University of Engineering & Technology Peshawar, Pakistan

CE301: Structure Analysis II

Module 08:

Analysis of S.I Frames Using stiffness method

By:
Prof. Dr. Bashir Alam
Civil Engineering Department
UET , Peshawar

Topics to be Covered

- Introduction
- Prerequisites for using stiffness method
- Types of frames
- Step wise procedure of stiffness method for frame analysis
- Analysis of Nonsway frames Example 1,2,3
- Assignment 04 (a)
- Analysis of sway frames Example 1,2,3
- Assignment 04 (b)

Stiffness Method for Frames Analysis

❑ Introduction:

Frames are analyzed with stiffness method due to

- To solve the problem in matrix notation, which is more systematic
- To compute reactions at all the supports.
- To compute internal resisting shear , axial & bending moment at any section of the frame.

Stiffness Method for Frames Analysis

- **Prerequisites for Analysis with stiffness method:**

  It is necessary that students must have strong background of the following concepts before starting analysis with stiffness or any other matrix method.

- Enough concept of Matrix Algebra
- Must be able to find the Kinematic Indeterminacy
- Must know the formulas & concept of fixed end actions

Stiffness Method for Frames Analysis

❑ Types of Frames :

On the basis of lateral displacement frames are classified in to following two types

- Frames with Sway
- Frames without Sway

Stiffness Method for Frames Analysis

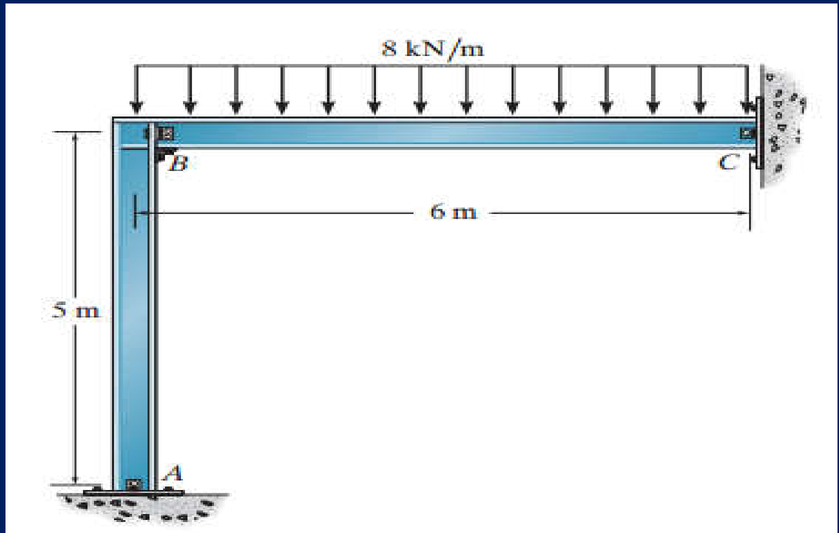
❖ **Sway:**

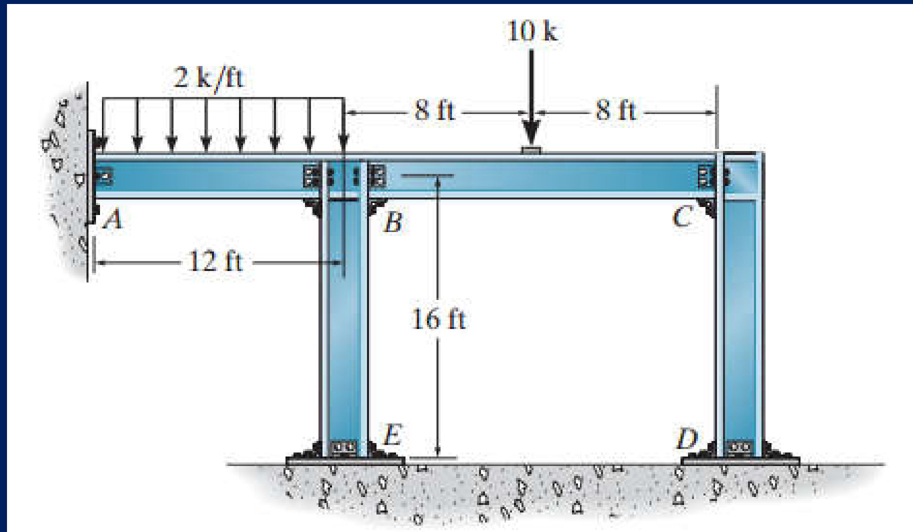
In statically indeterminate structures the structures the frames deflect laterally due the presence of lateral load or unsymmetrical vertical loads or where the frames themselves are unsymmetrical.

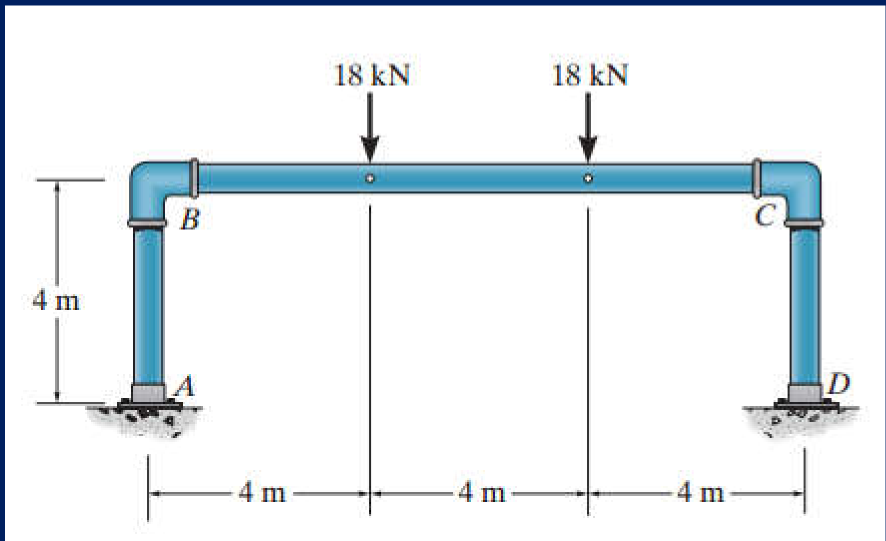
- Causes of side sway:
  - ✓ Unsymmetrical loading (eccentric loading)
  - ✓ Different end conditions of the columns of the frame
  - ✓ Non uniform sections of the members
  - ✓ Horizontal loading on column of the frame
  - ✓ Settlement of the supports of the frame

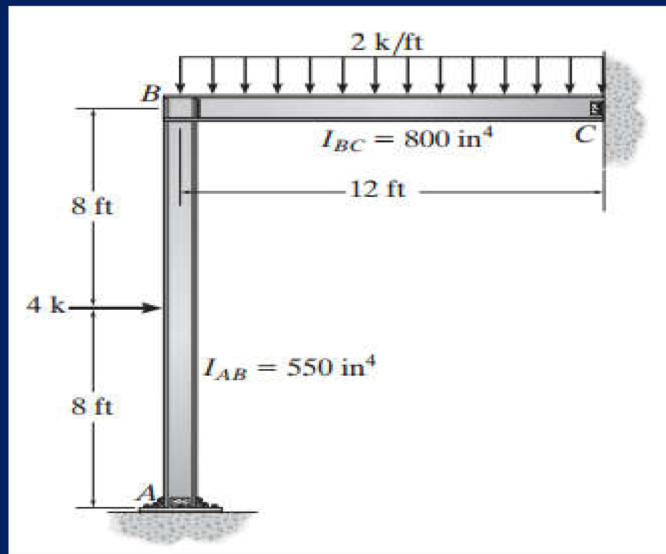
Stiffness Method for Frames Analysis

- **Frames without side sway:**



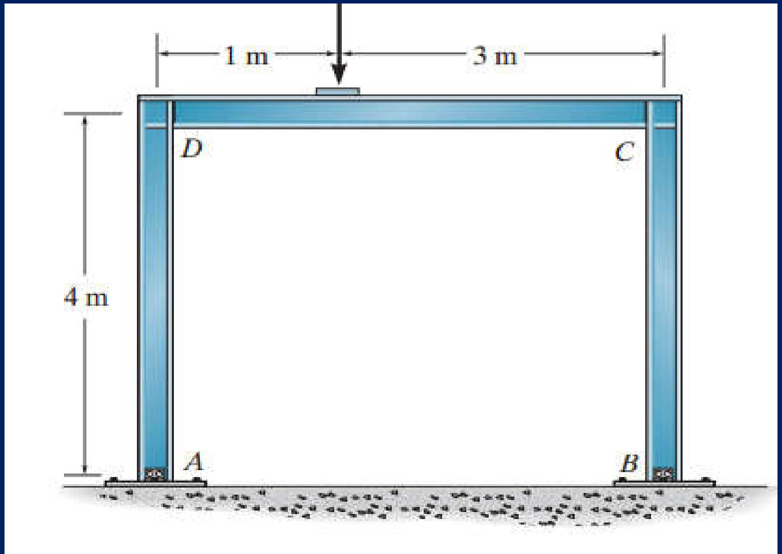


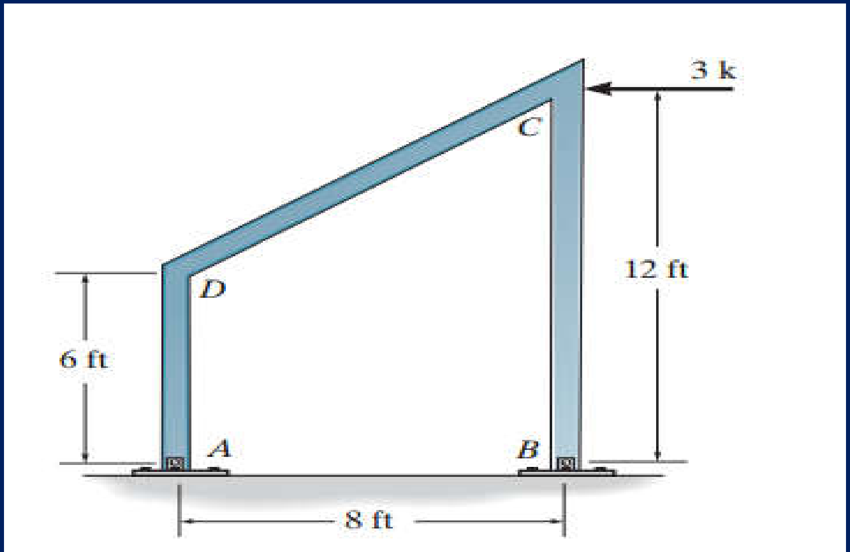


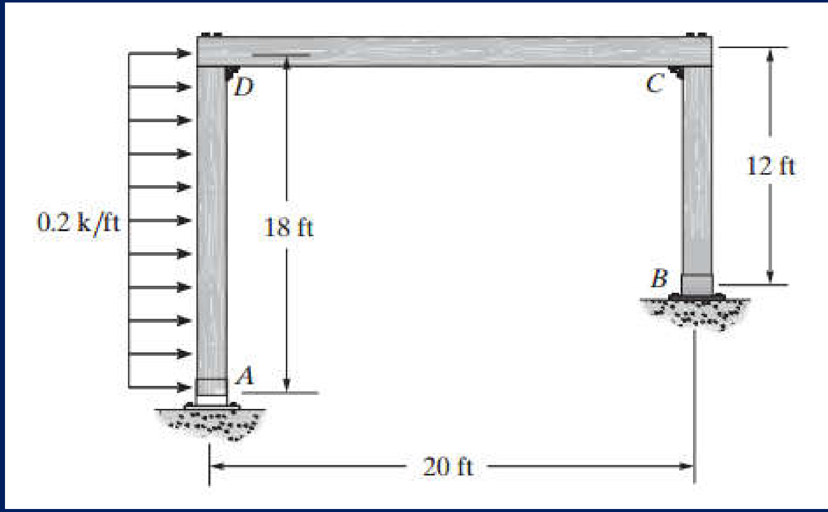


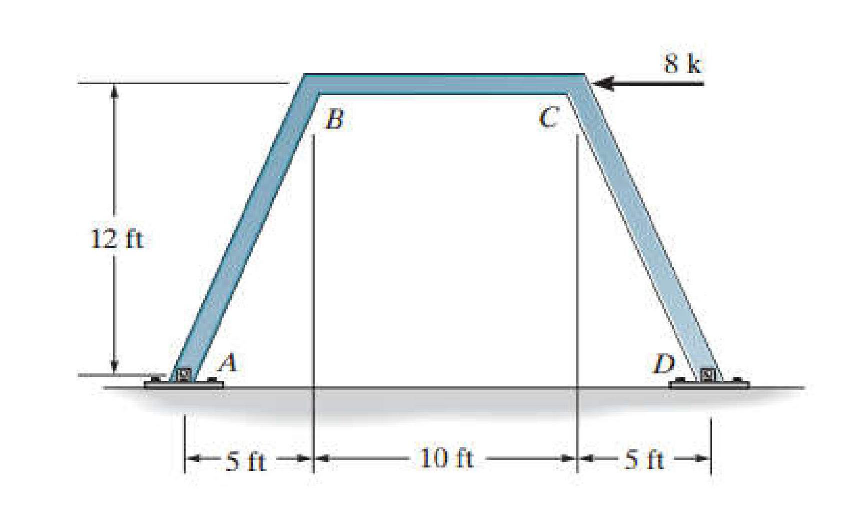
Stiffness Method for Frames Analysis

- **Frames with side sway:**









Stiffness Method for Frames Analysis

Part I

Analysis of Frames without side sway

Stiffness Method for Frames Analysis

❑ **Step wise Solution Procedure using Stiffness method method:**

The following steps must be followed while solving a structure using Stiffness method.

- **Step # 01:** Make the structure kinametically determinate, by restraining the joints i.e select the redundant joint displacement.

Stiffness Method for Frames Analysis

Step # 02: Apply the actual external loads on the BKDS (Basic kinametically determinate structure) and find the actions at the locations of redundant joints (compute fixed end actions) this will generate ADL matrix.

Step # 03: Apply the redundant joint displacement on the BKDS (To standardize the procedure, only a unit displacement is applied in the +ve direction) this will generate stiffness coefficient matrix.

Stiffness Method for Frames Analysis

Step # 04: Apply equilibrium condition at the location of the redundant joint displacement to write equilibrium equations and solve for unknown joint displacement.

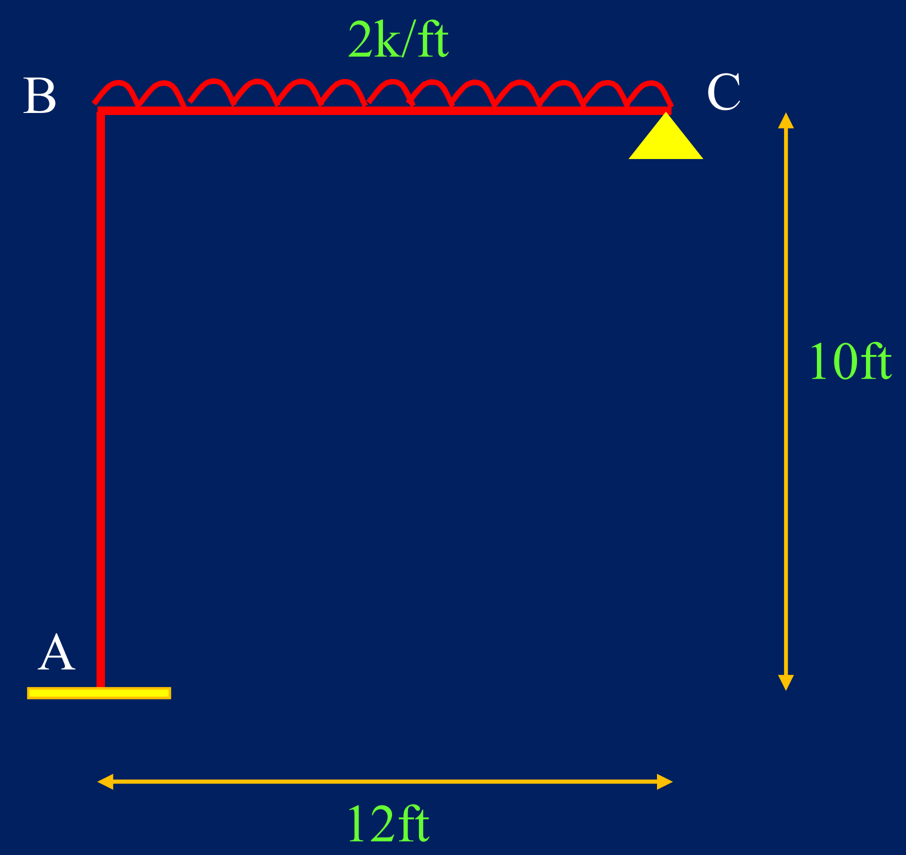
$$[AD] = [ADL] + [S] \bullet [D]$$

$$[D] = [S]^{-1} \bullet [AD - ADL]$$

Step # 05: Compute the member end actions .

Stiffness Method for Frames Analysis

Problem 01: Analyze the given Frame using stiffness method.

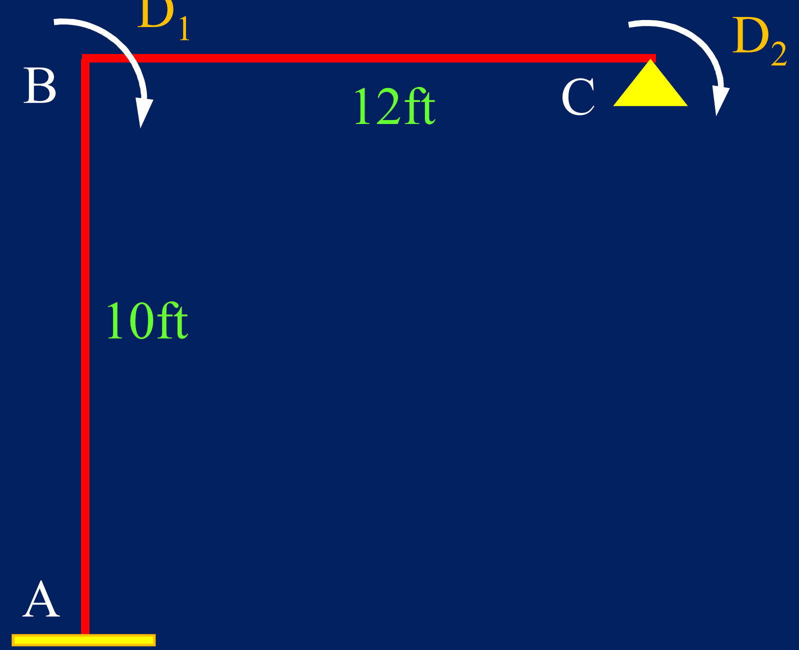


Take EI = constant

K.I = 2 degree (neglecting the axial effects)
So two redundant joint displacements should be chosen.

Stiffness Method for Frames Analysis

- **Step # 01:** Selection of redundant Joint displacements and assign coordinates at those locations. Also compute AD values.

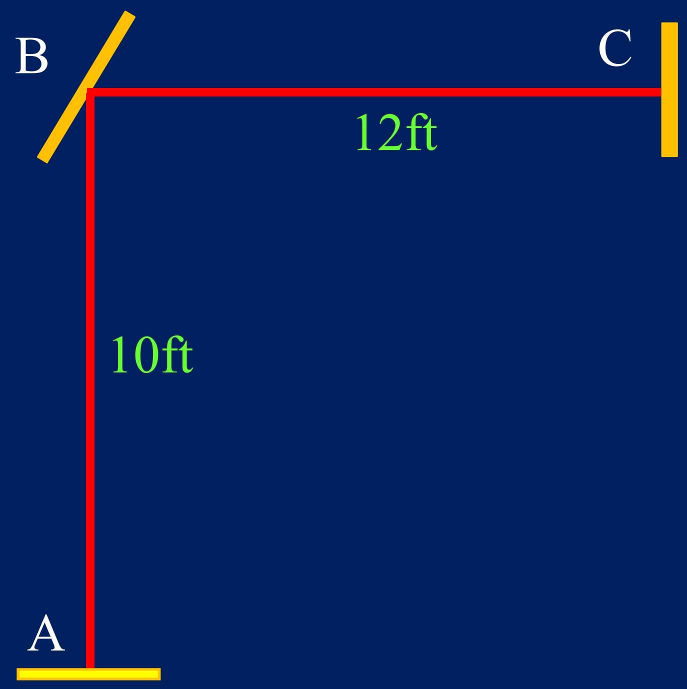


Rotation at B & C is taken as redundant joint displacement.

$$[D]_{2*1} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad [AD]_{2*1} = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Stiffness Method for Frames Analysis

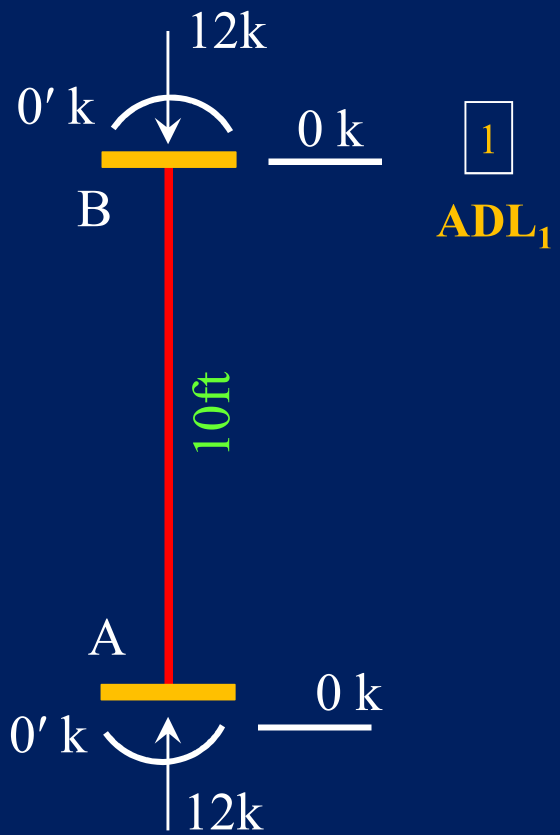
- Restrain all the degrees of freedom to get the restrained structure.

