chapter 10

Network Theorems

A network is a combination of components, such as resistances and voltage sources, interconnected to achieve a particular end result. However, networks generally need more than the rules of series and parallel circuits for analysis. Kirchhoff's laws can always be applied for any circuit connections. The network theorems, though, usually provide shorter methods for solving a circuit.

Some theorems enable us to convert a network into a simpler circuit, equivalent to the original. Then the equivalent circuit can be solved by the rules of series and parallel circuits. Other theorems enable us to convert a given circuit into a form that permits easier solutions.

Only the applications are given here, although all network theorems can be derived from Kirchhoff's laws. Note that resistance networks with batteries are shown as examples, but the theorems can also be applied to ac networks.

Chapter Objectives

After studying this chapter you should be able to

- *Apply* the superposition theorem to find the voltage across two points in a circuit containing more than one voltage source.
- *State* the requirements for applying the superposition theorem.
- *Determine* the Thevenin and Norton equivalent circuits with respect to any pair of terminals in a complex network.
- *Apply* Thevenin's and Norton's theorems in solving for an unknown voltage or current.
- *Convert* a Thevenin equivalent circuit to a Norton equivalent circuit and vice versa.
- *Apply* Millman's theorem to find the common voltage across any number of parallel branches.
- *Simplify* the analysis of a bridge circuit by using delta to wye conversion formulas.

Chapter Outline

Important Terms

active components
bilateral components
current source
linear component
Millman's theorem
Norton's theorem
passive components
superposition theorem
Thevenin's theorem
voltage source

Online Learning Center

Additional study aids for this chapter are available at the Online Learning Center: **www.mhhe.com/grob11e**.

10–1 Superposition Theorem

The superposition theorem is very useful because it extends the use of Ohm's law to circuits that have more than one source. In brief, we can calculate the effect of one source at a time and then superimpose the results of all sources. As a definition, the superposition theorem states: *In a network with two or more sources, the current or voltage for any component is the algebraic sum of the effects produced by each source acting separately.*

To use one source at a time, all other sources are "killed" temporarily. This means disabling the source so that it cannot generate voltage or current without changing the resistance of the circuit. A voltage source such as a battery is killed by assuming a short circuit across its potential difference. The internal resistance remains.

Voltage Divider with Two Sources

The problem in Fig. 10–1 is to find the voltage at P to chassis ground for the circuit in Fig. 10–1*a*. The method is to calculate the voltage at P contributed by each source separately, as in Fig. 10–1*b* and *c*, and then superimpose these voltages.

To find the effect of V_1 first, short-circuit V_2 as shown in Fig. 10–1*b*. Note that the bottom of R_1 then becomes connected to chassis ground because of the short circuit across V_2. As a result, R_2 and R_1 form a series voltage divider for the V_1 source.

Furthermore, the voltage across R_1 becomes the same as the voltage from P to ground. To find this V_{R_1} across R_1 as the contribution of the V_1 source, we use the voltage divider formula:

$$V_{R_1} = \frac{R_1}{R_1 + R_2} \times V_1 = \frac{60\text{ k}\Omega}{60\text{ k}\Omega + 30\text{ k}\Omega} \times 24\text{ V}$$

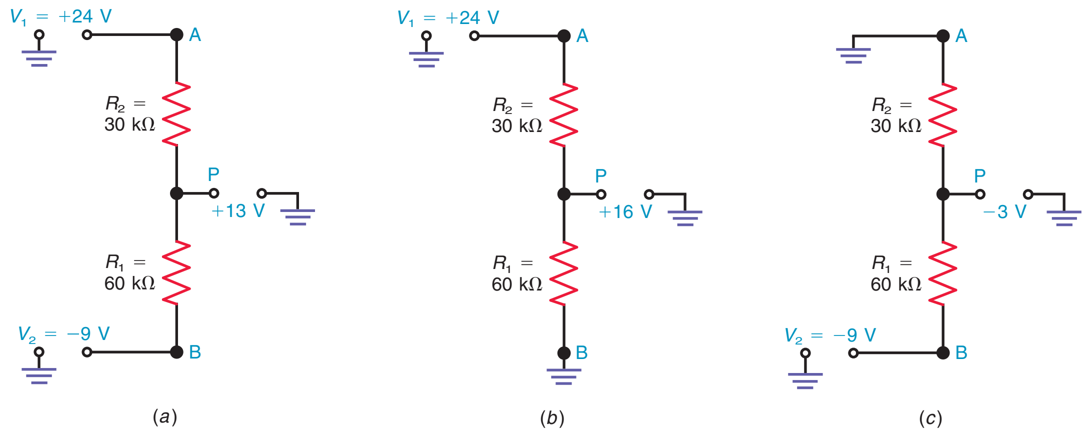
$$= \frac{60}{90} \times 24\text{ V}$$

$$V_{R_1} = 16\text{ V}$$

Next find the effect of V_2 alone, with V_1 short-circuited, as shown in Fig. 10–1*c*. Then point A at the top of R_2 becomes grounded. R_1 and R_2 form a series voltage divider again, but here the R_2 voltage is the voltage at P to ground.

With one side of R_2 grounded and the other side to point P, V_{R_2} is the voltage to calculate. Again we have a series divider, but this time for the negative voltage V_2. Using

MultiSim **Figure 10–1** Superposition theorem applied to a voltage divider with two sources V_1 and V_2. (*a*) Actual circuit with +13 V from point P to chassis ground. (*b*) V_1 alone producing +16 V at P. (*c*) V_2 alone producing −3 V at P.



the voltage divider formula for V_{R_2} as the contribution of V_2 to the voltage at P,

$$V_{R_2} = \frac{R_2}{R_1 + R_2} \times V_2 = \frac{30\ \text{k}\Omega}{30\ \text{k}\Omega + 60\ \text{k}\Omega} \times -9\ \text{V}$$

$$= \frac{30}{90} \times -9\ \text{V}$$

$$V_{R_2} = -3\ \text{V}$$

This voltage is negative at P because V_2 is negative.

