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3rd week lecture 2nd

Derivative by exponential function.

e^{-3x}

Sol:

if $f(x) = e^{-3x}$

Diff w.r.t "x"

$$\frac{d}{dx} f(x) = \frac{d}{dx}(e^{-3x})$$

$$f'(x) = e^{-3x} \cdot \frac{d}{dx}(-3x)$$

$$f'(x) = e^{-3x} \cdot -3\frac{d}{dx}(x)$$

$$f'(x) = e^{-3x}(-3)$$

$$f'(x) = -3 \cdot e^{-3x}$$ Ans

(4) $f(x) = x^3 \cdot e^{4x}$

$1^{st} \cdot \frac{d}{dx} 2^{nd} + 2^{nd} \cdot \frac{d}{dx}$

Sol:

$f(x) = x^3 \cdot e^{4x}$

Using product rule

$$\frac{d}{dx} f(x) = \frac{d}{dx}(x^3 \cdot e^{4x})$$

$$f'(x) = x^3 \cdot \frac{d}{dx} e^{4x} + e^{4x} \cdot \frac{d}{dx} x^3$$

$$f'(x) = x^3 \cdot e^{4x} \cdot \frac{d}{dx}(4x) + e^{4x} \cdot 3x^2$$

$$f'(x) = x^3 \cdot e^{4x} \cdot 4 + e^{4x} \cdot 3x^2$$

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$f(x,y,z) = xy^2z^3 + 3yz$

finding PD w.r.t "x"

$f_x = \frac{d}{dx} xy^2z^3 + \frac{d}{dx} 3yz$

$f_x = y^2z^3 \frac{d}{dx} x + 0$

$f_x = y^2z^3(1)$

$\boxed{f_x = y^2z^3}$ — (i)

∴ finding PD w.r.t "y"

$f_y = \frac{d}{dy} xy^2z^3 + \frac{d}{dy} 3yz$

$f_y = xz^3 \frac{d}{dy} y^2 + 3z \frac{d}{dy} y$

$f_y = xz^3(2y) + 3z(1)$

$\boxed{f_y = 2xyz^3 + 3z}$ — (ii)

∴ finding PD w.r.t to "z"

$f_z = \frac{d}{dz} xy^2z^3 + \frac{d}{dz} 3yz$

$f_z = xy^2 \frac{d}{dz} z^3 + 3y \frac{d}{dz} z$

$f_z = xy^2(3z^2) + 3y(1)$

$\boxed{f_z = 3xy^2z^2 + 3y}$ — (iii)

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$f'(x) = 4x^3 e^{4x} + 3x^2 \cdot e^{4x}$

$f'(x) = e^{4x}(4x^3 + 3x^2)$ Ans.

(14) $f(x) = e^{\tan x}$

Sol:

$f(x) = e^{\tan x}$

Diff w.r.t "x"

$\frac{d}{dx} f(x) = \frac{d}{dx}\left(e^{\tan x}\right)$

$f'(x) = e^{\tan x} \cdot \frac{d}{dx}(\tan x)$

$f'(x) = e^{\tan x} \cdot \sec^2 x \ \frac{d}{dx}(x)$

$f'(x) = e^{\tan x} \cdot \sec^2 x \ (1)$

$f'(x) = e^{\tan x} \cdot \sec^2 x$ Ans.

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$f(r,s) = r \cdot \ln(r^2+s^2)$

⇒ Find partial derivatives w.r.t r and s

∴ Finding w.r.t (r) :-

d of 1 × 2nd + 1st . d of 2nd

$$f_r = \left[\frac{d}{dr} r \cdot \ln(r^2+s^2) + r \cdot \frac{d}{dr} \ln(r^2+s^2)\right] \cdot \frac{d}{dr}(r^2+s^2)$$

$$f_r = 1 \cdot \ln(r^2+s^2) + r \cdot \frac{1}{r^2+s^2} \cdot \frac{d}{dr} r^2 + \frac{d}{dr} s^2$$

$$f_r = \ln(r^2+s^2) + r \cdot \frac{1}{r^2+s^2} \cdot 2r + 0$$

$$\boxed{f_r = \ln(r^2+s^2) + \frac{2r^2}{r^2+s^2}} \quad \text{— (i)}$$

∴ Finding w.r.t (s) :-

$$f_s = \frac{d}{ds} r \cdot \ln(r^2+s^2) + r \cdot \frac{d}{ds} \ln(r^2+s^2) \cdot \frac{d}{ds}$$