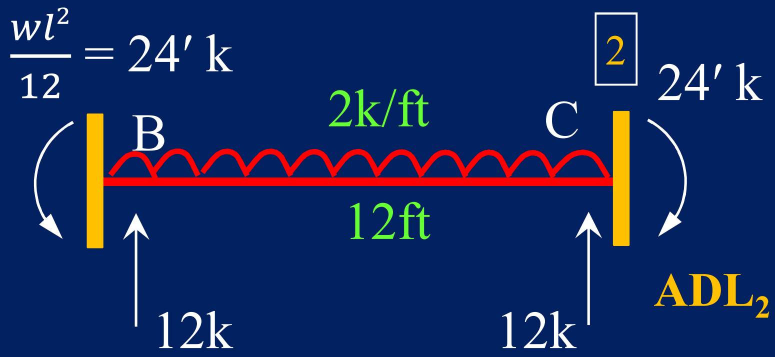


Basic kinematic determinate structure (BKDS) or restrained structure

Stiffness Method for Frames Analysis

- **Step # 02 :** Restrained structure acted upon by the actual loads. compute the values of actions in the restrained structure corresponding to the redundant locations. This will generate ADL matrix.(Fixed end actions)





$$ADL_1 = -24' k \qquad ADL_2 = 24' k$$

$$[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} = \begin{bmatrix} -24 \\ 24 \end{bmatrix}$$

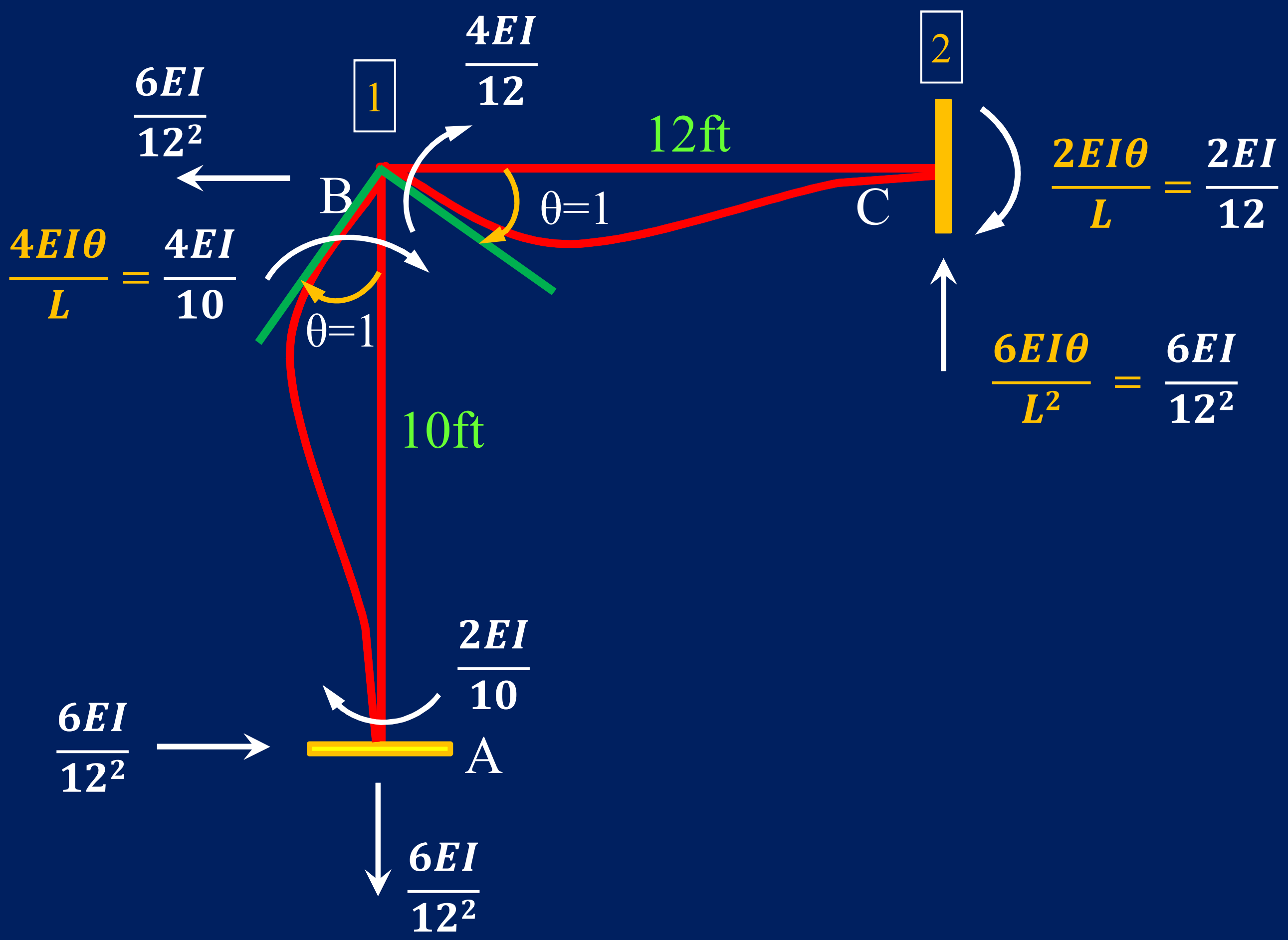
Stiffness Method for Frames Analysis

Step # 03 : Primary structure acted upon by a unit value of D & computation of stiffness coefficients " S" values in the BKDS corresponding to the redundant joint displacement locations.

- 1st a unit rotation is applied at location 1 & prevented at 2 as shown. Compute the values of S_{11} and S_{21} .
- Then apply a unit rotation at the redundant displacement location 2 and prevented at 1 as shown. Compute the values of S_{12} & S_{22} .

Stiffness Method for Frames Analysis

i. 1st a unit rotation is applied at location 1 & prevented at 2 (D_1=1 & D_2=0)as shown. Compute the values of S_{11} and S_{21} .



Stiffness Method for Frames Analysis

Step # 03 (i): Contd…

$$S_{11} = \frac{4EI}{12} + \frac{4EI}{10}$$

$$S_{11} = 0.733EI$$

$$S_{21} = \frac{2EI}{12}$$

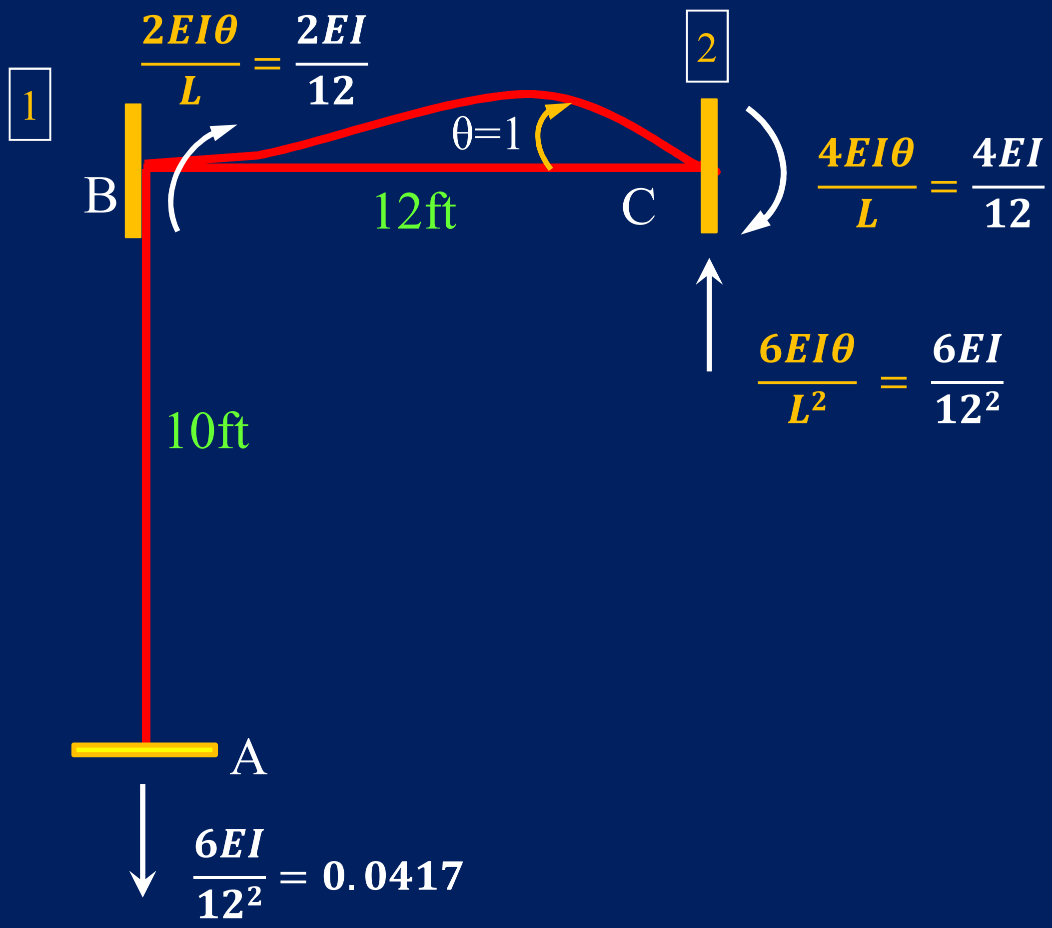
$$S_{21} = 0.1667EI$$

S_{11} = Action(sum of moments in this case) in the BKDS at redundant displacement location 1 due to unit rotation at that location.

S_{21} = Moment in the BDS at redundant displacement location 2 due to a unit rotation applied at location 1

Stiffness Method for Frames Analysis

ii. Now a unit rotation is applied at the redundant displacement location 2 and prevented at 1 (D_2=1 & D_1=0) as shown. Compute the values of S_{12} & S_{22} .



Stiffness Method for Frames Analysis

Step # 03 : Contd…

$$S_{12} = \frac{2EI}{12} \qquad S_{22} = \frac{4EI}{12}$$

$$S_{12} = 0.1667EI \qquad S_{22} = 0.333EI$$

S_{12} = Moment in the BKDS at redundant displacement location 1 due to unit rotation applied at redundant displacement location 2.

S_{22} = Moment in the BKDS at redundant displacement location 2 due to a unit rotation applied at that location.

Stiffness Method for Frames Analysis

Step # 03 : Contd…

$$S_{11} = 0.733EI \qquad S_{12} = 0.1667EI$$

$$S_{21} = 0.1667EI \qquad S_{22} = 0.333EI$$

$$[\,S\,] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$$[\,S\,] = EI\begin{bmatrix} 0.733 & 0.1667 \\ 0.1667 & 0.333 \end{bmatrix}$$

Stiffness coefficient matrix

Stiffness Method for Frames Analysis

Step # 04: Apply equilibrium condition at the location of the redundant joint displacement to write equilibrium equations and solve for unknown joint displacement.

$$AD_1 = ADL_1 + S_{11}D_1 + S_{12}D_2$$

$$AD_2 = ADL_2 + S_{21}D_1 + S_{22}D_2$$

$$\begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} + \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$[AD]_{2\,*\,1} = [ADL]_{2\,*\,1} + [S]_{2\,*\,2} \bullet [D]_{2\,*\,1}$$

$$[\boldsymbol{D}] = [\boldsymbol{S}]^{-1} \bullet [\boldsymbol{AD} - \boldsymbol{ADL}]$$

Stiffness Method for Frames Analysis

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix}$$

$$\begin{bmatrix} D1 \\ D2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.733 & 0.1667 \\ 0.1667 & 0.333 \end{bmatrix}^{-1} \begin{bmatrix} 0 - (-24) \\ 0 - 24 \end{bmatrix}$$

$$\begin{bmatrix} D1 \\ D2 \end{bmatrix} = \begin{bmatrix} 55.40 \\ -99.741 \end{bmatrix} \frac{1}{EI}$$

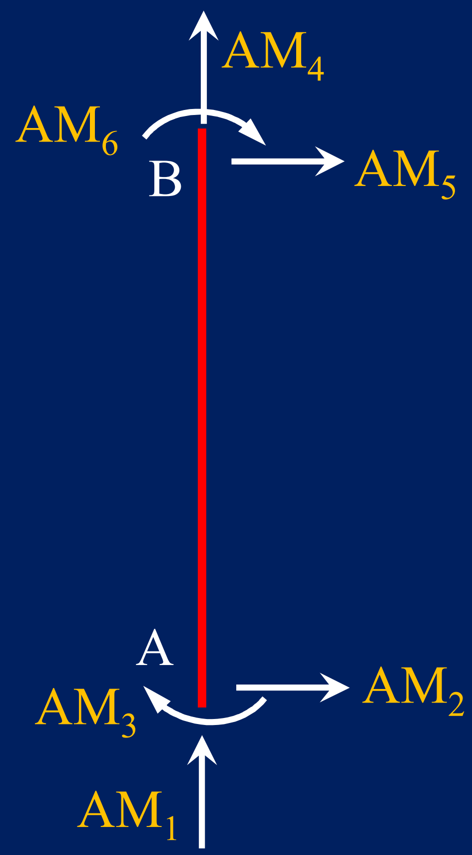
-ive sign shows that our assumed redundant joint displacement direction is wrong

Stiffness Method for Frames Analysis

Step # 05: Compute the member end actions. As we know that

$$[AM] = [AML] + [AMD][AD]$$

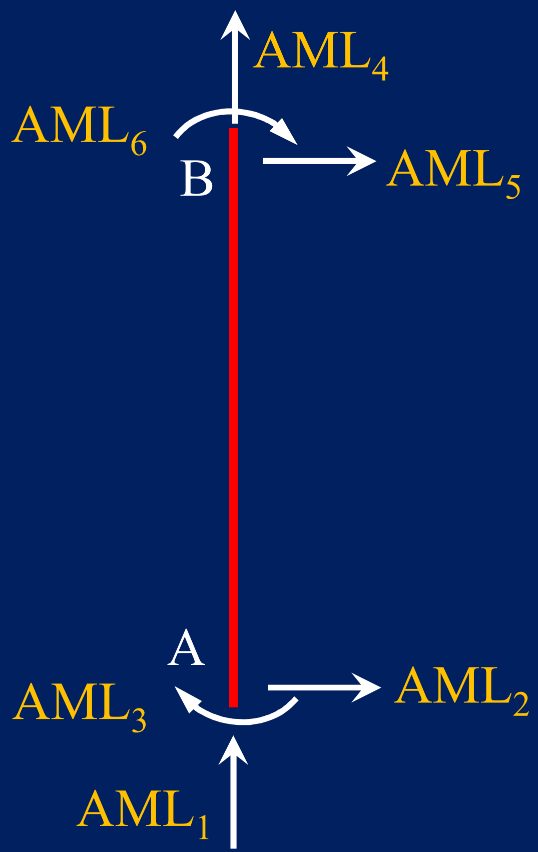
Member AB:

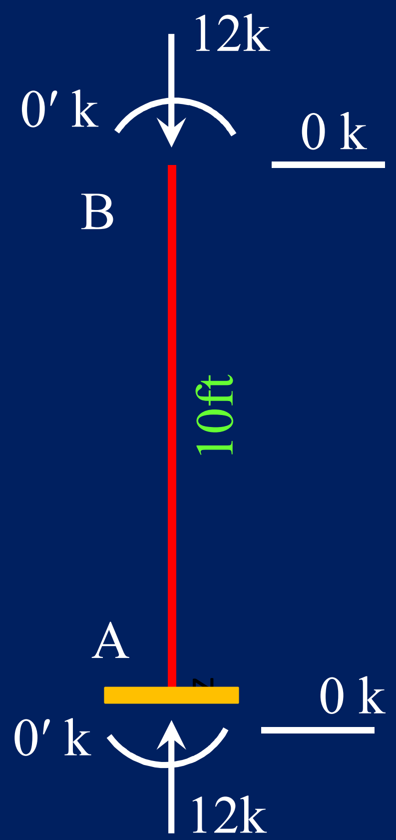


AM_n = is the member end action in the indeterminate structure at different location specified with a number "n". n shows the number of member end actions.

Stiffness Method for Frames Analysis

i. Compute AML values.



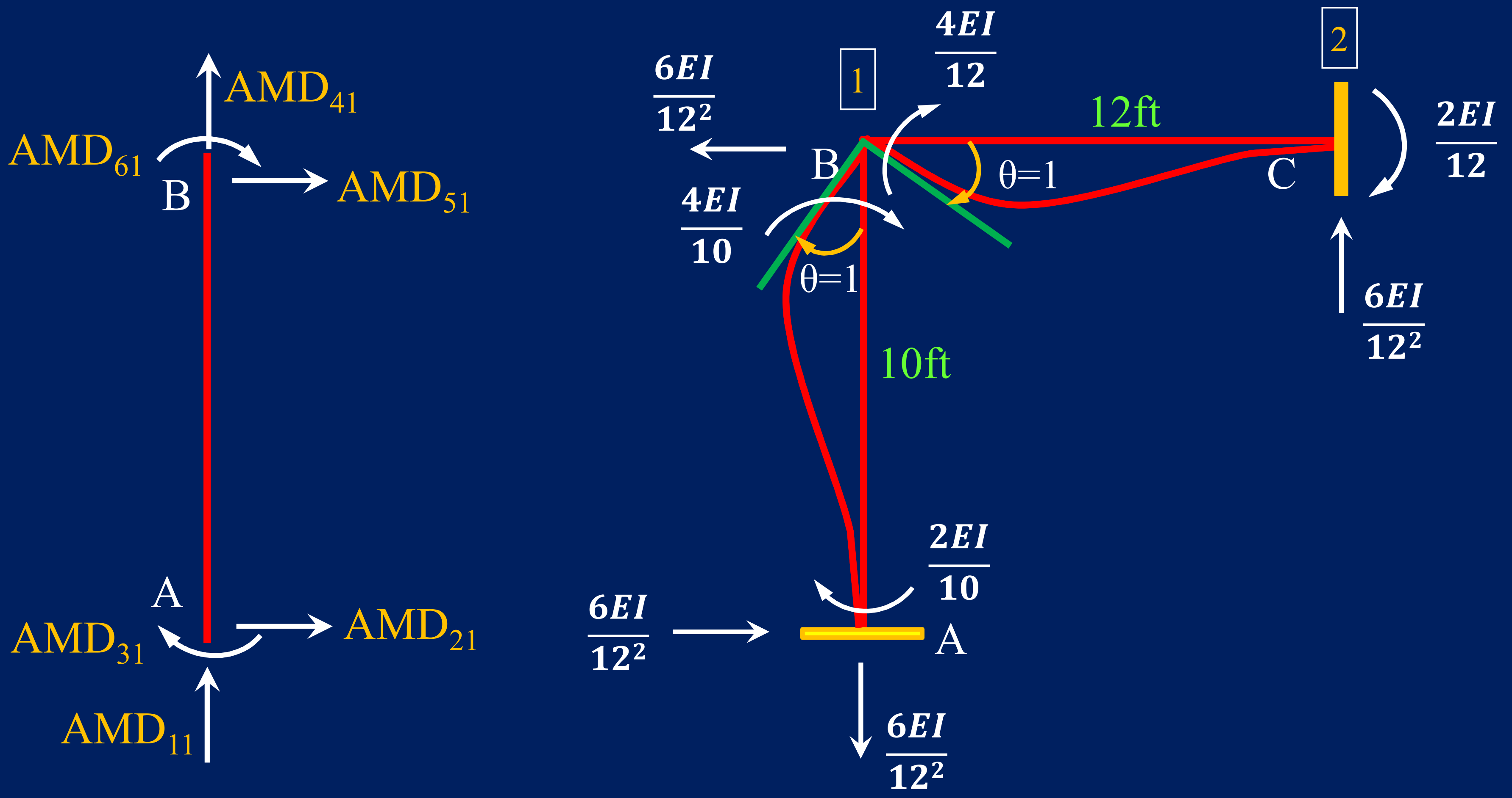


$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \\ -12 \\ 0 \\ 0 \end{bmatrix}$$

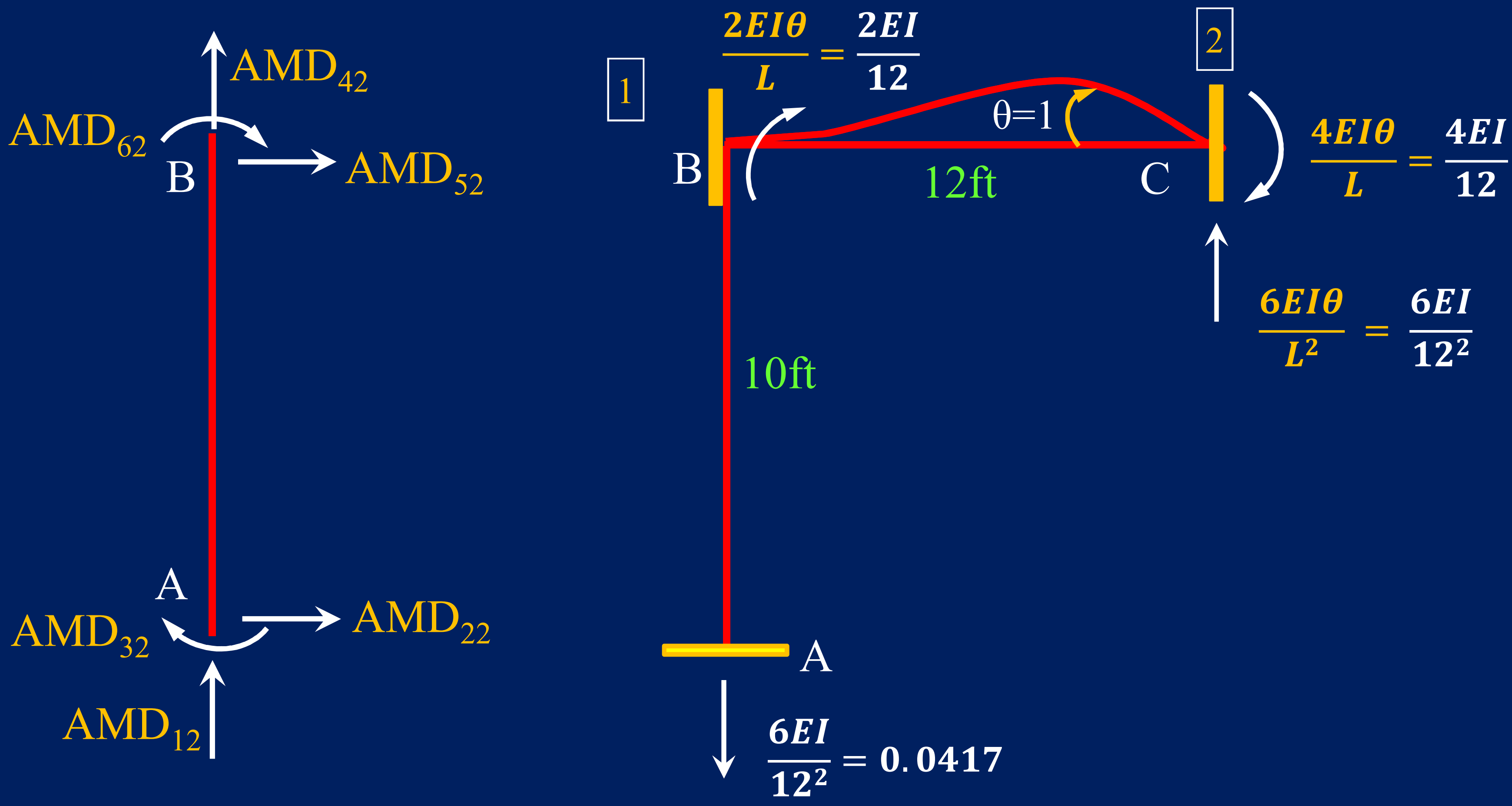
Stiffness Method for Frames Analysis

b) Compute the AMD values.

- 1st apply a unit rotation at redundant location 1 and then at 2 as shown below.



