ULTIMATE STRENGTH OF PRESTRESSED SECTIONS

ULTIMATE STRENGTH OF PRESTRESSED SECTIONS

Considerable emphasis is given to the ultimate strength of pre-stressed sections, the objective being to obtain a satisfactory factor of safety against collapse. There is tremendous change that occurs in a pre-stressed member's behavior after tensile cracks occur. Before the cracks begin to form, the entire cross section of a pre-stressed member is effective in resisting forces, but after the tensile cracks begin to develop, the cracked part is not effective in resisting tensile forces. Cracking is usually assumed to occur when calculated tensile stresses equal the modulus of rupture of the concrete (about $7.5\sqrt{f_c'}$).

ULTIMATE STRENGTH OF PRESTRESSED SECTIONS

Another question that might occur at this time is this: “What effect do the pre-stress forces have on the ultimate strength of a section?” The answer to the question is simple. An ultimate-strength analysis is based on the assumption that the pre-stressing strands are stressed above their yield point. If the strands have yielded, the tensile side of the section has cracked and the theoretical ultimate resisting moment is the same as for a non pre-stressed beam constructed with the same concrete and reinforcing.

The theoretical calculation of ultimate capacities for pre-stressed sections is not such a routine thing as it is for ordinary reinforced concrete members. The high-strength steels from which pre-stress tendons are manufactured do not have distinct yield points.

ULTIMATE STRENGTH OF PRESTRESSED SECTIONS

Despite this fact, the strength method for determining the ultimate moment capacities of sections checks rather well with load tests as long as the steel percentage is sufficiently small as to ensure a tensile failure and as long as bonded strands are being considered.

In the expressions used here, f_{ps} is the average stress in the pre-stressing steel at the design load. This stress is used in the calculations because the pre-stressing steels usually used in pre-stressed beams do not have well-defined yield points (that is, the flat portions that are common to stress–strain curves for ordinary structural steels). Unless the yield points of these steels are determined from detailed studies, their values are normally specified.

ULTIMATE STRENGTH OF PRESTRESSED SECTIONS

For instance, the ACI Code (18.7.2) states that the following approximate expression may be used for calculating f_{ps}. In this expression f_{pu} is the ultimate strength of the pre-stressing steel, p_p is the percentage of pre-stress reinforcing A_{ps}/bd, and fse is the effective stress in the pre-stressing steel after losses. If more accurate stress values are available, they may be used instead of the specified values. In no case may the resulting values be taken as more than the specified yield strength f_{py}, or $f_{se} + 60{,}000$. For bonded members,

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\left[\rho_p \frac{f_{pu}}{f'_c} + \frac{d}{d_p}(\omega - \omega')\right]\right) \qquad \text{if } f_{se} \geq 0.5 f_{pu}$$

(ACI Equation 18-3)

ULTIMATE STRENGTH OF PRESTRESSED SECTIONS

where γ_p is a factor for the type of pre-stress tendon whose values are specified in ACI Section 18.0 (γ_p = 0.55 for f_{py}/f_{pu} not less than 0.80, 0.40 for f_{py}/f_{pu} not less than 0.85, and 0.28 for f_{py}/f_{pu} not less than 0.90), d_p = distance from the extreme compression fiber to the centroid of the pre-stress reinforcement, $\omega = \rho f_y / f'_c$, $\omega' = \rho' f_y / f'_c$.

If any compression reinforcing is considered in calculating f_{ps}, the terms in brackets may not be taken less than 0.17 (see Commentary R18.7.2). Should compression reinforcing be taken into account and if the term in brackets is small, the depth to the neutral axis will be small and thus the compression reinforcing will not reach its yield stress. For this situation the results obtained with ACI Equation 18-3 are not conservative, thus explaining why the ACI provides the 0.17 limit.

ULTIMATE STRENGTH OF PRESTRESSED SECTIONS

Should the compression reinforcing be neglected in using the equation, will equal zero and the term in brackets may be less than 0.17. Should *d'* be large, the strain in the compression steel may be considerably less than the yield strain, and as a result the compression steel will not influence f_{ps} as favorably as implied by the equation. As a result, ACI Equation 18-3 may only be used for beams in which $d' \leq 0.15\, d_p$.

For un-bonded members with span to depth ≤ 35,

$$f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{100\rho_p} \text{ but not greater than } f_{py} \text{ nor } (f_{se} + 60{,}000)$$

(ACI Equation 18-4)

For unbonded members with span to depth > 35,

$$f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{300\rho_p} \qquad \text{(ACI Equation 18-5)}$$

However, f_{ps} may not exceed f_{py}, or $f_{se} + 30{,}000$.

ULTIMATE STRENGTH OF PRESTRESSED SECTIONS

As in reinforced concrete members, the amount of steel in pre-stressed sections is limited to ensure tensile failures. The limitation rarely presents a problem except in members with very small amounts of pre-stressing or in members that have not only pre-stress strands, but also some regular reinforcing bars.

The calculations involved in determining the permissible ultimate capacity of a rectangular pre-stressed beam.

ULTIMATE STRENGTH OF PRESTRESSED SECTIONS

EXAMPLE 19.3

Determine the permissible ultimate moment capacity of the prestressed bonded beam of Figure 19.9 if $f_{py} = 240{,}000$ psi, f_{pu} is 275,000 psi, and f'_c is 5000 psi.

SOLUTION

Approximate Value of f_{ps} from ACI Code

$$\rho_p = \frac{A_{ps}}{bd} = \frac{1.40}{(12)(21.5)} = 0.00543$$

$$\frac{f_{py}}{f_{pu}} = \frac{240{,}000}{275{,}000} = 0.873$$

$Y_p = 0.40$, as given in ACI Equation 18-3. f_{ps} = estimated stress in pre-stressed reinforcement at nominal strength. Note that $\beta_1 = 0.80$ for 5000 psi concrete and *d*, the distance from the extreme compression fiber of the beam to the centroid of any nonpre-stressed tension reinforcement is 0 since there is no such reinforcement in this beam.

ULTIMATE STRENGTH OF PRESTRESSED SECTIONS

$$f_{ps} = f_{pu}\left\{1 - \frac{\gamma_p}{\beta_1}\left[\rho_p \frac{f_{pu}}{f'_c} + \frac{d}{d_p}(\omega - \omega')\right]\right\} \qquad \text{(ACI Equation 18-3)}$$

$$= 275\left\{1 - \frac{0.40}{0.80}\left[0.00543\left(\frac{275}{5}\right) + 0\right]\right\} = 233.9 \text{ ksi}$$

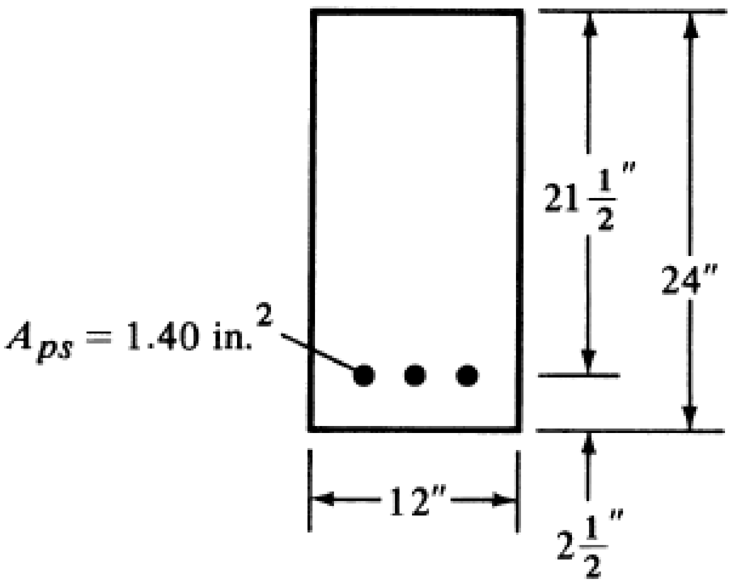


Figure 19.9

ULTIMATE STRENGTH OF PRESTRESSED SECTIONS

Moment Capacity

$$a = \frac{A_{ps} f_{ps}}{0.85 f'_c b} = \frac{(1.40)(233.9)}{(0.85)(5)(12)} = \underline{\underline{6.42''}}$$

