

PRE-STRESS CONCRETE

INTRODUCTION

INTRODUCTION

Pre-stressing can be defined as the **imposition of internal stresses into a structure that are of opposite character** to those that will be caused by the service or working loads. A rather common method used to describe pre-stressing is shown in next figure, where a row of books has been squeezed together by a person's hands. The resulting "beam" can carry a downward load **as long as the compressive stress due to squeezing at the bottom of the "beam" is greater than the tensile stress there due to the moment produced by the weight of the books and the superimposed loads. Such a beam has no tensile strength** and thus no moment resistance until it is squeezed together or pre-stressed.

INTRODUCTION

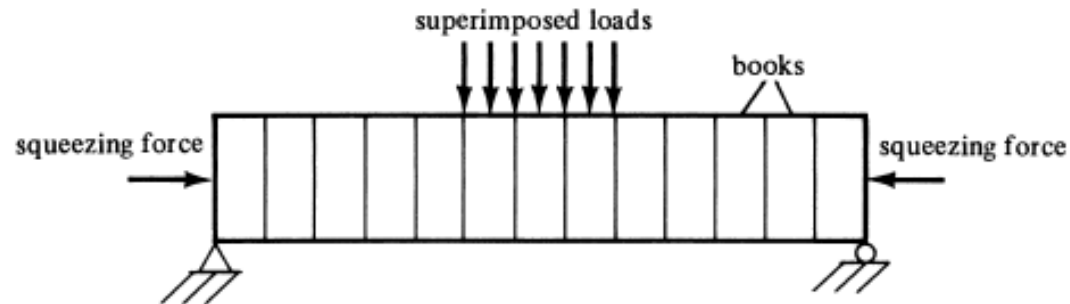


Figure 19.1 Prestressing.

From the preceding discussion it is easy to see why pre-stressing has captured the imagination of so many persons and why it has all sorts of possibilities now and in the future. In normal concrete, only a portion of the cross sections of members in bending could be considered effective in resisting loads because a large part of those cross sections were in tension and thus cracked. If, however, flexural members can be **pre-stressed** so that their entire cross sections are kept in compression, **then the properties of the entire sections are available to resist the applied forces.**

INTRODUCTION

The theory of pre-stressing is quite simple and has been used for many years in various kinds of structures. For instance, wooden barrels have long been made by putting **tightened metal bands around** them, thus compressing the staves together and making a tight container with resistance to the outward pressures of the enclosed liquids. Pre-stressing is primarily used for concrete beams to **counteract tension stresses** caused by the weight of the members and the superimposed loads. Should these loads cause a positive moment in a beam, it is possible by pre-stressing to introduce a negative moment that can counteract part or all of the positive moment. An ordinary beam has to have sufficient strength to support itself as well as the other loads, but it is possible with pre-stressing to produce a negative loading that will eliminate the effect of the beam's weight, thus producing a "**weightless beam.**"

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For a more detailed illustration of pre-stressing, reference is made to next figure. It is assumed that the following steps have been taken with regard to this beam:

1. Steel strands (represented by the dashed lines) were placed in the lower part of the beam form.
2. The strands were tensioned to a very high stress.
3. The concrete was placed in the form and allowed to gain sufficient strength for the pre-stressed strands to be cut.
4. The strands were cut.

INTRODUCTION

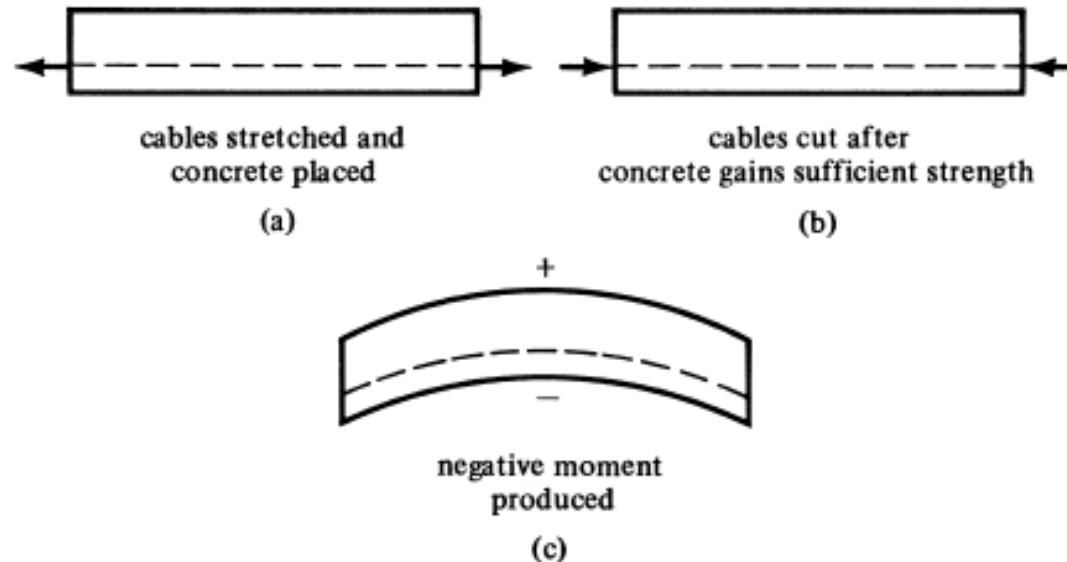


Figure 19.2

The cut strands tend to resume their original length, thus compressing the lower part of the beam and causing a negative bending moment. The positive moment caused by the beam weight and any superimposed gravity loads is directly opposed by the negative moment. Another way of explaining this is to say that a **compression stress** has been produced in the bottom of the beam **opposite in character to the tensile stress that is caused there by the working loads.**

INTRODUCTION



Prestressed concrete channels, John A. Denies Son Company Warehouse #4, Memphis, Tennessee. (Courtesy of Master Builders.)

ADVANTAGES AND DISADVANTAGES OF PRE-STRESSED CONCRETE

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Advantages

It is possible with pre-stressing to utilize the entire cross section of members to resist loads. Thus, smaller members can be used to support the same loads, or the same-size members can be used for longer spans. This is a particularly important advantage because member weights make up a substantial part of the total design loads of concrete structures.

Pre-stressed members are crack-free under working loads and, as a result, look better and are more watertight, providing better corrosion protection for the steel. Furthermore, crack-free pre-stressed members require less maintenance and last longer than cracked reinforced concrete members.

Advantages

Therefore, for a large number of structures, pre-stressed concrete provides the **lowest first-cost solution, and when its reduced maintenance is considered, pre-stressed concrete provides the lowest overall cost for many additional cases.**

The negative moments caused by pre-stressing produce camber in the members, with the result that total **deflections are reduced**. Other advantages of pre-stressed concrete include the following: reduction in diagonal tension stresses, sections with greater stiffness under working loads, and increased fatigue and impact resistance as compared to ordinary reinforced concrete.