Finally, the total voltage at P is

$$V_P = V_{R_1} + V_{R_2} = 16 - 3$$

$$V_p = 13\ \text{V}$$

This algebraic sum is positive for the net V_P because the positive V_1 is larger than the negative V_2.

By superposition, therefore, this problem was reduced to two series voltage dividers. The same procedure can be used with more than two sources. Also, each voltage divider can have any number of series resistances. Note that in this case we were dealing with ideal voltage sources, that is, sources with zero internal resistance. If the source did have internal resistance, it would have been added in series with R_1 and R_2.

GOOD TO KNOW

When applying the superposition theorem to a dc network, it is important to realize that the power dissipated by a resistor in the network is not equal to the sum of the power dissipation values produced by each source acting separately. The reason is that power is not linearly related to either voltage or current. Recall that $P = \frac{V^2}{R}$ and $P = I^2R$.

Requirements for Superposition

All components must be linear and bilateral to superimpose currents and voltages. *Linear* means that the current is proportional to the applied voltage. Then the currents calculated for different source voltages can be superimposed.

Bilateral means that the current is the same amount for opposite polarities of the source voltage. Then the values for opposite directions of current can be combined algebraically. Networks with resistors, capacitors, and air-core inductors are generally linear and bilateral. These are also *passive components,* that is, components that do not amplify or rectify. *Active components,* such as transistors, semiconductor diodes, and electron tubes, are never bilateral and often are not linear.

10–1 Self-Review

Answers at end of chapter.

a. In Fig. 10–1*b*, which *R* is shown grounded at one end?
b. In Fig. 10–1*c*, which *R* is shown grounded at one end?

10–2 Thevenin's Theorem

Named after M. L. Thevenin, a French engineer, Thevenin's theorem is very useful in simplifying the process of solving for the unknown values of voltage and current in a network. By Thevenin's theorem, many sources and components, no matter how they are interconnected, can be represented by an equivalent series circuit with respect to any pair of terminals in the network. In Fig. 10–2, imagine that the block at the left contains a network connected to terminals A and B. Thevenin's theorem states that the *entire* network connected to A and B can be replaced by a single voltage source V_{TH} in series with a single resistance R_{TH}, connected to the same two terminals.

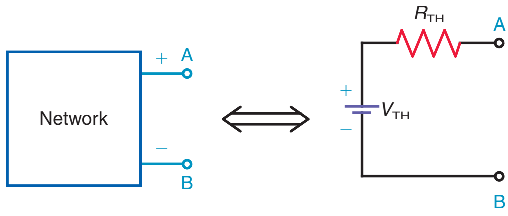
Voltage V_{TH} is the open-circuit voltage across terminals A and B. This means finding the voltage that the network produces across the two terminals with an open circuit between A and B. The polarity of V_{TH} is such that it will produce current from A to B in the same direction as in the original network.

Resistance R_{TH} is the open-circuit resistance across terminals A and B, but with all sources killed. This means finding the resistance looking back into the network from terminals A and B. Although the terminals are open, an ohmmeter across AB

GOOD TO KNOW

Of all the different theorems covered in this chapter, Thevenin's theorem is by far the most widely used.

Figure 10–2 Any network in the block at the left can be reduced to the Thevenin equivalent series circuit at the right.



would read the value of R_{TH} as the resistance of the remaining paths in the network without any sources operating.

Thevenizing a Circuit

As an example, refer to Fig. 10–3*a*, where we want to find the voltage V_L across the 2-Ω R_L and its current I_L. To use Thevenin's theorem, mentally disconnect R_L. The two open ends then become terminals A and B. Now we find the Thevenin equivalent of the remainder of the circuit that is still connected to A and B. In general, open the part of the circuit to be analyzed and "thevenize" the remainder of the circuit connected to the two open terminals.

Our only problem now is to find the value of the open-circuit voltage V_{TH} across AB and the equivalent resistance R_{TH}. The Thevenin equivalent always consists of a single voltage source in series with a single resistance, as in Fig. 10–3*d*.

The effect of opening R_L is shown in Fig. 10–3*b*. As a result, the 3-Ω R_1 and 6-Ω R_2 form a series voltage divider without R_L.

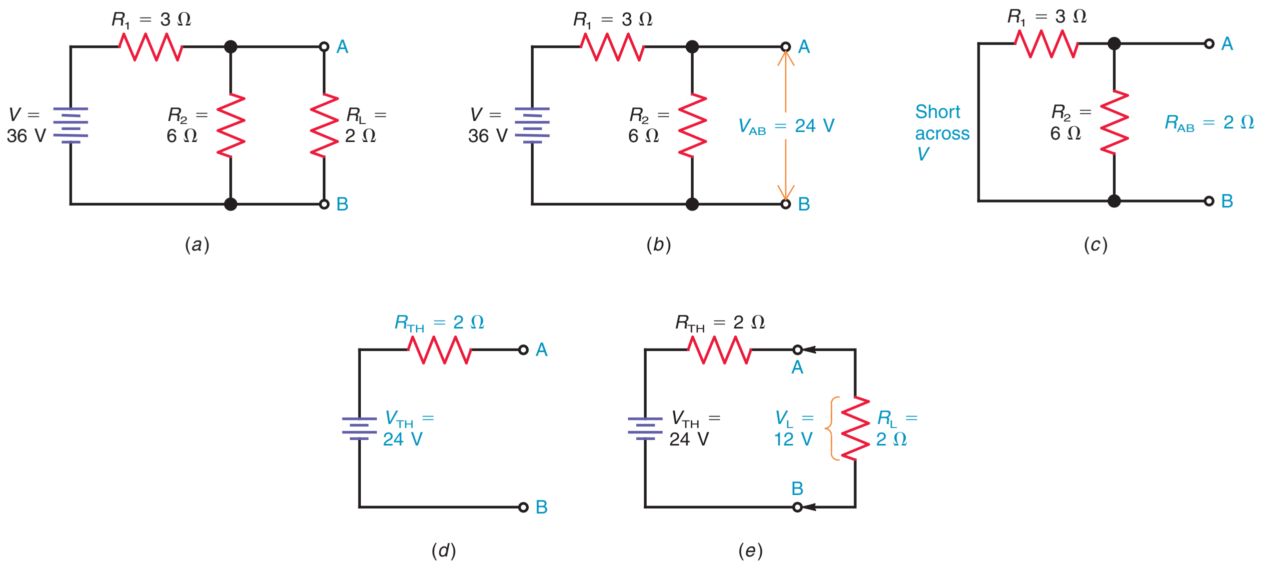
Furthermore, the voltage across R_2 now is the same as the open-circuit voltage across terminals A and B. Therefore V_{R_2} with R_L open is V_{AB}. This is the V_{TH} we need for the Thevenin equivalent circuit. Using the voltage divider formula,