$$c = \frac{a}{\beta_1} = \frac{6.42}{0.80} = 8.03''$$

$$\epsilon_t = \frac{d - c}{c} 0.003 = \frac{21.5 - 803}{8.03}(0.003)$$

$$= 0.00503 > 0.0050$$

$\underline{\underline{\therefore \text{ The member is}}}$
$\underline{\underline{\text{tension controlled and } \phi = 0.9.}}$

$$\phi M_n = \phi A_{ps} f_{ps}\left(d - \frac{a}{2}\right) = (0.9)(1.40)(233.9)\left(21.5 - \frac{6.42}{2}\right)$$

$$= 5390 \text{ in.-k} = \underline{\underline{449.2 \text{ ft-k}}}$$

ULTIMATE STRENGTH OF PRESTRESSED SECTIONS

The approximate value of f_{ps} obtained by the ACI formula is very satisfactory for all practical purposes. Actually, a slightly more accurate value of f_{ps} and thus of the moment capacity of the section can be obtained by calculating the strain in the pre-stress strands due to the pre-stress and adding to it the strain due to the ultimate moment. This latter strain can be determined from the values of *a* and the strain diagram, as frequently used in earlier chapters for checking to see if tensile failures control in reinforced concrete beams. With the total strain, a more accurate cable stress can be obtained by referring to the stress–strain curve for the pre-stressing steel being used. Such a curve is shown in next figure.

ULTIMATE STRENGTH OF PRESTRESSED SECTIONS

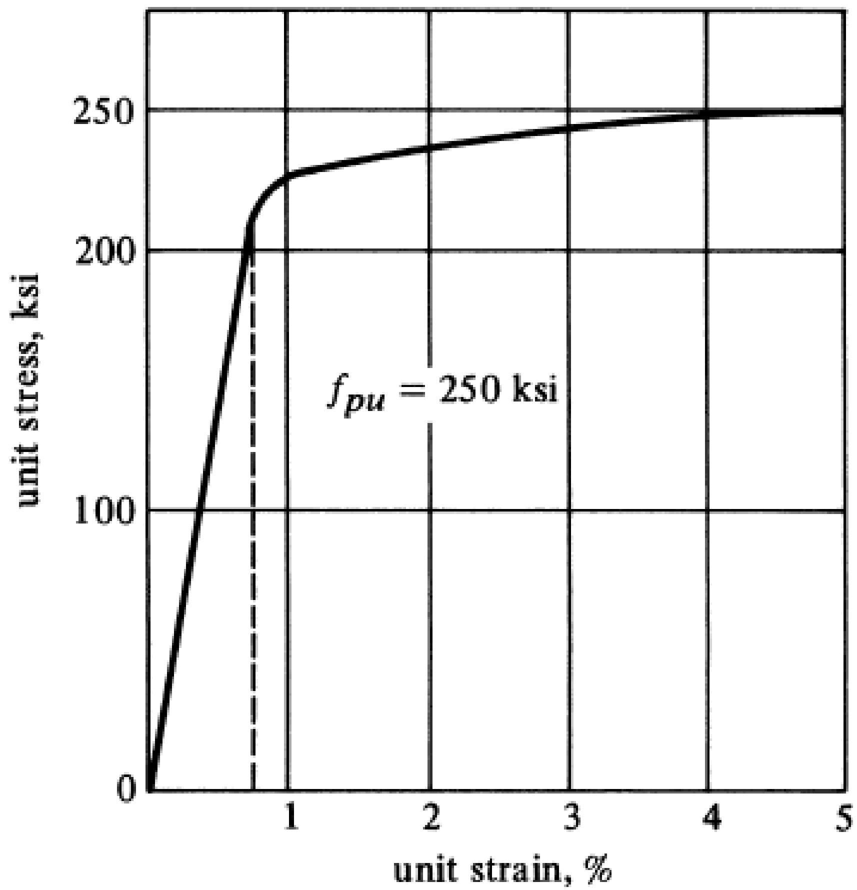


Figure 19.10 Typical stress–strain curve for high-tensile steel wire.

ULTIMATE STRENGTH OF PRESTRESSED SECTIONS

The analysis described herein is satisfactory for pre-tensioned beams or for bonded posttensioned beams, but is not so good for un-bonded posttensioned members. In these latter beams the steel can slip with respect to the concrete; as a result, the steel stress is almost constant throughout the member. The calculations for *Mu* for such members are less accurate than for bonded members. Unless some ordinary reinforcing bars are added to these members, large cracks may form which are not attractive and which can lead to some corrosion of the pre-stress strands.

If a pre-stressed beam is satisfactorily designed with service loads, then checked by strength methods and found to have insufficient strength to resist the factored loads ($M_u = 1.2M_D + 1.6M_L$), non pre-stressed reinforcement may be added to increase the factor of safety.

ULTIMATE STRENGTH OF PRESTRESSED SECTIONS

The increase in T due to these bars is assumed to equal $A_s f_y$ (Code 18.7.3). The Code (18.8.2) further states that the total amount of pre-stressed and non pre-stressed reinforcement shall be sufficient to develop an ultimate moment equal to at least 1.2 times the cracking moment of the section. This cracking moment is calculated with the modulus of rupture of the concrete, except for flexural members with a shear and flexural strength equal to at least twice that required to support the factored loads and for two-way, un-bonded posttensioned slabs. This additional steel also will serve to reduce cracks. (The 1.2 requirement may be waived for two-way un-bonded posttensioned slabs and for flexural members with shear and flexural strength at least equal to twice that required by ACI Section 9.2.)

DEFLECTIONS

The deflections of pre-stressed concrete beams must be calculated very carefully. Some members that are completely satisfactory in all other respects are not satisfactory for practical use because of the magnitudes of their deflections.

In previous chapters, one method used for limiting deflections was to specify minimum depths for various types of members. These minimum depths, however, are applicable only to non pre-stressed sections. The actual deflection calculations are made as they are for members made of other materials, such as structural steel, reinforced concrete, and so on. However, the same problem exists for reinforced concrete members, and that is the difficulty of determining the modulus of elasticity to be used in the calculations.

DEFLECTIONS

The modulus varies with age, with different stress levels, and with other factors. Usually the gross moments of inertia are used for immediate deflection calculations for members whose calculated extreme fiber stresses at service loads in the pre-compressed tensile zone are $\leq 7.5\sqrt{f_c'}$. (ACI 18.3.3). Transformed *I* values may be used for other situations as described in ACI Sections 18.3.3, 18.3.4, and 18.3.5.

