$f(r,s) = r \cdot \ln(r^2+s^2)$ (3)

⇒ Find partial derivatives w.r.t r and s

∴ Finding w.r.t (r) :- diff 1st and 1st. diff 2nd

$$f_r = \left[\frac{d}{dr} r \cdot \ln(r^2+s^2) + r \cdot \frac{d}{dr} \ln(r^2+s^2)\right] \cdot \frac{d}{dr}(r^2+s^2$$

$$f_r = 1 \cdot \ln(r^2+s^2) + r \cdot \frac{1}{r^2+s^2} \cdot \frac{d}{dr} r^2 + \frac{d}{dr} s^2$$

$$f_r = \ln(r^2+s^2) + r \cdot \frac{1}{r^2+s^2} \cdot 2r + 0$$

$$\boxed{f_r = \ln(r^2+s^2) + \frac{2r^2}{r^2+s^2}} \quad \text{— (i)}$$

∴ finding w.r.t (s) :-

$$f_s = \frac{d}{ds} r \cdot \ln(r^2+s^2) + r \cdot \frac{d}{ds} \ln(r^2+s^2) \cdot \frac{d}{ds}(r^2+s$$

$$f_s = 0 \cdot \ln(r^2+s^2) + r \cdot \frac{1}{r^2+s^2} \cdot \frac{d}{ds} r^2 + \frac{d}{ds} s^2$$

$$f_s = 0 + r \cdot \frac{1}{r^2+s^2} \cdot 0 + 2s$$

$$\boxed{f_s = \frac{2rs}{r^2+s^2}} \quad \text{— (ii)}$$

17.3 (9)

$f'(x) = 2 \tan 2x$

(12) $f(x) = (\ln x)^4$

Sol:

using Power rule

$\frac{d}{dx} f(x) = \frac{d}{dx} (\ln x)^4$

(3)

$f'(x) = 4 (\ln x)^{4-1} \frac{d}{dx} (\ln x)$

$f'(x) = 4 (\ln x)^3 \cdot \frac{1}{x}$

$f'(x) = \frac{4 (\ln x)^3}{x}$

$f(x) = x \cdot \ln x$

Sol:

$\Rightarrow \frac{-92}{18} + 2y = -5.$

Add $\frac{92}{18}$ on both sides.

$\Rightarrow \frac{-92}{18} + 2y + \frac{92}{18} = -5 + \frac{92}{18}$

$\Rightarrow 2y = -5 + \frac{92}{18}$

$\Rightarrow 2y = \frac{-90+92}{18}$

$\Rightarrow 2y = \frac{2}{18}$

Divide "2" on both sides

$\Rightarrow \frac{2y}{2} = \frac{2}{18 \times 2}$

$\Rightarrow y = \frac{2}{36}$

$\Rightarrow y = \frac{1}{18}$

$\left(\frac{-23}{18}, \frac{1}{18}\right)$ — Ans!

4) $8x - y = -1$, $7x - y = -2$ ... (3)

(6)

Multiply equ (1 with "2".

⇒ $3x - 3y = -4$.

⇒ $2(3x - 3y = -4)$.

⇒ $6x - 6y = -12 \rightarrow (3$.

Multiply equ (2 with "3".

⇒ $4x + 2y = -5$.

⇒ $3(4x + 2y = -5)$.

⇒ $12x + 6y = -15 \rightarrow (4$.

Now add equ (3 and equ (4.

$$\begin{array}{rl} & 6x - 6y = -12 \\ + & 12x + 6y = -15 \\ \hline & 18x = -23 \end{array}$$

⇒ $18x = -23$.

Divide '18' on both sides.

⇒ $\frac{18x}{18} = \frac{-23}{18}$.

⇒ $x = \frac{-23}{18}$

Put $x = \frac{-23}{18}$ in equ (2.

⇒ $4x + 2y = -5$.

⇒ $4\left(\frac{-23}{18}\right) + 2y = -5$

$5x^2 - 7x = 4x$ $= 4x + 3 - 4x$.

