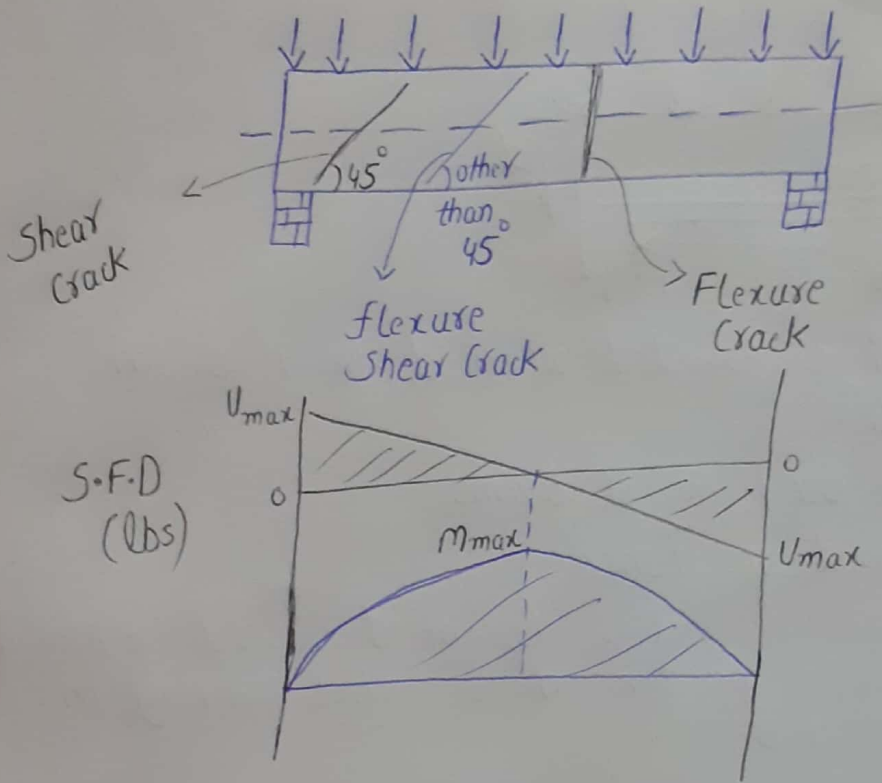


Shear Design of Rectangular Beams:

→ Starting Point of Shear Cracks $\begin{cases} \rightarrow \text{Material Weak} \\ \rightarrow \text{Maximum Shear.} \end{cases}$



$$\text{Max. Bending Stress} = \frac{M \cdot y}{I}$$

↓
It will be max. at mid section at the outer most fibre.

* To resist the moment due to externally applied loads, longitudinal or, main reinforcement is provided.

* To resist the shear, stirrups or, web reinforcement is provided.

* Types of Web Reinforcement:

→ For stirrup, generally #3 or, #4 bar can be used because larger bar cannot bent easily.

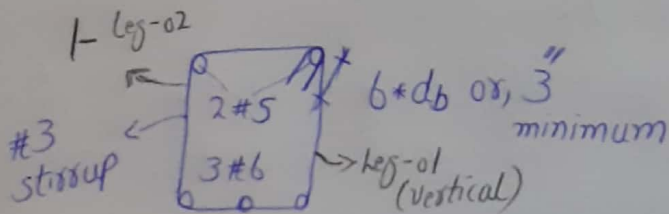


Figure-01: Two Leg U-Shape stirrup.

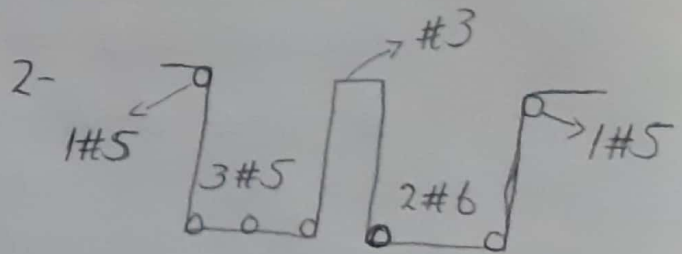
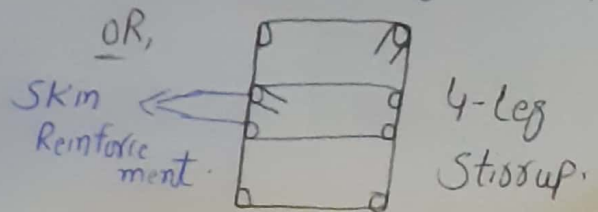