The deflection due to the force in a set of straight tendons is considered first in this section, with reference being made to Figure 19.11(a). The pre-stress forces cause a negative moment equal to P_e and thus an upward deflection or camber of the beam. This ℄ deflection can be calculated by taking moments at the point desired when the conjugate beam is loaded with the *M*/*EI* diagram. At the ℄ the deflection equals

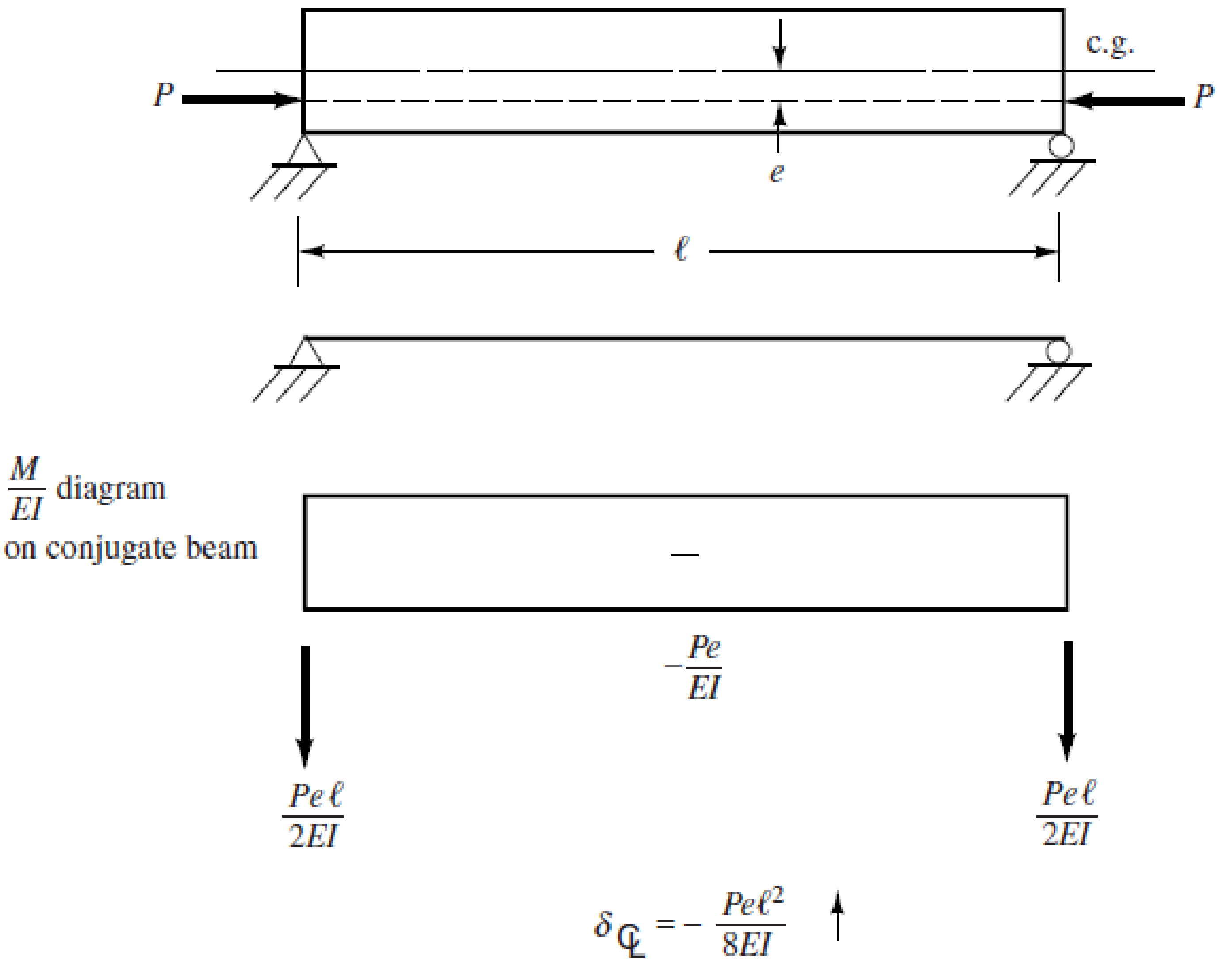
DEFLECTIONS

$$-\left(\frac{Pe\ell}{2EI}\right)\left(\frac{\ell}{2}-\frac{\ell}{4}\right)=-\frac{Pe\ell^2}{8EI}\uparrow$$

Should the cables not be straight, the deflection will be different due to the different negative moment diagram produced by the cable force. If the cables are bent down or curved, as shown in parts (b) and (c) of Figure 19.11, the conjugate beam can again be applied to compute the deflections. The resulting values are shown in the figure.

The deflections due to the tendon stresses will change with time. First of all, the losses in stress in the pre-stress tendons will reduce the negative moments they produce and thus the upward deflections. On the other hand, the long-term compressive stresses in the bottom of the beam due to the pre-stress negative moments will cause creep and therefore increase the upward deflections.

DEFLECTIONS



(a)