$5x^2 - 7x = 3$.

Subtract 3 from both sides.

$5x^2 - 7x - 3 = 3 - 3$.

$5x^2 - 7x - 3 = 0$.

According to Quadratic Equation;

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here $a = 5$, $b = -7$ and $c = -3$.

$$\Rightarrow x = \frac{-(-7) \pm \sqrt{(-7) - 4(5)(-3)}}{2(5)}$$

$$\Rightarrow x = \frac{+7 \pm \sqrt{-7 + 20(3)}}{10}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{-7 + 60}}{10}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{53}}{10}$$

$$\Rightarrow x = \frac{7 \pm (7.2)}{10}$$

$$\Rightarrow x = \frac{7 + 7.2}{10}, \quad x = \frac{7 - 7.2}{10}$$

$$\Rightarrow x = \frac{14.2}{10}, \quad x = \frac{0.2}{10}$$

$$\Rightarrow x = 1.42, \quad x = 0.028.$$ Ans!

Add $\frac{2}{5}$ on both sides.

$\Rightarrow -y - \frac{2}{5} + \frac{2}{5} = 0 + \frac{2}{5}$

$\Rightarrow -y = \frac{2}{5}$

$\Rightarrow y = -\frac{2}{5}$ ... value of (y)

$\left(\frac{9}{5}, \frac{-2}{5}\right)$. Ans!

2) $-4x + 3y = 0$, $3x - y = -4$.

Sol:-

$-4x + 3y = 0 \rightarrow (1)$.

$3x - y = -4 \rightarrow (2)$.

Multiply equ (2) with "3".

$3(3x - y = -4)$.

$\Rightarrow 9x - 3y = -12 \rightarrow (3)$.

Now add equ (1) and equ (3).

$\Rightarrow$

$$\begin{array}{r} -4x + 3y = 0 \\ 9x - 3y = -12 \\ \hline 5x \quad\quad = -12 \end{array}$$

$\Rightarrow 5x = -12$.

Divide "5" on both sides.

$\Rightarrow \frac{5x}{5} = \frac{-12}{5}$

Abdullah Shawkat.

"Functions with Several Variables." (4)

Q1) Find domain and range of the following.

	Functions	Domain	Range
1)	$f(x,y) = x-y$	All real no's	All real no's.
2)	$f(x,y) = y-x$	All real no's	All real no's.
3)	$f(x,y) = 4x^2+9y^2$	All real no's	All real no's.
4)	$f(x,y) = \sqrt{y-x}$	All real no's but $y \geq x$.	All real no's.
5)	$f(x,y) = y^2-x^2$	All real no's but $y \geq x$.	All real no's.
6)	$f(x,y) = \frac{1}{xy}$	$xy \in R \neq 0$.	All real no's.
7)	$f(x,y) = \frac{y}{x^2}$	$x \in R \neq 0$.	All real no's.
8)	$f(x,y) = \sqrt{16-x^2-y^2}$	Real no's b/w $[-2,+2]$.	All real no's.

Solve the following functions.

1) $x-3 = 4-3x$.

Sol:-

$$x-3 = 4-3x.$$

$$\Rightarrow x-3-x = 4-3x-x$$

$$\Rightarrow -3 = 4-3x-x.$$

Q4 (4)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Rightarrow a = -3$, $b = 1$ and $c = 8$.

$$\Rightarrow x = \frac{-(1) \pm \sqrt{(1) - 4(-3)(8)}}{2(-3)}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 + 96}}{-6}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{97}}{-6}$$

$$\Rightarrow x = \frac{1 \pm \sqrt{97}}{6}$$

$$\Rightarrow x = \frac{1 + \sqrt{97}}{6}, \quad x = \frac{1 - \sqrt{97}}{6}$$

$$\Rightarrow x = \frac{1 + 9.8}{6}, \quad x = \frac{1 - 9.8}{6}$$

$$\Rightarrow x = \frac{10.8}{6}, \quad x = \frac{8.84}{6}$$

$\Rightarrow x = 1.8$, $x = 1.47$. Ans!