$$V_{R_2} = \frac{6}{9} \times 36\text{ V} = 24\text{ V}$$

$$V_{R_2} = V_{AB} = V_{TH} = 24\text{ V}$$

This voltage is positive at terminal A.

MultiSim **Figure 10–3** Application of Thevenin's theorem. (*a*) Actual circuit with terminals A and B across R_L. (*b*) Disconnect R_L to find that V_{AB} is 24 V. (*c*) Short-circuit *V* to find that R_{AB} is 2 Ω. (*d*) Thevenin equivalent circuit. (*e*) Reconnect R_L at terminals A and B to find that V_L is 12 V.



To find R_{TH}, the 2-Ω R_L is still disconnected. However, now the source V is short-circuited. So the circuit looks like Fig. 10–3*c*. The 3-Ω R_1 is now in parallel with the 6-Ω R_2 because both are connected across the same two points. This combined resistance is the product over the sum of R_1 and R_2.

$$R_{TH} = \frac{18}{9} = 2\ \Omega$$

Again, we assume an ideal voltage source whose internal resistance is zero.

As shown in Fig. 10–3*d*, the Thevenin circuit to the left of terminals A and B then consists of the equivalent voltage V_{TH}, equal to 24 V, in series with the equivalent series resistance R_{TH}, equal to 2 Ω. This Thevenin equivalent applies for any value of R_L because R_L was disconnected. We are actually thevenizing the circuit that feeds the open AB terminals.

To find V_L and I_L, we can finally reconnect R_L to terminals A and B of the Thevenin equivalent circuit, as shown in Fig. 10–3*e*. Then R_L is in series with R_{TH} and V_{TH}. Using the voltage divider formula for the 2-Ω R_{TH} and 2-Ω R_L, $V_L = 1/2 \times 24\ \text{V} = 12\ \text{V}$. To find I_L as V_L/R_L, the value is 12 V/2 Ω, which equals 6 A.

These answers of 6 A for I_L and 12 V for V_L apply to R_L in both the original circuit in Fig. 10–3*a* and the equivalent circuit in Fig. 10–3*e*. Note that the 6-A I_L also flows through R_{TH}.

The same answers could be obtained by solving the series-parallel circuit in Fig. 10–3*a*, using Ohm's law. However, the advantage of thevenizing the circuit is that the effect of different values of R_L can be calculated easily. Suppose that R_L is changed to 4 Ω. In the Thevenin circuit, the new value of V_L would be $4/6 \times 24\ \text{V} = 16\ \text{V}$. The new I_L would be 16 V/4 Ω, which equals 4 A. If we used Ohm's law in the original circuit, a complete, new solution would be required each time R_L was changed.

Looking Back from Terminals A and B

The way we look at the resistance of a series-parallel circuit depends on where the source is connected. In general, we calculate the total resistance from the outside terminals of the circuit in toward the source as the reference.

When the source is short-circuited for thevenizing a circuit, terminals A and B become the reference. Looking back from A and B to calculate R_{TH}, the situation becomes reversed from the way the circuit was viewed to determine V_{TH}.

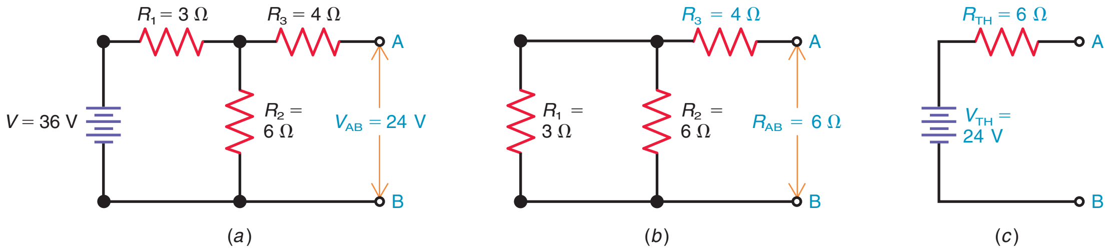
For R_{TH}, imagine that a source could be connected across AB, and calculate the total resistance working from the outside in toward terminals A and B. Actually, an ohmmeter placed across terminals A and B would read this resistance.

This idea of reversing the reference is illustrated in Fig. 10–4. The circuit in Fig. 10–4*a* has terminals A and B open, ready to be thevenized. This circuit is similar to that in Fig. 10–3 but with the 4-Ω R_3 inserted between R_2 and terminal A. The interesting point is that R_3 does not change the value of V_{AB} produced by the source V, but R_3 does increase the value of R_{TH}. When we look back from

GOOD TO KNOW

The Thevenin equivalent circuit driving terminals A and B does not change even though the value of R_L may change.

Figure 10–4 Thevenizing the circuit of Fig. 10–3 but with a 4-Ω R_3 in series with the A terminal. (*a*) V_{AB} is still 24 V. (*b*) Now the R_{AB} is 2 + 4 = 6 Ω. (*c*) Thevenin equivalent circuit.



terminals A and B, the 4 Ω of R_3 is in series with 2 Ω to make R_{TH} 6 Ω, as shown for R_{AB} in Fig. 10–4*b* and R_{TH} in Fig. 10–4*c*.