DEFLECTIONS

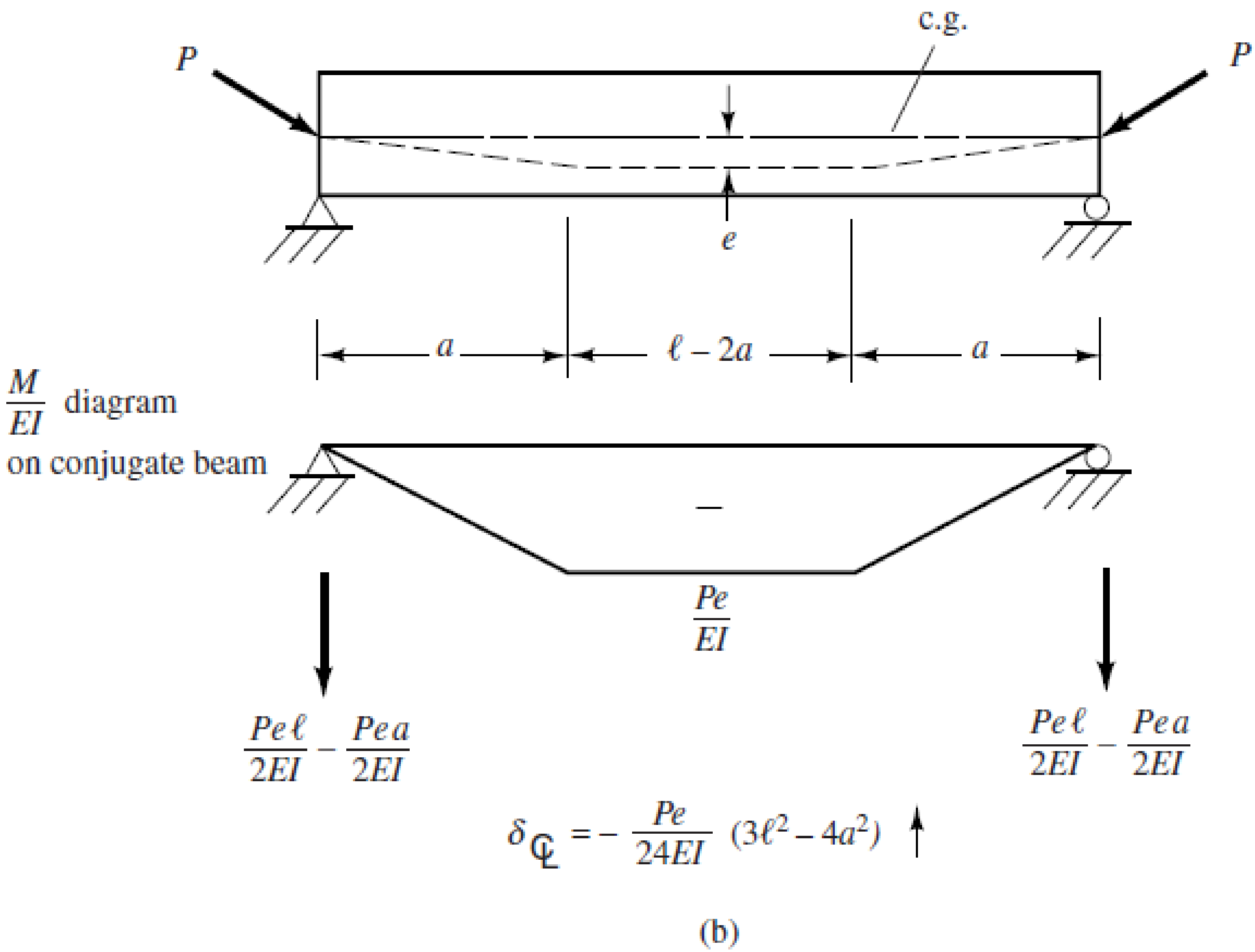
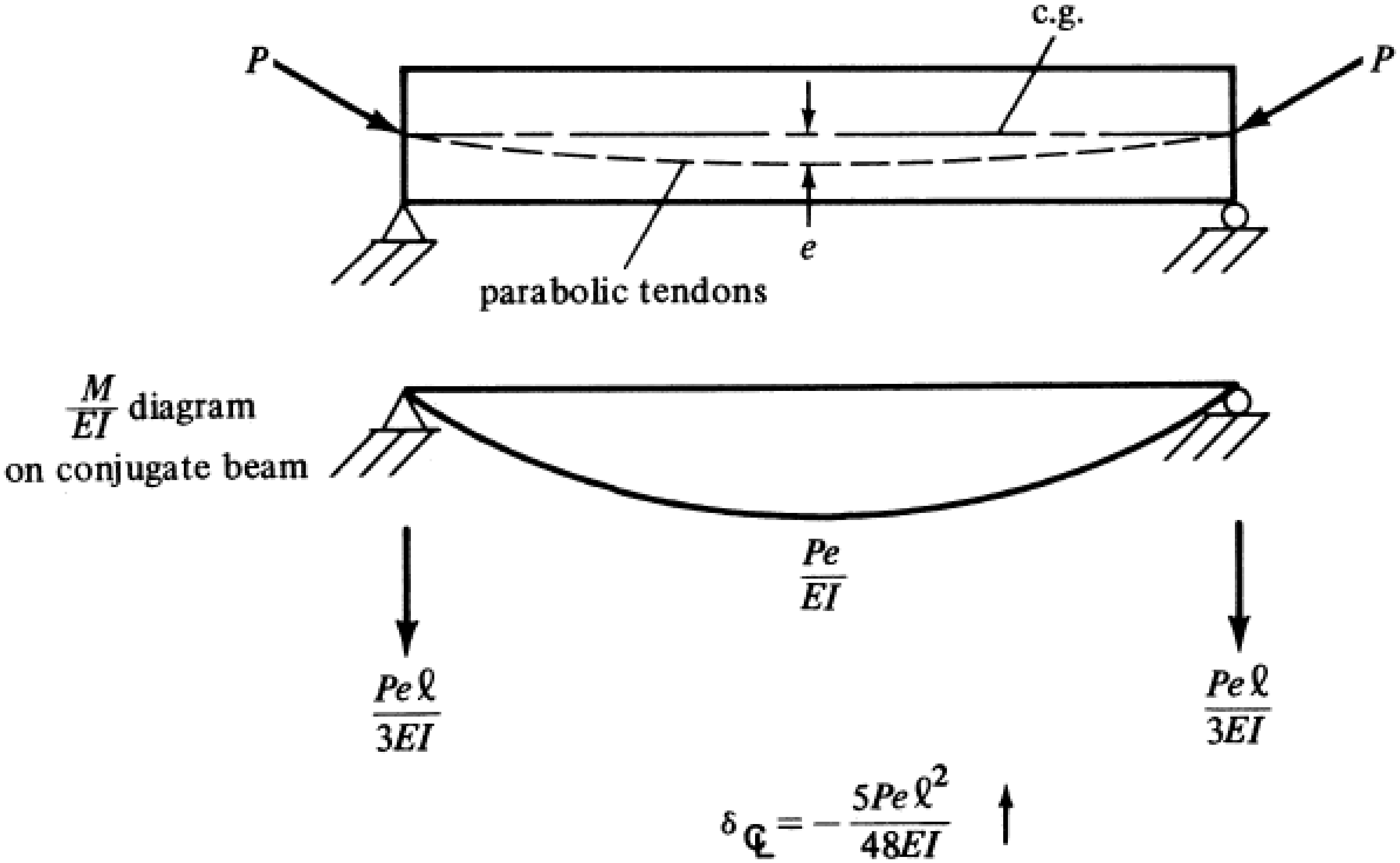


Figure 19.11 Deflections in prestressed beams.

DEFLECTIONS



(c)

DEFLECTIONS

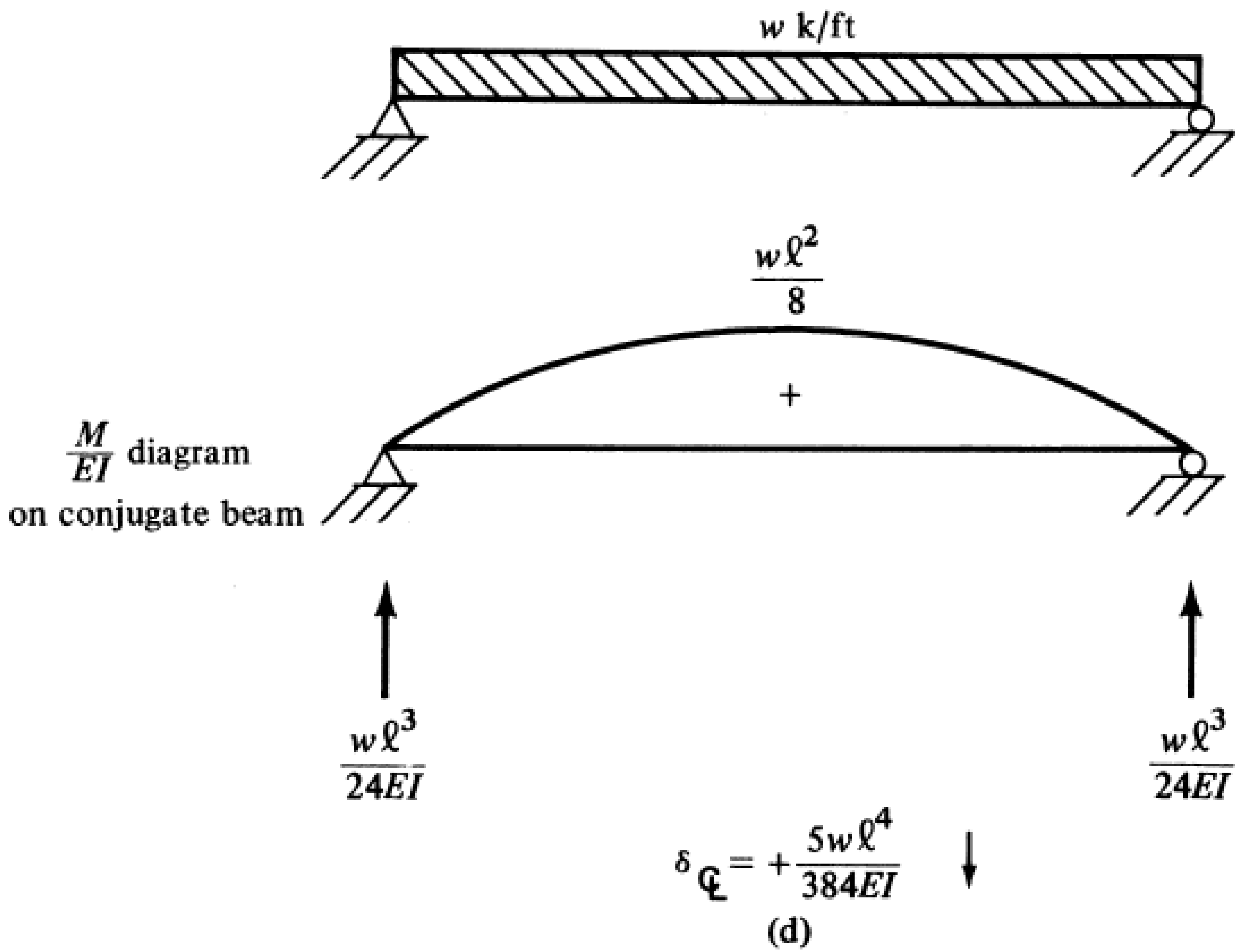


Figure 19.11 (*continued*)

DEFLECTIONS

In addition to the deflections caused by the tendon stresses, there are deflections due to the beam’s own weight and due to the additional dead and live loads subsequently applied to the beam. These deflections can be computed and superimposed on the ones caused by the tendons. Figure 19.11(d) shows the ℄ deflection of a uniformly loaded simple beam obtained by taking moments at the ℄ when the conjugate beam is loaded with the *M*/*EI* diagram.