Disadvantages

Pre-stressed concrete requires the use of higher-strength concretes and steels and the use of more complicated formwork, with resulting higher labor costs. Other disadvantages include the following:

1. Closer **control** required in manufacture.
2. **Losses in the initial pre-stressing forces.** When the compressive forces due to pre-stressing are applied to the concrete, it will shorten somewhat, partially relaxing the cables. The result is some reduction in cable tension with a resulting loss in pre-stressing forces. Shrinkage and creep of the concrete add to this effect.

Disadvantages

3. Additional stress conditions must be checked in design, such as the stresses occurring when pre-stress forces are first applied and then after pre-stress losses have taken place, as well as the stresses occurring for different loading conditions.
4. Cost of **end anchorage devices and end-beam plates** that may be required.

PRE-TENSIONING AND POST-TENSIONING

The two general methods of pre-stressing are pre-tensioning and posttensioning. *Pre-tensioning* has already been illustrated, where the pre-stress tendons were tensioned before the concrete was placed. After the concrete had hardened sufficiently, the tendons were cut and the pre-stress force was transmitted to the concrete by bond. This method is particularly well suited for mass production because the casting beds can be constructed several hundred feet long. The tendons can be run for the entire bed lengths and used for casting several beams in a line at the same time.

In *post-tensioned* construction, the tendons are stressed after the concrete is placed and has gained the desired strength. Plastic or metal tubes, conduits, sleeves, or similar devices with unstressed tendons inside are located in the form and the concrete is placed.

PRETENSIONING AND POSTTENSIONING

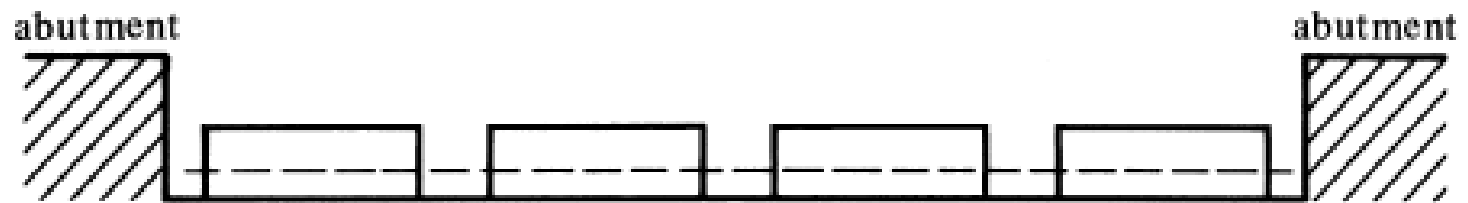


Figure 19.3 Prestress bed.

hollow ducts or tubes for prestress tendons

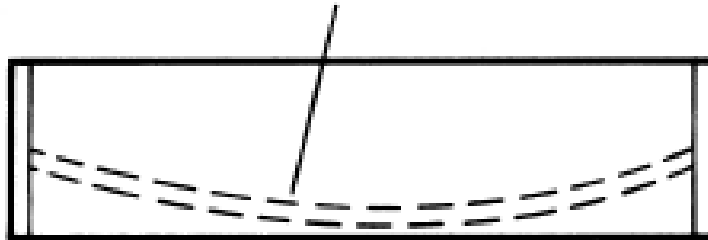


Figure 19.4 Posttensioned beam.

PRETENSIONING AND POSTTENSIONING

After the concrete has sufficiently hardened, the tendons are stretched and **mechanically attached to end anchorage devices to keep the tendons in their stretched positions**. Thus by posttensioning, the pre-stress forces are transferred to the concrete **not by bond, but by end bearing**.

It is actually possible in post-tensioning to have either bonded or un-bonded tendons. If bonded, the conduits are often made of aluminum, steel, or other metal sheathing. In addition, it is possible to use steel tubing or rods or rubber cores that are cast in the concrete and removed a few hours after the concrete is placed. After the steel is tensioned, **cement grout** is injected into the duct for bonding. The grout is also useful in protecting the steel from corrosion. If the tendons are to be un-bonded, they should be greased to facilitate tensioning and to protect them from corrosion.

MATERIALS USED FOR PRE-STRESSED CONCRETE

The materials ordinarily used for pre-stressed concrete are concrete and high-strength steels. The concrete used is probably of a higher strength than that used for reinforced concrete members, for several reasons, including the following:

1. The **modulus of elasticity of such concretes is higher**, with the result that the **elastic strains in the concrete are smaller** when the tendons are cut. Thus the relaxations or losses in the tendon stresses are smaller.
2. In pre-stressed concrete, the **entire members are kept in compression**, and thus all the concrete is effective in resisting forces. Hence it is reasonable to pay for a more expensive but stronger concrete if all of it is going to be used.

MATERIALS USED FOR PRE-STRESSED CONCRETE

(In ordinary reinforced concrete beams, more than half of the cross sections are in tension and thus assumed to be cracked. As a result, more than half of a higher-strength concrete used there would be wasted.)

3. Most pre-stressed work is of the **precast, pre-tensioned** type done at the pre-stress yard where the work can be carefully controlled; consequently, dependable higher-strength concrete can readily be obtained.
4. For pre-tensioned work, the higher-strength concretes permit the use **of higher bond stresses between the cables and the concrete.**

MATERIALS USED FOR PRE-STRESSED CONCRETE



Prestressed concrete segmental bridge over the River Trent near Scunthorpe, Lincolnshire, England. (Courtesy of Cement and Concrete Association.)

MATERIALS USED FOR PRE-STRESSED CONCRETE

High-strength steels are necessary to produce and keep satisfactory pre-stress forces in members. The strains that occur in these steels during stressing are much greater than those that can be obtained with ordinary reinforcing steels. As a result, when the concrete elastically shortens in compression and also shortens due to creep and shrinkage, **the losses in strain in the steel (and thus stress) represent a smaller percentage of the total stress.** Another reason for using high-strength steels is that a **large pre-stress force can be developed in a small area.**