Figure: 02: 4-leg stirrup

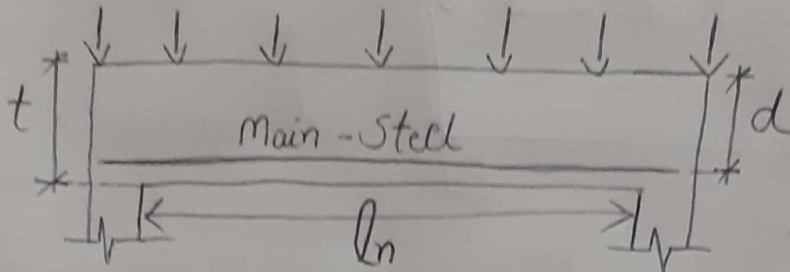
$$A_v = \text{Area of stirrup} = 2 \cdot \frac{\pi \cdot d_b^2}{4}$$

↳ diameter of bar/stirrup



→ Deep Beam: If the ratio of clear span to the overall depth/thickness is less than or equal to 4.

$$l_n/t \leq 4$$



→ Skin Reinforcement: It is provided when depth of beam exceeding 30" or 36".

- * To prevent lateral buckling of beams.
- * To enhance torsional resistance.

→ Skin reinforcement or additional reinforcement should be placed near the vertical faces in the tension to control cracking in the web.

ACI code Provisions for Shear Design:

→ Critical section is at a distance 'd' from the face of support.

a- Shear strength capacity of concrete = $V_c = 2 * \sqrt{f'_c} * b_w * d$

* U_u = Total factored Shear force applied at a given section.

b- Minimum Web Reinforcement:

If $U_u \leq \phi * V_c$ → Theoretically no web reinforcement is required. However, ACI code require provision of atleast a minimum area of web reinforcement equal to:

$$\rightarrow A_{u_{min}} = \frac{0.75 * \sqrt{f'_c} * b_w * S}{f_y} \quad \text{OR,} \quad \rightarrow A_{u_{min}} = \frac{S_o * b_w * S}{f_y}$$

Also, from these two formulas, you can find Max. Spacing.

$$S_{max} = \frac{A_u * f_y}{0.75 * \sqrt{f'_c} * b_w} \quad \text{OR,} \quad S_{max} = \frac{A_u * f_y}{S_o * b_w}$$

* If $V_u < \frac{1}{2} * \phi * V_c$ \rightarrow Then No Web Reinforcement is needed

where; S = longitudinal Spacing of web Reinf = inches

f_y = Yield Strength of Steel (web)

A_v = Total cross-section area of web steel with in distance 'S'

* First stirrup is provided at a distance $\frac{S}{2}$.

* Shear at critical section is represented by ' V_u '.

* Between critical section ' V_u ' and ' ϕV_c ', Spacing b/w web reinforcement can be find by the following formula;

$$S = \frac{\phi * A_v * f_y * d}{V_u - \phi V_c}$$

* Preferably $S \neq 4''$, because of proper compaction, vibration and Pouring of concrete etc.

$\rightarrow V_s$ = Shear force carried by web reinf. / stirrups.

According to ACI code;

If $V_s \leq 4 * \sqrt{f'_c} * b_w * d$, Then Max. spacing of stirrups will be smallest of the following four conditions.

1- $24''$

2- $\frac{d}{2}$

3- $S_{max} = \frac{A_v * f_y}{0.75 * \sqrt{f'_c} * b_w}$

4- $S_{max} = \frac{A_v * f_y}{50 * b_w}$

And, if $V_s > 4 * \sqrt{f'_c} * b_w * d$ \rightarrow Then Maximum spacing will be halved.

