Design Problem-01- Doubly Reinforced Beam;

A section of beam is restricted to 16" * 26" in dimensions. Compute suitable reinforcement to the section to carry safety factored moment of 6600 kips-inch. Take $f'_c = 3$ ksi and $f_y = 40$ ksi.

Sol: $b = 16''$, $h = 26''$, $M_u = 6600$ kip-inches,
$f'_c = 3$ ksi, $f_y = 40$ ksi, Let, $d = h - 3 = 26 - 3 = 23''$.

Assume; $d' = 2.5''$.

Step-01: First check the capacity of section as singly reinf. beam.

$$\rho_{max} = 0.85 * \beta_1 * \frac{f'_c}{f_y} * \left(\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}\right)$$

$$= 0.85 * 0.85 * \frac{3}{40} * \left(\frac{0.003}{0.003 + 0.005}\right)$$

Reinforcement Ratio $= \frac{A_{st}}{b * d}$ ← $\rho_{max} = 0.0203$

As, we know that;

Step-02: $\rho_{max} = \frac{A_{st}}{b * d}$

So, $A_{st} = \rho_{max} * b * d = 0.0203 * 16 * 23 = 7.47 \text{ in}^2$.

Step-03: $M_{u_2} = \phi * A_{st} * f_y * \left(d - \frac{a}{2}\right)$

But first, $a = \frac{A_{st} * f_y}{0.85 * f'_c * b} = \frac{7.47 * 40}{0.85 * 3 * 16} = 7.32''$.

$$M_{u_2} = 0.90 * 7.47 * 40 * \left(23 - \frac{7.32}{2}\right)$$

$= 5200.65 \text{ k}'' < M_u = 6600 \text{ k}''$.

⇒ Design a Section as Doubly Reinforced

Doubly Reinforced Beam:

→ Doubly Reinf. sections contain reinf. both at tension and at compression zones.

→ They become necessary when the cross-section of beam is restricted.

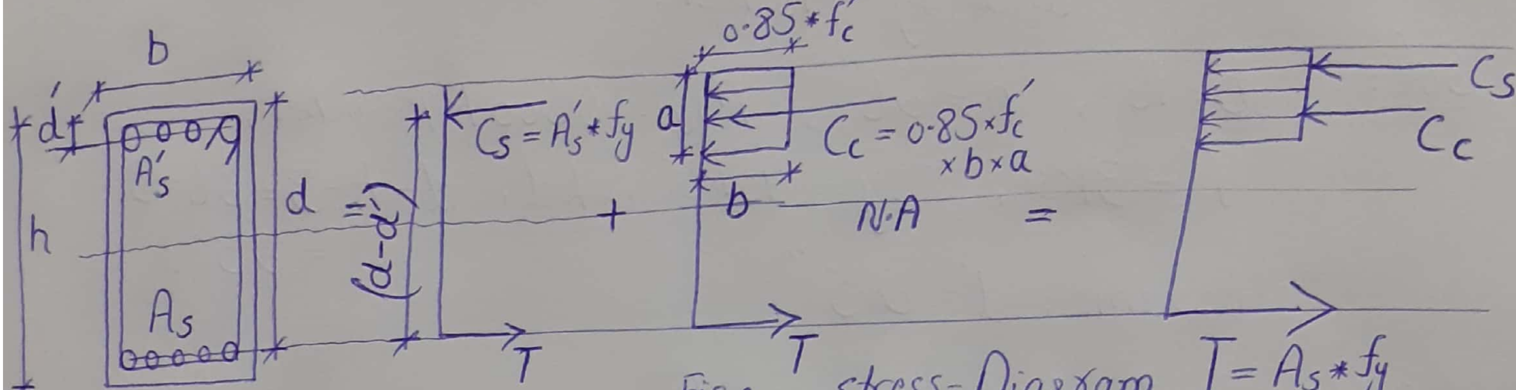


Fig: Beam-X Section

Fig: stress-Diagram

where; b = Width of beam

h = thickness of beam

d = Effective depth = distance from top fibre of beam to center of tensile steel.

d' = Effective depth OR, Effective cover = Distance from top fibre of beam to centre of Comp. steel.

a = Depth of Compression block.

f'_c = Concrete compression strength

N.A = Neutral Axis

T = Tensile force = $A_s * f_y$ → Yield strength of steel

A_s → Area of steel (Tensile) = No. of Bars * Area of One Bar

A'_s = Area of Compression steel

C_s = Compression force developed in steel $= A'_s * f_y$

C_c = Compression force developed in concrete.