$$f_s = 0 \cdot \ln(r^2+s^2) + r \cdot \frac{1}{r^2+s^2} \cdot \frac{d}{ds} r^2 + \frac{d}{ds} s$$

$$f_s = 0 + r \cdot \frac{1}{r^2+s^2} \cdot 0 + 2s$$

$$\boxed{f_s = \frac{2rs}{r^2+s^2}} \quad \text{— (ii)}$$

$z = x^4 e^{y^5}$ 5

$\Rightarrow$ find P.D w.r.t x and y 13

$\therefore$ finding P.D w.r.t (x) :-

$$\frac{dz}{dx} = \frac{dz}{dx} x^4 . e^{y^5}$$

$$\frac{dz}{dx} = e^{y^5} . \frac{dz}{dx} x^4$$

$$\frac{dz}{dx} = e^{y^5} . 4x^3$$

$$\boxed{\frac{dz}{dx} = 4x^3 . e^{y^5}} \quad \text{— (i)}$$

note

$x^4 e^{y^5}$

take derivative of x^4

constant of

$\therefore$ finding P.D w.r.t (y) :-

$$\frac{dz}{dy} = \frac{dz}{dy} x^4 . e^{y^5}$$

$$\frac{dz}{dy} = x^4 . e^{y^5} (5y^4)$$

$$\boxed{\frac{dz}{dy} = x^4 . e^{y^5} (5y^4)} \quad \text{— (ii)}$$

note

$x^4 e^{y^5}$

now finding w.r.

x^4 is the con

and for e^{y^5}

the rule ;

$e^u \to e^u . u$

Hur.

(2)

Q2) Evaluate :-

$$\int_1^2 \int_1^3 xy^2 \, dx \, dy$$

Sol-

let $I = \int_1^2 \left[\int_1^3 xy^2 \, dx \right] dy$

$= \int_1^2 y^2 \left| \frac{x^2}{2} \right|_1^3 dy = \int_1^2 \frac{y^2}{2} \left((3)^2 - (1)^2 \right) dy$

$= \int_1^2 \frac{y^2}{2} \cdot (9-1) \, dy = \int_1^2 \frac{y^2}{\cancel{2}_1} \cdot \cancel{8}^4 \, dy$

$= 4 \int_1^2 y^2 \, dy = 4 \left| \frac{y^3}{3} \right|_1^2$

$= \frac{4}{3} \left((2)^3 - (1)^3 \right) = \frac{4}{3} (8-1)$

$= \frac{4}{3} \cdot 7 = \frac{28}{3}$ Ans

① DOUBLE INTEGRALS / 14 4

MULTIPLE INTEGRALS

Evaluate, $\int_0^5 \int_0^{x^2} x\left(x^2+y^2\right)\, dy\, dx$

Sol.-

$$\text{let } I = \int_0^5 \int_0^{x^2} \left(x^3 + xy^2\right) dy\, dx$$

integrate w.r.t y

$$= \int_0^5 \left[\int_0^{x^2} \left(x^3+xy^2\right) dy\right] dx$$

$$= \int_0^5 \left| x^3 y + \frac{xy^3}{3} \right|_0^{x^2} dx$$

$$\left(x-0\right)^3$$

$$= \int_0^5 \left\{ x^3\left(x^2-0\right) + \frac{x}{3} \, \left(x^2\right)^3 - (0)^3 \right\} dx$$

$$\left(x^2-0\right)^3$$

$$= \int_0^5 x^5 + \frac{x}{3}\left(x^6\right) dx$$

$$= \int_0^5 \left(x^5 + \frac{x^7}{3}\right) dx$$

now integrate w.r.t x

$$= \left| \frac{x^6}{6} + \frac{x^8}{24} \right|_0^5 = \frac{(5)^6}{6} + \frac{(5)^8}{24}$$

$8-6=2$

Getting 5^6 common $\longleftarrow$ $= 5^6\left(\frac{1}{6} + \frac{25}{24}\right) = 5^6\left(\frac{4+25}{24}\right)$ taking LCM

$$= 5^6 \cdot \frac{29}{24} = 18,880.208$$

Ans

Multivariate Final

(1) (6) (1) (18)