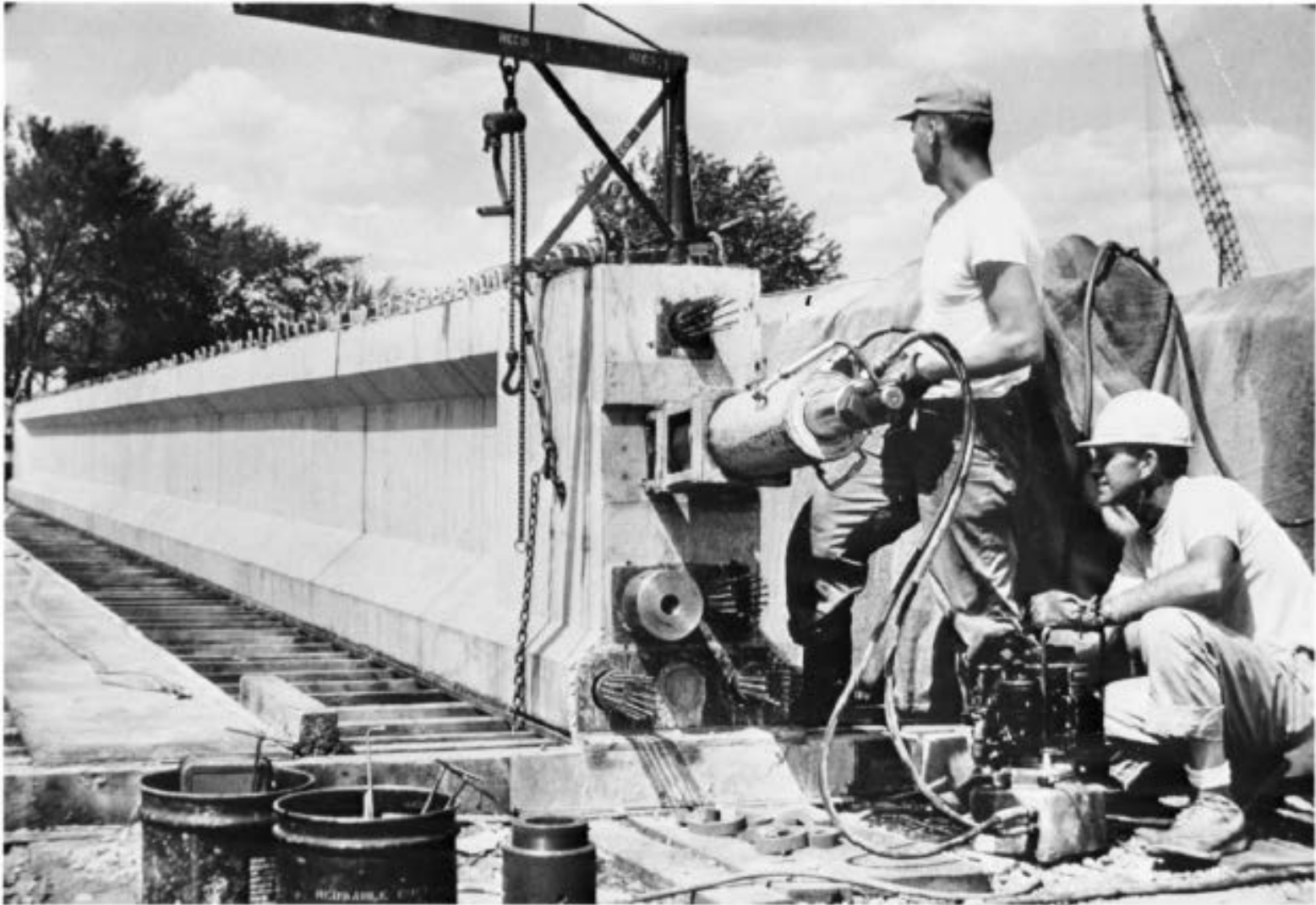
Early work with pre-stressed concrete using ordinary-strength bars to induce the pre-stressing forces in the concrete **resulted in failure because the low stresses that could be put into the bars were completely lost due to the concrete's shrinkage and creep.**

MATERIALS USED FOR PRE-STRESSED CONCRETE

Should a pre-stress of 20,000 psi be put into such rods, the resulting strains would be equal to $20,000 / (29 \times 10^6) = 0.00069$. This value is less than the long-term creep and shrinkage strain normally occurring in concrete, roughly 0.0008, which would completely relieve the stress in the steel. Should a high-strength steel be stressed to about 150,000 psi and have the same creep and shrinkage, the stress reduction will be of the order of $(0.0008)(29 \times 10^6) = 23,200$ psi, leaving $150,000 - 23,200 = 126,800$ psi in the steel (a loss of only 15.47% of the steel stress).

Three forms of pre-stressing steel are used: single wires, wire strands, and bars. The greater the diameter of the wires, the smaller become their strengths and bond to the concrete.

MATERIALS USED FOR PRE-STRESSED CONCRETE



Prestressing girders for bridge in Butler, Pennsylvania. (Courtesy of Portland Cement Association.)

MATERIALS USED FOR PRE-STRESSED CONCRETE

As a result, wires are manufactured with diameters from 0.192 in. up to a maximum of 0.276 in. (about $9/32$ in.). In posttensioning work, large numbers of wires are grouped in parallel into tendons. Strands that are made by twisting wires together are used for most pre-tensioned work. They are of the seven-wire type, where a center wire is tightly surrounded by twisting the other six wires helically around it. Strands are manufactured with diameters from $1/4$ to $1/2$ in. sometimes large-size, high-strength, heat-treated alloy steel bars are used for post-tensioned sections. they are available with diameters running from $3/4$ to $1\frac{3}{8}$ in.

High-strength pre-stressing steels do not have distinct yield points as do the structural carbon reinforcing steels.

MATERIALS USED FOR PRE-STRESSED CONCRETE

The practice of considering yield points, however, is so firmly embedded in the average designer's mind that high-strength steels are probably given an arbitrary yield point anyway. The yield stress for wires and strands is usually assumed to be the stress that causes a total elongation of 1% to occur in the steel. For high-strength bars the yield stress is assumed to occur when a 0.2% permanent strain occurs.

STRESS CALCULATIONS

For a consideration of stresses in a pre-stressed rectangular beam, reference is made to next slide. For this example the pre-stress tendons are assumed to be straight, although it will later be shown **that a curved shape is more practical for most beams**. The tendons are assumed to be located an eccentric distance e below the centroidal axis of the beam. As a result, the beam is subjected to a combination of direct compression and a moment due to the eccentricity of the pre-stress. In addition, there will be a moment due to the external load, including the beam's own weight. The resulting stress at any point in the beam caused by these three factors can be written as follows where P is the pre-stressing force:

$$f = -\frac{P}{A} \pm \frac{Pec}{I} \pm \frac{Mc}{I}$$

STRESS CALCULATIONS

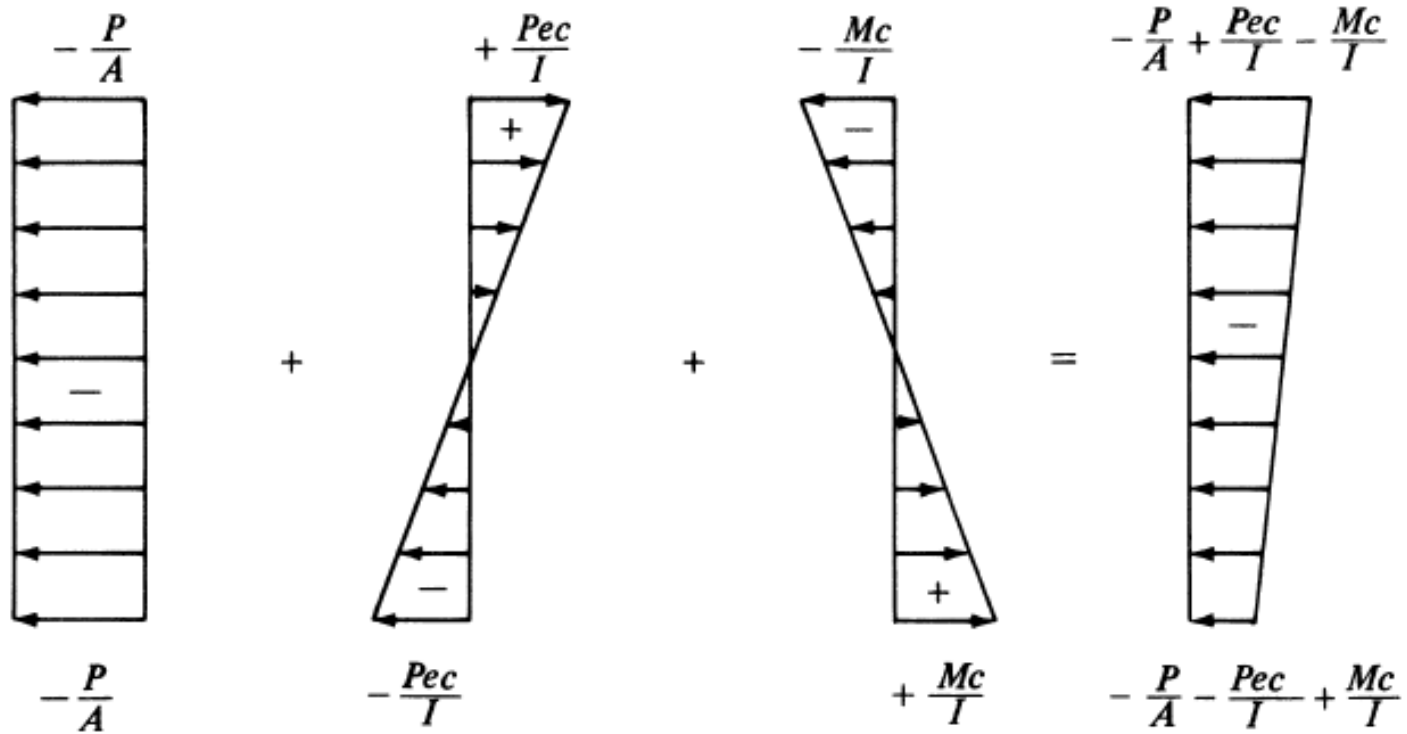
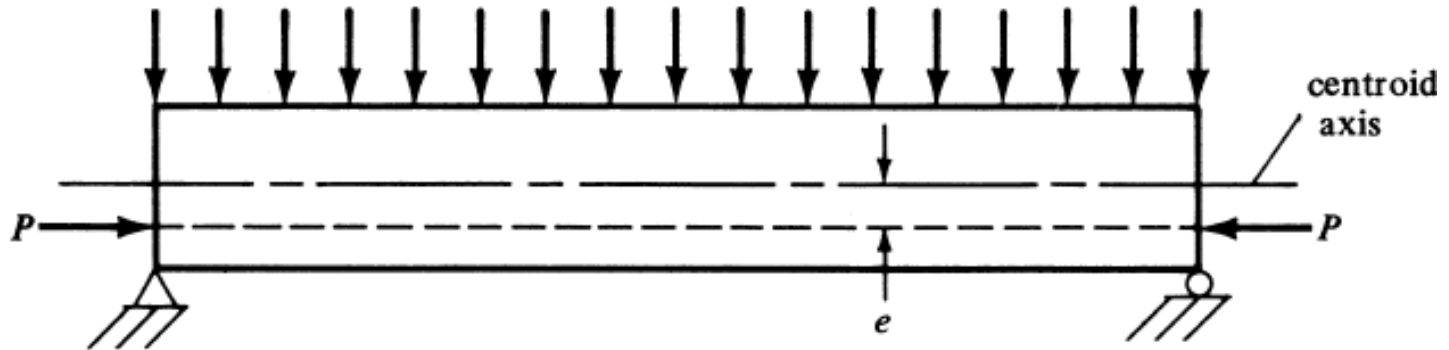


Figure 19.5

STRESS CALCULATIONS

In the previous figure, a stress diagram is drawn for each of these three items, and all three are combined to give the final stress diagram.

The usual practice is to base the stress calculations in the elastic range on the properties of the gross concrete section. **The gross section consists of the concrete external dimensions with no additions made for the transformed area of the steel tendons nor subtractions made for the duct areas in post-tensioning.** This method is considered to give satisfactory results because the changes in stresses obtained if net or transformed properties are used are usually not significant.

STRESS CALCULATIONS

The next example illustrates the calculations needed to determine the stresses at various points in a simple-span pre-stressed rectangular beam. It will be noted that, as there are no moments at the ends of a simple beam due to the external loads or to the beams own weight, the Mc/I part of the stress equation is zero there and the equation reduces to

$$f = -\frac{P}{A} \pm \frac{Pec}{I}$$

STRESS CALCULATIONS

EXAMPLE 19.1

Calculate the stresses in the top and bottom fibers at the centerline and ends of the beam.