P.T.O:

(3) (2)

Add '112x' on both sides.

$\Rightarrow 84 - 112x + 112x = 14x^2 - 20x + 112x.$

$\Rightarrow 84 = 14x^2 - 20x + 112x$

Subtract '84' from both sides.

$\Rightarrow 84 - 84 = 14x^2 - 20x + 112x - 84.$

$\Rightarrow 0 = 14x^2 - 20x + 112x - 84.$

$\Rightarrow 0 = 14x^2 + 92x - 84.$

$\Rightarrow 14x^2 + 92x - 84 = 0.$

Taking '2' as common.

$\Rightarrow 2(7x^2 + 46x - 42) = 0.$

Dividing '2' on both sides.

$\Rightarrow \frac{2(7x^2 + 46x - 42)}{2} = \frac{0}{2}$

$\Rightarrow 7x^2 + 46x - 42 = 0.$

Through Quadratic Equation we have :-

$$x = \frac{-b \pm \sqrt{b - 4ac}}{2a} \longrightarrow \text{equation}$$

Here $ax^2 + bx + c = 0 \longrightarrow$

$\Rightarrow 7x^2 + 46x - 42 = 0.$

$\Rightarrow a = 7, \quad b = 46 \quad \text{and} \quad c = -42.$

Put the values.

(4)

$\Rightarrow x = \frac{-(46) \pm \sqrt{(46)^2 - 4(7)(-4}}{2(7)}$

$\Rightarrow x = \frac{-46 \pm \sqrt{46 - 28(-42)}}{14}$

$\Rightarrow x = \frac{-46 \pm \sqrt{46 + 1176}}{14}$

$\Rightarrow x = \frac{-46 \pm \sqrt{1222}}{14}$ (23, 7)

$\Rightarrow x = \frac{-23 \pm \sqrt{1222}}{7}$

$\Rightarrow x = \frac{-23 + (34.9)}{7}$, $x = \frac{-23 - (34.9)}{7}$

$\Rightarrow x = \frac{11.95}{7}$, $x = \frac{-57.9}{7}$

$\Rightarrow x = 1.708$ ✓ , $x = -8.27$ ✓ Ans!

3) $x(5x-3) = 4x + 3$ (3)

Sol:-

$\Rightarrow x(5x-3) = 4x + 3.$

$\Rightarrow 5x^2 - 3x = 4x + 3.$

Subtract $4x$ from both sides.

(4) $x\left[\frac{5}{x} - 3\right] = 4x - 8$

Sol:-

$\Rightarrow x\left[\frac{5}{x} - 3\right] = 4x - 8.$

$\Rightarrow \frac{5x^2}{x} - 3x^2 = 4x - 8.$

Subtract '4x' from both sides.

$\Rightarrow \frac{5x^2}{x} - 3x^2 - 4x = 4x - 8 - 4x.$

$\Rightarrow \frac{5x^2}{x} - 3x^2 - 4x = -8.$

$\Rightarrow \frac{5x^2 - 3x^3 - 4x^2}{x} = -8$

$\Rightarrow \frac{5x^2 - 4x^2 - 3x^3}{x} = -8$

$\Rightarrow \frac{1x^2 - 3x^3}{x} = -8.$

Taking 'x' common.

$\Rightarrow \frac{x(1x - 3x^2)}{x} = -8$

$\Rightarrow 1x - 3x^2 = -8$

Add '8' on both sides

$\Rightarrow 1x - 3x^2 + 8 = -8 + 8.$

$\Rightarrow 1x - 3x^2 + 8 = 0.$

$\Rightarrow -3x^2 + 1x + 8 = 0.$

Through Quadratic Equation;

Note

(2)

$\Rightarrow -3-4 = 4-3x-x-4$

$\Rightarrow -3-4 = -3x+x$

$\Rightarrow -7 = -4x$

$\Rightarrow \frac{-7}{-4} = \frac{-4x}{-4}$

$\Rightarrow \frac{+7}{+4} = x$

$\Rightarrow \frac{7}{4} = x$

$\Rightarrow x = \frac{7}{4}$ Ans!