Next example shows the initial and long-term deflection calculations for a rectangular pre-tensioned beam.

DEFLECTIONS

EXAMPLE 19.4

The pretensioned rectangular beam shown in Figure 19.12 has straight cables with initial stresses of 175 ksi and final stresses after losses of 140 ksi. Determine the deflection at the beam ℄ immediately after the cables are cut. $E = 4 \times 10^6$ psi. Assume concrete is uncracked.

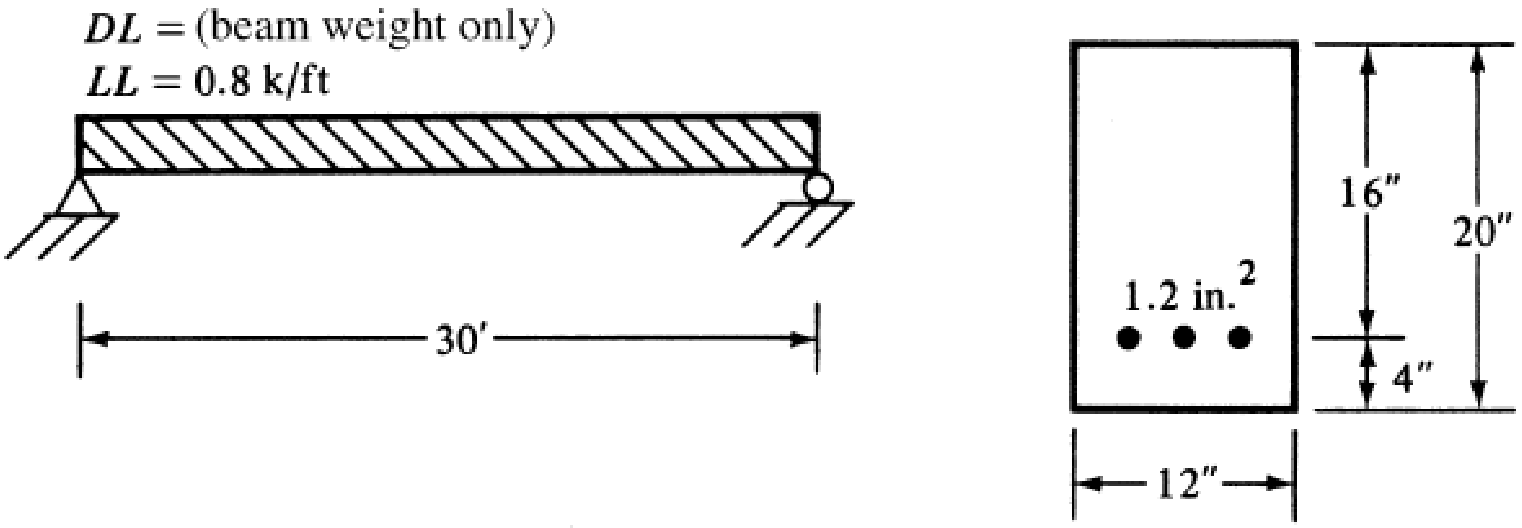


Figure 19.12

DEFLECTIONS

EXAMPLE 19.4

$$I_g = \left(\frac{1}{12}\right)(12)(20)^3 = 8000 \text{ in.}^4$$

$$e = 6''$$

$$Bm \text{ weight} = \frac{(12)(20)}{144}(150) = 250 \text{ lb/ft}$$

Deflection Immediately after Cables Are Cut

$$\delta \text{ due to cable} = -\frac{Pe\ell^2}{8EI} = -\frac{(1.2 \times 175{,}000)(6)(12 \times 30)^2}{(8)(4 \times 10^6)(8000)} = -0.638'' \uparrow$$

$$\delta \text{ due to beam weight} = +\frac{5w\ell^4}{384EI} = \frac{(5)\left(\frac{250}{12}\right)(12 \times 30)^4}{(384)(4 \times 10^6)(8000)} = \underline{+0.142'' \downarrow}$$

$$\text{Total deflection} = -0.496'' \uparrow$$

DEFLECTIONS

Long-term deflections can be computed as previously described. From the preceding example it can be seen that, not counting external loads, the beam is initially cambered upward by 0.496 in.; as time goes by, this camber increases due to creep in the concrete. Such a camber is often advantageous in offsetting deflections caused by the superimposed loads. In some members, however, the camber can be quite large, particularly for long spans and where lightweight aggregates are used. If this camber is too large, the results can be quite detrimental to the structure (warping of floors, damage to roofing, cracking and warping of partitions, and so on).

DEFLECTIONS

The resulting cambers may be rather large, and, worse, they may not be equal in the different sections. It then becomes necessary to force the different sections to the same deflection and tie them together in some fashion so that a smooth surface is provided for roofing. Once the surface is even, the members may be connected by welding together metal inserts, such as angles that were cast in the edges of the different sections for this purpose.

Both reinforced concrete members and pre-stressed members with overhanging or cantilevered ends will often have rather large deflections. The total deflections at the free end of these members are due to the sum of the normal deflections plus the effect of support rotations.

DEFLECTIONS

This latter effect may frequently be the larger of the two, and, as a result, the sum of the two deflections may be so large as to affect the appearance of the structure detrimentally. For this reason, many designers try to avoid cantilevered members in pre-stressed construction.

SHEAR IN PRESTRESSED SECTIONS

Web reinforcement for pre-stressed sections is handled in a manner similar to that used for a conventional reinforced concrete beam. In the expressions that follow, b_w is the web width or the diameter of a circular section, and d is the distance from the extreme fiber in compression to the centroid of the tensile reinforcement. Should the reaction introduce compression into the end region of a pre-stressed member, sections of the beam located at distances less than $h/2$ from the face of the support may be designed for the shear computed at $h/2$ where h is the overall thickness of the member.

$$v_u = \frac{V_u}{\phi b_w d}$$

SHEAR IN PRESTRESSED SECTIONS

The Code (11.4.1) provides two methods for estimating the shear strength that the concrete of a pre-stressed section can resist. There is an approximate method, which can be used only when the effective pre-stress force is equal to at least 40% of the tensile strength of the flexural reinforcement f_{pu}, and a more detailed analysis, which can be used regardless of the magnitude of the effective pre-stress force. These methods are discussed in the paragraphs to follow.