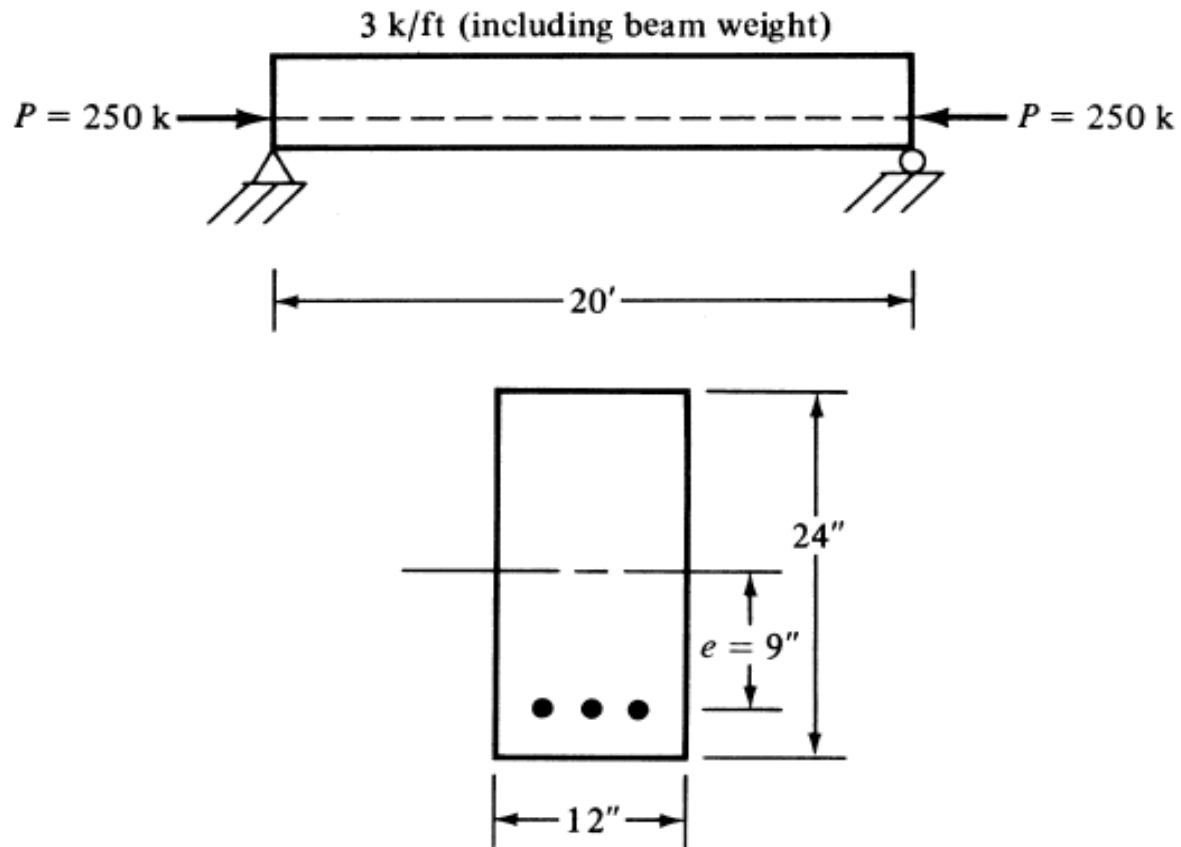


Figure 19.6

STRESS CALCULATIONS

Section Properties

$$I = \left(\frac{1}{12}\right)(12)(24)^3 = 13,824 \text{ in.}^4$$

$$A = (12)(24) = 288 \text{ in.}^2$$

$$M = \frac{(3)(20)^2}{8} = 150 \text{ ft-k}$$

Stresses at Beam Centerline

$$\begin{aligned} f_{\text{top}} &= -\frac{P}{A} + \frac{Pec}{I} - \frac{Mc}{I} = -\frac{250}{288} + \frac{(250)(9)(12)}{13,824} - \frac{(12)(150)(12)}{13,824} \\ &= -0.868 + 1.953 - 1.562 = -0.477 \text{ ksi} \end{aligned}$$

$$f_{\text{bottom}} = -\frac{P}{A} - \frac{Pec}{I} + \frac{Mc}{I} = -0.868 - 1.953 + 1.562 = -1.259 \text{ ksi}$$

Stresses at Beam Ends

$$f_{\text{top}} = -\frac{P}{A} + \frac{Pec}{I} = -0.868 + 1.953 = +1.085 \text{ ksi}$$

$$f_{\text{bottom}} = -\frac{P}{A} - \frac{Pec}{I} = -0.868 - 1.953 = -2.821 \text{ ksi}$$

STRESS CALCULATIONS

In the example it was shown that when the pre-stress tendons are straight, the tensile stress at the top of the beam at the ends will be quite high. If, however, the tendons are **draped**, as shown in next figure, it is possible to reduce or even eliminate the tensile stresses. Out in the span, the centroid of the strands may be below the lower kern point, but if at the ends of the beam, where there is no stress due to dead-load moment, it is below the kern point, tensile stresses in the top will be the result. If the tendons are draped so that at the ends they are located **at or above this point**, tension will not occur in the top of the beam.

In post-tensioning, the sleeve or conduit is placed in the forms in the curved position desired. The tendons in pre-tensioned members can be placed at or above the lower kern points and then can be pushed down to the desired depth at the centerline or at other points.

STRESS CALCULATIONS

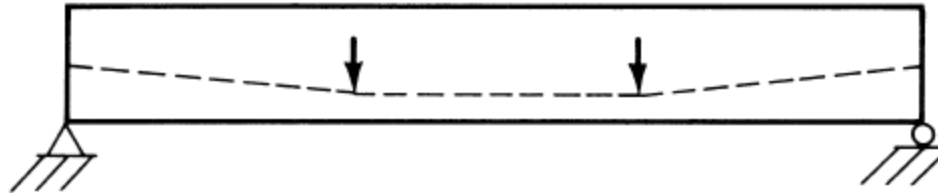


Figure 19.7 Draped tendons.

Actually, however, the tendons at the beam ends do not have to be as high as the kern points because the ACI Code (18.4.1) permits some tension in the top of the beam when the tendons are cut. This value is $3\sqrt{f'_{ci}}$ where f'_{ci} is the strength of the concrete at the time the tendons are cut, as determined by testing concrete cylinders. This permissible value equals about 40% of the cracking strength or modulus of rupture of the concrete ($7.5\sqrt{f'_{ci}}$) at that time. The stress at the bottom of the beam, which is compressive, is permitted to go as high as $0.60 f'_{ci}$.

STRESS CALCULATIONS

In previous figure the tendons were shown held down at the one-third points. Two alternatives to draped tendons that have been used are to use straight tendons, located below the lower kern point but which are encased in tubes at their ends, or have their ends greased. Both methods are used to prevent the development of negative moments at the beam ends.

Next example shows the calculations necessary to locate the **kern point** for the beam of earlier example. In addition, the stresses at the top and bottom of the beam ends are computed. It will be noted that, according to these calculations, the kern point is 4 in. below the mid-depth of the beam, and it would thus appear that the pre-stress tendons should be located at the kern point at the beam ends and pushed down to the desired depth farther out in the beam.

STRESS CALCULATIONS

The Code actually permits tensile stresses at the ends of simple beams to go as high as $6\sqrt{f'_{ci}}$. These allowable tensile values are applicable to the stresses that occur immediately after the transfer of the pre-stressing forces and after the losses occur due to elastic shortening of the concrete and relaxation of the tendons and anchorage seats. It is further assumed that the time-dependent losses of creep and shrinkage have not occurred. If the calculated tensile stresses are greater than the permissible values, it is necessary to use some additional bonded reinforcing (pre-stressed or unpre-stressed) to resist the *total* tensile force in the concrete computed on the basis of an un-cracked section.

STRESS CALCULATIONS

Section 18.4.2 of the Code provides allowable stresses at service loads after all pre-stress losses have occurred. An extreme fiber compression stress equal to $0.45 f'_c$ is permitted for pre-stress plus sustained loads. The allowable compression stress for pre-stress plus total loads is $0.60 f'_c$. In effect, the ACI here provides a one-third increase in allowable compression stress when a large percentage of the service loads are deemed to be transient or of short duration.