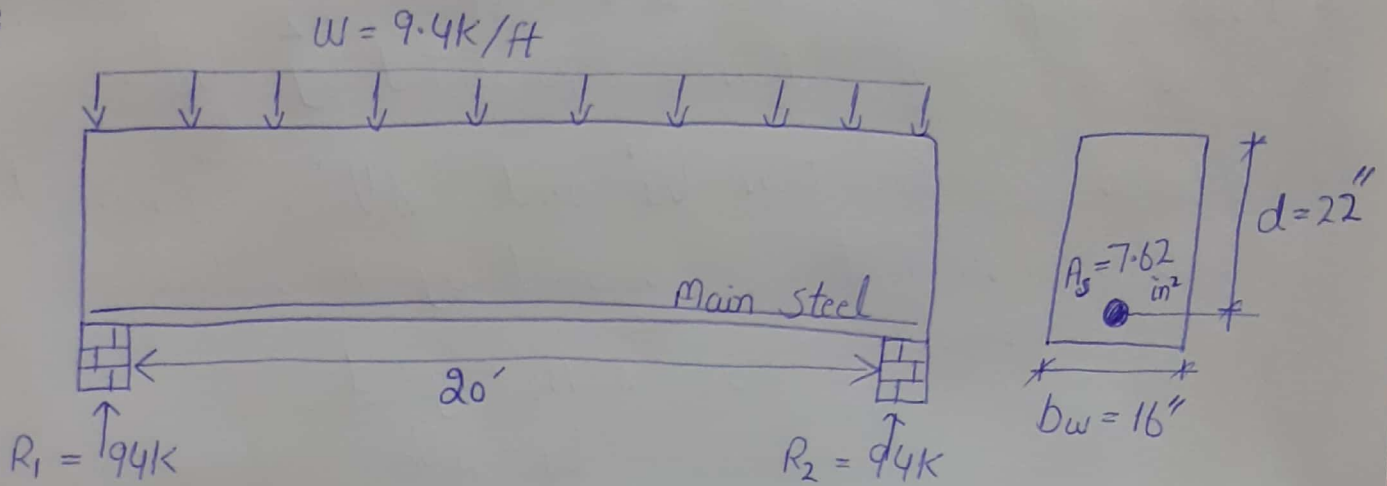
And if $V_s > 8 * \sqrt{f'_c} * b_w * d$

Then either increase cross-sectional dimensions.

OR, increase f'_c .

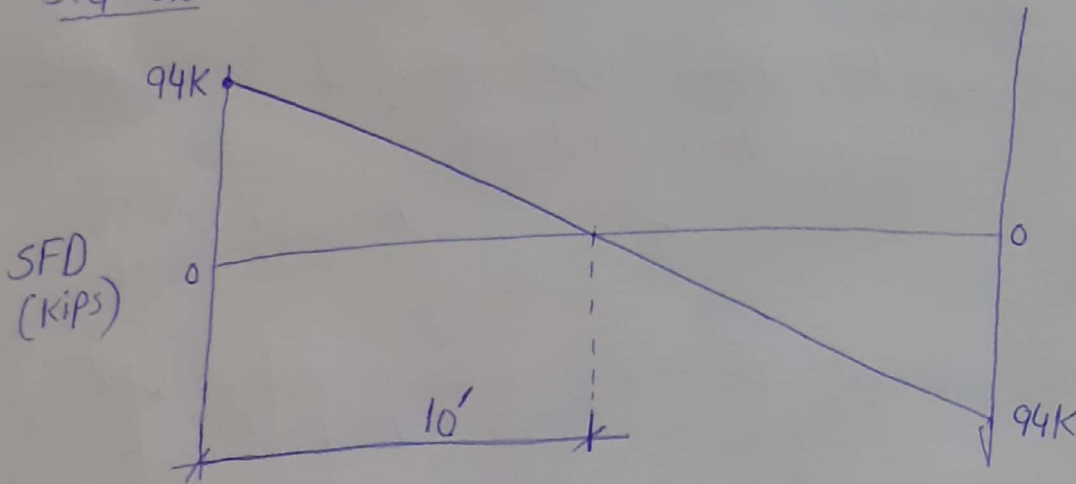
Design Problem-01: A simply supported rectangular beam 16" wide having an effective depth of 22" carries a total factored load of 9.4k/ft on a 20 ft. clear span. It is reinforced with 7.62 in² of tensile steel which continues uninterrupted into the supports. If $f'_c = 4000$ psi and $f_y = 60000$ psi. Using vertical U-stirrup. Design the web/shear reinforcement.

Sol:



Step-01: Find values of ' R_1 ' and ' R_2 '.
Total load = $9.4 \times 20 = 188 \frac{1}{2} = 94 \text{ k}$.

