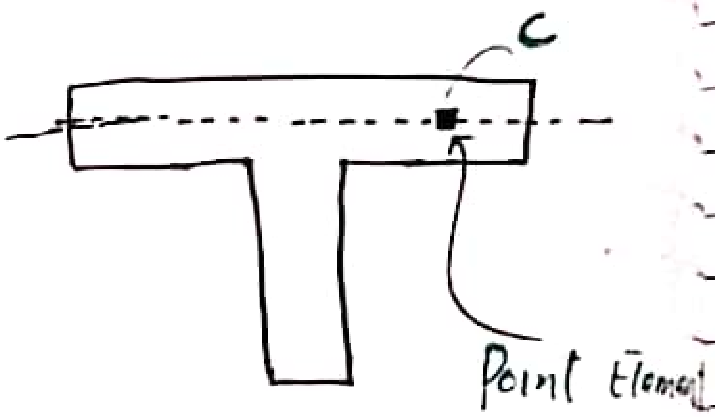
$$\boxed{\sigma_{min} = 834.6 \text{ psi}}$$



Cas -7

Stress State of a Point Element

Find Stress State of a Point Element located at a distance of 4ft frome left Support and 0.25in below the Top fibre



As to Find the Condition of Stressed element at Point 'C' in this given T Section. It is requires to find all the applied Stresses at this Point

In This given problem the Stresses acting on Point 'C' is flexural and Shear Stress Their is no Torsional ~~Stress~~ force acting on this Beam due to The load Symmetry along the Beam axis (Longitudinal axis)

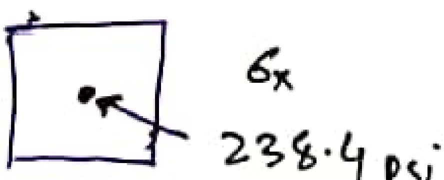
Flexural Stress at Point 'c'

$\sigma = 238.4$ psi [see page #12 Case-2)
point 'c' exactly lies in this case i.e 4ft from left Support and 0.25 below the top fibre

Shear Stress at point 'c'

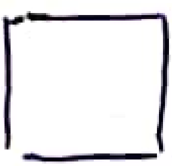
$\tau = 0.6$ psi [see Cas-8 on page # 11]

Consider this point 'c' is a planner element



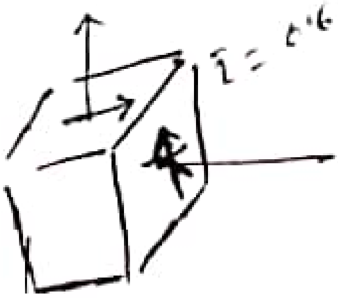
As the flexural Stress is perpendicular to the cross section it can be represented by normal Stress

238.4 psi is Compressive
Because point c lies in Compression zone of Beam cross section

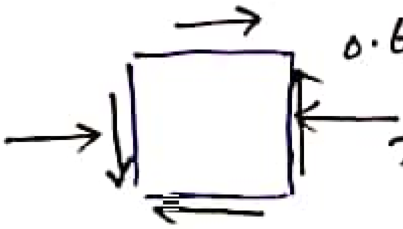


If point c lies below the Centroid Than Stress would be Tensile

Note All faces have same stresses



Combine Stress on 2D element



—x—x—x—x—x

Question **Note** Find the Stress State Condition of Point 'C' at a 20° clockwise Orientation

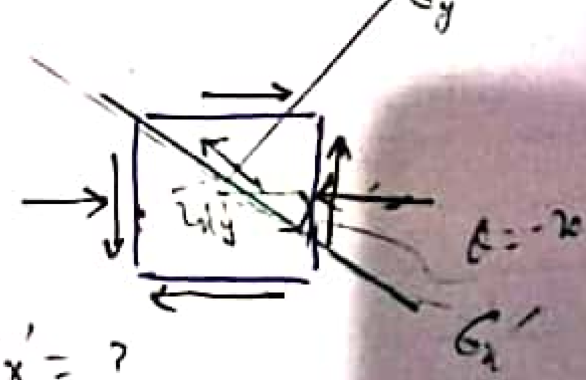
$\theta = -20°$

Solve Given Stress state

$\sigma_x = -238.4$ psi

$\sigma_y = 0$

$\tau_{xy} = 0.6$ psi



$\sigma_x' = ?$

$\sigma_y' = ?$

$\tau_{x'y'} = ?$

As we Derive the following equations for Stress transformation

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

for $\sigma_{x'}$

$$\sigma_{x'} = \frac{-238.4 + 0}{2} + \frac{-238.4 - 0}{2} \cos(2(-20)) + (0.6) \sin(2(-20))$$