Let us consider why V_{AB} is the same 24 V with or without R_3. Since R_3 is connected to the open terminal A, the source *V* cannot produce current in R_3. Therefore, R_3 has no *IR* drop. A voltmeter would read the same 24 V across R_2 and from A to B. Since V_{AB} equals 24 V, this is the value of V_{TH}.

Now consider why R_3 does change the value of R_{TH}. Remember that we must work from the outside in to calculate the total resistance. Then, A and B are like source terminals. As a result, the 3-Ω R_1 and 6-Ω R_2 are in parallel, for a combined resistance of 2 Ω. Furthermore, this 2 Ω is in series with the 4-Ω R_3 because R_3 is in the main line from terminals A and B. Then R_{TH} is 2 + 4 = 6 Ω. As shown in Fig. 10–4*c*, the Thevenin equivalent circuit consists of V_{TH} = 24 V and R_{TH} = 6 Ω.

10–2 Self-Review

Answers at end of chapter.

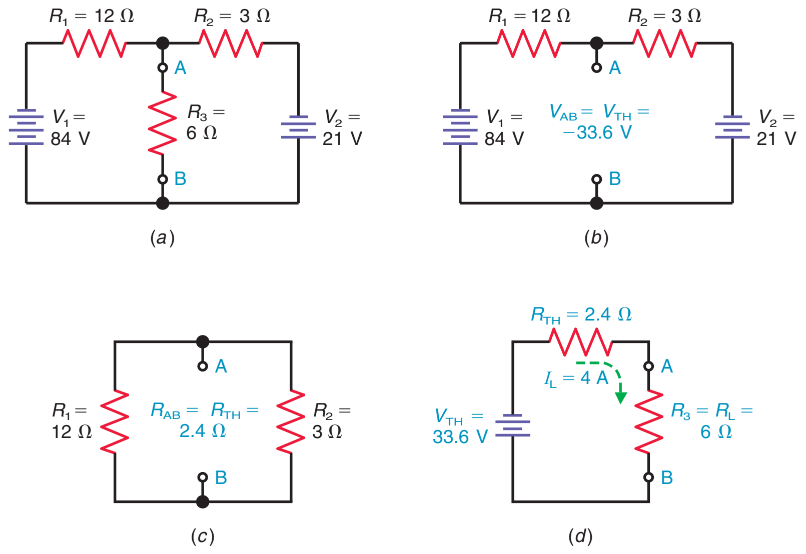
a. For a Thevenin equivalent circuit, terminals A and B are open to find both V_{TH} and R_{TH}. (True/False)

b. For a Thevenin equivalent circuit, the source voltage is short-circuited only to find R_{TH}. (True/False)

10–3 Thevenizing a Circuit with Two Voltage Sources

The circuit in Fig. 10–5 has already been solved by Kirchhoff's laws, but we can use Thevenin's theorem to find the current I_3 through the middle resistance R_3. As shown in Fig. 10–5*a*, first mark the terminals A and B across R_3. In Fig. 10–5*b*, R_3 is disconnected. To calculate V_{TH}, find V_{AB} across the open terminals.

Figure 10–5 Thevenizing a circuit with two voltage sources V_1 and V_2. (*a*) Original circuit with terminals A and B across the middle resistor R_3. (*b*) Disconnect R_3 to find that V_{AB} is −33.6 V. (*c*) Short-circuit V_1 and V_2 to find that R_{AB} is 2.4 Ω. (*d*) Thevenin equivalent with R_L reconnected to terminals A and B.



Superposition Method

With two sources, we can use superposition to calculate V_{AB}. First short-circuit V_2. Then the 84 V of V_1 is divided between R_1 and R_2. The voltage across R_2 is between terminals A and B. To calculate this divided voltage across R_2,

$$V_{R_2} = \frac{R_2}{R_{1-2}} \times V_1 = \frac{3}{15} \times (-84)$$

$$V_{R_2} = -16.8 \text{ V}$$

This is the only contribution of V_1 to V_{AB}. The polarity is negative at terminal A.

To find the voltage that V_2 produces between A and B, short-circuit V_1. Then the voltage across R_1 is connected from A to B. To calculate this divided voltage across R_1,

$$V_{R_1} = \frac{R_1}{R_{1-2}} \times V_2 = \frac{12}{15} \times (-21)$$

$$V_{R_2} = -16.8 \text{ V}$$

Both V_1 and V_2 produce −16.8 V across the AB terminals with the same polarity. Therefore, they are added.

The resultant value of $V_{AB} = -33.6$ V, shown in Fig. 10–5*b*, is the value of V_{TH}. The negative polarity means that terminal A is negative with respect to B.

To calculate R_{TH}, short-circuit the sources V_1 and V_2, as shown in Fig. 10–5*c*. Then the 12-Ω R_1 and 3-Ω R_2 are in parallel across terminals A and B. Their combined resistance is 36/15, or 2.4 Ω, which is the value of R_{TH}.

The final result is the Thevenin equivalent in Fig. 10–5*d* with an R_{TH} of 2.4 Ω and a V_{TH} of 33.6 V, negative toward terminal A.

To find the current through R_3, it is reconnected as a load resistance across terminals A and B. Then V_{TH} produces current through the total resistance of 2.4 Ω for R_{TH} and 6 Ω for R_3:

$$I_3 = \frac{V_{TH}}{R_{TH} + R_3} = \frac{33.6}{2.4 + 6} = \frac{33.6}{8.4} = 4 \text{ A}$$

This answer of 4 A for I_3 is the same value calculated before, using Kirchhoff's laws, in Fig. 9–5.

It should be noted that this circuit can be solved by superposition alone, without using Thevenin's theorem, if R_3 is not disconnected. However, opening terminals A and B for the Thevenin equivalent simplifies the superposition, as the circuit then has only series voltage dividers without any parallel current paths. In general, a circuit can often be simplified by disconnecting a component to open terminals A and B for Thevenin's theorem.

