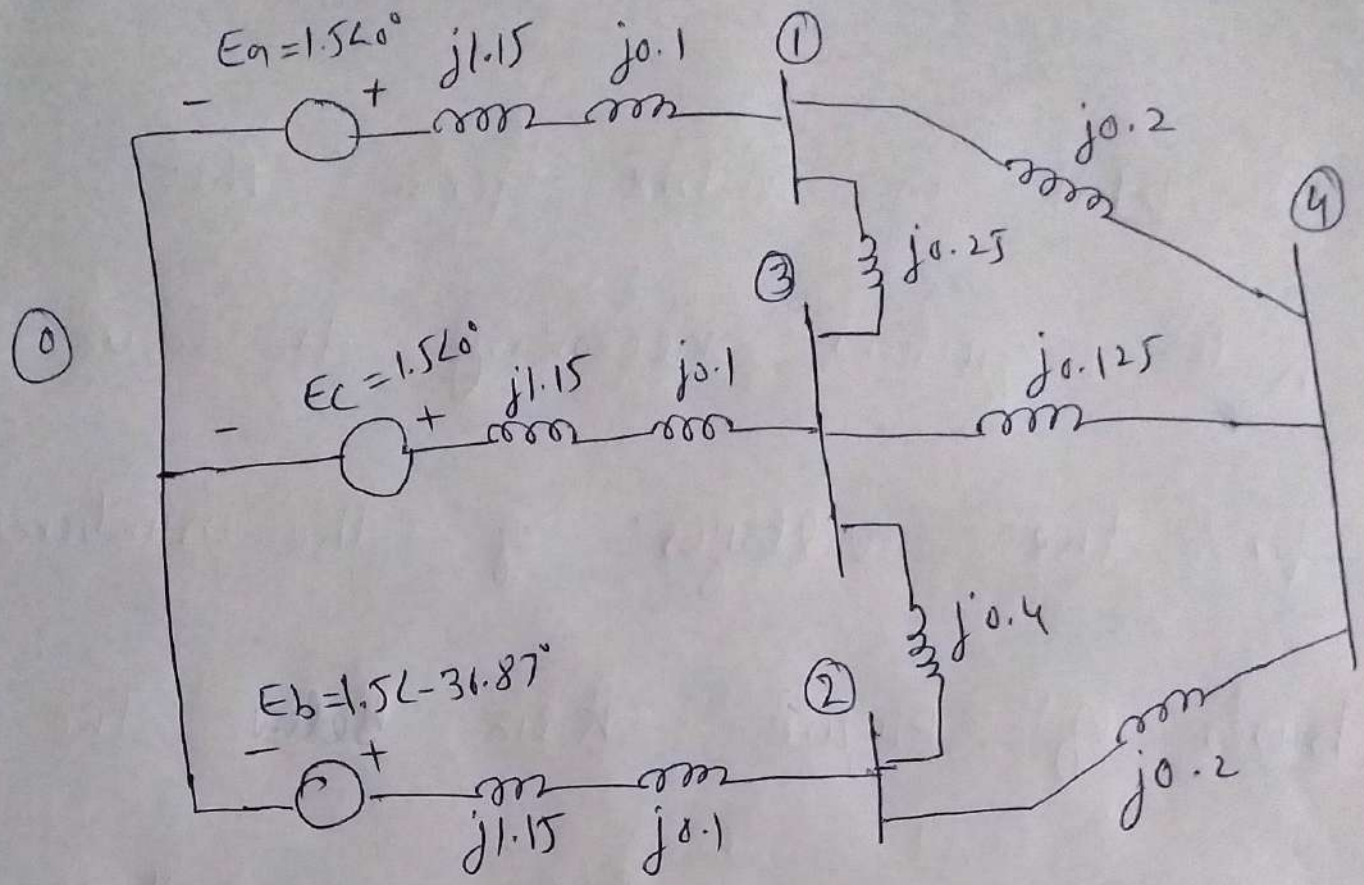
(8) Write in Matrix form the node equations necessary to solve for the voltages of the numbered buses of Figure shown. The network is equivalent as follows. The emfs shown in figure are

$$E_a = 1.5 \angle 0^\circ$$

$$E_b = 1.5 \angle -36.87^\circ$$

and $E_c = 1.5 \angle 0^\circ$, all in per unit.