$$\sigma_{x'} = -119 \quad -91.3 \quad -0.385$$

$$\boxed{\sigma_{x'} = -210.8 \text{ Psi}}$$

Compressive

for $\sigma_{y'}$

$$\sigma_{y'} = \frac{-238.4 + 0}{2} + \frac{-238.4}{2} \cos(2(-20)) + (0.6) \sin 2(-20)$$

$$\sigma_{y'} = -119 \quad + 91.3 \quad + 0.385$$

$$\boxed{\sigma_y' = -27.315 \text{ psi}}$$ Compressive

for $\tau_{x'y'}$

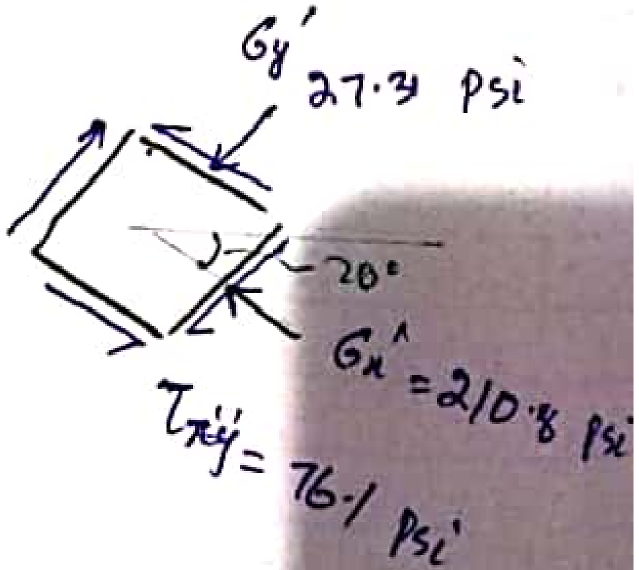
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\tau_{x'y'} = -\frac{(-238.4) - 0}{2} \sin(2(-20)) + 0.6 \cos(2(-20))$$

$$= -76.6 \qquad + 0.4596$$

$$\boxed{\tau_{x'y'} = -76.1}$$

New Stress State after 20° clockwise Orientation is Shown



Find its Principle Stresses

We know that principle Stress equation is

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{-238.4 + 0}{2} \pm \sqrt{\left(\frac{-238.4 - 0}{2}\right)^2 + (0.6)^2}$$

$$\sigma_{1,2} = -119.2 \pm \sqrt{14389.45}$$

$$\sigma_{1,2} = -119.2 \pm 119.7$$

$\sigma_y = \sigma_1 = -119.2 + 119.7 = 0.501$ psi

$\sigma_x = \sigma_2 = -119.2 - 119.7 = 238.9$

Or First Find $\theta_p = ?$

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} =$$

$$\boxed{\theta_p = -0.144}$$

Put in General Equation

$$\sigma_{x_{max}}' = \frac{-238.4+0}{2} + \frac{-238.4-0}{2} \cos(2(-0.144)) + 0.6 \sin(\ldots)$$

$$\sigma_{p_{max}}' = -119.2 - 119.2 = 238.4$$

$$\sigma_{p_{max}}' = 238.4 \text{ psi}$$

These Calculation Shows That this given State of Stress is itself a principle Stress Condition in which the Shear Stress is approximately equal to zero

- Max in plane Shear Stress

In This Case

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$\tan 2\theta_s = \frac{-(-238.4 - 0)/2}{0.6}$$

$$\tan 2\theta_s =$$

$$\boxed{\theta_s = 44.85}$$ Anticlockwise

Put This in the General Equation for $\tau_{x'y'}$

$$\tau_{x'y'} = -\left[\frac{\sigma_x - \sigma_y}{2}\right]\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_{x'y'} = \frac{-(-238.4 - 0)}{2}\sin(2(44.8)) + 0.6\cos(2(4$$

$$\boxed{\tau_{x'y'} = 119.2 \text{ psi}}$$ Max in plane Shear Stress

To Draw Mohr's Circle for the Given Problem

Solve

As we know To Draw a Circle we need the coordinate of ~~Centre~~ centre of circle as well as the radius of Circle

We also know that for mohr's Circle the coordinate of Centre is $\left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$

Centre coordinate

$$(h, k) = \left[\frac{-238.4 + 0}{2}, 0\right]$$

$$= [-119.2, 0]$$

Radius of Mohr's Circle is

$$r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-238.4 - 0}{2}\right)^2 + (0.6)^2}$$

$r = 119.2$ psi

Scale

~~? psi = 1cm~~

25psi = 1cm