The allowable tensile stress at ends of simply supported members immediately after pre-stress transfer is $6\sqrt{f'_{ci}}$. Section 18.4.3 of the Code permits under certain conditions higher permissible stresses.

STRESS CALCULATIONS

EXAMPLE 19.2

Determine the location of the lower kern point at the ends of the beam of Example 19.1. Calculate the stresses at the top and bottom of the beam ends, assuming the tendons are placed at the kern point.

Top fiber stress = + 1.085 ksi

SOLUTION

Locating the Kern Point

$$\begin{aligned}f_{\text{top}} &= -\frac{P}{A} + \frac{Pec}{I} = 0 \\-\frac{250}{288} + \frac{(250)(e)(12)}{13,824} &= 0 \\-0.868 + 0.217e &= 0 \\e &= 4''\end{aligned}$$

Computing Stresses

$$\begin{aligned}f_{\text{top}} &= -\frac{P}{A} - \frac{Pec}{I} = -\frac{250}{288} + \frac{(250)(4)(12)}{13,824} \\&= -0.868 + 0.868 = 0 \\f_{\text{bottom}} &= -\frac{P}{A} - \frac{Pec}{I} = -0.868 - 0.868 = -1.736 \text{ ksi}\end{aligned}$$

SHAPES OF PRE-STRESSED SECTIONS

For simplicity in introducing pre-stressing theory, rectangular sections are used for most of the examples. From the viewpoint of formwork alone, rectangular sections are the most economical, but more complicated shapes, such as I's and T's, will require smaller quantities of concrete and pre-stressing steel to carry the same loads and, as a result, they frequently have the lowest overall costs.

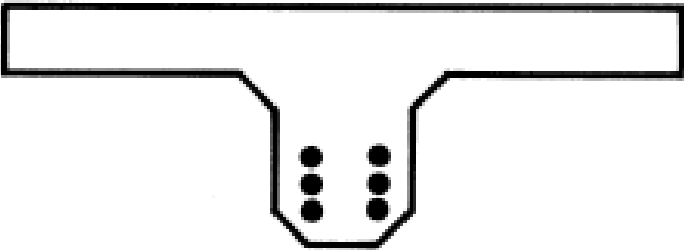
If a member is to be **made only one time**, a cross section requiring simple formwork (thus often rectangular) will probably be used. For instance, simple formwork is essential for most cast-in-place work. Should, however, the forms be used a large number of times to make many identical members, more complicated cross sections, such as I's and T's, channels, or boxes, will be used.

SHAPES OF PRE-STRESSED SECTIONS

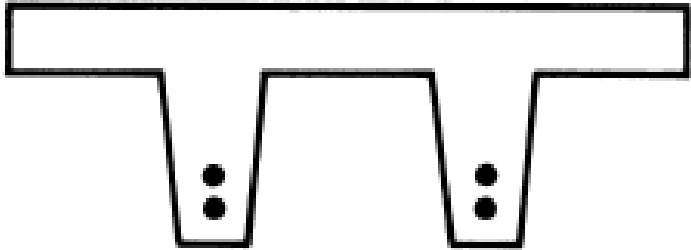
For such sections the cost of the formwork as a percentage of each member's total cost will be much reduced. Several types of commonly used pre-stressed sections are shown. The same general theory used for the determination of stresses and flexural strengths applies to shapes such as these, as it does to rectangular sections.

The usefulness of a particular section depends on the simplicity and reusability of the formwork, the appearance of the sections, the degree of difficulty of placing the concrete, and the theoretical properties of the cross section. The greater the amount of concrete located near the extreme fibers of a beam, **the greater will be the lever arm between the C and T forces and thus the greater the resisting moment.** Of course, there are some limitations on the widths and thicknesses of the flanges.

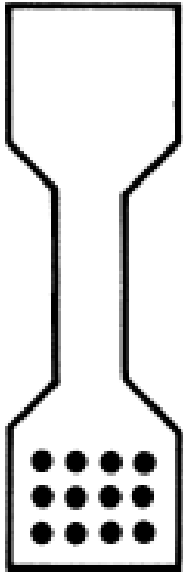
SHAPES OF PRE-STRESSED SECTIONS



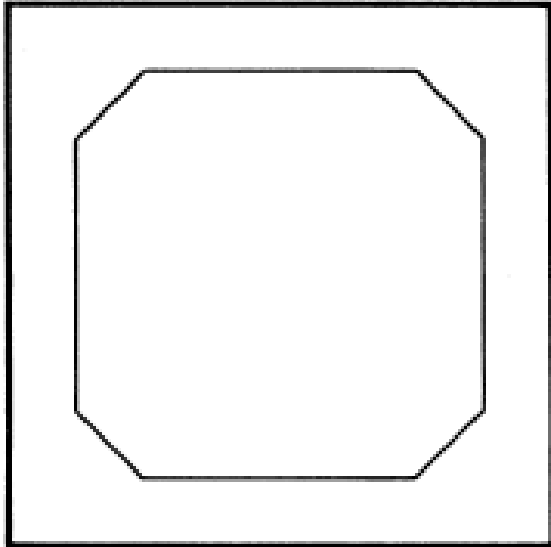
single T
(a)



double T
(b)



I section
(c)



box section
(d)

SHAPES OF PRE-STRESSED SECTIONS

In addition, the webs must be sufficiently large to resist shear and to allow the proper placement of the concrete and at the same time be sufficiently thick to avoid buckling.

A pre-stressed T such as the one shown in Figure 19.8(a) is often a very economical section because a large proportion of the concrete is placed in the compression flange, where it is quite effective in resisting compressive forces. The double T shown in Figure 19.8(b) is used for schools, office buildings, stores, and so on and is probably the most used pre-stressed section. The total width of the flange provided by a double T is in the range of about 5 to 8 ft, and spans of 30 to 50 ft are common. You can see that a floor or roof system can be erected easily and quickly by placing a series of precast double T's side by side $\Pi\Pi\Pi\Pi$.

SHAPES OF PRE-STRESSED SECTIONS

The sections serve as both the beams and slabs for the floor or roof system. Single T's are normally used for heavier loads and longer spans up to as high as 100 or 120 ft. Double T's for such spans would be very heavy and difficult to handle. The single T is not used as much today as it was in the recent past due to stability difficulties in both shipping and erection.

The I and box sections, shown in parts (c) and (d), have a larger proportion of their concrete placed in their flanges, with the result that larger moments of inertia are possible (as compared to rectangular sections with the same amounts of concrete and pre-stressing tendons). The formwork, however, is complicated, and the placing of concrete is difficult. Box girders are frequently used for bridge spans. Their properties are the same as for I sections.