Stiffness Method for Frames Analysis



Stiffness Method for Frames Analysis

So the AMD values are

$$[AMD]_{6\,*\,2} = \begin{bmatrix} AMD_{11} & AMD_{12} \\ AMD_{21} & AMD_{22} \\ AMD_{31} & AMD_{32} \\ AMD_{41} & AMD_{42} \\ AMD_{51} & AMD_{52} \\ AMD_{61} & AMD_{62} \end{bmatrix} = \begin{bmatrix} -0.0417 & -0.0417 \\ 0.06 & 0 \\ 0.2 & 0 \\ 0.0417 & 0.0417 \\ -0.06 & 0 \\ 0.4 & 0 \end{bmatrix}$$

Stiffness Method for Frames Analysis

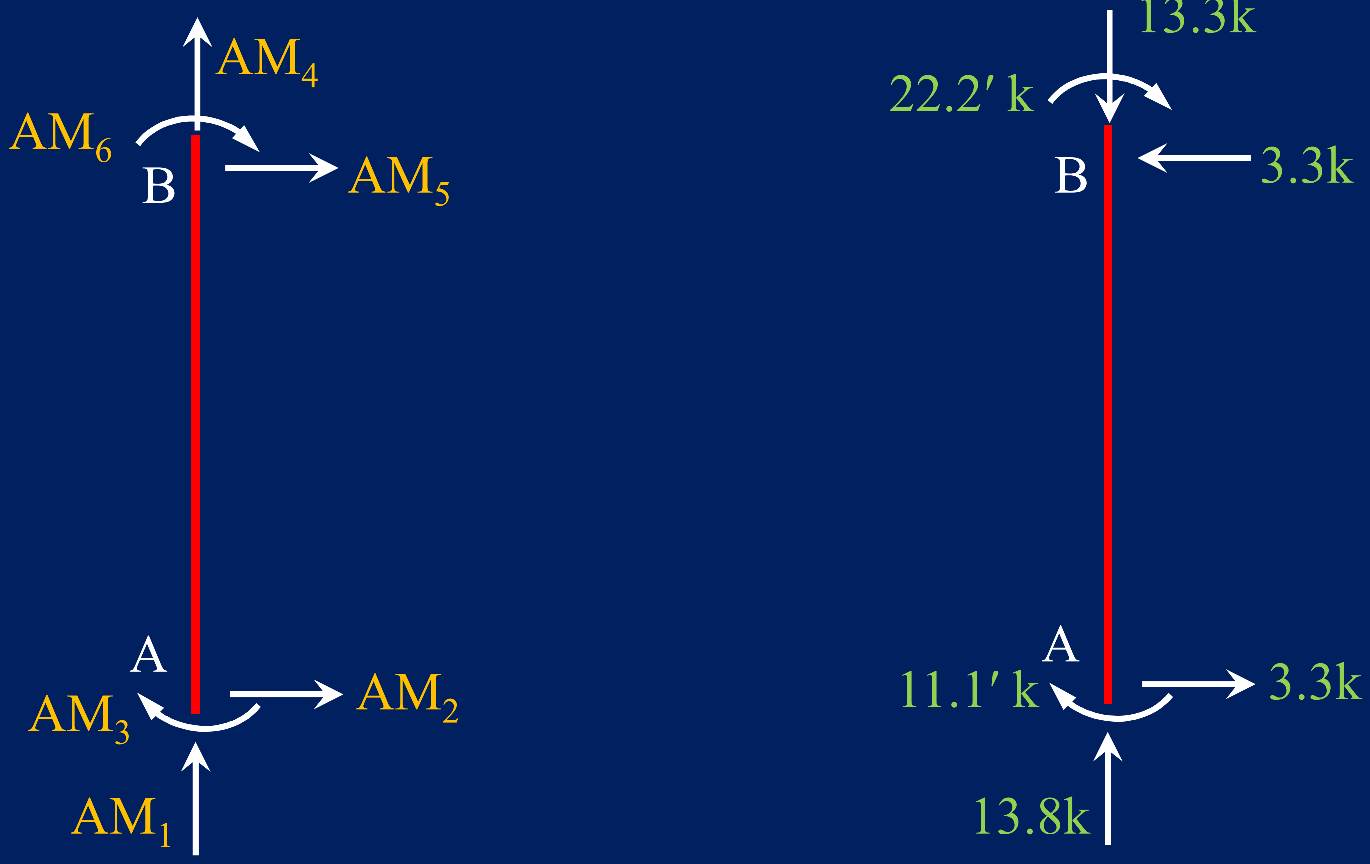
Now member end actions will be computed as given below

$$[AM] = [AML] + [AMD][AD]$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \\ -12 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.0417 & -0.0417 \\ 0.06 & 0 \\ 0.2 & 0 \\ 0.0417 & 0.0417 \\ -0.06 & 0 \\ 0.4 & 0 \end{bmatrix} \begin{bmatrix} 55.40 \\ -99.714 \end{bmatrix} = \begin{bmatrix} 13.8\ \text{k} \\ 3.3\ \text{k} \\ 11.1'\ \text{k} \\ -13.8\ \text{k} \\ -3.3\ \text{k} \\ 22.2'\ \text{k} \end{bmatrix}$$

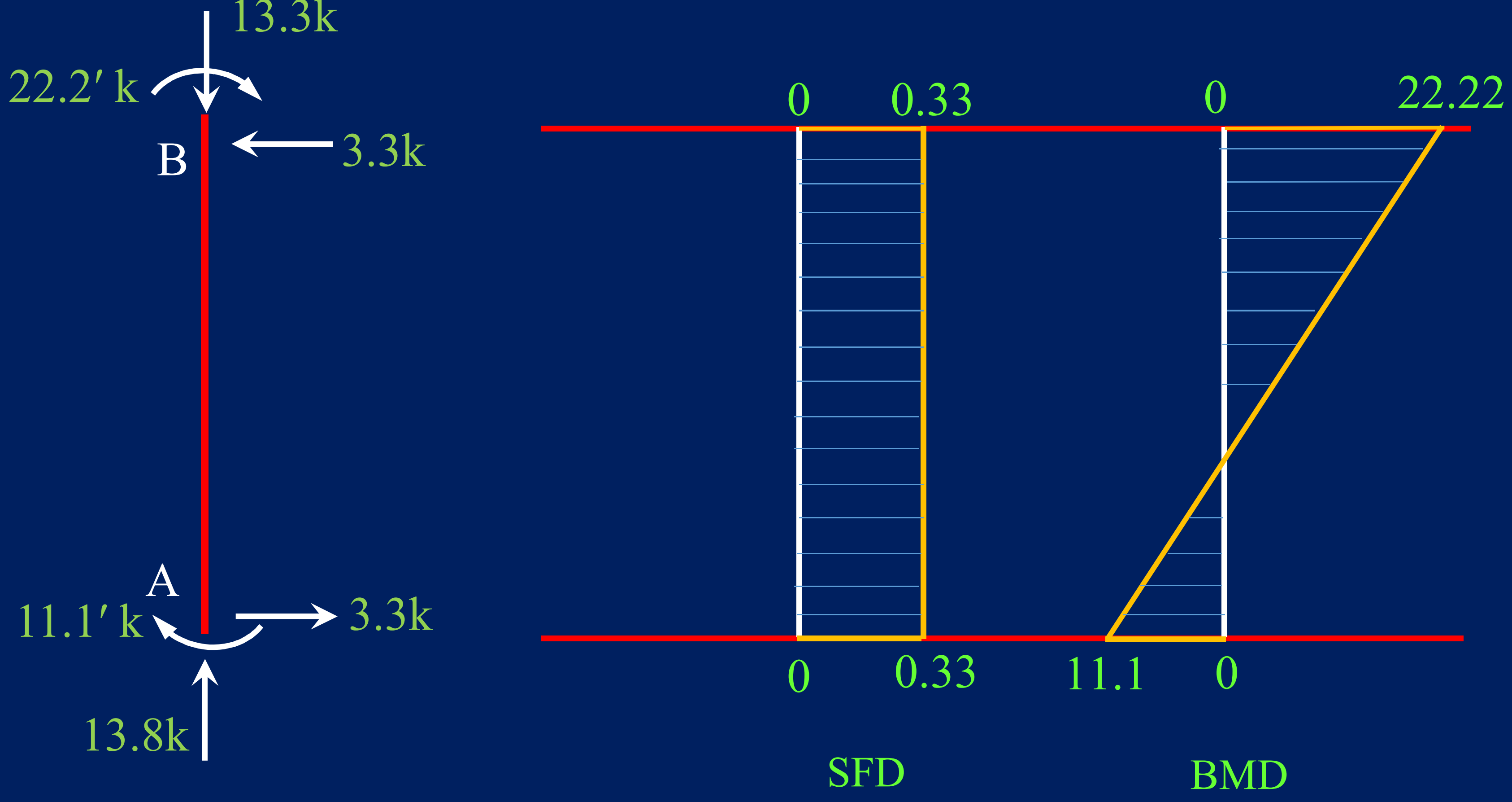
Stiffness Method for Frames Analysis

Final Analyzed Member



Stiffness Method for Frames Analysis

Now Shear force and bending moment diagrams



Stiffness Method for Frames Analysis

Step # 05: Compute the member end actions. As we know that

$$[AM] = [AML] + [AMD][AD]$$

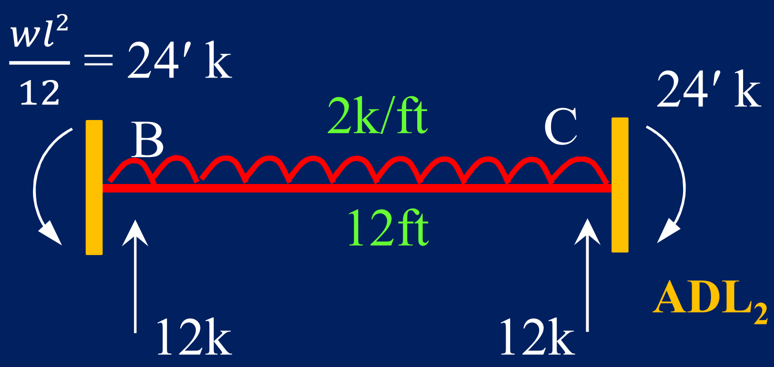
Member BC:

AM_n = is the member end action in the indeterminate structure at different location specified with a number “n”. n shows the number of member end actions.

Stiffness Method for Frames Analysis

i. Compute AML values.

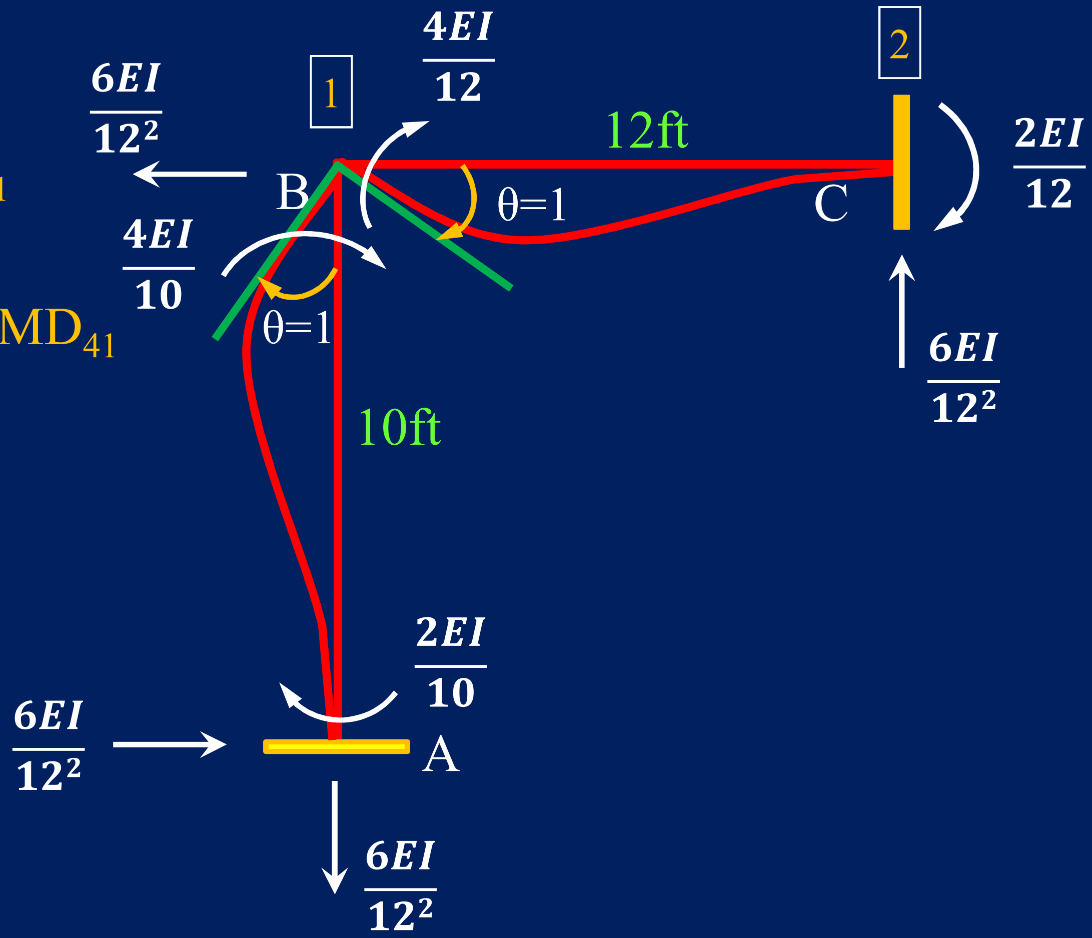
$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ -24 \\ 0 \\ 12 \\ 24 \end{bmatrix}$$



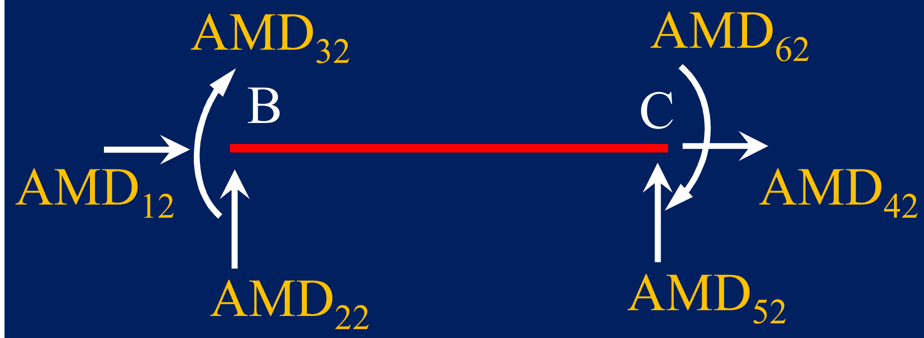
Stiffness Method for Frames Analysis

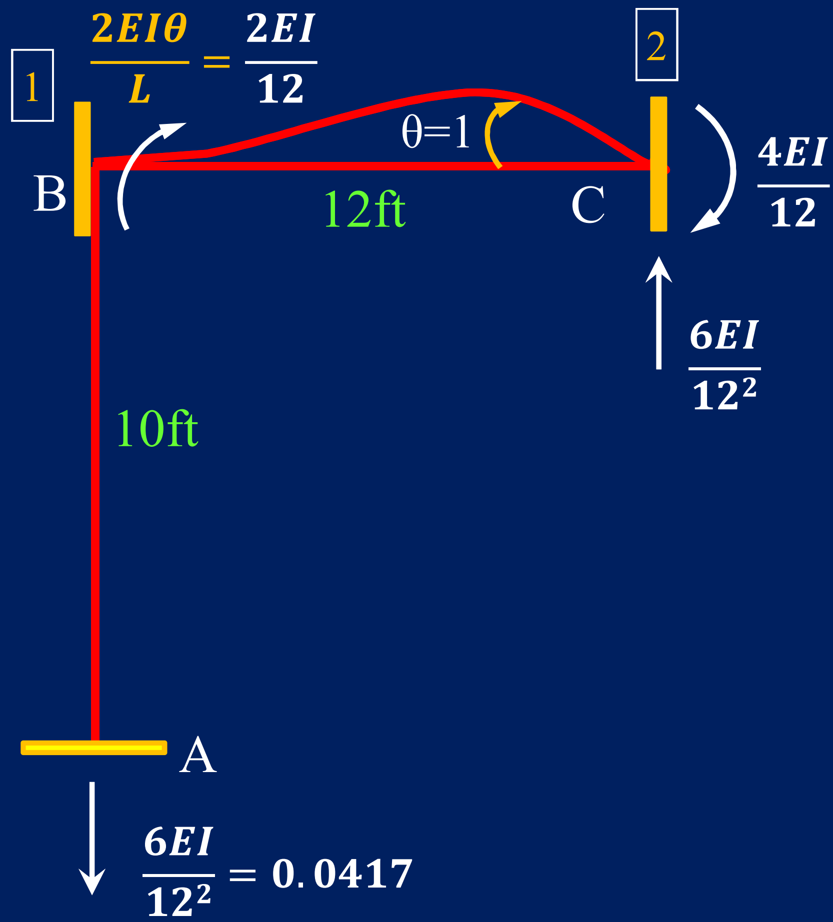
b) Compute the AMD values.

- 1^{st} apply a unit rotation at redundant location 1 and then at 2 as shown below.



Stiffness Method for Frames Analysis





Stiffness Method for Frames Analysis

So the AMD values are

$$[AMD]_{6\,*\,2} = \begin{bmatrix} AMD_{11} & AMD_{12} \\ AMD_{21} & AMD_{22} \\ AMD_{31} & AMD_{32} \\ AMD_{41} & AMD_{42} \\ AMD_{51} & AMD_{52} \\ AMD_{61} & AMD_{62} \end{bmatrix} = \begin{bmatrix} 0.06 & 0 \\ -0.0417 & -0.0417 \\ 0.333 & 0.167 \\ -0.06 & 0 \\ 0.0417 & 0.0417 \\ 0.0167 & 0.333 \end{bmatrix}$$

Stiffness Method for Frames Analysis

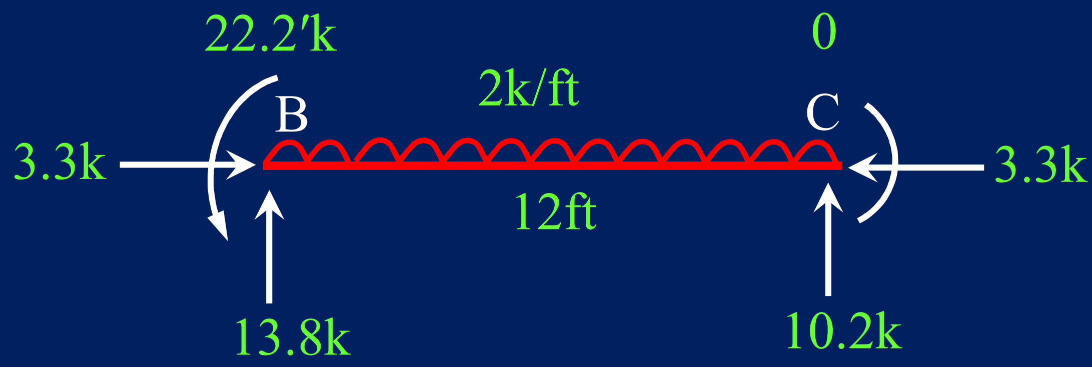
Now member end actions will be computed as given below

$$[AM] = [AML] + [AMD][AD]$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ -24 \\ 0 \\ 12 \\ 24 \end{bmatrix} + \begin{bmatrix} 0.06 & 0 \\ -0.0417 & -0.0417 \\ 0.333 & 0.167 \\ -0.06 & 0 \\ 0.0417 & 0.0417 \\ 0.0167 & 0.333 \end{bmatrix} \begin{bmatrix} 55.40 \\ -99.714 \end{bmatrix} = \begin{bmatrix} 3.3\ \text{k} \\ 13.8\ \text{k} \\ -22.2'\ \text{k} \\ -3.3\ \text{k} \\ 10.2\ \text{k} \\ 0'\ \text{k} \end{bmatrix}$$