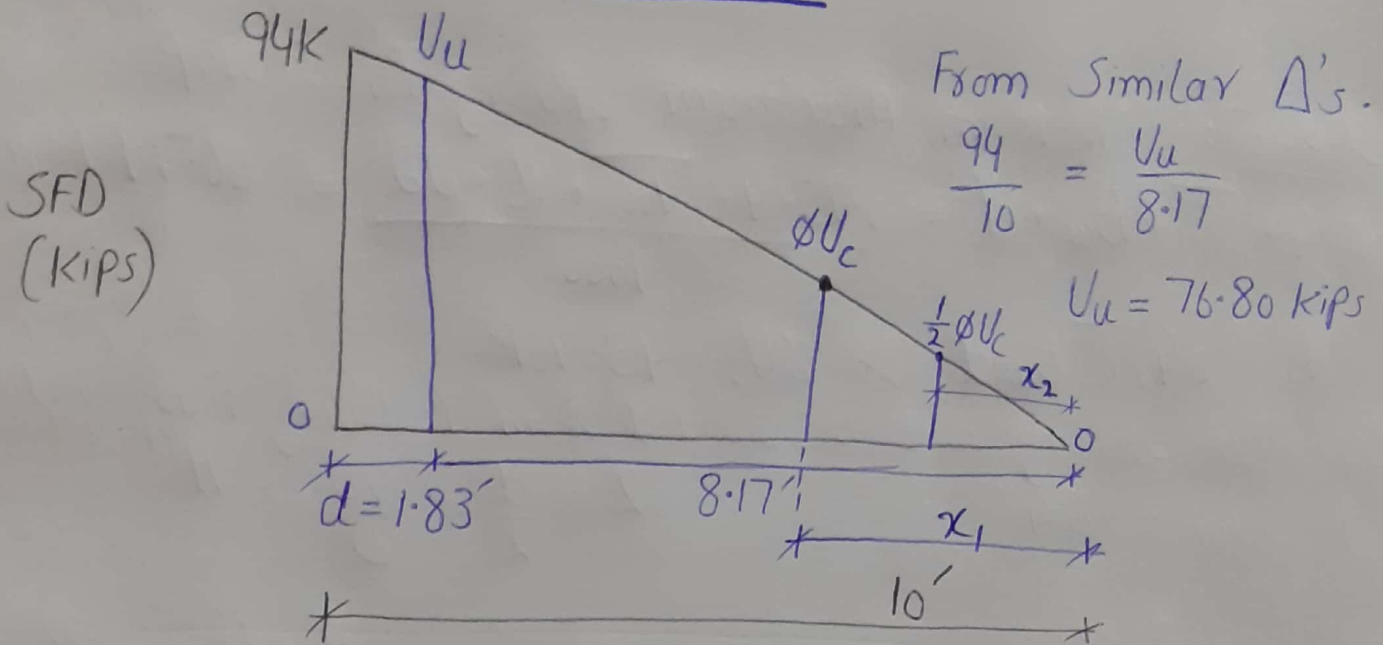
Step-02: Draw its Shear force diagram.



Step-03: Find value of critical shear ' V_u ' and its location.

As, we know that critical section is located at distance ' d ' from face of support = $d = 22 \text{ inches} = 1.83 \text{ feet}$.

Value of Critical Shear at distance ' d ' by similarity of Triangles.



Step-04: Find Value of ' ϕU_c ' and ' $\frac{1}{2} \phi U_c$ ' and also its distances from zero shear to right side.

$$\phi U_c = \phi * 2 * \sqrt{f'_c} * b_w * d = \frac{0.75 * 2 * \sqrt{4000} * 16 * 22}{1000} = 33.40 \text{ k}$$

Location of ϕU_c by Similarity of Δ 's.

$$\frac{94}{10} = \frac{33.40}{x_1} \Rightarrow x_1 = 3.55'$$

Now; $\frac{1}{2} \phi U_c = \frac{33.40}{2} = 16.70 \text{ k}$

Location of $\frac{1}{2} \phi U_c \Rightarrow \frac{94}{10} = \frac{16.70}{x_2} \Rightarrow x_2 = 1.78'$

Step-05: Value of ϕU_s . ($U_u = \phi U_s + \phi U_c$)

So, $\phi U_s = U_u - \phi U_c = 76.80 - 33.40 = 43.40 \text{ k}$.

Step-06: Check on section adequacy.

$$\phi * 8 * \sqrt{f'_c} * b_w * d = \frac{0.75 * 8 * \sqrt{4000} * 16 * 22}{1000} = 133.57 \text{ k}$$

As, $\phi U_s < \phi * 8 * \sqrt{f'_c} * b_w * d \Rightarrow$ It means section is adequate.

Step-07: Check on Maximum spacing for stirrups.

$$\phi * 4 * \sqrt{f'_c} * b_w * d = \frac{0.75 * 4 * \sqrt{4000} * 16 * 22}{1000} = 66.79 \text{ kip}$$

As, $\phi 4 \sqrt{f'_c} b_w d > \phi U_s = 43.40 \text{ k}$.

So, Max. Spacing will be selected from following four conditions:

1- $S_{max} = 24''$

2- $\frac{d}{2} = \frac{22}{2} = 11''$

3- $S_{max} = \frac{A_v * f_y}{0.75 * \sqrt{f'_c} * b_w}$
 $= \frac{0.22 * 60000}{0.75 * \sqrt{4000} * 16} = 17.40''$

4- $S_{max} = \frac{A_v * f_y}{50 * b_w} = \frac{0.22 * 60000}{50 * 16} = 16.50''$

From above four conditions, Least value of spacing for #3, 2 legged stirrup will be selected.