SHAPES OF PRE-STRESSED SECTIONS

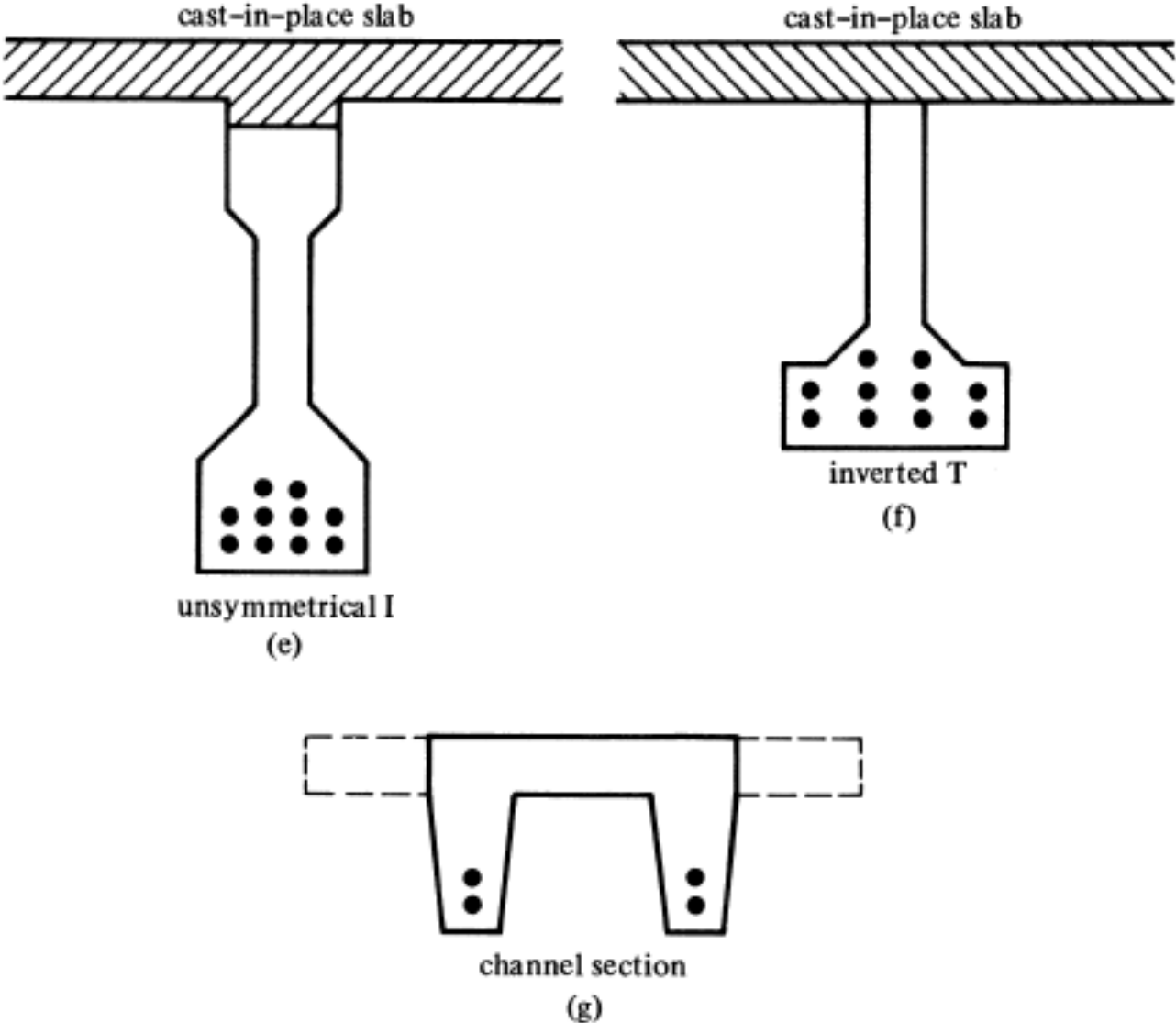


Figure 19.8 Commonly used prestressed sections.

SHAPES OF PRE-STRESSED SECTIONS

Unsymmetrical I's, with large bottom flanges to contain the tendons and small top flanges, may be economical for certain composite sections where they are used together with a slab poured in place to provide the compression flange. A similar situation is shown in figure (f), where an inverted T is used with a cast-in-place slab.

Many variations of these sections are used, such as the channel section shown in figure (g). Such a section might be made by blocking out the flanges of a double- T form as shown, and the resulting members might be used for stadium seats or similar applications.

SHAPES OF pre-stressed SECTIONS



Posttensioned segmental precast concrete for East Moors Viaduct, Lanbury Way, Cardiff, South Wales. (Courtesy of Cement and Concrete Association.)

PRE-STRESS LOSSES

The flexural stresses calculated for the beams earlier were based on initial stresses in the pre-stress tendons. These stresses, however, become smaller with time (over a period of roughly five years) due to several factors. These factors, which are discussed in the paragraphs to follow, include:

- 1.** Elastic shortening of the concrete
- 2.** Shrinkage and creep of the concrete
- 3.** Relaxation or creep in the tendons
- 4.** Slippage in posttensioning end anchorage systems
- 5.** Friction along the ducts used in posttensioning

PRE-STRESS LOSSES

Although it is possible to calculate pre-stress losses individually for each of the factors listed above, it is usually more practical and just as satisfactory to use single lump-sum estimates for all the items together. There are just too many interrelated factors affecting the estimates to achieve accuracy. For 5000 psi concrete, the AASHTO Specification provides an estimated total loss of pre-stress in pre-tensioning strands equal to 45,000 psi or 30,000 to 35,000 psi for post-tensioned tendons consisting of stress-relieved 270 ksi strands and 240 ksi stress-relieved wire.

Such lump-sum estimates of total pre-stress losses are applicable only to average pre-stress members made with normal concrete, construction procedures, and quality control.

PRE-STRESS LOSSES

Should conditions be decidedly different from these and/or if the project is extremely significant, it would be well to consider making detailed loss estimates such as those introduced in the next few paragraphs.

The ultimate strength of a pre-stressed member is almost completely controlled by the cross-sectional area of the cables. Consequently, losses in pre-stress will have very little effect on its ultimate flexural strength. However, **losses in pre-stress will cause more cracking to occur under working loads**, with the result that deflections will be larger. Furthermore, the member's shear and fatigue strength will be somewhat reduced.

pre-stress LOSSES

Elastic Shortening of the Concrete

When the tendons are cut for a pre-tensioned member, the pre-stress force is transferred to the concrete, with the result that the concrete is put in compression and shortens, thus **permitting some relaxation or shortening of the tendons**. The stress in the concrete adjacent to the tendons can be computed. The strain in the concrete, ϵ_c , which equals f_c/E_c , is assumed due to bond to equal the steel strain ϵ_s . Thus the loss in pre-stress can be computed as $\epsilon_s E_s$. An average value of pre-stress loss in pre-tensioned members due to elastic shortening is about **3% of the initial value**.