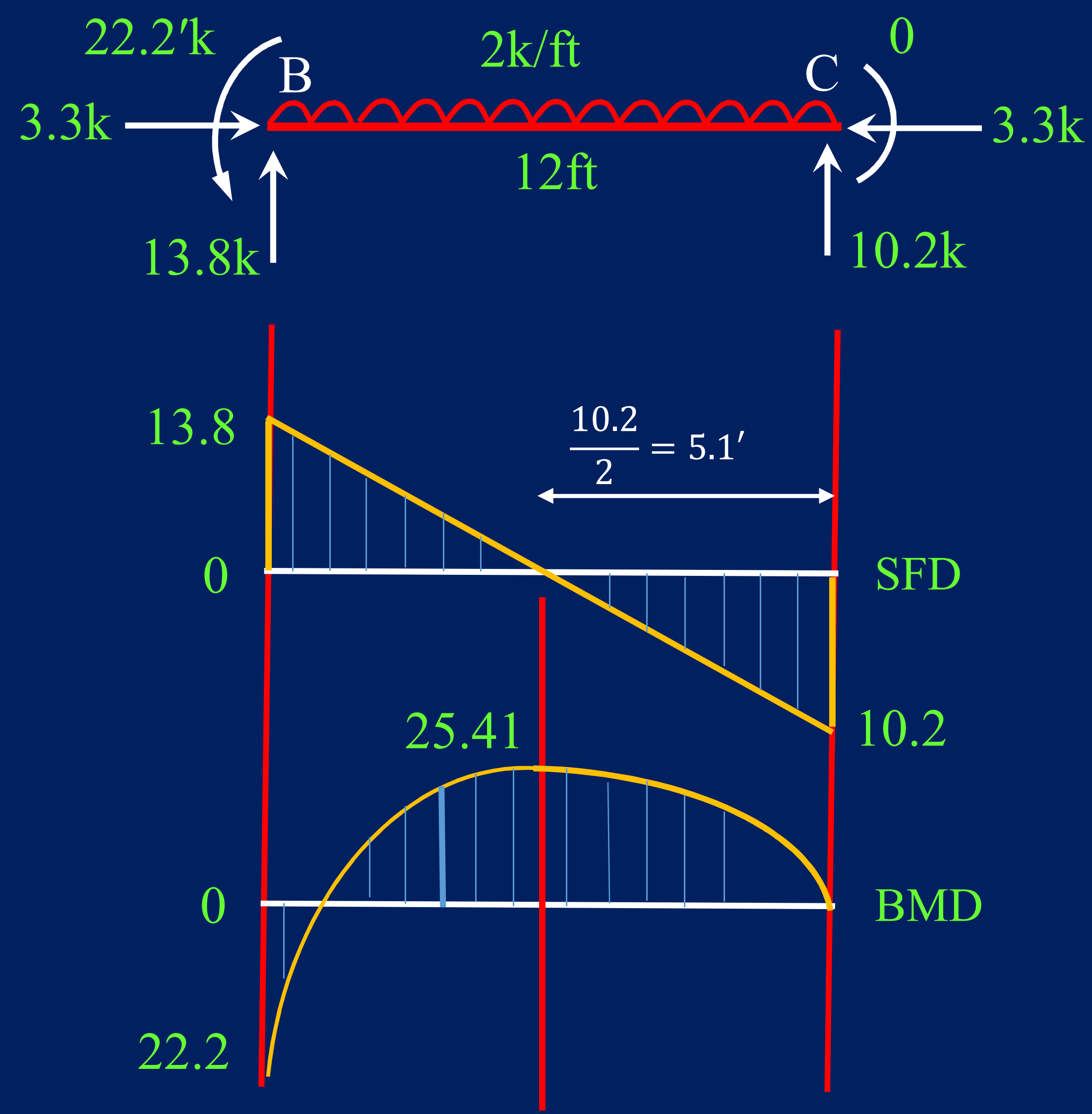
Stiffness Method for Frames Analysis

Final Analyzed Member



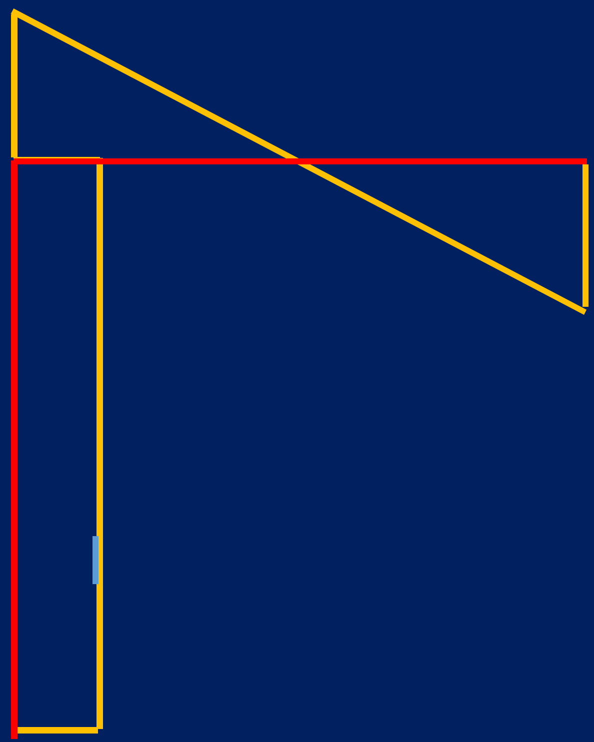
Stiffness Method for Frames Analysis

Now Shear force and bending moment diagrams

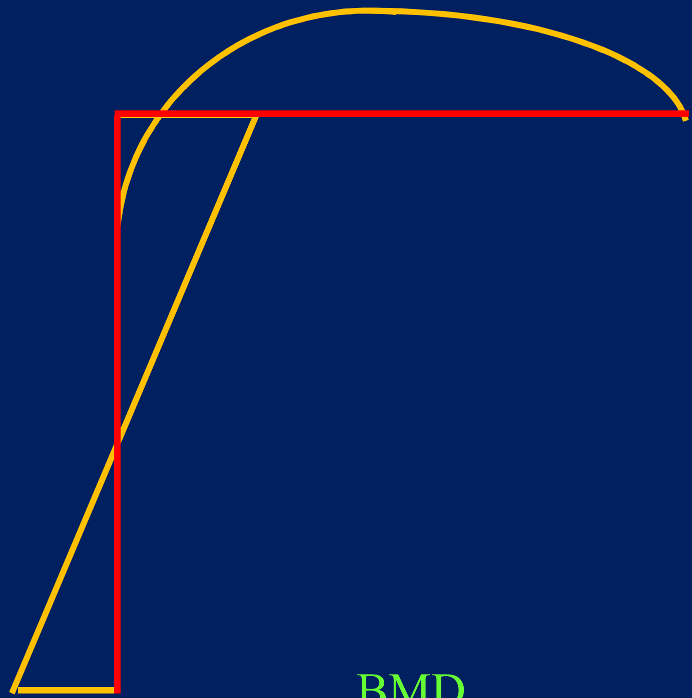


Stiffness Method for Frames Analysis

Combined shear force & bending moment diagrams:



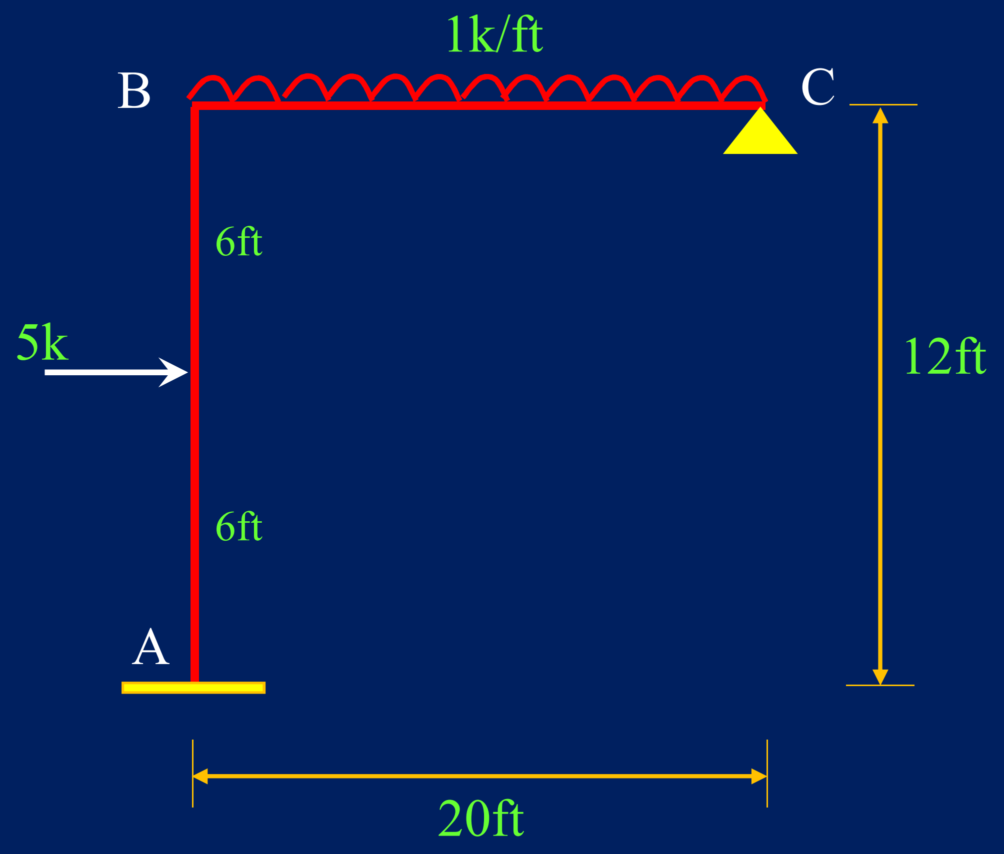
SFD



BMD

Stiffness Method for Frames Analysis

Problem 02: Analyze the given Frame using stiffness method.

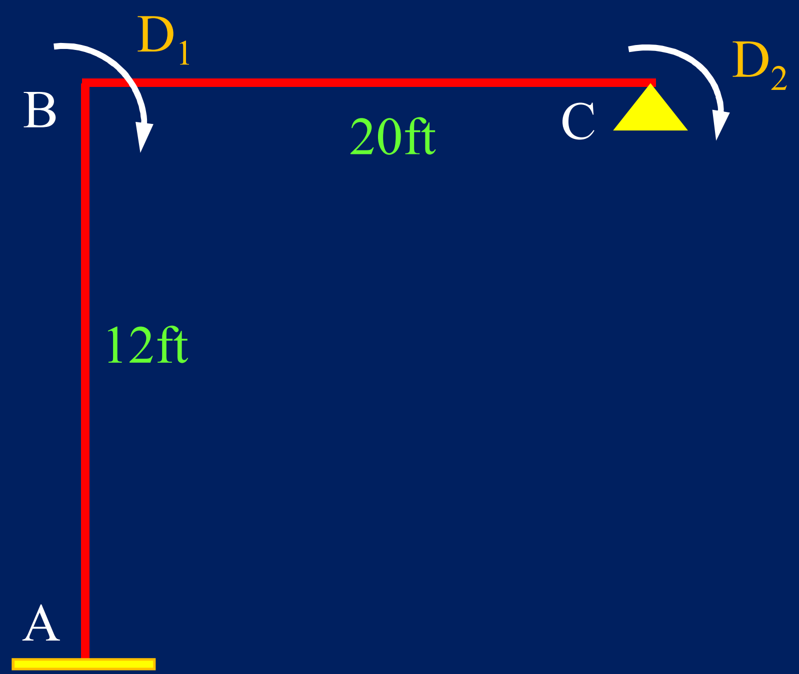


Take EI = constant

K.I = 2 degree (neglecting the axial effects)
So two redundant joint displacements should be chosen.

Stiffness Method for Frames Analysis

- **Step # 01:** Selection of redundant Joint displacements and assign coordinates at those locations. Also compute AD values.

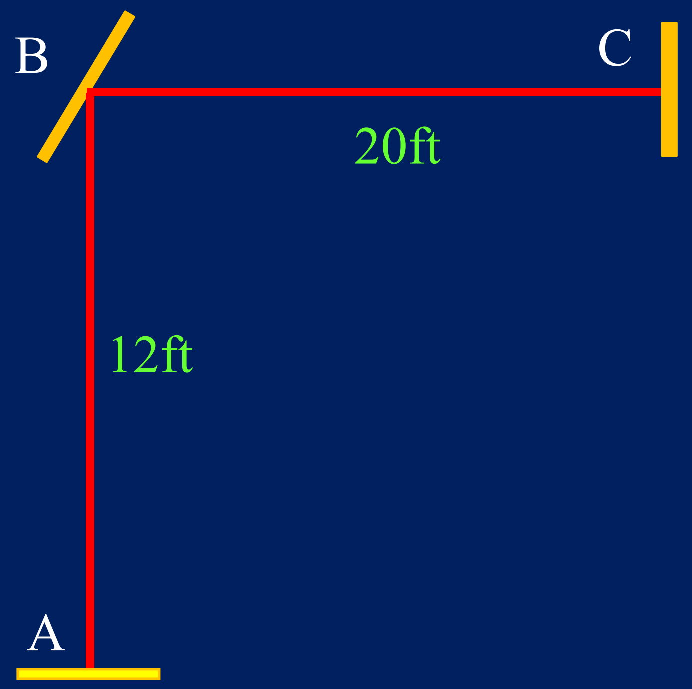


Rotation at B & C is taken as redundant joint displacement.

$$[D]_{2*1} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad [AD]_{2*1} = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Stiffness Method for Frames Analysis

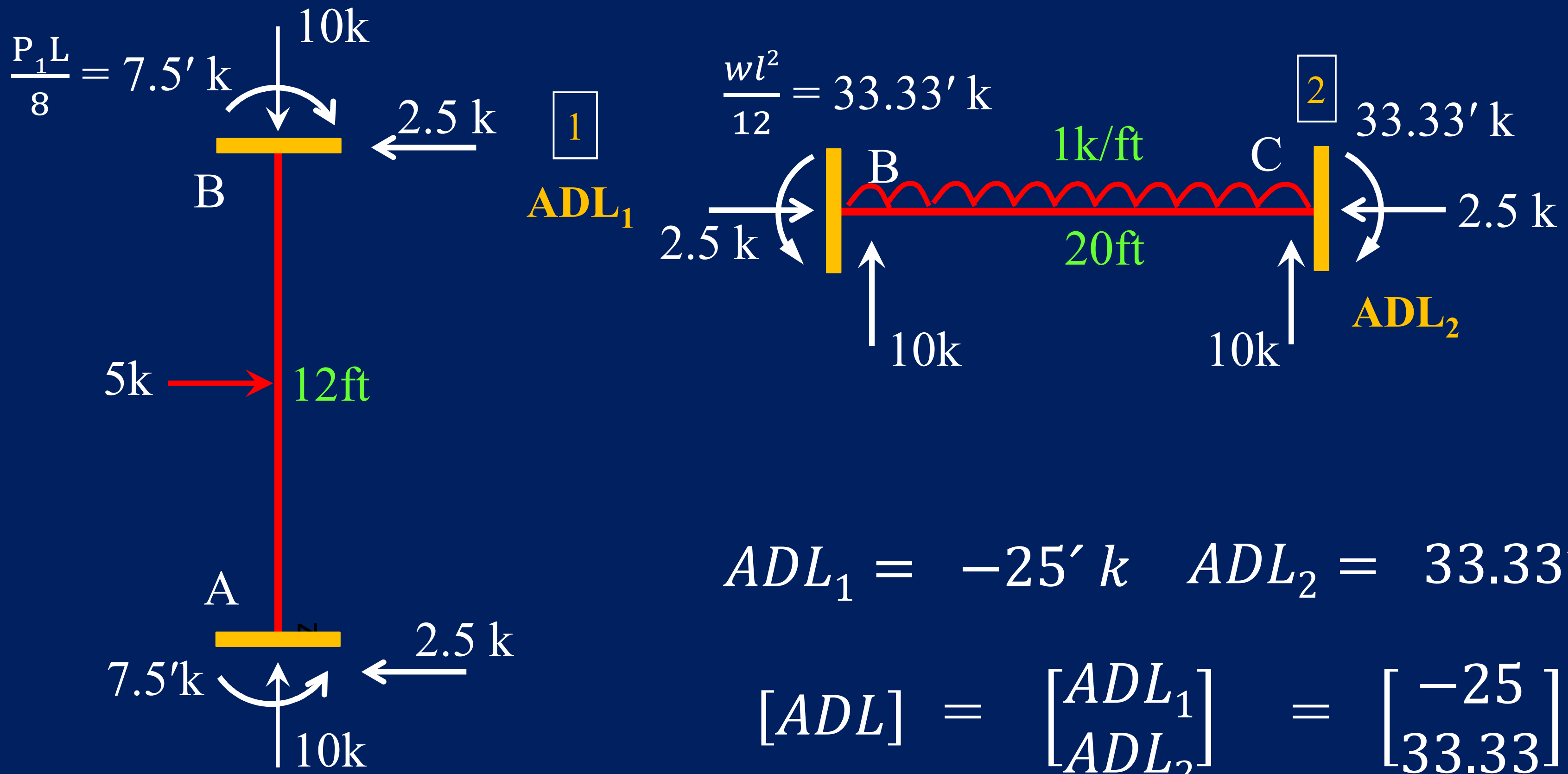
- Restrain all the degrees of freedom to get the restrained structure.



Basic kinematic determinate structure (BKDS) or restrained structure

Stiffness Method for Frames Analysis

- **Step # 02 :** Restrained structure acted upon by the actual loads. Compute ADL matrix.(Fixed end actions)



$$ADL_1 = -25' \, k \quad ADL_2 = 33.33' \, k$$

$$[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} = \begin{bmatrix} -25 \\ 33.33 \end{bmatrix}$$

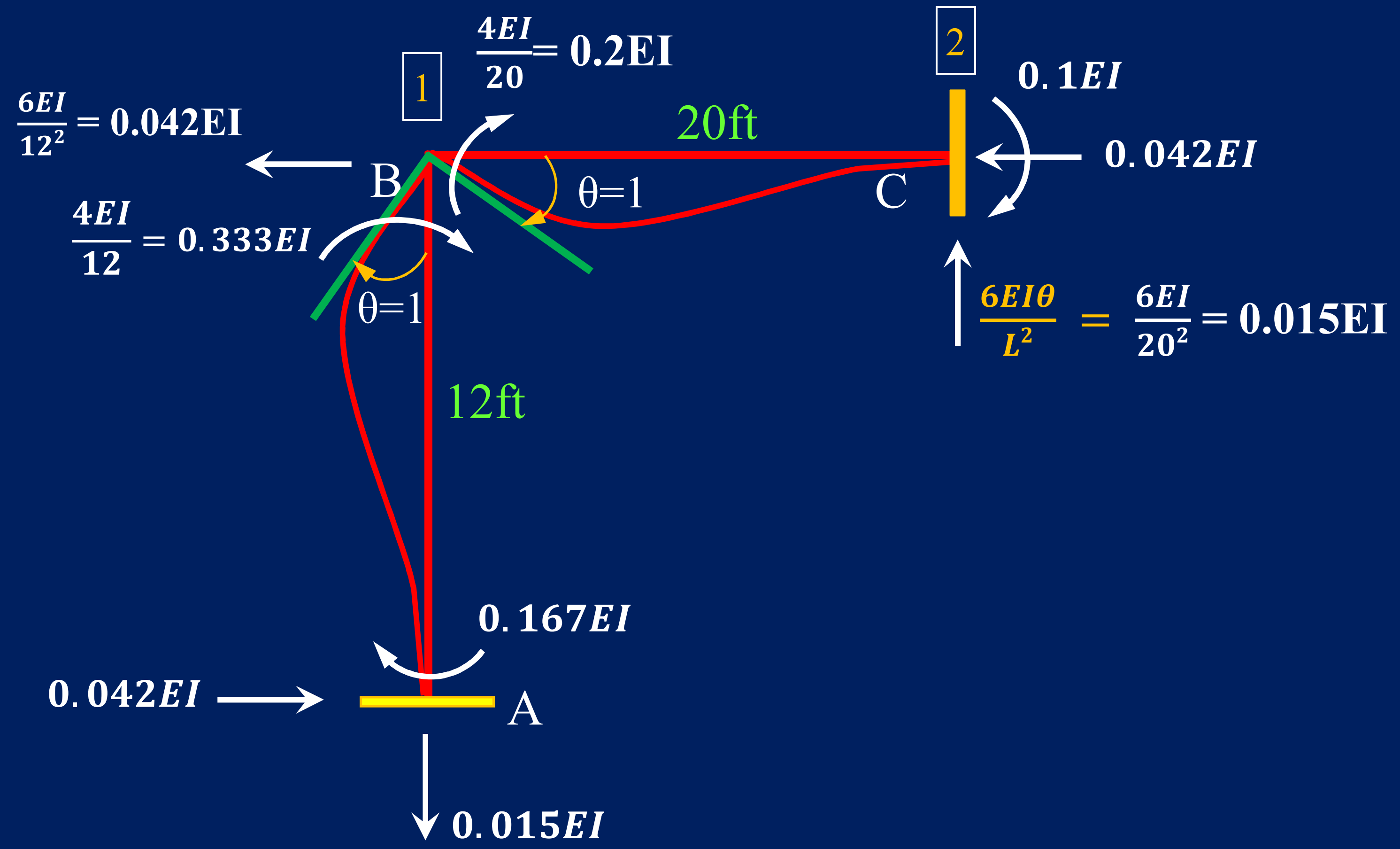
Stiffness Method for Frames Analysis

Step # 03 : Primary structure acted upon by a unit value of D & computation of stiffness coefficients " S" values in the BKDS corresponding to the redundant joint displacement locations.

- 1st a unit rotation is applied at location 1 & prevented at 2 as shown. Compute the values of S_{11} and S_{21} .
- Then apply a unit rotation at the redundant displacement location 2 and prevented at 1 as shown. Compute the values of S_{12} & S_{22} .

Stiffness Method for Frames Analysis

i. 1st a unit rotation is applied at location 1 & prevented at 2 (D_1=1 & D_2=0)as shown. Compute the values of S_{11} and S_{21} .



Stiffness Method for Frames Analysis

Step # 03 (i): Contd…

$$S_{11} = \frac{4EI}{12} + \frac{4EI}{20} \qquad S_{21} = \frac{2EI}{12}$$

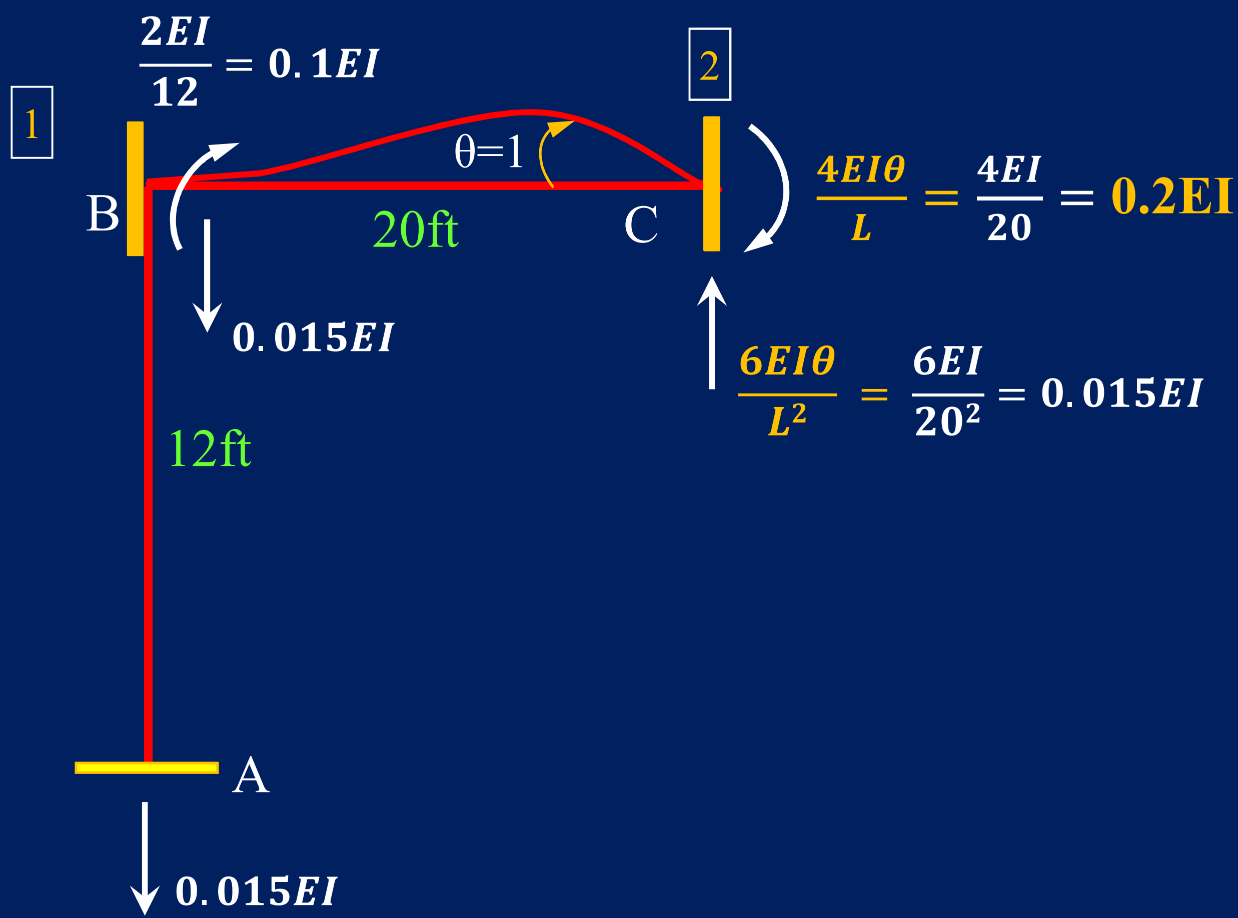
$$S_{11} = 0.533EI \qquad S_{21} = 0.1EI$$

S_{11} = Action(sum of moments in this case) in the BKDS at redundant displacement location 1 due to unit rotation at that location.

S_{21} = Moment in the BDS at redundant displacement location 2 due to a unit rotation applied at location 1

Stiffness Method for Frames Analysis

ii. Now a unit rotation is applied at the redundant displacement location 2 and prevented at 1 (D_2=1 & D_1=0) as shown. Compute the values of S_{12} & S_{22} .