SHEAR IN PRESTRESSED SECTIONS

Approximate Method

With this method, the nominal shear capacity of a pre-stressed section can be taken as

$$V_c = \left(0.6\sqrt{f'_c} + \frac{700V_u d}{M_u}\right)b_w d \qquad \text{(ACI Equation 11-9)}$$

The Code (11.4.1) states that regardless of the value given by this equation, V_c need not be taken as less than $\boxed{2\sqrt{f'_c}b_w d,}$ nor may it be larger than $\boxed{5\sqrt{f'_c}b_w d.}$ In this expression, V_u is the maximum design shear at the section being considered, M_u is the design moment at the same section occurring simultaneously with V_u, and d is the distance from the extreme compression fiber to the centroid of the pre-stressed tendons. The value of $V_u d/M_u$ is limited to a maximum value of 1.0. (At sections near the ends of members where pre-stressing forces may not have been fully developed or transferred, V_c may not be larger than the values given in ACI Sections 11.4.3)

SHEAR IN PRESTRESSED SECTIONS

More Detailed Analysis

If a more detailed analysis is desired (it will have to be used if the effective pre-stressing force is less than 40% of the tensile strength of the flexural reinforcement), the nominal shear force carried by the concrete is considered to equal the smaller of V_{ci} or V_{cw}, to be defined here. The term V_{ci} represents the nominal shear strength of a member provided by the concrete when diagonal cracking results from combined shear and moment. The term V_{cw} represents the nominal shear strength of the member provided by the concrete when diagonal cracking results from excessive principal tensile stress in the concrete. In both expressions to follow, d is the distance from the extreme compression fiber to the centroid of the pre-stressed tendons or is $0.8h$, whichever is greater (Code 11.4.2.3).

SHEAR IN PRESTRESSED SECTIONS

The estimated shear capacity V_{ci} can be computed by the following expression, given by the ACI Code (11.4.2.1):

$$V_{ci} = 0.6\sqrt{f'_c}\,b_w d + V_d + \frac{V_i M_{cr}}{M_{\max}} \quad \text{but need not be taken as less than } 1.7\sqrt{f'_c}\,b_w d$$

(ACI Equation 11-10)

In this expression, V_d is the shear at the section in question due to service dead load, M_{max} is the factored maximum bending moment at the section due to externally applied design loads, V_i is the shear that occurs simultaneously with M_{max}, and M_{cr} is the cracking moment, which is to be determined as follows:

$$M_{cr} = \left(\frac{I}{y_t}\right)\left(6\sqrt{f'_c} + f_{pe} - f_d\right)$$

(ACI Equation 11-11)

SHEAR IN PRESTRESSED SECTIONS

Where

I = the moment of inertia of the section that resists the externally applied loads

Y_t = the distance from the centroidal axis of the gross section (neglecting the reinforcing) to the extreme fiber in tension

F_{pe} = the compressive stress in the concrete due to pre-stress after all losses at the extreme fiber of the section where the applied loads cause tension

f_d = the stress due to unfactored dead load at the extreme fiber where the applied loads cause tension

SHEAR IN PRESTRESSED SECTIONS

From a somewhat simplified principal tension theory, the shear capacity of a beam is equal to the value given by the following expression but need not be less than $1.7\sqrt{f'_c}b_w d$.

$$V_{cw} = (3.5\sqrt{f'_c} + 0.3f_{pc})b_w d + V_p \qquad \text{(ACI Equation 11-12)}$$

In this expression, f_{pc} is the calculated compressive stress (in pounds per square inch) in the concrete at the centroid of the section resisting the applied loads due to the effective pre-stress after all losses have occurred. (Should the centroid be in the flange, f_{pc} is to be computed at the junction of the web and flange.) V_p is the vertical component of the effective pre-stress at the section under consideration.

SHEAR IN PRESTRESSED SECTIONS

Alternately, the Code (11.4.2.2) states that V_{cw} may be taken as the shear force that corresponds to a multiple of dead load plus live load, which results in a calculated principal tensile stress equal to $4\sqrt{f'_c}$ at the centroid of the member or at the intersection of the flange and web if the centroid falls in the web.

A further comment should be made here about the computation of f_{pc} for pre-tensioned members, since it is affected by the transfer length. The Code (11.4.4) states that the transfer length can be taken as 50 diameters for strand tendons and 100 diameters for wire tendons. The pre-stress force may be assumed to vary linearly from zero at the end of the tendon to a maximum at the aforesaid transfer distance. If the value of $h/2$ is less than the transfer length, it is necessary to consider the reduced pre-stress when V_{cw} is calculated (ACI 11.4.3).

DESIGN OF SHEAR REINFORCEMENT

Should the computed value of V_u exceed V_c, the area of vertical stirrups (the Code not permitting inclined stirrups or bent-up bars in pre-stressed members) must not be less than A_v as determined by the following expression from the Code (11.5.6.2):

$$V_s = \frac{A_v f_y d}{s} \qquad \text{(ACI Equation 11-15)}$$

As in conventional reinforced concrete design, a minimum area of shear reinforcing is required at all points where V_u is greater than $\frac{1}{2}\phi V_c$ This minimum area is to be determined from the expression to follow if the effective pre-stress is less than 40% of the tensile strength of the flexural reinforcement (ACI Code 11.5.3):

$$A_v = 0.75\sqrt{f'_c}\,\frac{b_w s}{f_y} \text{ but shall not be less than } \frac{50 b_w s}{f_y} \quad \text{(ACI Equation 11-13)}$$

where b_w and s are in inches.

DESIGN OF SHEAR REINFORCEMENT

If the effective pre-stress is equal to or greater than 40% of the tensile strength of the flexural reinforcement, the following expression, in which A_{ps} is the area of pre-stressed reinforcement in the tensile zone, is to be used to calculate A_v:

$$A_v = \left(\frac{A_{ps}}{80}\right)\left(\frac{f_{pu}}{f_y}\right)\left(\frac{s}{d}\right)\sqrt{\left(\frac{d}{b_w}\right)} \qquad \text{(ACI Equation 11-14)}$$

Section 11.5.4.1 of the ACI Code states that in no case may the maximum spacing exceed 0.75*h* or 24 in. Examples 19.5 and 19.6 illustrate the calculations necessary for determining the shear strength and for selecting the stirrups for a pre-stressed beam.

DESIGN OF SHEAR REINFORCEMENT

EXAMPLE 19.5

Calculate the shearing strength of the section shown in Figure 19.13 at 4 ft from the supports, using both the approximate method and the more detailed method allowed by the ACI Code. Assume that the area of the pre-stressing steel is 1.0 in.2, the effective pre-stress force is 250 k, and f'_c = 4000 psi.