An expression for the loss of pre-stress due to elastic shortening of the concrete can be derived as:

PRE-STRESS LOSSES

Elastic Shortening of the Concrete

It can be seen that the compressive strain in the concrete due to pre-stress must equal the lessening of the steel strain

$$\epsilon_c = \Delta\epsilon_s$$

These values can be written in terms of stresses as follows:

$$\frac{f_c}{E_c} = \frac{\Delta f_s}{E_s}$$

Thus we can write

$$\Delta f_s = \frac{E_s f_c}{E_c} = n f_c$$

where f_c is the stress in the concrete after transfer of stresses from the cables.

PRE-STRESS LOSSES

Elastic Shortening of the Concrete

If we express Δf_s as being the initial tendon stress f_{si} minus the tendon stress after transfer, we can write

$$f_{si} - f_s = n f_c$$

Then letting P_0 be the initial total cable stress and P_f the stress afterward, we obtain

$$P_0 - P_f = n \frac{P_f}{A_c} A_{ps}$$

$$P_0 = n \frac{P_f}{A_c} A_{ps} + P_f$$

$$P_0 = P_f \left(\frac{n A_{ps}}{A_c} + 1 \right) = \frac{P_f}{A_c} (n A_{ps} + A_c)$$

Then

$$f_c = \frac{P_0}{A_c + A_{ps}} = \text{approximately } \frac{P_0}{A_g}$$

PRE-STRESS LOSSES

Elastic Shortening of the Concrete

and finally

$$\Delta f_s = n f_c = \frac{n P_0}{A_g}$$

a value that can easily be calculated.

For post-tensioned members, the situation is a little more involved because it is rather common to stress a few of the strands at a time and connect them to the end plates. As a result, the losses vary, with the greatest losses occurring in the first strands stressed and the least losses occurring in the last strands stressed. For this reason, an average loss may be calculated for the different strands. Losses due to elastic shortening average about for posttensioned members. It is, by the way, often possible to calculate the expected losses in each set of tendons and overstress them by that amount so the net losses will be close to zero.

PRE-STRESS LOSSES

Shrinkage and Creep of the Concrete

The losses in pre-stressing due to the shrinkage and creep in the concrete are quite variable. For one thing, the amount of shrinkage that occurs in concrete varies from almost zero up to about 0.0005 in./in. (depending on dampness and on the age of the concrete when it is loaded), with an average value of about 0.0003 in./in. being the usual approximation.

The loss in pre-stress due to shrinkage can be said to equal $\epsilon_{sh}E_s$, where ϵ_{sh} is the shrinkage strain of the concrete. A recommended value of ϵ_{sh} is to be determined by taking the basic shrinkage strain times a correction factor based on the volume (V)-to-surface (S) ratio times a relative humidity correction (H).

$$\epsilon_{sh} = (0.00055) \left(1 - 0.06 \frac{V}{S} \right) (1.5 - 0.15H)$$

PRE-STRESS LOSSES

Shrinkage and Creep of the Concrete

Should the member be posttensioned, an additional multiplier is to be taken into account; between the end of the moist curing until the prestressing forces are applied.

The amount of creep in the concrete depends on several factors and can vary from 1 to 5 times the instantaneous elastic shortening. Pre-stress forces are usually applied to pre-tensioned members much earlier in the age of the concrete than for posttensioned members. Pre-tensioned members are normally cast in a bed at the pre-stress yard, where the speed of production of members is an important economic matter.

PRE-STRESS LOSSES

Shrinkage and Creep of the Concrete

It is desired to tension the steel, place the concrete, and take the members out of the pre-stress bed as quickly as the concrete gains sufficient strength so that work can start on the next set of members. As a result, **creep and shrinkage are larger, as are the resulting losses.**

Average losses are about **6% for pre-tensioned members and about 5% for posttensioned members.** The losses in cable stresses due to concrete creep strain can be determined by multiplying an experimentally determined creep coefficient C_t by nf_c .

$$\Delta f_s = C_t n f_c$$

PRE-STRESS LOSSES

Shrinkage and Creep of the Concrete

A value of $C_t = 2.0$ is recommended for pre-tensioned sections, while 1.6 is recommended for post-tensioned ones. These values should be reduced by 20% if lightweight concrete is used. The value f_c is defined as the stress in the concrete adjacent to the centroid of the tendons due to the initial pre-stress ($-P/A$) and due to the permanent dead loads that are applied to the member after pre-stressing ($-Pec/I$), where e is measured from the centroid of the section to the centroid of the tendons.

PRE-STRESS LOSSES

Relaxation or Creep in the Tendons

The plastic flow or relaxation of steel tendons is quite small when the stresses are low, but the percentage of relaxation increases as stresses become higher. In general, the **estimated losses run from about 2 to 3%** of the initial stresses. The amount of these losses actually varies quite a bit for different steels and should be determined from test data available from the steel manufacturer in question. A formula is available with which this loss can be computed.

PRE-STRESS LOSSES

Slippage in Posttensioning End Anchorage Systems

When the jacks are released and the pre-stress forces are transferred to the end anchorage system, a little slippage of the tendons occurs. The amount of the **slippage depends on the system used and tends to vary from about 0.10 in. to 0.20 in.** Such deformations are quite important if the members and thus the tendons are short, but if they are long, the percentage is much less important.

PRE-STRESS LOSSES

Friction along the Ducts Used in Post-tensioning

There are losses in post-tensioning due to friction between the tendons and the surrounding ducts. In other words, the stress in the tendons gradually falls off as the distance from the tension points increases due to friction between the tendons and the surrounding material. These losses are due to the so-called **length and curvature effects**.

The *length effect* is the friction that would have existed if the cable had been straight and not curved. Actually, it is impossible to have a perfectly straight duct in post-tensioned construction, and the result is friction, called the length effect or sometimes the *wobble effect*. The magnitude of this friction is dependent on the stress in the tendons, their length, the workmanship for the particular member in question, and the coefficient of friction between the materials.

PRE-STRESS LOSSES

Friction along the Ducts Used in Post-tensioning

The *curvature effect* is the amount of friction that occurs in addition to the unplanned wobble effect. The resulting loss is due to the coefficient of friction between the materials caused by the pressure on the concrete from the tendons, which is dependent on the stress and the angle change in the curved tendons. It is possible to reduce frictional losses substantially in pre-stressing by several methods. These include jacking from both ends, overstressing the tendons initially, and lubricating un-bonded cables.

The ACI Code (18.6.2) requires that frictional losses for post-tensioned members be computed with wobble and curvature coefficients experimentally obtained and verified during the pre-stressing operation.

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