GOOD TO KNOW

The polarity of V_{TH} is extremely critical because it allows us to determine the actual direction of I_3 through R_3.

10–3 Self-Review

Answers at end of chapter.

In the Thevenin equivalent circuit in Fig. 10–5*d*,

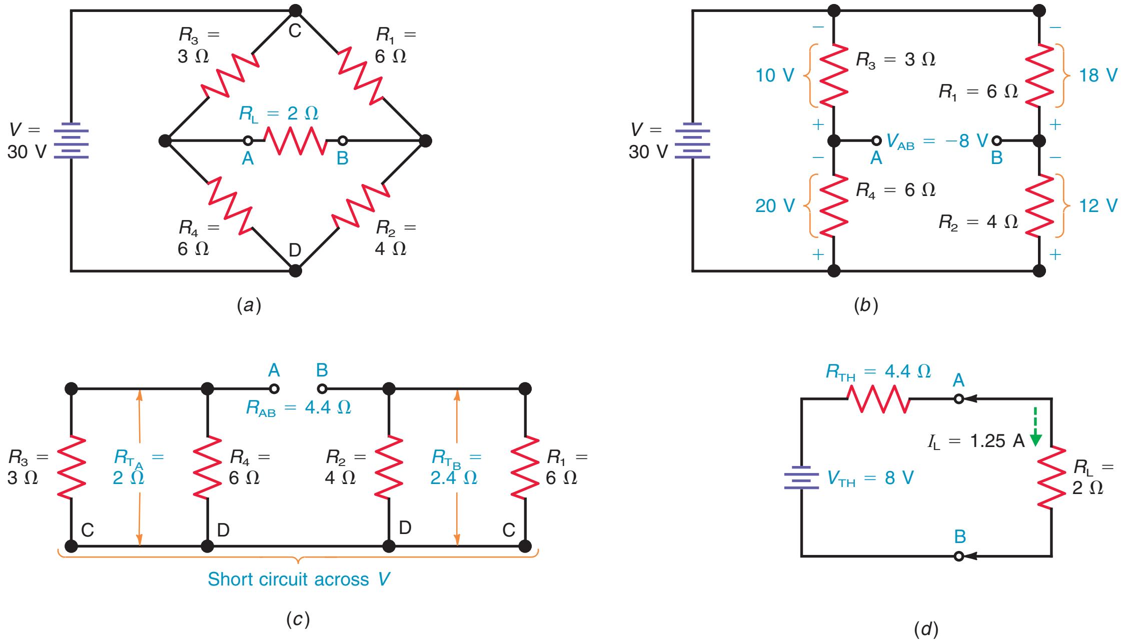
a. How much is R_T?

b. How much is V_{R_L}?

10–4 Thevenizing a Bridge Circuit

As another example of Thevenin's theorem, we can find the current through the 2-Ω R_L at the center of the bridge circuit in Fig. 10–6*a*. When R_L is disconnected to open terminals A and B, the result is as shown in Fig. 10–6*b*. Notice how the circuit has become simpler because of the open. Instead of the unbalanced bridge

Figure 10–6 Thevenizing a bridge circuit. (*a*) Original circuit with terminals A and B across middle resistor R_L. (*b*) Disconnect R_L to find V_{AB} of −8 V. (*c*) With source *V* short-circuited, R_{AB} is 2 + 2.4 = 4.4 Ω. (*d*) Thevenin equivalent with R_L reconnected to terminals A and B.



in Fig. 10–6*a* which would require Kirchhoff's laws for a solution, the Thevenin equivalent in Fig. 10–6*b* consists of just two voltage dividers. Both the R_3–R_4 divider and the R_1–R_2 divider are across the same 30-V source.

Since the open terminal A is at the junction of R_3 and R_4, this divider can be used to find the potential at point A. Similarly, the potential at terminal B can be found from the R_1–R_2 divider. Then V_{AB} is the difference between the potentials at terminals A and B.

Note the voltages for the two dividers. In the divider with the 3-Ω R_3 and 6-Ω R_4, the bottom voltage V_{R_4} is $6/9 \times 30 = 20$ V. Then V_{R_3} at the top is 10 V because both must add up to equal the 30-V source. The polarities are marked negative at the top, the same as the source voltage *V*.

Similarly, in the divider with the 6-Ω R_1 and 4-Ω R_2, the bottom voltage V_{R_2} is $4/10 \times 30 = 12$ V. Then V_{R_1} at the top is 18 V because the two must add up to equal the 30-V source. The polarities are also negative at the top, the same as *V*.

Now we can determine the potentials at terminals A and B with respect to a common reference to find V_{AB}. Imagine that the positive side of the source *V* is connected to a chassis ground. Then we would use the bottom line in the diagram as our reference for voltages. Note that V_{R_4} at the bottom of the R_3-R_4 divider is the same as the potential of terminal A with respect to ground. This value is −20 V, with terminal A negative.

Similarly, V_{R_2} in the R_1-R_2 divider is the potential at B with respect to ground. This value is −12 V with terminal B negative. As a result, V_{AB} is the difference between the −20 V at A and the −12 V at B, both with respect to the common ground reference.

The potential difference V_{AB} then equals

$$V_{AB} = -20 - (-12) = -20 + 12 = -8 \text{ V}$$

Terminal A is 8 V more negative than B. Therefore, V_{TH} is 8 V, with the negative side toward terminal A, as shown in the Thevenin equivalent in Fig. 10–6*d*.

The potential difference V_{AB} can also be found as the difference between V_{R_3} and V_{R_1} in Fig. 10–6*b*. In this case, V_{R_3} is 10 V and V_{R_1} is 18 V, both positive with respect to the top line connected to the negative side of the source V. The potential difference between terminals A and B then is $10 - 18$, which also equals -8 V. Note that V_{AB} must have the same value no matter which path is used to determine the voltage.