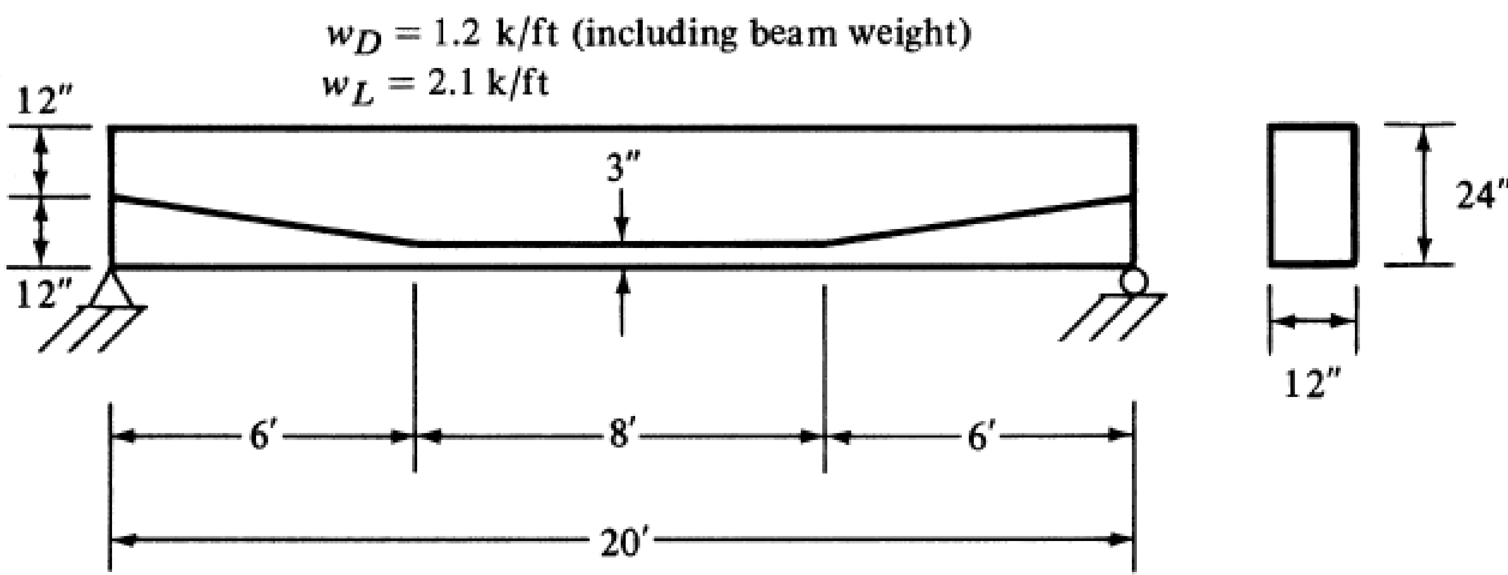


Figure 19.13

DESIGN OF SHEAR REINFORCEMENT

SOLUTION

Approximate Method

$$w_u = (1.2)(1.2) + (1.6)(2.1) = 4.8 \text{ k/ft}$$

$$V_u = (10)(4.8) - (4)(4.8) = 28.8 \text{ k}$$

$$M_u = (10)(4.8)(4) - (4)(4.8)(2) = 153.6 \text{ ft-k}$$

$$\frac{V_u d}{M_u} = \frac{(28.8)(24 - 3 - 3)}{(12)(153.6)} = 0.281 < 1.0 \qquad \underline{\underline{\text{OK}}}$$

$$V_c = \left(0.6\sqrt{f'_c} + 700\frac{V_u d}{M_u}\right)b_w d \qquad \text{(ACI Equation 11-9)}$$

$$= [0.6\sqrt{4000} + (700)(0.281)](12)(18) = 50{,}684 \text{ lb}$$

$$\text{Minimum } V_c = (2\sqrt{4000})(12)(18) = 27{,}322 \text{ lb} < 50{,}684 \text{ lb}$$

$$\text{Maximum } V_c = (5\sqrt{4000})(12)(18) = 68{,}305 \text{ lb} > 50{,}684 \text{ lb}$$

$$V_c = 50{,}684 \text{ lb}$$

DESIGN OF SHEAR REINFORCEMENT

More Detailed Method

$$I = \left(\frac{1}{12}\right)(12)(24)^3 = 13{,}824 \text{ in.}^4$$

$$y_t = 12''$$

$$f_{pe} = \text{compressive stress in concrete due to prestress after all losses}$$

$$= \frac{P}{A} + \frac{Pec}{I}$$

$$f_{pe} = \frac{250{,}000}{(12)(24)} + \frac{(250{,}000)(6)(12)}{13{,}824} = 2170 \text{ psi}$$

$$M_d = \text{dead load moment at } 4' \text{ point} = (10)(1.2)(4) - (4)(1.2)(2)$$

$$= 38.4 \text{ ft-k}$$

$$f_d = \text{stress due to the dead load moment} = \frac{(12)(38{,}400)(12)}{13{,}824}$$

$$= 400 \text{ psi}$$

DESIGN OF SHEAR REINFORCEMENT

$$M_{cr} = \text{cracking moment} = \left(\frac{I}{y_t}\right)(6\sqrt{f'_c} + f_{pe} - f_d) \qquad \text{(ACI Equation 11-11)}$$

$$= \left(\frac{13{,}824}{12}\right)(6\sqrt{4000} + 2170 - 400) = 2{,}476{,}193 \text{ in.}-\text{lb}$$

$$= 206{,}349 \text{ ft-lb}$$

$$\text{Beam weight} = \frac{(12)(24)}{144}(150) = 300 \text{ lb/ft}$$

$$w_u \text{ not counting beam weight} = (1.2)(1.2 - 0.3) + (1.6)(2.1) = 4.44 \text{ k/ft}$$

$$M_{max} = (10)(4.44)(4) - (4)(4.44)(2) = 142.08 \text{ ft-k} = 142{,}080 \text{ ft-lb}$$

$$V_i \text{ due to } w_u \text{ occurring same time as } M_{max} = (10)(4.44) - (4)(4.44)$$

$$= 26.64 \text{ k} = 26{,}640 \text{ lb}$$

$$V_d = \text{dead load shear} = (10)(1.2) - (4)(1.2) = 7.2 \text{ k} = 7200 \text{ lb}$$

$$d = 24 - 3 - 3 = 18'' \quad \text{or} \quad (0.8)(24) = \underline{\underline{19.2''}}$$

$$V_{ci} = 0.6\sqrt{f'_c}b_w d + V_d + \frac{V_i M_{cr}}{M_{max}} \qquad \text{(ACI Equation 11-10)}$$

$$= (0.6\sqrt{4000})(12)(19.2) + 7200 + \frac{(26{,}640)(206{,}349)}{42{,}080} = 54{,}634 \text{ lb}$$

$$\text{but need not be less than } (1.7\sqrt{4000})(12)(19.2) = 24{,}772 \text{ lb}$$

DESIGN OF SHEAR REINFORCEMENT

Computing V_{cw}

f_{pc} = calculated compressive stress in psi at the centroid of the concrete due to the effective prestress

$$= \frac{250{,}000}{(12)(24)} = 868 \text{ psi}$$

$$V_p = \text{vertical component of effective prestress at section} = \frac{9}{\sqrt{9^2 + 72^2}}(250{,}000)$$

$$= \left(\frac{9}{72.56}\right)(250{,}000) = 31{,}009 \text{ lb}$$

$$V_{cw} = (3.5\sqrt{f_c'} + 0.3f_{pc})b_w d + V_p \qquad \text{(ACI Equation 11-12)}$$

$$= (3.5\sqrt{4000} + 0.3 \times 868)(12)(19.2) + 31{,}009 = 142{,}006 \text{ lb}$$

Using Lesser of V_{ci} or V_{cw}

$$V_c = 54{,}634 \text{ lb}$$

DESIGN OF SHEAR REINFORCEMENT

EXAMPLE 19.6

Determine the spacing of #3 ⊔ stirrups required for the beam of Example 19.5 at 4 ft from the end support if f_{pu} is 250 ksi for the prestressing steel and f_y for the stirrups is 40 ksi. Use the value of V_c obtained by the approximate method, 50,684 lb.

SOLUTION

$$w_u = (1.2)(1.2) + (1.6)(2.1) = 4.8 \text{ k/ft}$$

$$V_u = (10)(4.8) - (4)(4.8) = 28.8 \text{ k}$$

$$\phi V_c = (0.75)(50{,}684) = 38{,}013 \text{ lb}$$

$$> V_u = 28{,}800 \text{ lb}$$

$$V_u > \frac{\phi V_c}{2} = 19{,}006.5 \text{ lb} < \phi V_c$$

A minimum amount of reinforcement is needed.