Stiffness Method for Frames Analysis

Step # 03 : Contd…

$$S_{12} = \frac{2EI}{12}$$

$$S_{12} = 0.1EI$$

$$S_{22} = \frac{4EI}{12}$$

$$S_{22} = 0.2EI$$

S_{12} = Moment in the BKDS at redundant displacement location 1 due to unit rotation applied at redundant displacement location 2.

S_{22} = Moment in the BKDS at redundant displacement location 2 due to a unit rotation applied at that location.

Stiffness Method for Frames Analysis

Step # 03 : Contd…

$S_{11} = 0.533EI$ $\quad\quad$ $S_{12} = 0.1EI$

$S_{21} = 0.1EI$ $\quad\quad$ $S_{22} = 0.2EI$

$$[\,S\,] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$$[\,S\,] = EI \begin{bmatrix} 0.533 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$$

Stiffness coefficient matrix

Stiffness Method for Frames Analysis

Step # 04: Apply equilibrium condition at the location of the redundant joint displacement to write equilibrium equations and solve for unknown joint displacement.

$$AD_1 = ADL_1 + S_{11}D_1 + S_{12}D_2$$

$$AD_2 = ADL_2 + S_{21}D_1 + S_{22}D_2$$

$$\begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} + \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$[AD]_{2*1} = [ADL]_{2*1} + [S]_{2*2} \bullet [D]_{2*1}$$

$$[\boldsymbol{D}] = [\boldsymbol{S}]^{-1} \bullet [\boldsymbol{AD} - \boldsymbol{ADL}]$$

Stiffness Method for Frames Analysis

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.533 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}^{-1} \begin{bmatrix} 0 - (-25.83) \\ 0 - 33.33 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 87.93 \\ -210.61 \end{bmatrix} \frac{1}{EI}$$

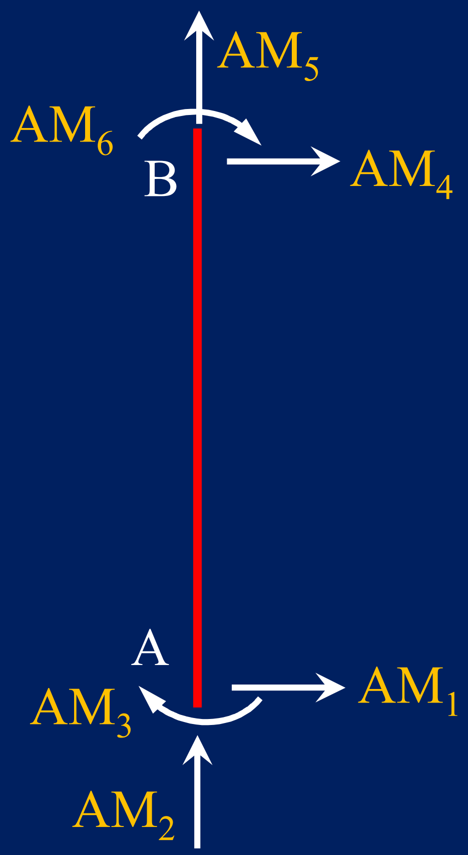
-ive sign shows that our assumed redundant joint displacement direction is wrong

Stiffness Method for Frames Analysis

Step # 05: Compute the member end actions. As we know that

$$[AM] = [AML] + [AMD][D]$$

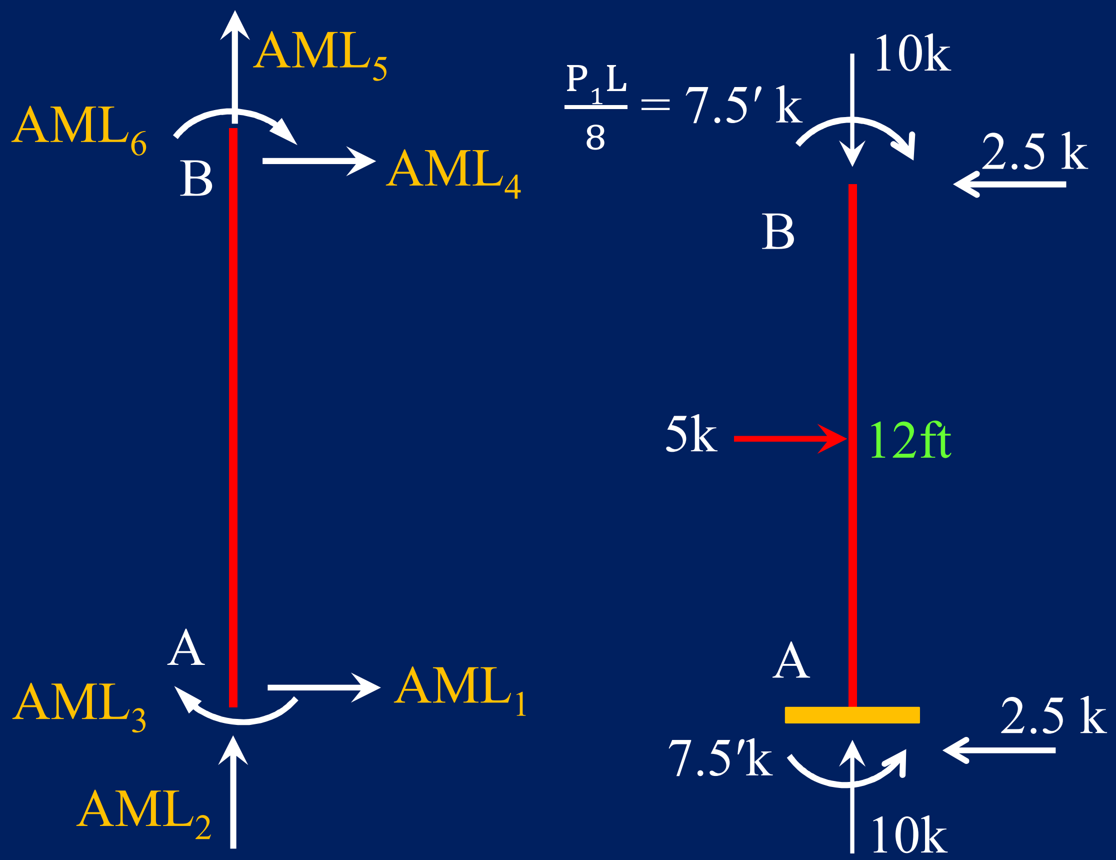
Member AB:



AM_n = is the member end action in the indeterminate structure at different location specified with a number "n". n shows the number of member end actions.

Stiffness Method for Frames Analysis

i. Compute AML values.

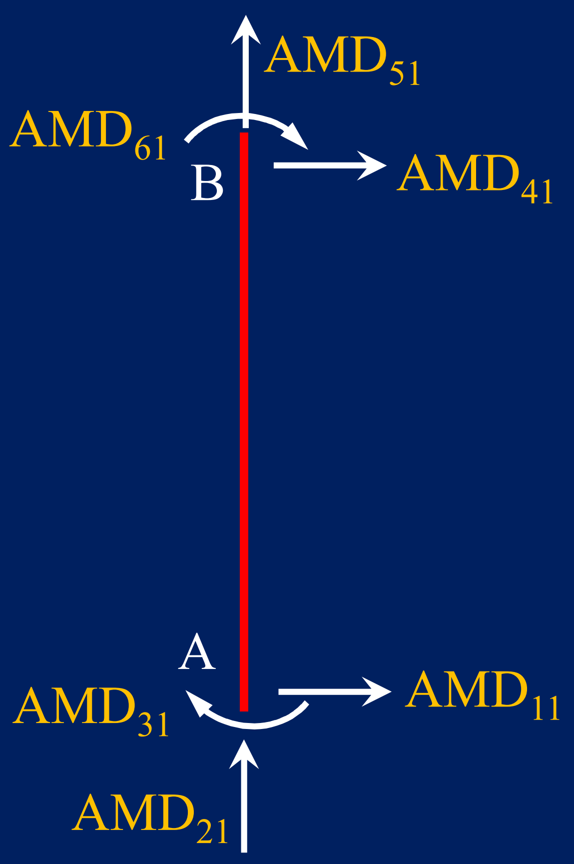


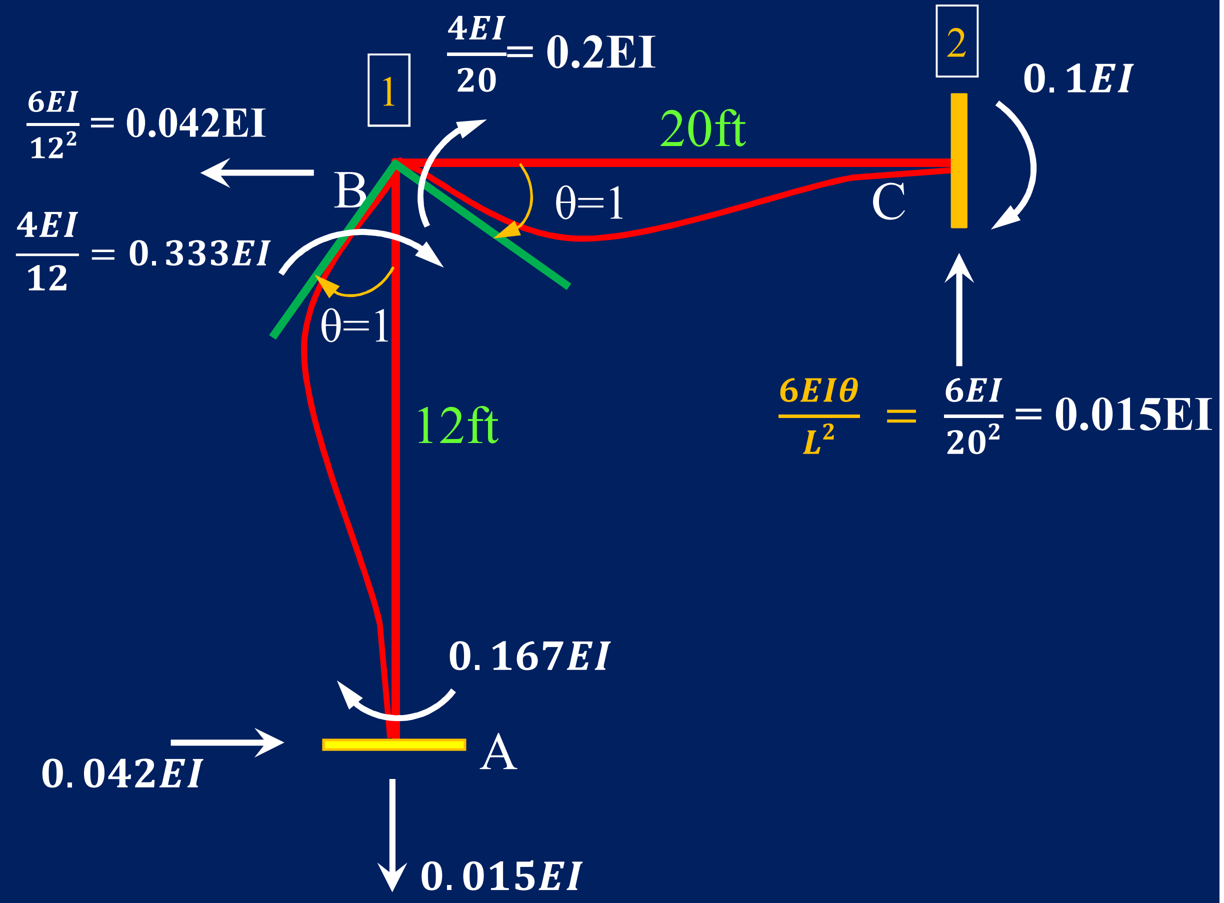
$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} = \begin{bmatrix} -2.5 \\ 10 \\ -7.5 \\ -2.5 \\ -10 \\ 7.5 \end{bmatrix}$$

Stiffness Method for Frames Analysis

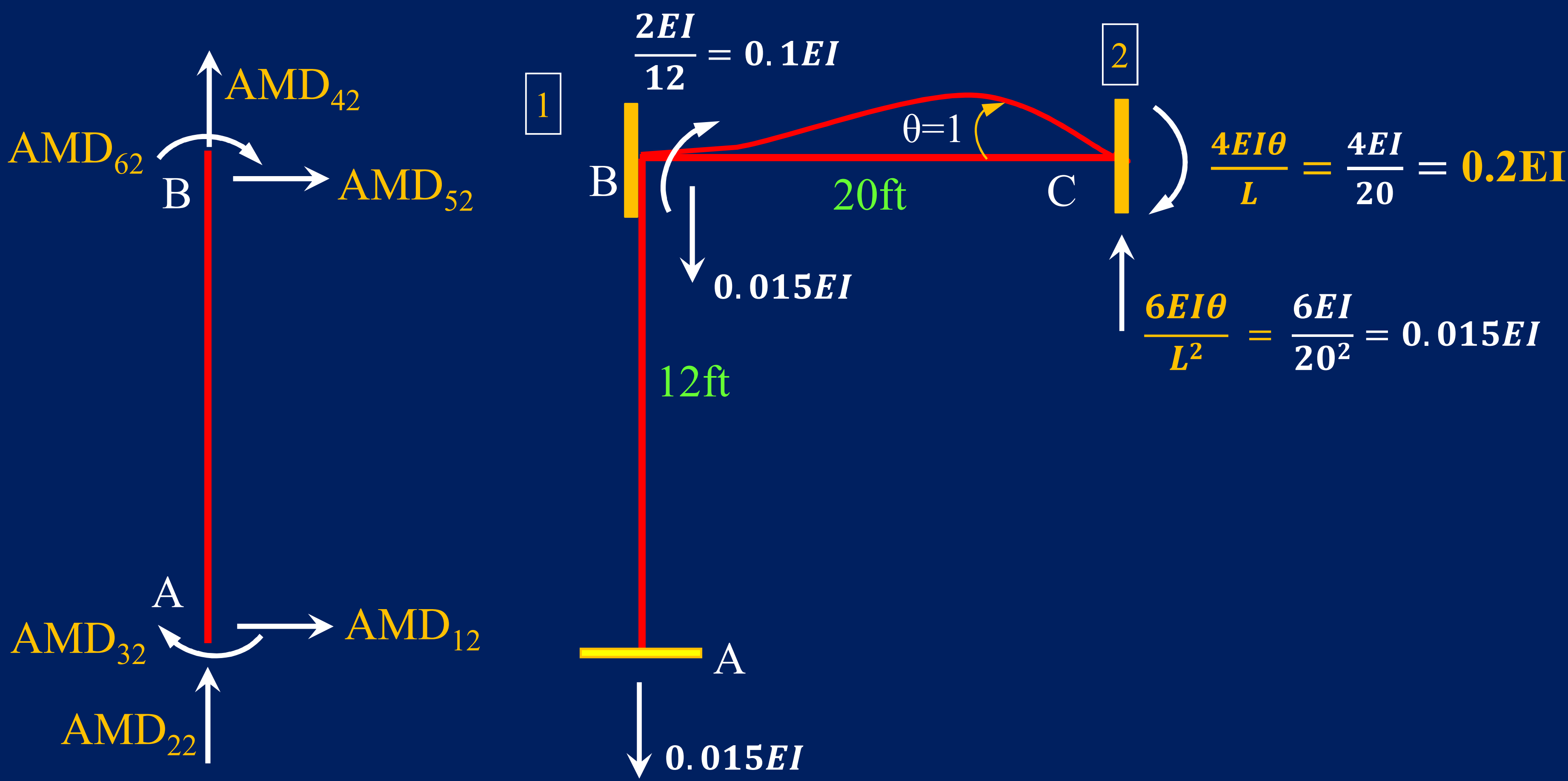
b) Compute the AMD values.

- 1^{st} apply a unit rotation at redundant location 1 and then at 2 as shown below.





Stiffness Method for Frames Analysis



Stiffness Method for Frames Analysis

So the AMD values are

$$[AMD]_{6\,*\,2} = \begin{bmatrix} AMD_{11} & AMD_{12} \\ AMD_{21} & AMD_{22} \\ AMD_{31} & AMD_{32} \\ AMD_{41} & AMD_{42} \\ AMD_{51} & AMD_{52} \\ AMD_{61} & AMD_{62} \end{bmatrix} = \begin{bmatrix} 0.42 & 0 \\ -0.015 & -0.015 \\ 0.167 & 0 \\ -0.042 & 0 \\ 0.015 & 0.015 \\ 0.333 & 0 \end{bmatrix}$$

Stiffness Method for Frames Analysis

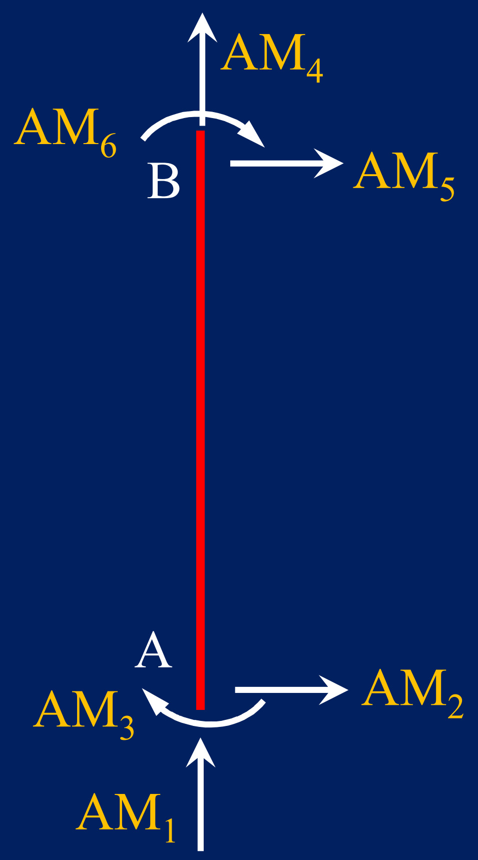
Now member end actions will be computed as given below

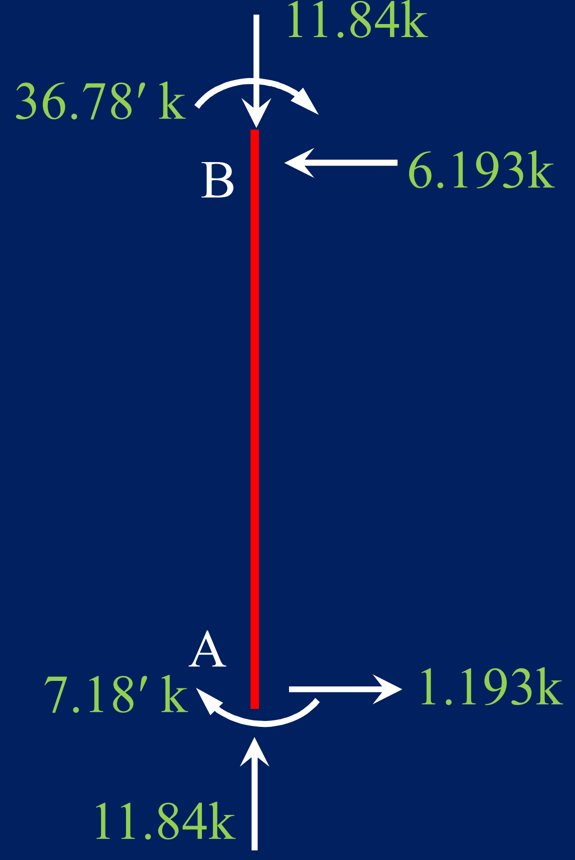
$$[AM] = [AML] + [AMD][AD]$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} -2.5 \\ 10 \\ -7.5 \\ -2.5 \\ -10 \\ 7.5 \end{bmatrix} + \begin{bmatrix} 0.42 & 0 \\ -0.015 & -0.015 \\ 0.167 & 0 \\ -0.042 & 0 \\ 0.015 & 0.015 \\ 0.333 & 0 \end{bmatrix} \begin{bmatrix} 87.93 \\ -210.61 \end{bmatrix} = \begin{bmatrix} 1.193 \text{ k} \\ 11.84 \text{ k} \\ 7.18' \text{ k} \\ -6.193 \text{ k} \\ -11.84 \text{ k} \\ 36.78' \text{ k} \end{bmatrix}$$