To find R_{TH}, the 30-V source is short-circuited while terminals A and B are still open. Then the circuit looks like Fig. 10–6*c*. Looking back from terminals A and B, the 3-Ω R_3 and 6-Ω R_4 are in parallel, for a combined resistance R_{T_A} of $18/9$ or 2 Ω. The reason is that R_3 and R_4 are joined at terminal A, while their opposite ends are connected by the short circuit across the source V. Similarly, the 6-Ω R_1 and 4-Ω R_2 are in parallel for a combined resistance R_{T_B} of $24/10 = 2.4$ Ω. Furthermore, the short circuit across the source now provides a path that connects R_{T_A} and R_{T_B} in series. The entire resistance is $2 + 2.4 = 4.4$ Ω for R_{AB} or R_{TH}.

The Thevenin equivalent in Fig. 10–6*d* represents the bridge circuit feeding the open terminals A and B with 8 V for V_{TH} and 4.4 Ω for R_{TH}. Now connect the 2-Ω R_L to terminals A and B to calculate I_L. The current is

$$I_L = \frac{V_{TH}}{R_{TH} + R_L} = \frac{8}{4.4 + 2} = \frac{8}{6.4}$$

$$I_L = 1.25 \text{ A}$$

This 1.25 A is the current through the 2-Ω R_L at the center of the unbalanced bridge in Fig. 10–6*a*. Furthermore, the amount of I_L for any value of R_L in Fig. 10–6*a* can be calculated from the equivalent circuit in Fig. 10–6*d*.

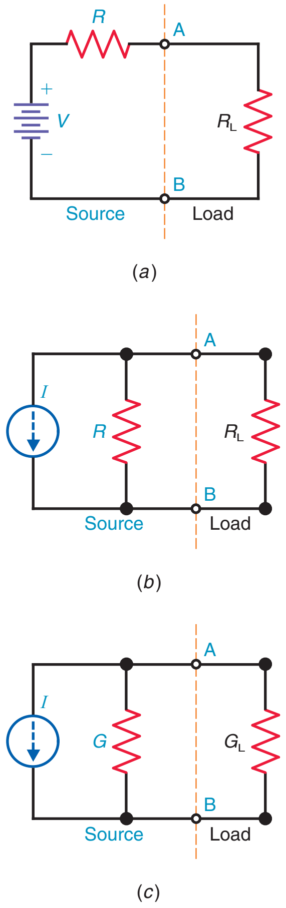
10–4 Self-Review

Answers at end of chapter.

In the Thevenin equivalent circuit in Fig. 10–6*d*,

a. How much is R_T?
b. How much is V_{R_L}?

Figure 10–7 General forms for a voltage source or current source connected to a load R_L across terminals A and B. (*a*) Voltage source V with series R. (*b*) Current source I with parallel R. (*c*) Current source I with parallel conductance G.



10–5 Norton's Theorem

Named after E. L. Norton, a scientist with Bell Telephone Laboratories, Norton's theorem is used to simplify a network in terms of currents instead of voltages. In many cases, analyzing the division of currents may be easier than voltage analysis. For current analysis, therefore, Norton's theorem can be used to reduce a network to a simple parallel circuit with a current source. The idea of a *current source* is that it supplies a total line current to be divided among parallel branches, corresponding to a *voltage source* applying a total voltage to be divided among series components. This comparison is illustrated in Fig. 10–7.

Example of a Current Source

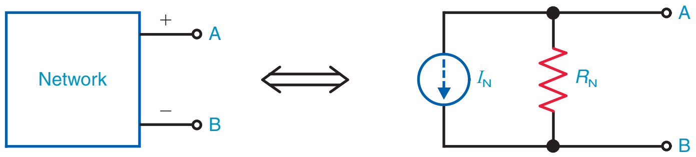
A source of electric energy supplying voltage is often shown with a series resistance that represents the internal resistance of the source, as in Fig. 10–7*a*. This method corresponds to showing an actual voltage source, such as a battery for dc circuits. However, the source may also be represented as a current source with a parallel resistance, as in Fig. 10–7*b*. Just as a voltage source is rated at, say, 10 V, a current source may be rated at 2 A. For the purpose of analyzing parallel branches, the concept of a current source may be more convenient than the concept of a voltage source.

If the current I in Fig. 10–7*b* is a 2-A source, it supplies 2 A no matter what is connected across the output terminals A and B. Without anything connected across A and B, all 2 A flows through the shunt R. When a load resistance R_L is connected across A and B, then the 2-A I divides according to the current division rules for parallel branches.

GOOD TO KNOW

A current source symbol that uses a solid arrow indicates the direction of conventional current flow. A dashed or broken arrow indicates the direction of electron flow.

Figure 10–8 Any network in the block at the left can be reduced to the Norton equivalent parallel circuit at the right.



Remember that parallel currents divide inversely to branch resistances but directly with conductances. For this reason it may be preferable to consider the current source shunted by the conductance G, as shown in Fig. 10–7*c*. We can always convert between resistance and conductance because $1/R$ in ohms is equal to G in siemens.

The symbol for a current source is a circle with an arrow inside, as shown in Fig. 10–7*b* and *c*, to show the direction of current. This direction must be the same as the current produced by the polarity of the corresponding voltage source. Remember that a source produces electron flow out from the negative terminal.

An important difference between voltage and current sources is that a current source is killed by making it open, compared with short-circuiting a voltage source. Opening a current source kills its ability to supply current without affecting any parallel branches. A voltage source is short-circuited to kill its ability to supply voltage without affecting any series components.

The Norton Equivalent Circuit

As illustrated in Fig. 10–8, Norton's theorem states that the entire network connected to terminals A and B can be replaced by a single current source I_N in parallel with a single resistance R_N. The value of I_N is equal to the short-circuit current through the AB terminals. This means finding the current that the network would produce through A and B with a short circuit across these two terminals.

The value of R_N is the resistance looking back from open terminals A and B. These terminals are not short-circuited for R_N but are open, as in calculating R_{TH} for Thevenin's theorem. Actually, the single resistor is the same for both the Norton and Thevenin equivalent circuits. In the Norton case, this value of R_{AB} is R_N in parallel with the current source; in the Thevenin case, it is R_{TH} in series with the voltage source.