Since effective prestress is greater than 40% of tensile strength of reinforcing,

$$A_v = \left(\frac{A_{ps}}{80}\right)\left(\frac{f_{pu}}{f_y}\right)\left(\frac{s}{d}\right)\sqrt{\left(\frac{d}{b_w}\right)} \qquad \text{(ACI Equation 11-14)}$$

DESIGN OF SHEAR REINFORCEMENT

EXAMPLE 19.6

SOLUTION

$$(2)(0.11) = \left(\frac{1.0}{80}\right)\left(\frac{250{,}000}{40{,}000}\right)\left(\frac{s}{18}\right)\sqrt{\frac{18}{12}}$$

$$s = 41.38'', \text{ but maximum } s = \left(\frac{3}{4}\right)(24) = 18'' \qquad \underline{\underline{\text{Use } 18''}}$$

ADDITIONAL TOPICS

Stresses in End Blocks

The part of a pre-stressed member around the end anchorages of the steel tendons is called the *end block*. In this region the pre-stress forces are transferred from very concentrated areas out into the whole beam cross section. It has been found that the length of transfer for posttensioned members is less than the height of the beam and in fact is probably much less.

ADDITIONAL TOPICS

Stresses in End Blocks

For posttensioned members, there is direct-bearing compression at the end anchorage; therefore, solid end blocks are usually used there to spread out the concentrated pre-stress forces. To prevent bursting of the block, either wire mesh or a grid of vertical and horizontal reinforcing bars is placed near the end face of the beam. In addition, both vertical and horizontal reinforcing is placed throughout the block.

For pre-tensioned members where the pre-stress is transferred to the concrete by bond over a distance approximately equal to the beam depth, a solid end block is probably not necessary, but spaced stirrups are needed. A great deal of information on the subject of end block stresses for posttensioned and pre-tensioned members is available.

ADDITIONAL TOPICS

Composite Construction

Precast pre-stressed sections are frequently used in buildings and bridges in combination with cast-in-place concrete. Should such members be properly designed for shear transfer so the two parts will act together as a unit, they are called *composite sections*. Examples of such members were previously shown in parts (e) and (f) of Figure 19.8. In composite construction, the parts that are difficult to form and that contain most of the reinforcing are precast, whereas the slabs and perhaps the top of the beams, which are relatively easy to form, are cast in place.

The precast sections are normally designed to support their own weights plus the green cast-in-place concrete in the slabs plus any other loads applied during construction.

ADDITIONAL TOPICS

Continuous Members

Continuous pre-stressed sections may be cast in place completely with their tendons running continuously from one end to the other. It should be realized for such members that where the service loads tend to cause positive moments, the tendons should produce negative moments and vice versa. This means that the tendons should be below the member's center of gravity in normally positive moment regions and above the center of gravity in normally negative moment regions. To produce the desired stress distributions, it is possible to use curved tendons and members of constant cross section or straight tendons with members of variable cross section. In Figure 19.14 several continuous beams of these types are shown.

ADDITIONAL TOPICS

Continuous Members

Another type of continuous section that has been used very successfully in the United States, particularly for bridge construction, involves the use of precast pre-stressed members made into continuous sections with cast-in-place concrete and regular reinforcing steel. Figure 19.14(d) shows such a case. For such construction the precast section resists a portion of the dead load, while the live load and the dead load that is applied after the cast-in-place concrete hardens are resisted by the continuous member.

ADDITIONAL TOPICS

Partial Pre-stressing

During the early days of pre-stressed concrete, the objective of the designer was to proportion members that could never be subject to tension when service loads were applied. Such members are said to be *fully pre-stressed*. Subsequent investigations of fully pre-stressed members have shown that they often have an appreciable amount of extra strength. As a result, many designers now believe that certain amounts of tensile stresses can be permitted under service loads. Members that are permitted to have some tensile stresses are said to be *partially pre-stressed*.

A major advantage of a partially pre-stressed beam is a decrease in camber. This is particularly important when the beam load or the dead load is quite low compared to the total design load.

ADDITIONAL TOPICS

Partial Pre-stressing

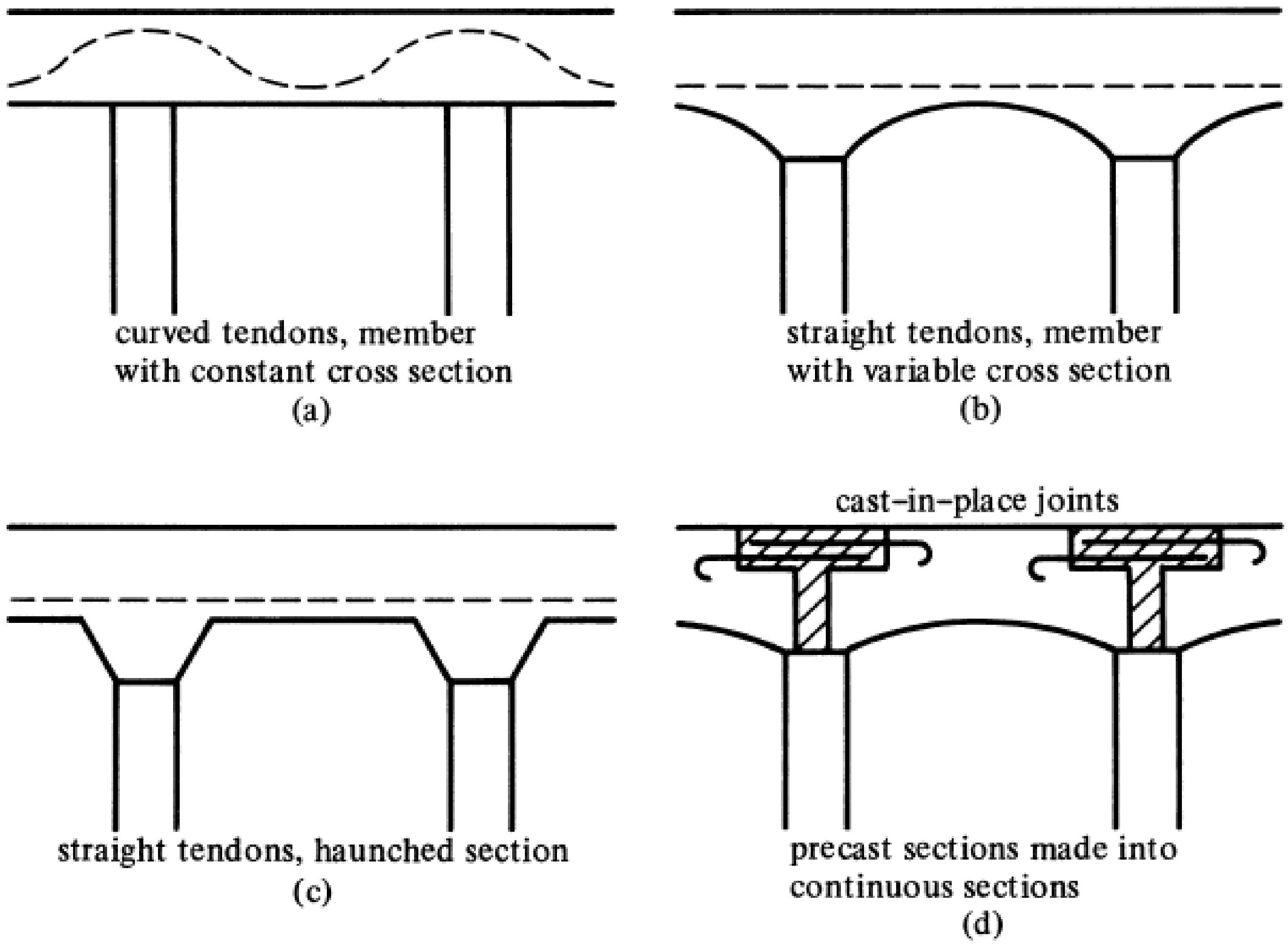


Figure 19.14 Continuous beams.

ADDITIONAL TOPICS

Partial Pre-stressing

To provide additional safety for partially pre-stressed beams, it is common practice to add some conventional reinforcement. This reinforcement will increase the ultimate flexural strength of the members as well as help to carry the tensile stresses in the beam.

Any Questions?

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