2) $\frac{3}{x} - 4 = \frac{2x}{4} - \frac{5}{7}$ (2)

Soli:-

$\frac{3}{x} - 4 = \frac{2x}{4} - \frac{5}{7}$

taking l.c.m

$\Rightarrow \frac{3-4x}{x} = \frac{14x-20}{28}$

$\Rightarrow \frac{3-4x}{x} \times \frac{14x-20}{28}$

By cross multiplication.

$\Rightarrow 28(3-4x) = x(14x-20)$

$\Rightarrow 84-112x = 14x^2-20x$

$\Rightarrow x = \frac{-12}{5}$ ∴ (x) value

Put $x = \frac{-12}{5}$ in eqn (1)

$\Rightarrow -4x + 3y = 0$.

$\Rightarrow -4\left(\frac{-12}{5}\right) + 3y = 0$.

$\Rightarrow \frac{48}{5} + 3y = 0$. ——— (1)

Subtract $\frac{48}{5}$ from both sides

$\Rightarrow \frac{48}{5} + 3y - \frac{48}{5} = 0 - \frac{48}{5}$

$\Rightarrow 3y = -\frac{48}{5}$.

Divide "3" on both sides.

$\Rightarrow \frac{3y}{3} = \frac{-48}{5 \times 3}$

$\Rightarrow y = -\frac{48}{15}$ ——— (y) value

$\left(-\frac{12}{5}, -\frac{48}{15}\right)$. Ans!

3) $3(x-y) = -4$, $4x + 2y = -5$ ——— (2)

Sol:-

$3(x-y) = -4$

$\Rightarrow 3x - 3y = -4 \rightarrow (1)$.

$\Rightarrow 4x + 2y = -5 \rightarrow (2)$

(7)

Sol:-

$8x - y = -1 \rightarrow (1)$

$7x - y = -2 \rightarrow (2)$

Subtract equ (2) from equ (1).

$$\begin{array}{r} 8x - y = -1 \\ \ominus\ 7x \ominus y = \ominus 2 \\ \hline 1x \quad = 1 \end{array}$$

$\Rightarrow 1x = 1$

$\Rightarrow x = 1.$

Put $x = 1$ in equ (1).

$8x - y = -1.$

$\Rightarrow 8(1) - y = -1.$

$\Rightarrow 8 - y = -1.$

$\Rightarrow$ Subtract '8' from both sides.

$\Rightarrow 8 - y - 8 = -1 - 8.$

$\Rightarrow -y = -9.$

Divide '-1' on both sides.

$\Rightarrow \frac{-y}{-1} = \frac{-9}{-1}$

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③ $f(x) = \ln(x^2 - 3x + 1)$

Sol:

① $\frac{d}{dx} f(x) = \frac{1}{x^2-3x+1} \cdot \frac{d}{dx}(x^2 - 3x + 1)$

$f'(x) = \frac{1}{x^2-3x+1}\left(\frac{d}{dx}x^2 - \frac{d}{dx}3x + \frac{d}{dx}(1)\right)$

$f'(x) = \frac{1}{x^2-3x+1}(2x - 3)$

$f'(x) = \frac{2x-3}{x^2-3x+1}$ Ans.

④ $f(x) = \ln(\sec 2x)$

Sol:

Formula: $\frac{d}{dx}(\sec u) = \sec u \cdot \tan u \frac{d}{dx}(u)$

② Diff w.r.t x^n

$\frac{d}{dx}(f(x)) = \frac{d}{dx}(\ln \sec 2x)$

$f'(x) = \frac{1}{\sec 2x} \frac{d}{dx}(\sec 2x)$

$f'(x) = \frac{1}{\sec 2x} \cdot \sec 2x \cdot \tan 2x \frac{d}{dx}(2x)$