Nortonizing a Circuit

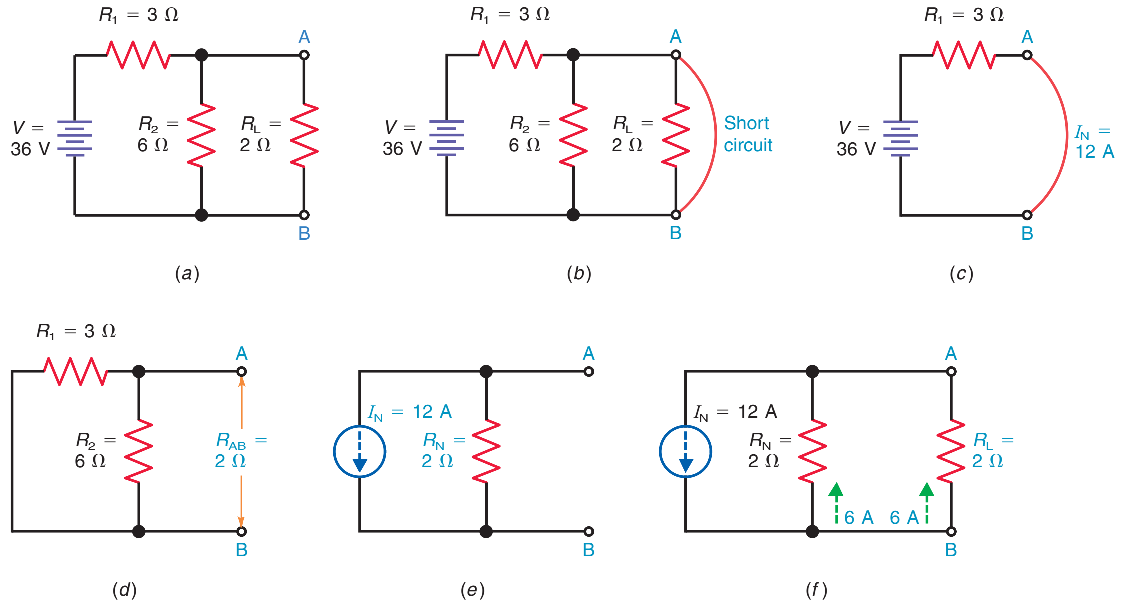
As an example, let us recalculate the current I_L in Fig. 10–9*a*, which was solved before by Thevenin's theorem. The first step in applying Norton's theorem is to imagine a short circuit across terminals A and B, as shown in Fig. 10–9*b*. How much current is flowing in the short circuit? Note that a short circuit across AB short-circuits R_L and the parallel R_2. Then the only resistance in the circuit is the 3-Ω R_1 in series with the 36-V source, as shown in Fig. 10–9*c*. The short-circuit current, therefore, is

$$I_N = \frac{36\ \text{V}}{3\ \Omega} = 12\ \text{A}$$

This 12-A I_N is the total current available from the current source in the Norton equivalent in Fig. 10–9*e*.

To find R_N, remove the short circuit across A and B and consider the terminals open without R_L. Now the source V is considered short-circuited. As shown in Fig. 10–9*d*, the resistance seen looking back from terminals A and B is 6 Ω in parallel with 3 Ω, which equals 2 Ω for the value of R_N.

Figure 10–9 Same circuit as in Fig. 10–3, but solved by Norton's theorem. (*a*) Original circuit. (*b*) Short circuit across terminals A and B. (*c*) The short-circuit current I_N is $^{36}/_{3} = 12$ A. (*d*) Open terminals A and B but short-circuit V to find R_{AB} is 2 Ω, the same as R_{TH}. (*e*) Norton equivalent circuit. (*f*) R_L reconnected to terminals A and B to find that I_L is 6 A.



The resultant Norton equivalent is shown in Fig. 10–9*e*. It consists of a 12-A current source I_N shunted by the 2-Ω R_N. The arrow on the current source shows the direction of electron flow from terminal B to terminal A, as in the original circuit.

Finally, to calculate I_L, replace the 2-Ω R_L between terminals A and B, as shown in Fig. 10–9*f*. The current source still delivers 12 A, but now that current divides between the two branches of R_N and R_L. Since these two resistances are equal, the 12-A I_N divides into 6 A for each branch, and I_L is equal to 6 A. This value is the same current we calculated in Fig. 10–3, by Thevenin's theorem. Also, V_L can be calculated as $I_L R_L$, or 6 A × 2 Ω, which equals 12 V.

Looking at the Short-Circuit Current

In some cases, there may be a question of which current is I_N when terminals A and B are short-circuited. Imagine that a wire jumper is connected between A and B to short-circuit these terminals. Then I_N must be the current that flows in this wire between terminals A and B.

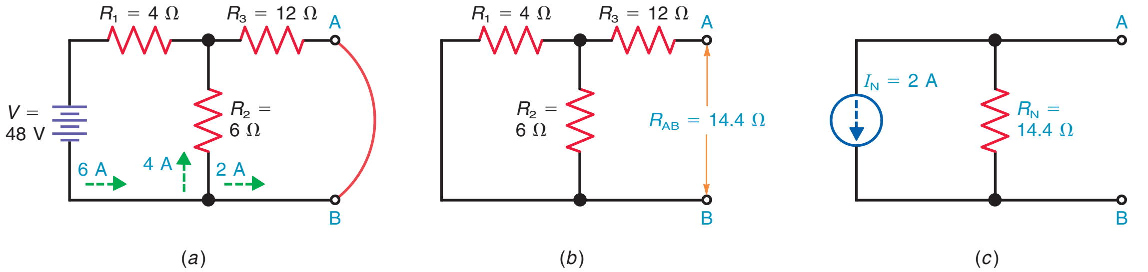
Remember that any components directly across these two terminals are also short-circuited by the wire jumper. Then these parallel paths have no effect. However, any components in series with terminal A or terminal B are in series with the wire jumper. Therefore, the short-circuit current I_N also flows through the series components.