Stiffness Method for Frames Analysis

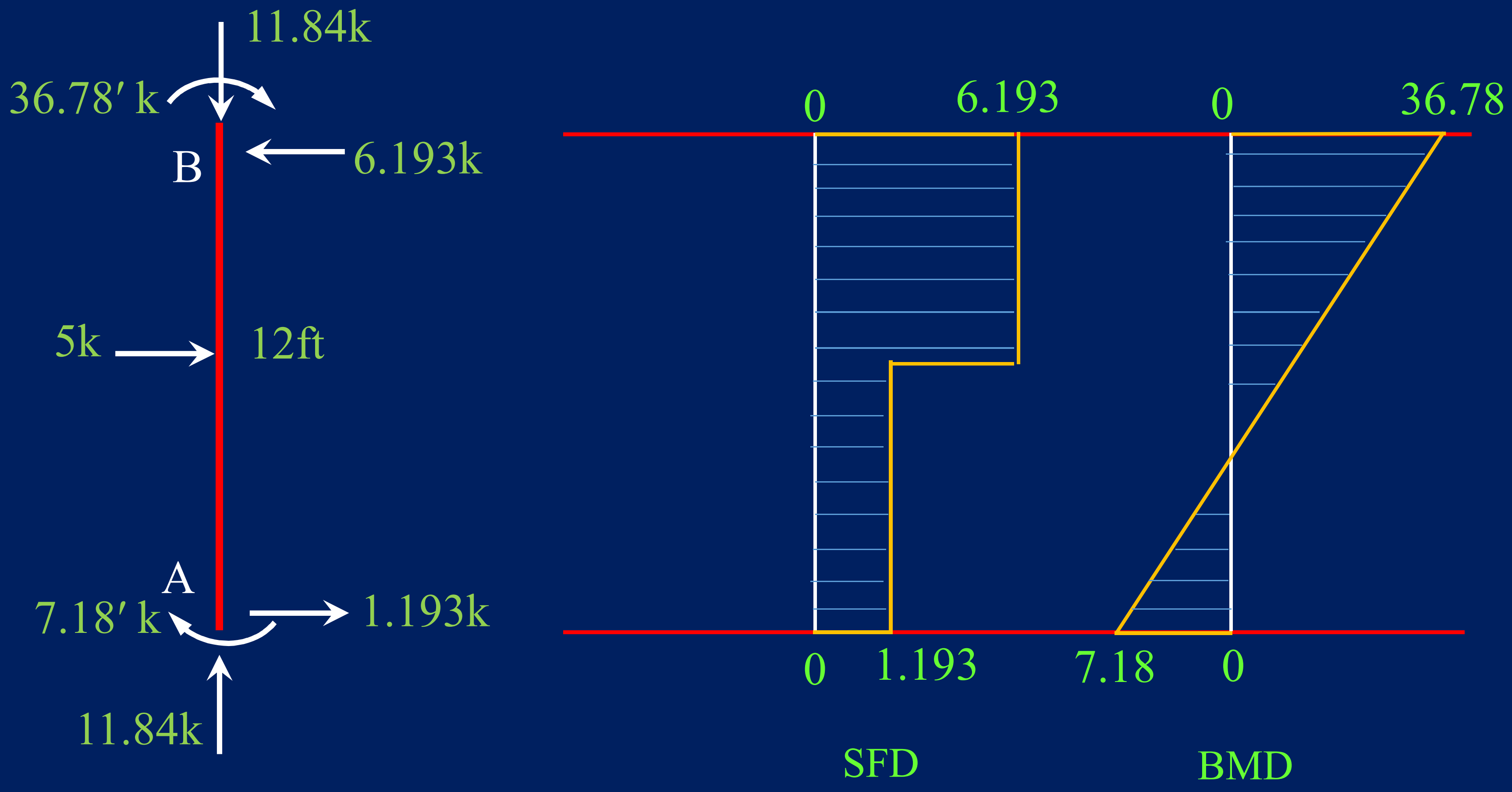
Final Analyzed Member





Stiffness Method for Frames Analysis

Now Shear force and bending moment diagrams



Stiffness Method for Frames Analysis

Step # 05: Compute the member end actions. As we know that

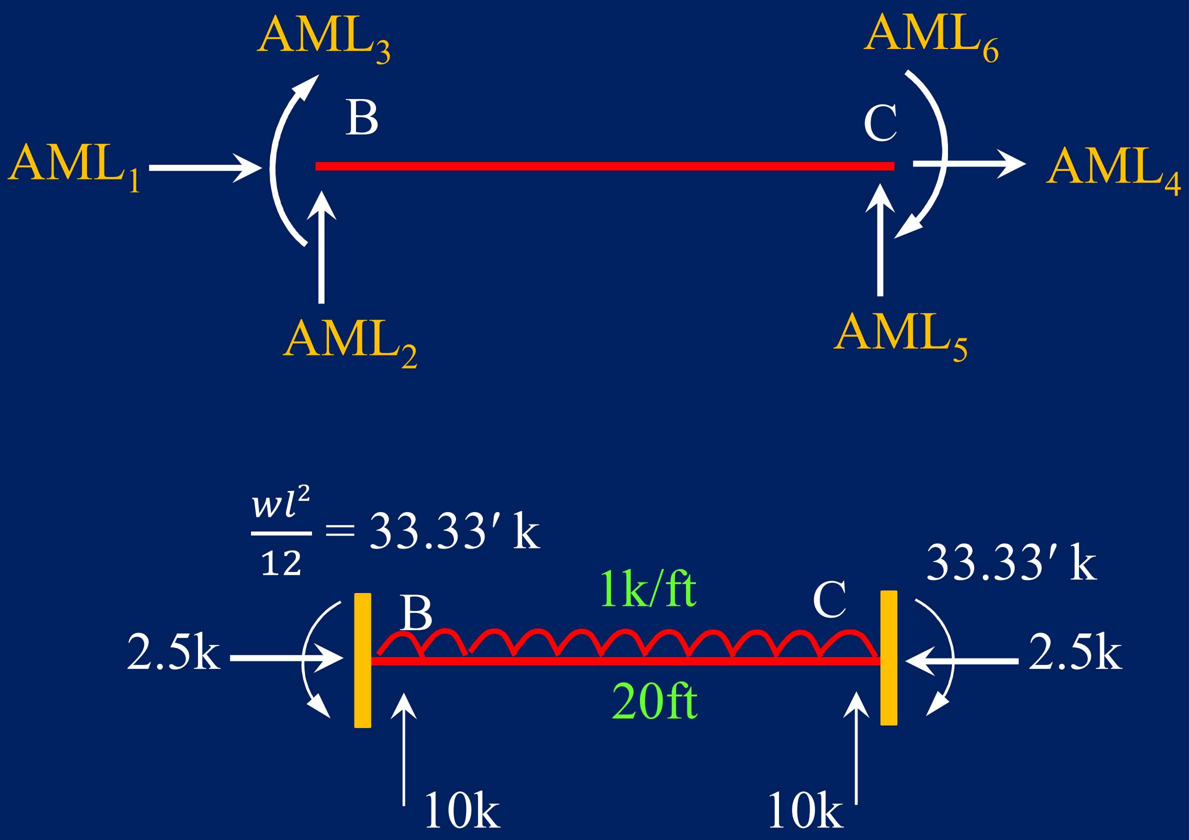
$$[AM] = [AML] + [AMD][D]$$

Member BC:

AM_n = is the member end action in the indeterminate structure at different location specified with a number "n". n shows the number of member end actions.

Stiffness Method for Frames Analysis

i. Compute AML values.

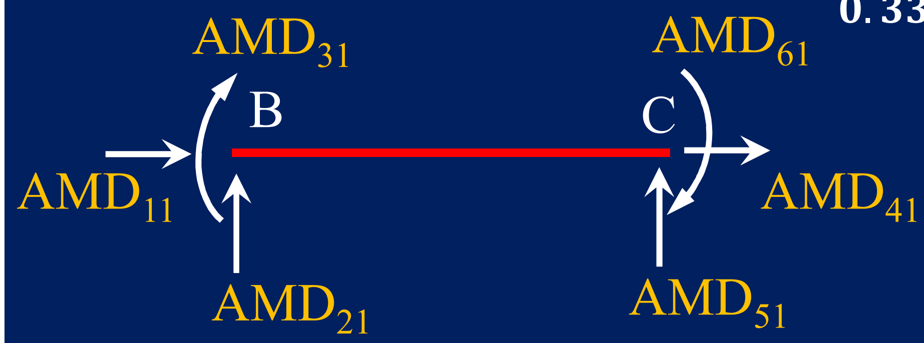


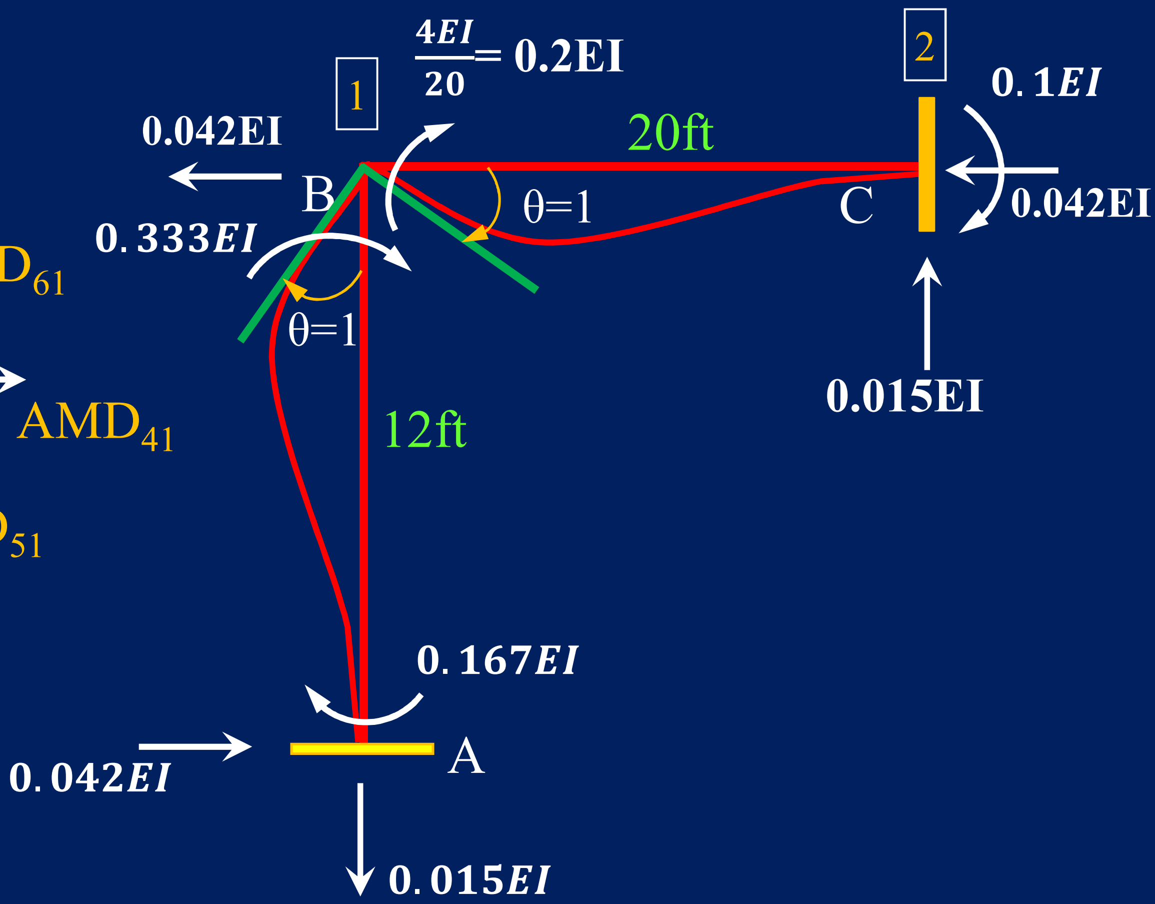
$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 10 \\ -33.33 \\ -2.5 \\ 10 \\ 33.33 \end{bmatrix}$$

Stiffness Method for Frames Analysis

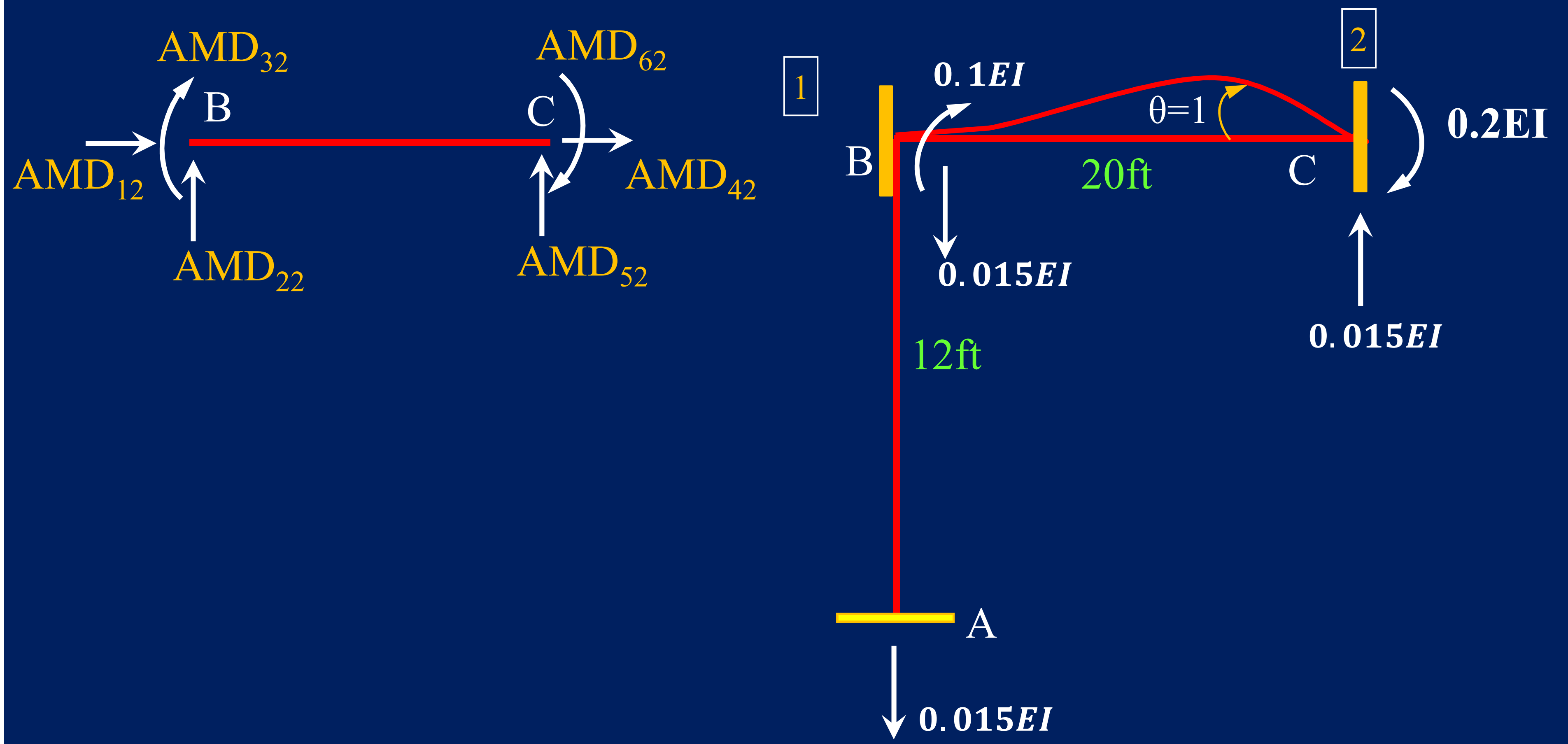
b) Compute the AMD values.

- 1^{st} apply a unit rotation at redundant location 1 and then at 2 as shown below.





Stiffness Method for Frames Analysis



Stiffness Method for Frames Analysis

So the AMD values are

$$[AMD]_{6\,*\,2} = \begin{bmatrix} AMD_{11} & AMD_{12} \\ AMD_{21} & AMD_{22} \\ AMD_{31} & AMD_{32} \\ AMD_{41} & AMD_{42} \\ AMD_{51} & AMD_{52} \\ AMD_{61} & AMD_{62} \end{bmatrix} = \begin{bmatrix} 0.042 & 0 \\ -0.015 & -0.015 \\ 0.2 & 0.1 \\ -0.042 & 0 \\ 0.015 & 0.015 \\ 0.1 & 0.2 \end{bmatrix}$$

Stiffness Method for Frames Analysis

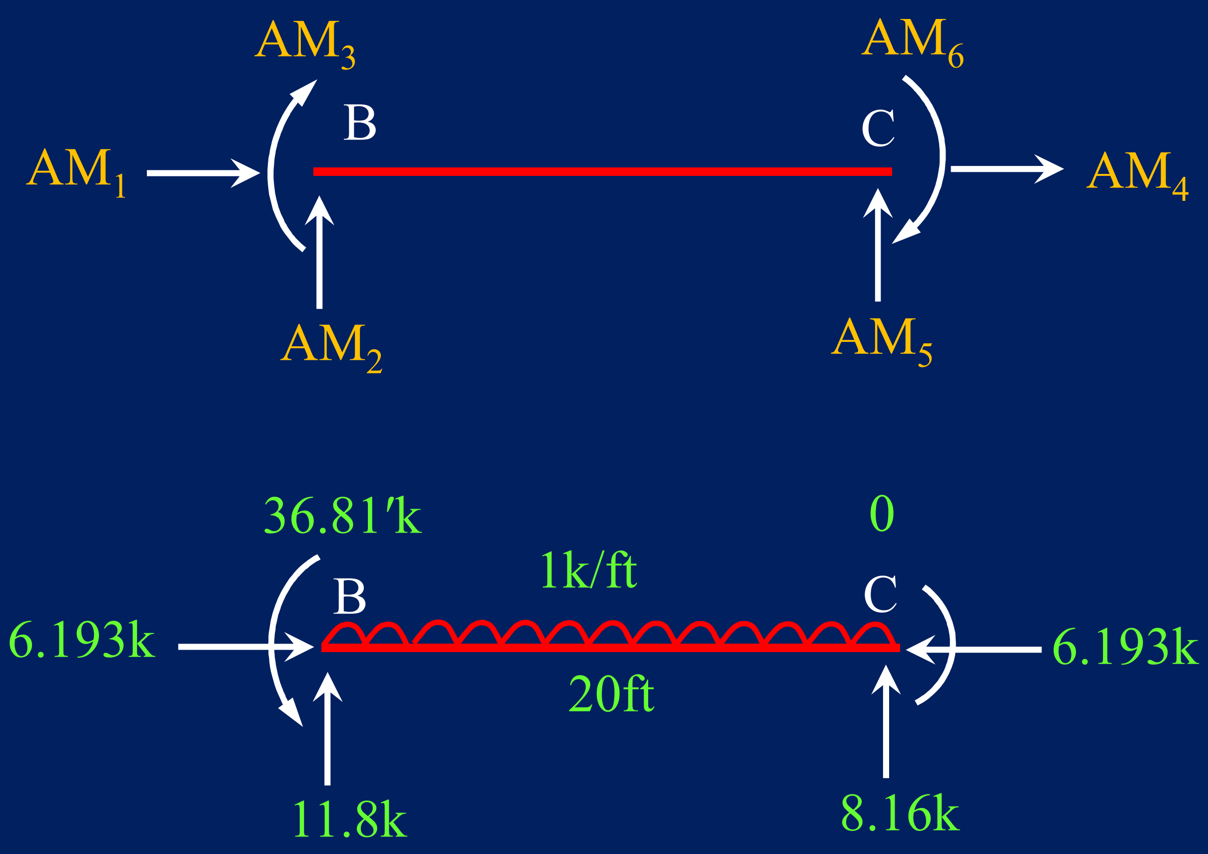
Now member end actions will be computed as given below

$$[AM] = [AML] + [AMD][D]$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 10 \\ -33.33 \\ -2.5 \\ 10 \\ 33.33 \end{bmatrix} + \begin{bmatrix} 0.042 & 0 \\ -0.015 & -0.015 \\ 0.2 & 0.1 \\ -0.042 & 0 \\ 0.015 & 0.015 \\ 0.1 & 0.2 \end{bmatrix} \begin{bmatrix} 87.93 \\ -210.61 \end{bmatrix} = \begin{bmatrix} 6.193\ \text{k} \\ 11.8\ \text{k} \\ -36.8'\ \text{k} \\ -6.193\ \text{k} \\ 8.16\ \text{k} \\ 0'\ \text{k} \end{bmatrix}$$

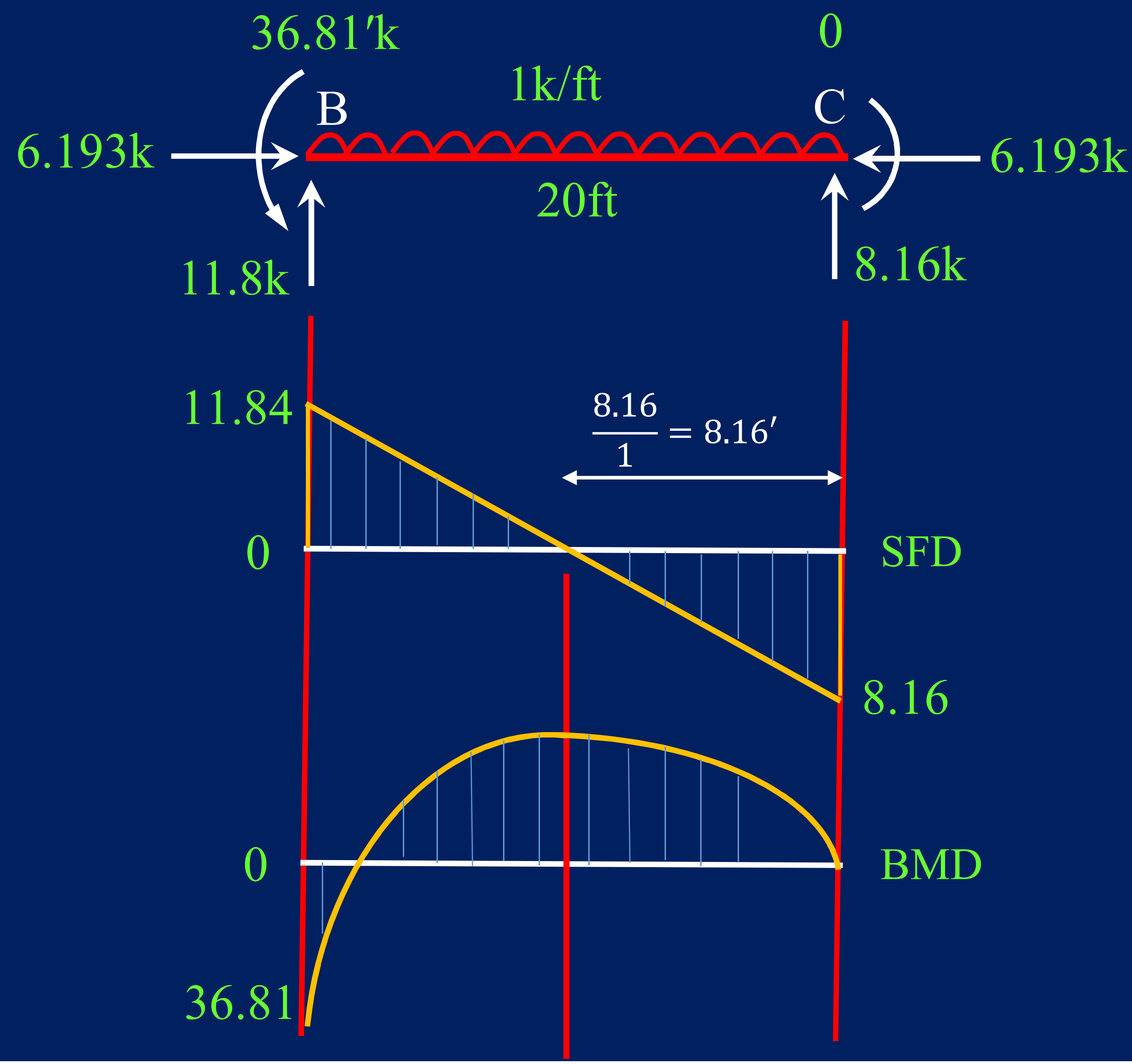
Stiffness Method for Frames Analysis

Final Analyzed Member



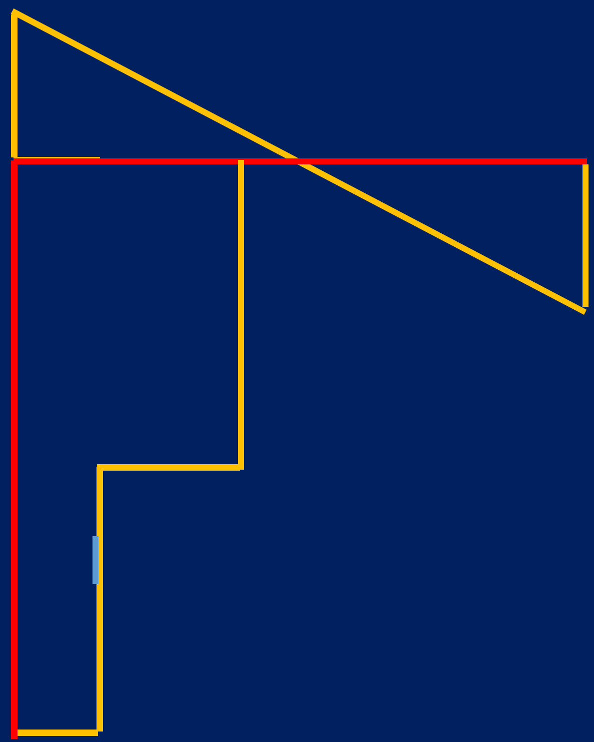
Stiffness Method for Frames Analysis

Now Shear force and bending moment diagrams

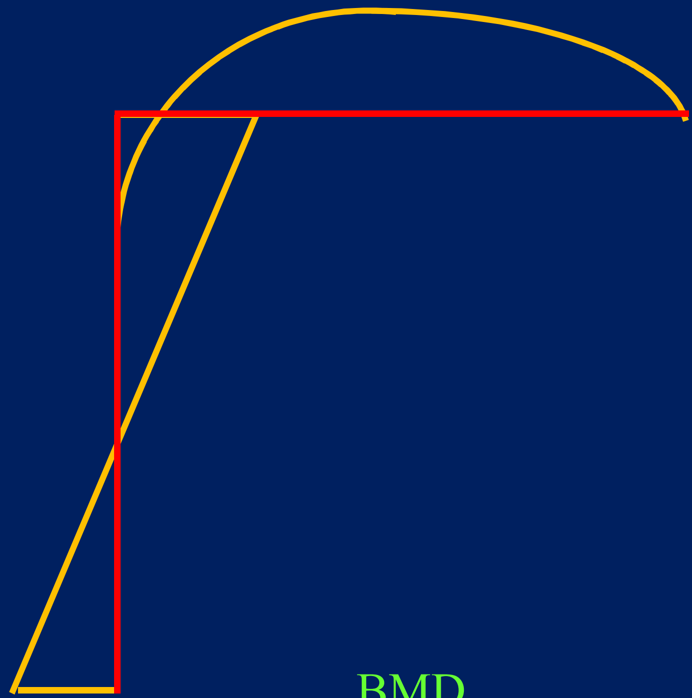


Stiffness Method for Frames Analysis

Combined shear force & bending moment diagrams:



SFD

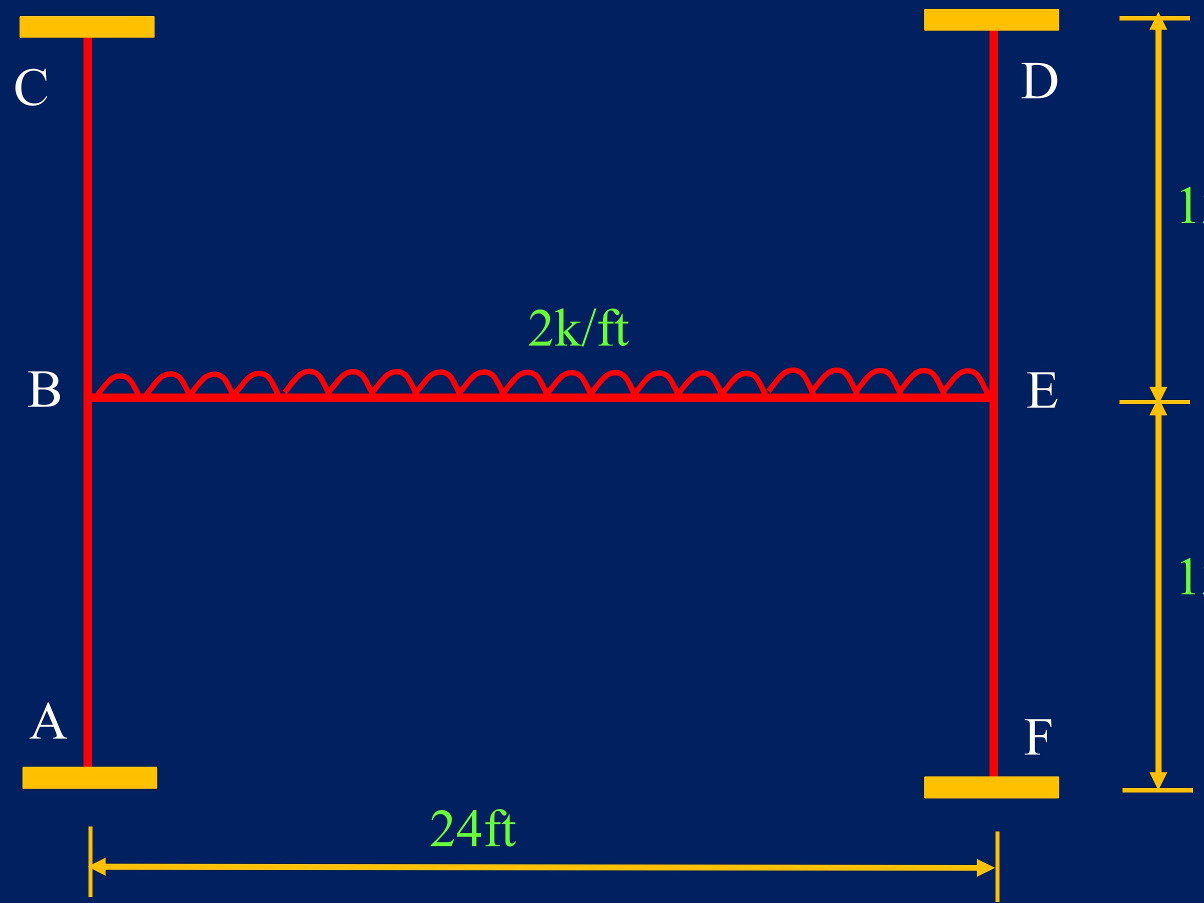


BMD

Stiffness Method for Frames Analysis

Problem 03: Analyze the given Frame using stiffness method.

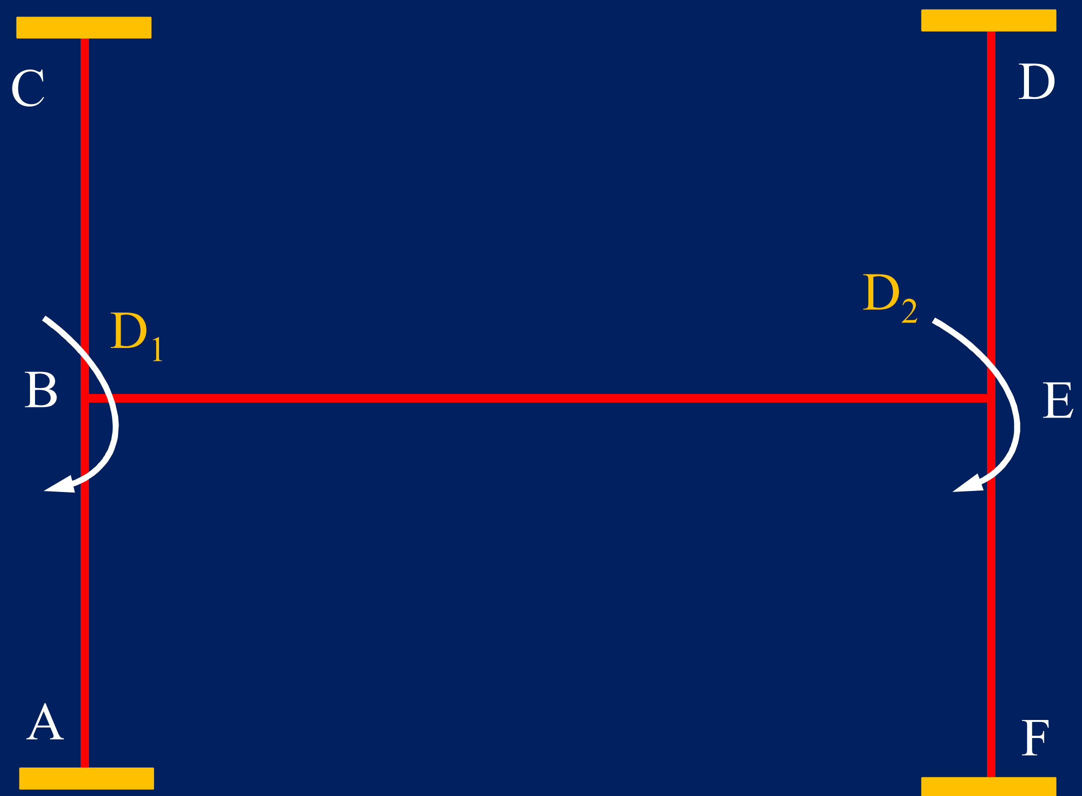
Take
EI = constant



K.I = 2 degree (neglecting the axial effects)

Stiffness Method for Frames Analysis

- **Step # 01:** Selection of redundant Joint displacements and assign coordinates at those locations. Also compute AD values.

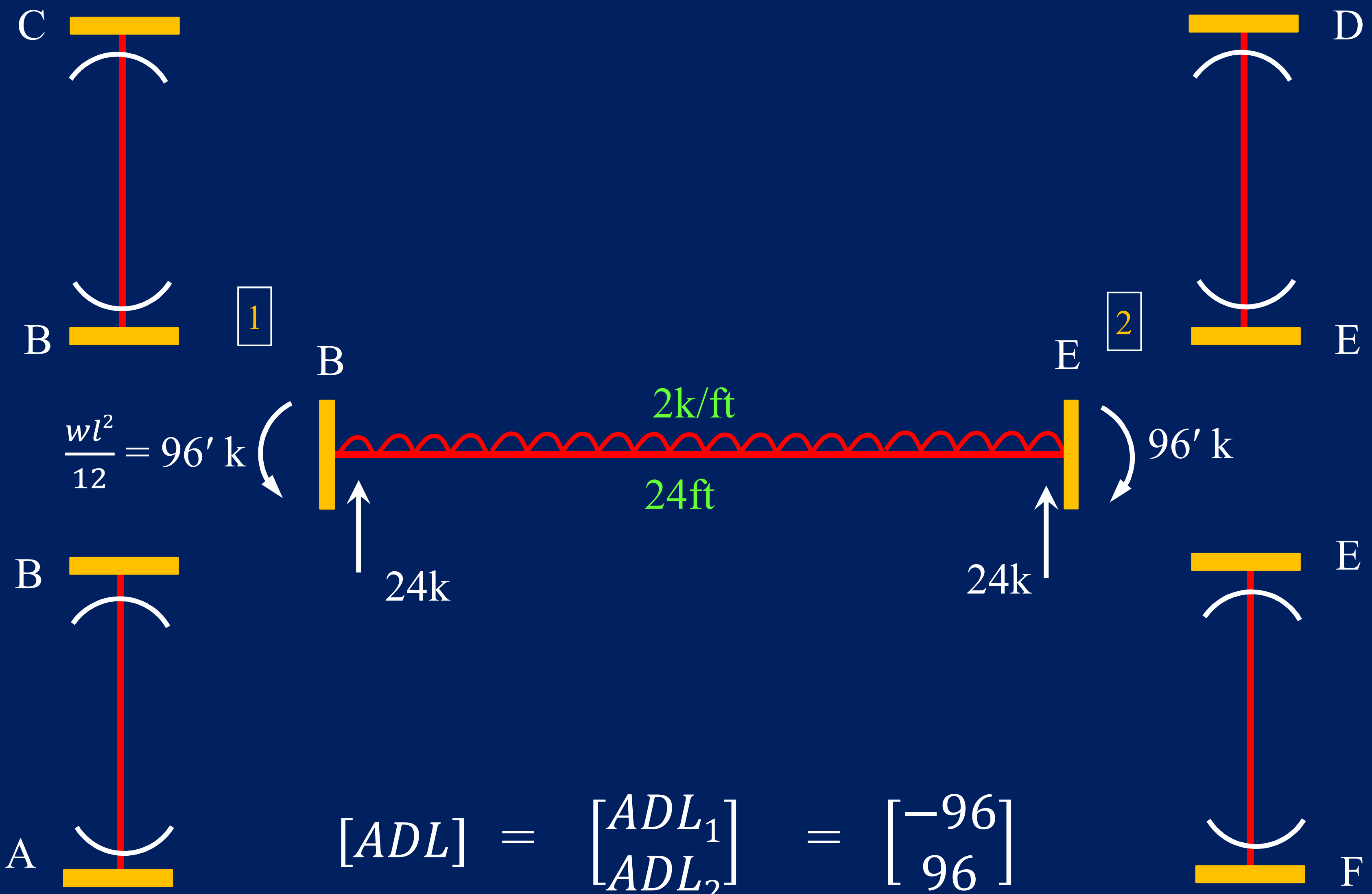


Rotation at B & C is taken as redundant joint displacement.

$$[D]_{2*1} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad [AD]_{2*1} = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Stiffness Method for Frames Analysis

- **Step # 02 :** Compute ADL matrix.(Fixed end actions)



$$[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} = \begin{bmatrix} -96 \\ 96 \end{bmatrix}$$

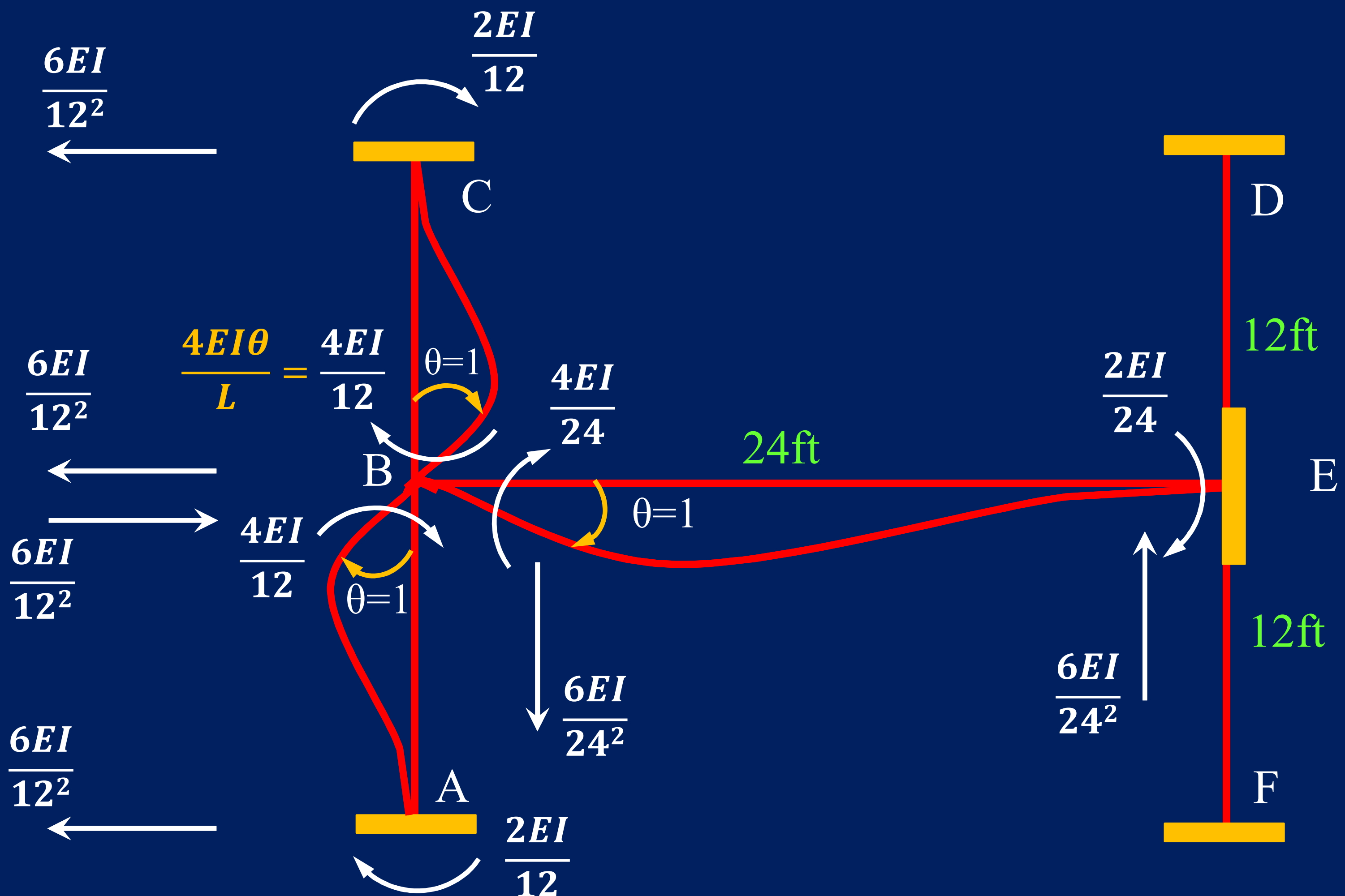
Stiffness Method for Frames Analysis

Step # 03 : Primary structure acted upon by a unit value of D & computation of stiffness coefficients " S" values in the BKDS corresponding to the redundant joint displacement locations.

- 1st a unit rotation is applied at location 1 & prevented at 2 as shown. Compute the values of S_{11} and S_{21} .
- Then apply a unit rotation at the redundant displacement location 2 and prevented at 1 as shown. Compute the values of S_{12} & S_{22} .

Stiffness Method for Frames Analysis

i. 1^{st} $D_1 = 1$ & $D_2 = 0$ & Compute the values of S_{11} and S_{21}.



Stiffness Method for Frames Analysis

Step # 03 (i): Contd…

$$S_{11} = \frac{4EI}{12} + \frac{4EI}{12} + \frac{4EI}{24} \qquad S_{21} = \frac{2EI}{24}$$

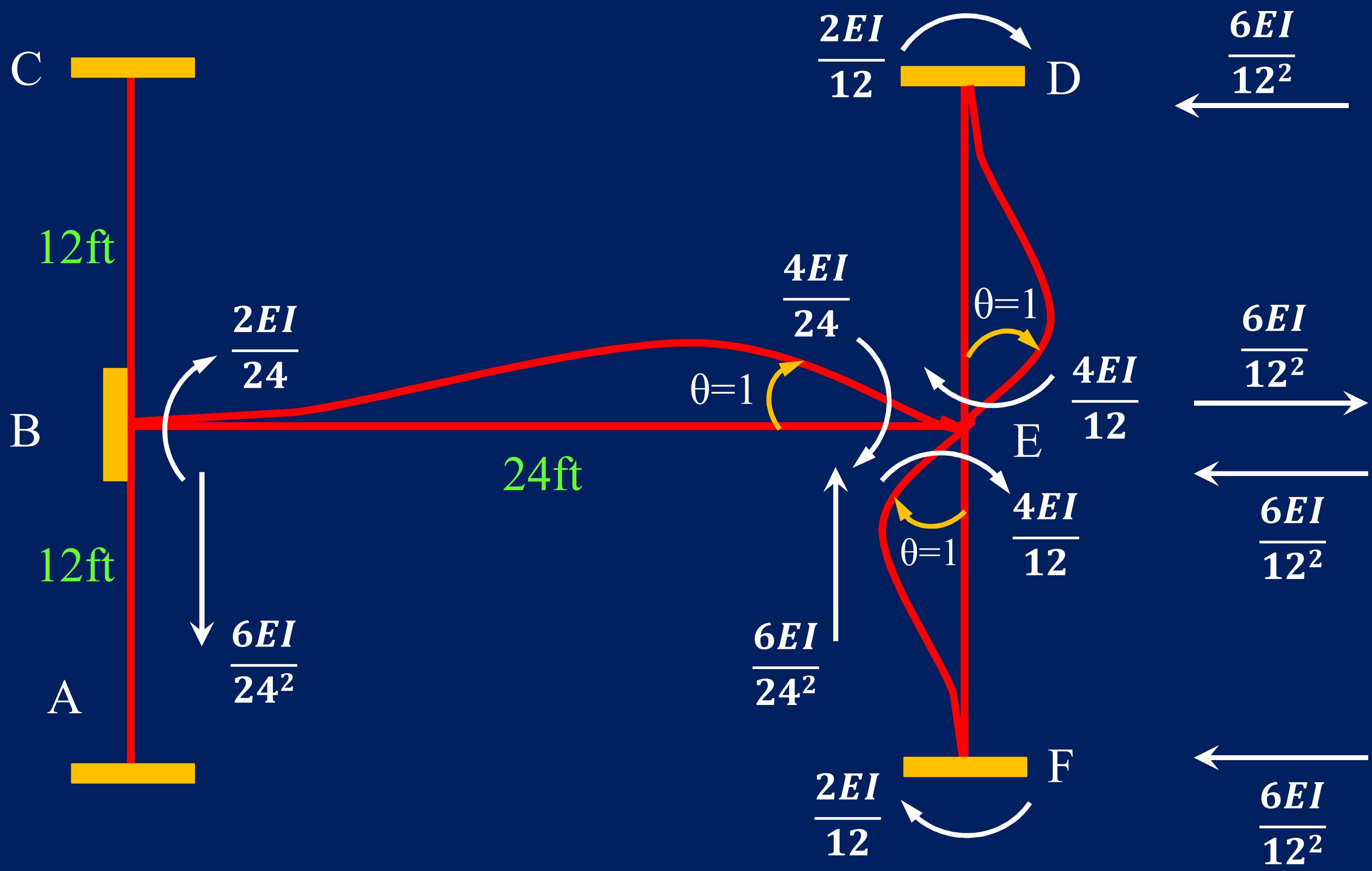
$$S_{11} = 0.833EI \qquad S_{21} = 0.0833EI$$

S_{11} = Action(sum of moments in this case) in the BKDS at redundant displacement location 1 due to unit rotation at that location.

S_{21} = Moment in the BDS at redundant displacement location 2 due to a unit rotation applied at location 1

Stiffness Method for Frames Analysis

ii. Now $D_2 = 1$ & $D_1 = 0$ as shown. Compute the values of S_{12} & S_{22}.



Stiffness Method for Frames Analysis

Step # 03 : Contd…

$$S_{12} = \frac{2EI}{24}$$

$$S_{12} = 0.083EI$$

$$S_{22} = \frac{4EI}{24} + \frac{4EI}{12} + \frac{4EI}{12}$$

$$S_{22} = 0.833EI$$

S_{12} = Moment in the BKDS at redundant displacement location 1 due to unit rotation applied at redundant displacement location 2.

S_{22} = Moment in the BKDS at redundant displacement location 2 due to a unit rotation applied at that location.