Sol:-

The current sources are

$$I_1 = I_3 = \frac{1.5\angle 0^\circ}{j1.25}$$

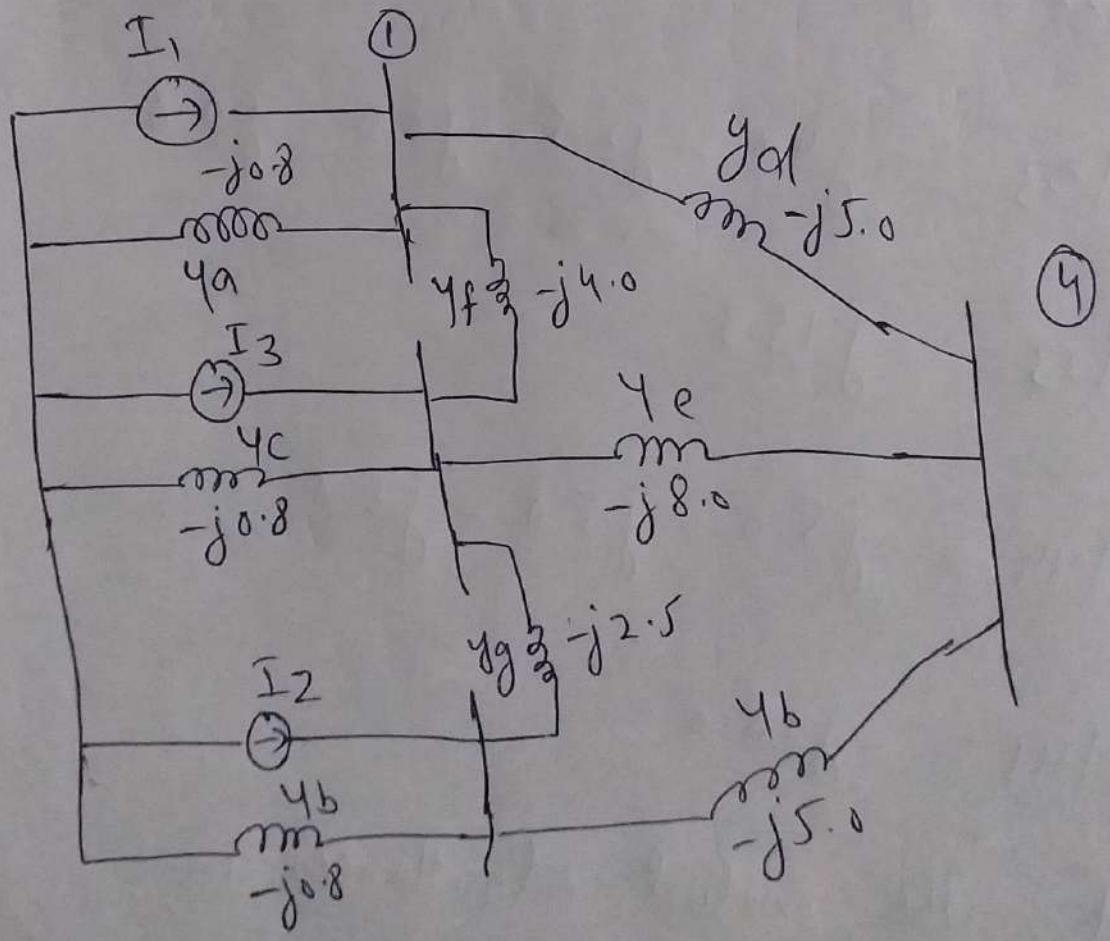
$$= 1.2\angle -90^\circ$$

$$= 0 - j120 \text{ p.u}$$

$$I_2 = \frac{1.5 \angle -36.87^\circ}{j1.25}$$

$$= 1.2 \angle -126.87^\circ$$

$$= 0.72 - j0.96 \text{ P.U}$$



self - admittances in P.U are:-

$$Y_{11} = -j5.0 - j4.0 - j0.8$$

$$Y_{11} = -j9.8$$

$$Y_{22} = -j5.0 - j2.5 - j0.8$$

$$Y_{22} = -j8.3$$

$$Y_{33} = -j4.0 - j2.5 - j8.0 - j0.8$$

$$Y_{33} = -j15.3$$

$$Y_{44} = -j5.0 - j5.0 - j8.0$$

$$Y_{44} = -j18.0$$

and the mutual admittances in P.U are:-

$$Y_{12} = Y_{21} = 0$$

$$Y_{13} = Y_{31} = -(-j4.0)$$

$$= +j4.0$$

$$Y_{14} = Y_{41} = +j5.0$$

$$Y_{23} = Y_{32} = +j2.5$$

$$Y_{24} = Y_{42} = +j5.0$$

$$Y_{34} = Y_{43} = +j8.0$$

The node equations in Matrix form are:-

$$\begin{bmatrix} 0 - j1.20 \\ -0.72 - j0.96 \\ 0 - j1.20 \\ 0 \end{bmatrix} = \begin{bmatrix} -j9.8 & j0.0 & j4.0 & j5.0 \\ j0.0 & -j8.3 & j2.5 & j5.0 \\ j4.0 & j2.5 & -j15.3 & j8.0 \\ j5.0 & j5.0 & j8.0 & -j18.0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Multiplying b/s of matrix by the Inverse of bus admittance Matrix.

i-e $A^{-1} = \frac{1}{\det A} \text{adj } A$

$$\begin{bmatrix} j0.4774 & j0.376 & j0.4020 & j0.4142 \\ j0.3706 & j0.4872 & j0.3922 & j0.4126 \\ j0.4020 & j0.3922 & j0.4558 & j0.4232 \\ j0.4142 & j0.4126 & j0.4232 & j0.4733 \end{bmatrix} \begin{bmatrix} 0-j1.20 \\ -0.72-j0.96 \\ 0-j1.20 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$\begin{bmatrix} 1.4111 - j0.2668 \\ 1.3830 - j0.3508 \\ 1.4059 - j0.2824 \\ 1.4009 - j0.2971 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

and so the voltages are

$$V_1 = 1.4111 - j\,0.2668$$

$$V_1 = 1.436 \angle -10.71^\circ \text{ P.U}$$

$$V_2 = 1.3830 - j0.3508$$

$$V_2 = 1.427 \angle -14.24^\circ \text{ P.U}$$

$$V_3 = 1.4059 - j0.2824$$

$$V_3 = 1.434 \angle -11.36^\circ \text{ P.U}$$

$V_4 = 1.4009 - j0.2971$

$V_4 = 1.432 < 11.97°$ P.U

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