Step-04: $M_{u_1} = M_u - M_{u_2} = 6600 - 5200.65 = 1399.35 K''$

Step-05: $M_{u_1} = \phi * A_s' * f_y * (d - d')$.

So, $A_s' = \frac{M_{u_1}}{\phi * f_y * (d - d')} = \frac{1399.35}{0.90 * 40 * (23 - 2.5)}$

$A_s' = 1.90 \text{ in}^2$

Step-06: Total Steel Area:

$A_s = A_{st} + A_s' = 7.47 + 1.90 = 9.37 \text{ in}^2$.

This total steel Area should/must be provided in the tensile zone as tension reinf.

Step-07: Selection of Bars:

A- For tensile steel:

Let try #8 bar, having Area = 0.785 in^2

↳ $A_b = \frac{\pi}{4} * (d_b)^2$

where; d_b = diameter of Bar

Number of Bars = $\frac{A_s}{A_b} = \frac{9.37}{0.785}$

= 11.80 ≃ 12 #8 Bars

And for #8 Bar;

$d_b = \frac{8}{8} = 1''$

So, $A_b = \frac{3.14}{4} * (1)^2$

$A_b = 0.785 \text{ in}^2$

B- For Compression Steel:

Let try #6 Bar, having Area = 0.44 in^2.

Number of Bars = $\frac{A_s'}{A_b} = \frac{1.90}{0.44} = 4.32$

≃ 5 #6 Bars

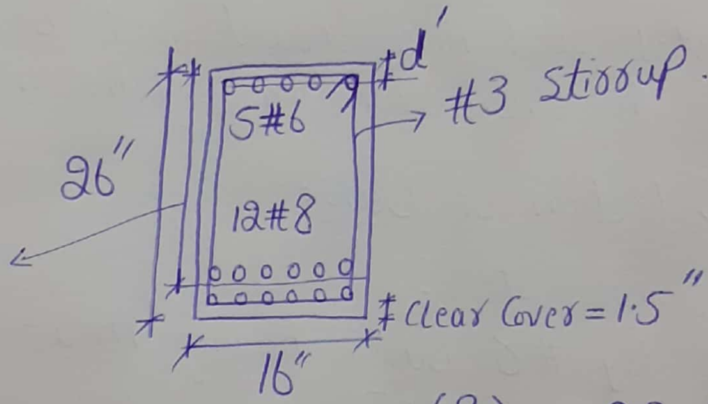
Step-08: Check on minimum width of beam.

b_{min} = 2 * Clear Cover + 2 * diameter of stirrup
+ No. of Main Bars * diameter of main bar
+ No. of spaces between main bars * dia. of main bar

$$= 2*1.5 + 2*\frac{3}{8} + 12*\frac{8}{8} + 11*\frac{8}{8}$$

$b_{min} = 26.75'' > 16''$ ⇒ Not good in one layer.

So, Main bars should be provided in two layers.



$$d = 26 - 1.5 - \frac{3}{8} - \frac{8}{8} - \frac{1}{2}\left(\frac{8}{8}\right) = 22.63''.$$

$$d' = 1.5 + \frac{3}{8} + \frac{1}{2}*\left(\frac{6}{8}\right) = 2.25''.$$

Step-09; Design Moment;

$$M_d = \phi * \left[A_s' * f_y * (d-d') + (A_s - A_s') * f_y * \left(d - \frac{a}{2}\right)\right]$$

where; $a = \frac{(A_s - A_s') * f_y}{0.85 * f_c' * b} = \frac{(12*0.785 - 5\times0.44)*40}{0.85*3*16} = 7.14''.$

$$M_d = 0.90*\left[(5*0.44)*40*(22.63-2.5) + (12*0.785-5*0.44)*40*\left(22.63-\frac{7.14}{2}\right)\right]$$

$M_d = 6608K'' > M_u = 6600K''$

⇒ Design is O.K.

Assignment # 02:

Book: Design of concrete structures
(13th edition by Nilson, Darwin, Dolan).

Page - 112: Problem - 3.10;

A rectangular concrete beam of width $b = 24$ inches is limited by architectural considerations to a maximum total depth $h = 16$ inches. It must carry a total factored load moment $M_u = 400$ ft-kips. Design the flexural reinforcement for this member, using compression steel if necessary. Material strengths are $f_y = 60,000$ Psi and $f_c' = 4000$ Psi. Show a sketch of your final design