Green's Theorem :-

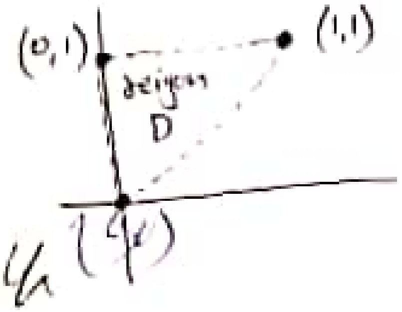
Let "C" be the positively oriented, piecewise-smooth simple closed curve in the plane and let "D" be the region bounded by "C". If 'P' and 'Q' has continuous partial derivatives on an open region that contains 'D', then:

Note

$$\oint_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$\Rightarrow \text{Evaluate } \oint_C \underbrace{xy\,dx}_{\text{Existing in front of } dx \text{ is our (P) function}} - \overbrace{x^2y^2}^{\text{what ever infront } dy \text{ is our (Q) function}}\,dy \quad \text{--- (i)}$$

where C is the triangle with vertices (0,0), (0,1), (1,1) positively oriented.



$\Rightarrow$ from line integral to double integral (dxdy)

$$\boxed{\iint\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA} \quad \text{--- (ii)}$$

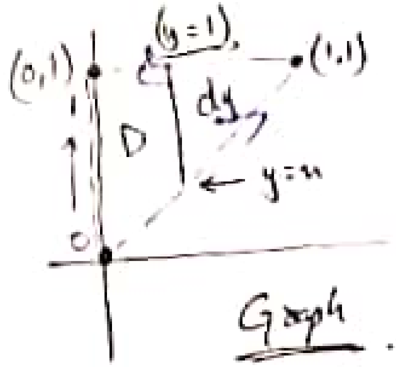
$$\frac{\text{Partial of Q}}{\text{w.r.t } x} - \frac{\text{Partial of P}}{\text{w.r.t } y}$$

$\Rightarrow$ Now see eq (i) and (ii)

first find partial of Q w.r.t 'x' and P w.r.t 'y'.

$$\boxed{= \iint(-2xy^2 - 0)\,dy\,dx}$$

$$= \int_0^1\int_{y=x}^{y=1}(-2xy^2 - 0)\,dy\,dx$$



Graph

P.T.O

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Evaluate

$$\int_1^3\int_0^2 (xy + x^2y^3)\, dy\, dx$$

Sol

let $I = \int_1^3\int_0^2 (xy + x^2y^3)\, dy\, dx$

$$= \int_1^3 \left[\int_0^2 (xy + x^2y^3)\, dy\right] dx$$

$$= \int_1^3 \left| \frac{xy^2}{2} + \frac{x^2y^4}{4} \right|_0^2 dx$$

$$= \int_1^3 \left[\frac{x(2)^2}{2} + \frac{x^2(2)^4}{4}\right] dx$$

$$= \int_1^3 \left[\frac{\cancel{4}^{2}x}{\cancel{2}_1} + \frac{\cancel{16}^{4}x^2}{\cancel{4}_1}\right] dx$$

$$= \int_1^3 \left[2x + 4x^2\right] dx$$

$$\therefore \Rightarrow \left| x^2 + \frac{4x^3}{3} \right|_1^3$$

$$= \left((3)^2 + \frac{4(3)^3}{3}\right) - \left((1)^2 + \frac{4(1)^3}{3}\right)$$

$$= \left(9 + \frac{4}{\cancel{3}_1}(\cancel{27}^{9})\right) - \left(1 + \frac{4}{3}\right)$$

$$= (9+36) - (1 + 4/3)$$

$$= 45 - 1 + 4/3 = 44 - 4/3$$

$$= \boxed{44 - 4/3} \quad \text{Ans}$$

$$= \int_0^1 \left[-\frac{2xy^3}{3} \Big|_{y=x}^{y=1} \right] dx$$

$$= \int_0^1 \left(-\frac{2x(1)^3}{3} + \frac{2x(x)^3}{3} \right) dx$$

$$= \int_0^1 \left(-\frac{2x}{3} + \frac{2x^4}{3} \right) dx$$

$$= \frac{-\cancel{2}}{3} \cdot \frac{x^2}{\cancel{2}} + \frac{2}{3} \cdot \frac{x^5}{5} \Big|_0^1$$

$$= \frac{-x^2}{3} + \frac{2}{15} x^5 \Big|_0^1$$

$$= \frac{-(1)^2}{3} + \frac{2}{15}(1)^5 - \frac{(0)^2}{3} + \frac{2}{15}(0)^5$$

$$= \left(\frac{-1}{3} + \frac{2}{15} \right) - (0)$$

$$= \frac{-5+2}{15} = \frac{-3}{15}$$

$$= \boxed{\frac{-1}{5}}$$

Ans