An example of a resistor in series with the short circuit across terminals A and B is shown in Fig. 10–10. The idea here is that the short-circuit I_N is a branch current, not the main-line current. Refer to Fig. 10–10*a*. Here the short circuit connects R_3 across R_2. Also, the short-circuit current I_N is now the same as the current I_3 through R_3. Note that I_3 is only a branch current.

To calculate I_3, the circuit is solved by Ohm's law. The parallel combination of R_2 with R_3 equals $^{72}/_{18}$ or 4 Ω. The R_T is 4 + 4 = 8 Ω. As a result, the I_T from the source is 48 V/ 8 Ω = 6 A.

This I_T of 6 A in the main line divides into 4 A for R_2 and 2 A for R_3. The 2-A I_3 for R_3 flows through short-circuited terminals A and B. Therefore, this current of 2 A is the value of I_N.

Figure 10–10 Nortonizing a circuit where the short-circuit current I_N is a branch current. (*a*) I_N is 2 A through short-circuited terminals A and B and R_3. (*b*) $R_N = R_{AB} = 14.4\ \Omega$. (*c*) Norton equivalent circuit.



To find R_N in Fig. 10–10*b*, the short circuit is removed from terminals A and B. Now the source *V* is short-circuited. Looking back from open terminals A and B, the 4-Ω R_1 is in parallel with the 6-Ω R_2. This combination is ${}^{24}\!/_{10} = 2.4\ \Omega$. The 2.4 Ω is in series with the 12-Ω R_3 to make $R_{AB} = 2.4 + 12 = 14.4\ \Omega$.

The final Norton equivalent is shown in Fig. 10–10*c*. Current I_N is 2 A because this branch current in the original circuit is the current that flows through R_3 and short-circuited terminals A and B. Resistance R_N is 14.4 Ω looking back from open terminals A and B with the source *V* short-circuited the same way as for R_{TH}.

10–5 Self-Review

Answers at end of chapter.

a. For a Norton equivalent circuit, terminals A and B are short-circuited to find I_N. (True/False)

b. For a Norton equivalent circuit, terminals A and B are open to find R_N. (True/False)

10–6 Thevenin–Norton Conversions

Thevenin's theorem says that any network can be represented by a voltage source and series resistance, and Norton's theorem says that the same network can be represented by a current source and shunt resistance. It must be possible, therefore, to convert directly from a Thevenin form to a Norton form and vice versa. Such conversions are often useful.

Norton from Thevenin

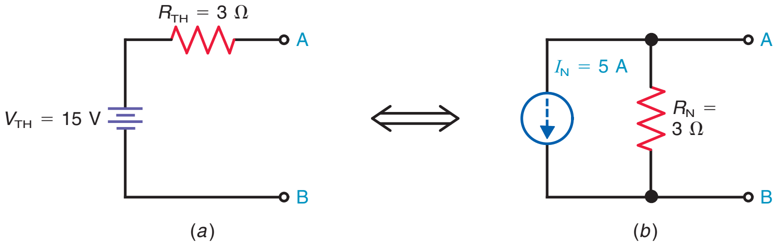
Consider the Thevenin equivalent circuit in Fig. 10–11*a*. What is its Norton equivalent? Just apply Norton's theorem, the same as for any other circuit. The short-circuit current through terminals A and B is

$$I_N = \frac{V_{TH}}{R_{TH}} = \frac{15\ \text{V}}{3\ \Omega} = 5\ \text{A}$$

GOOD TO KNOW

An ideal current source is assumed to have an internal resistance of infinite ohms. Therefore, when calculating the Thevenin resistance, R_{TH}, it is only practical to consider a current source as an open circuit.

Figure 10–11 Thevenin equivalent circuit in (*a*) corresponds to the Norton equivalent in (*b*).



The resistance, looking back from open terminals A and B with the source V_{TH} short-circuited, is equal to the 3 Ω of R_{TH}. Therefore, the Norton equivalent consists of a current source that supplies the short-circuit current of 5 A, shunted by the same 3-Ω resistance that is in series in the Thevenin circuit. The results are shown in Fig. 10–11*b*.

Thevenin from Norton

For the opposite conversion, we can start with the Norton circuit of Fig. 10–11*b* and get back to the original Thevenin circuit. To do this, apply Thevenin's theorem, the same as for any other circuit. First, we find the Thevenin resistance by looking back from open terminals A and B. An important principle here, though, is that, although a voltage source is short-circuited to find R_{TH}, a current source is an open circuit. In general, a current source is killed by opening the path between its terminals. Therefore, we have just the 3-Ω R_N, in parallel with the infinite resistance of the open current source. The combined resistance then is 3 Ω.

In general, the resistance R_N always has the same value as R_{TH}. The only difference is that R_N is connected in parallel with I_N, but R_{TH} is in series with V_{TH}.

Now all that is required is to calculate the open-circuit voltage in Fig. 10–11*b* to find the equivalent V_{TH}. Note that with terminals A and B open, all current from the current source flows through the 3-Ω R_N. Then the open-circuit voltage across the terminals A and B is

$$I_N R_N = 5 \text{ A} \times 3\ \Omega = 15 \text{ V} = V_{TH}$$

As a result, we have the original Thevenin circuit, which consists of the 15-V source V_{TH} in series with the 3-Ω R_{TH}.

Conversion Formulas

In summary, the following formulas can be used for these conversions:

Thevenin from Norton:

$$R_{TH} = R_N$$
$$V_{TH} = I_N \times R_N$$

Norton from Thevenin:

$$R_N = R_{TH}$$
$$I_N = V_{TH}/R_{TH}$$

Another example of these conversions is shown in Fig. 10–12.

Figure 10–12 Example of Thevenin-Norton conversions. (*a*) Original circuit, the same as in Figs. 10–3*a* and 10–9*a*. (*b*) Thevenin equivalent. (*c*) Norton equivalent.