So, $S_{max} = 11'' \text{ c/c}$

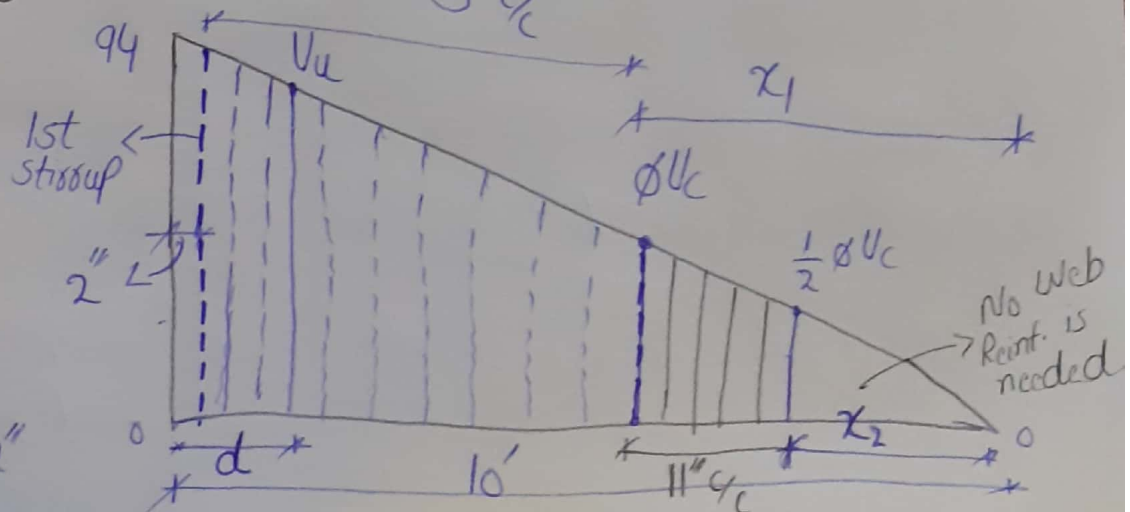
Step-08: Spacing of stirrup from/at critical section.

$$S \geq \frac{\phi * A_v * f_y * d}{U_u - \phi U_c} = \frac{0.75 * 0.22 * 60 * 22}{76.80 - 33.44}$$

$S = 5'' \text{ c/c}$

Step-09: Final Sketch. $5'' \text{ c/c}$

SFD (KIPS)

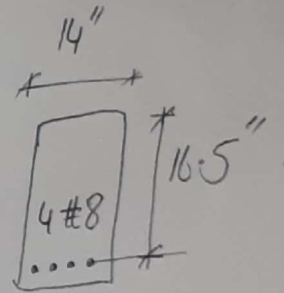
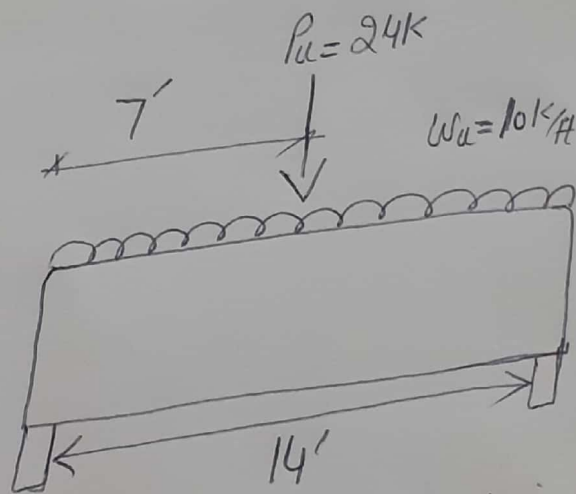


*As we know that first stirrup from face of support = $\frac{S}{2} = \frac{5}{2} \approx 2''$

Home Work;

Pb-01: A rectangular beam having $b=12''$ and $d=22''$ spans 20ft. face to face of simple supports. It is reinforced for flexure with three No. 11 bars that continue uninterrupted to the ends of the span. It is to carry service D.L of 1.63 kips/ft (including self wt.) and service L.L of 3.26 kips/ft, both uniformly distributed along the span. Design the shear reinforcement using No. 3 vertical U-stirrups. Material strengths are $f'_c=4000$ Psi and $f_y=60000$ Psi.

Pb-02:



$f'_c = 4 \text{ ksi}$, $f_y = 60 \text{ ksi}$

Design for shear at;

- i) - distance 'd' from face of support.
- ii) - at distance 4'-0" from face of support.