Stiffness Method for Frames Analysis

Step # 03 : Contd…

$$S_{11} = 0.833EI \qquad S_{12} = 0.0833EI$$

$$S_{21} = 0.0833EI \qquad S_{22} = 0.833EI$$

$$[\,S\,] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$$[\,S\,] = EI \begin{bmatrix} 0.833 & 0.0833 \\ 0.0833 & 0.833 \end{bmatrix}$$

Stiffness coefficient matrix

Stiffness Method for Frames Analysis

Step # 04: Apply equilibrium condition at the location of the redundant joint displacement to write equilibrium equations and solve for unknown joint displacement.

$$AD_1 = ADL_1 + S_{11}D_1 + S_{12}D_2$$

$$AD_2 = ADL_2 + S_{21}D_1 + S_{22}D_2$$

$$\begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} + \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$[AD]_{2\,*\,1} = [ADL]_{2\,*\,1} + [S]_{2\,*\,2} \bullet [D]_{2\,*\,1}$$

$$[\boldsymbol{D}] = [\boldsymbol{S}]^{-1} \bullet [\boldsymbol{AD} - \boldsymbol{ADL}]$$

Stiffness Method for Frames Analysis

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix}$$

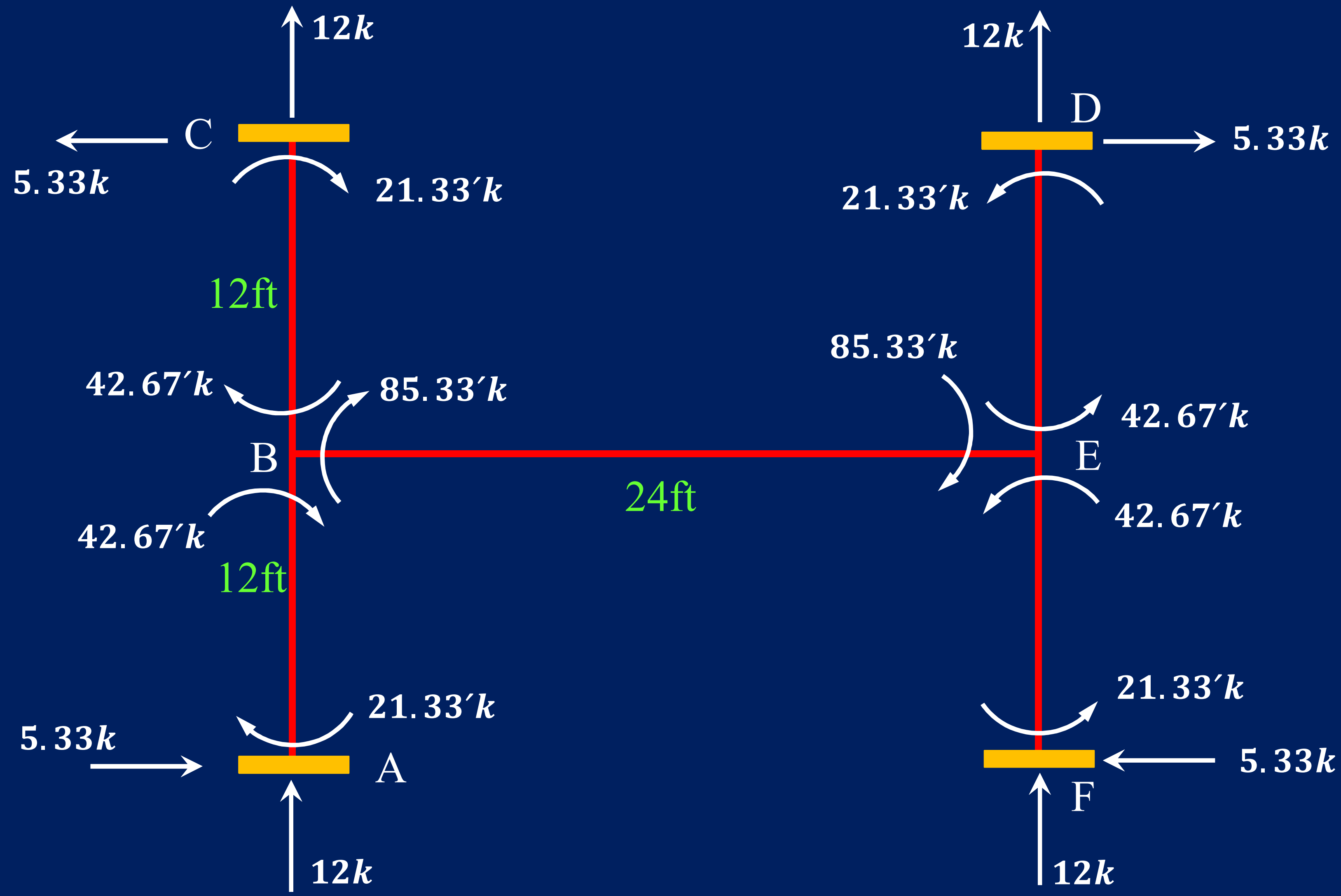
$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.833 & 0.0833 \\ 0.0833 & 0.833 \end{bmatrix}^{-1} \begin{bmatrix} 0 - (-96) \\ 0 - 96 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 128 \\ -128 \end{bmatrix} \frac{1}{EI}$$

-ive sign shows that our assumed redundant joint displacement direction is wrong

Stiffness Method for Frames Analysis

Step # 05: Compute the member end actions & you will get.

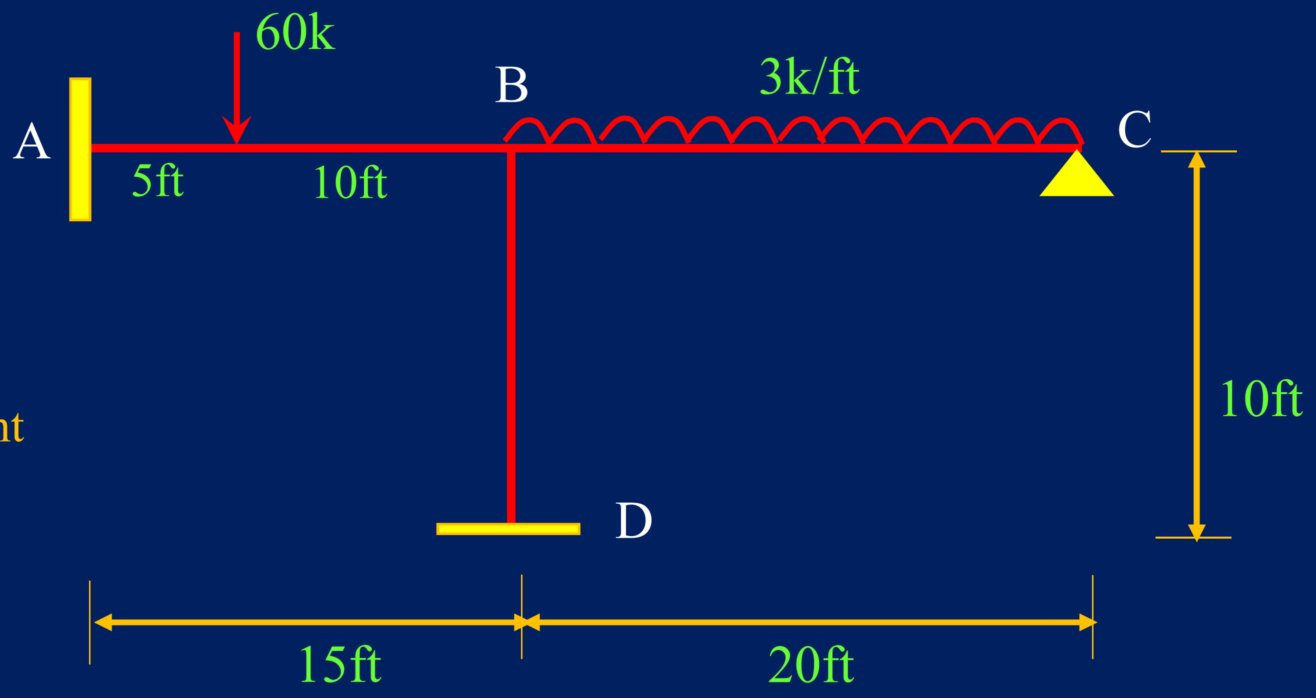


Stiffness Method for Frames Analysis

Step # 06: Draw Shear force & bending moment diagram.

Stiffness Method for Frames Analysis

Problem 04: Analyze the given Frame using stiffness method.

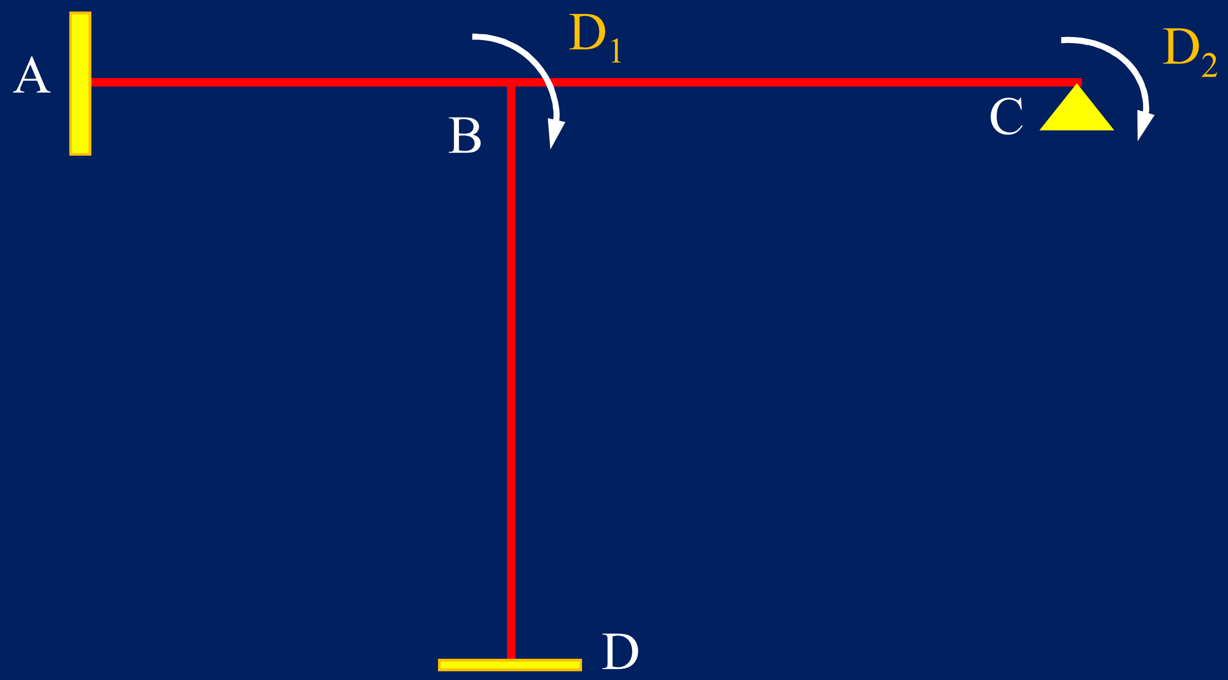


Take EI = constant

K.I = 2 degree (neglecting the axial effects)

Stiffness Method for Frames Analysis

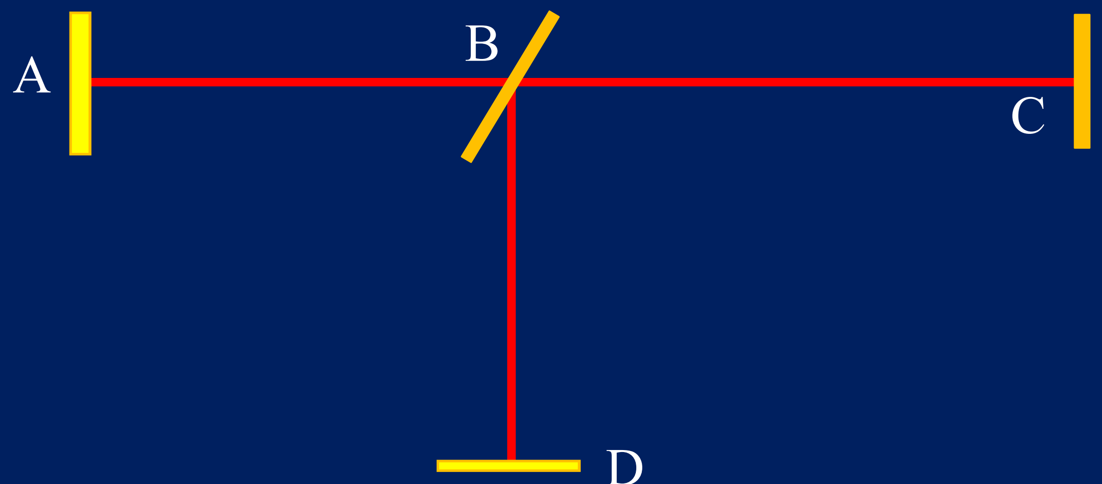
- **Step # 01:** Selection of redundant Joint displacements and assign coordinates at those locations. Also compute AD values.



$$[D]_{2*1} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad [AD]_{2*1} = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Stiffness Method for Frames Analysis

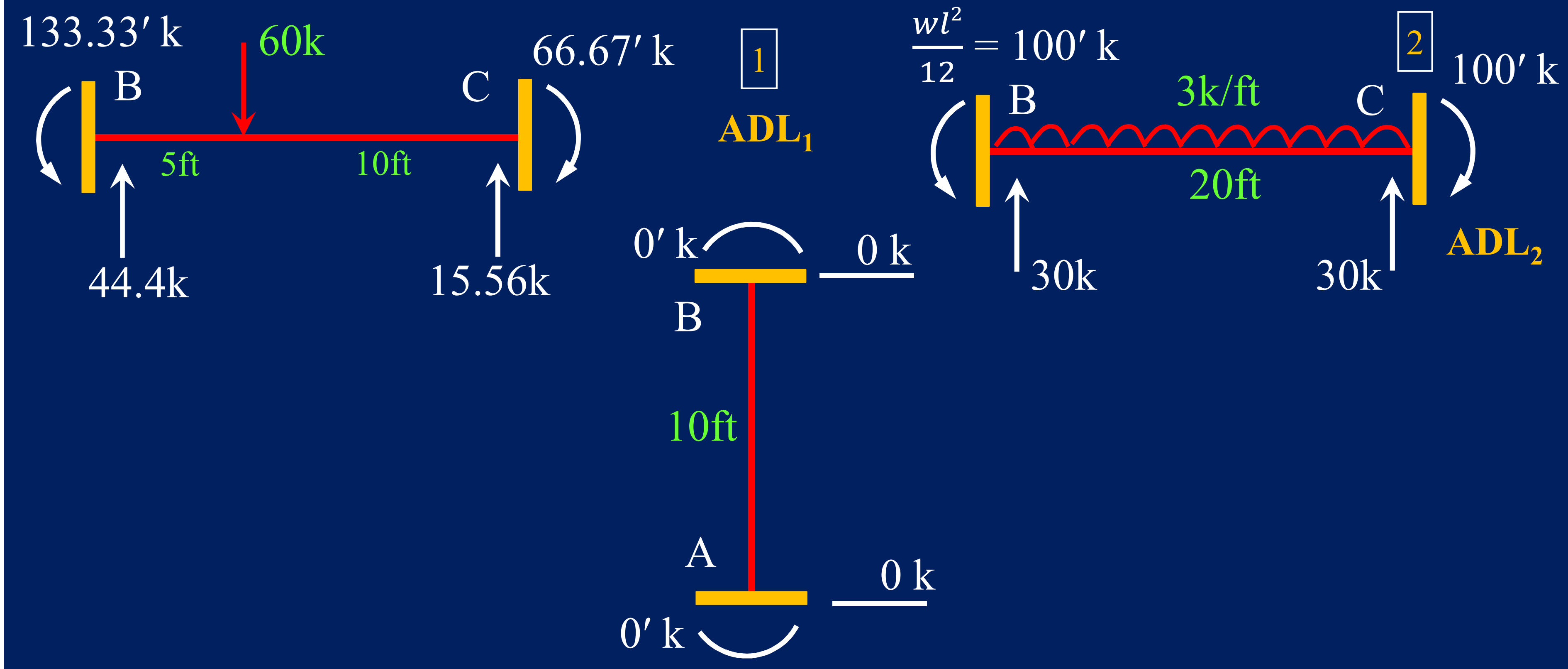
- Restrain all the degrees of freedom to get the restrained structure.



Basic kinematic determinate structure (BKDS) or
restrained structure

Stiffness Method for Frames Analysis

- **Step # 02 :** Compute ADL matrix.(Fixed end actions)



$$[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} = \begin{bmatrix} -33.33 \\ 100 \end{bmatrix}$$

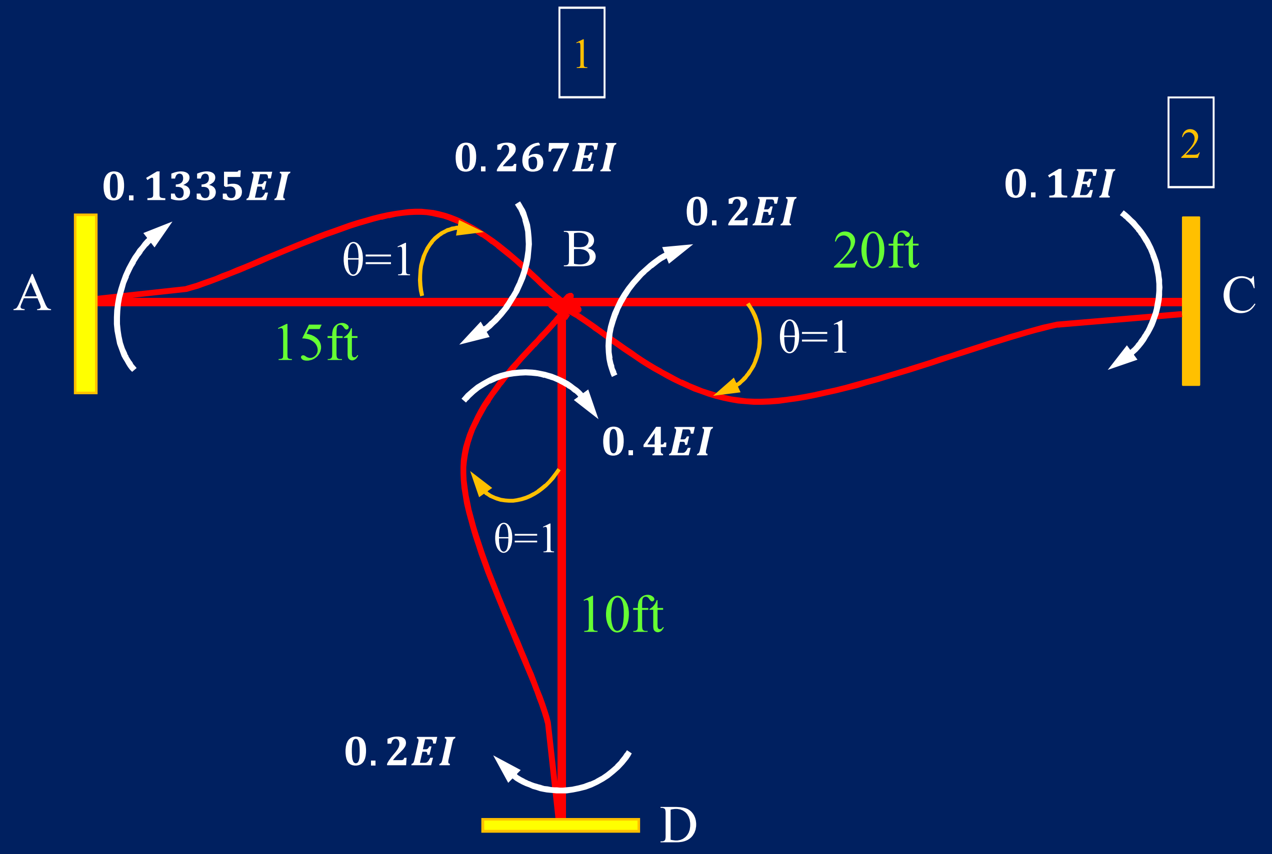
Stiffness Method for Frames Analysis

Step # 03 : Primary structure acted upon by a unit value of D & computation of stiffness coefficients " S" values in the BKDS corresponding to the redundant joint displacement locations.

- 1st a unit rotation is applied at location 1 & prevented at 2 as shown. Compute the values of S_{11} and S_{21} .
- Then apply a unit rotation at the redundant displacement location 2 and prevented at 1 as shown. Compute the values of S_{12} & S_{22} .

Stiffness Method for Frames Analysis

i. $D_1 = 1$ & $D_2 = 0$) as shown. Compute the values of S_{11} and S_{21}.



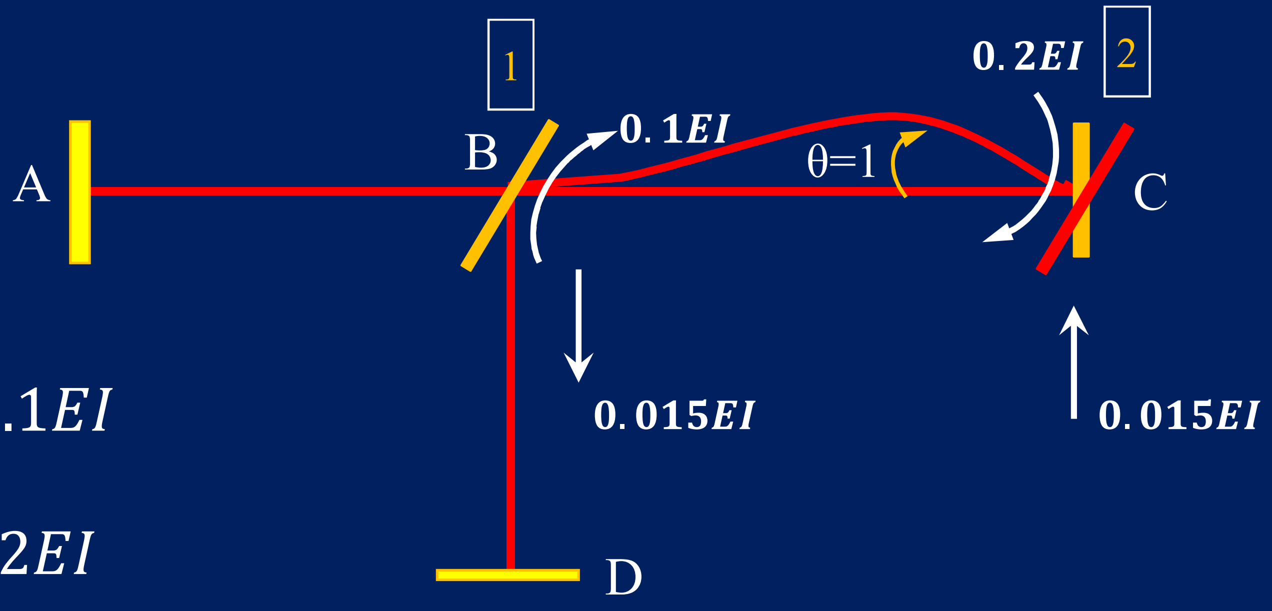
$$S_{11} = (0.2 + 0.4 + 0.267)EI$$

$$S_{11} = 0.867EI$$

$$S_{21} = 0.1EI$$

Stiffness Method for Frames Analysis

ii. Now $D_2 = 1$ & $D_1 = 0$ & Compute the values of S_{12} & S_{22} .



$$S_{12} = 0.1EI$$

$$S_{22} = 0.2EI$$

$$[\,S\,] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \qquad [\,S\,] = EI \begin{bmatrix} 0.867 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$$

Stiffness coefficient matrix

Stiffness Method for Frames Analysis

Step # 04: Apply equilibrium condition at the location of the redundant joint displacement to write equilibrium equations and solve for unknown joint displacement.

$$AD_1 = ADL_1 + S_{11}D_1 + S_{12}D_2$$

$$AD_2 = ADL_2 + S_{21}D_1 + S_{22}D_2$$

$$\begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} + \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$[AD]_{2 * 1} = [ADL]_{2 * 1} + [S]_{2 * 2} \bullet [D]_{2 * 1}$$

$$[\boldsymbol{D}] = [\boldsymbol{S}]^{-1} \bullet [\boldsymbol{AD} - \boldsymbol{ADL}]$$

Stiffness Method for Frames Analysis

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix}$$

$$\begin{bmatrix} D1 \\ D2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.867 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}^{-1} \begin{bmatrix} 0 - (-33.3) \\ 0 - 100 \end{bmatrix}$$

$$\begin{bmatrix} D1 \\ D2 \end{bmatrix} = \begin{bmatrix} 102.6 \\ -551 \end{bmatrix} \frac{1}{EI}$$

-ive sign shows that our assumed redundant joint displacement direction is wrong

Step # 05: Compute the member end actions & then draw shear force and bending moment diagram.

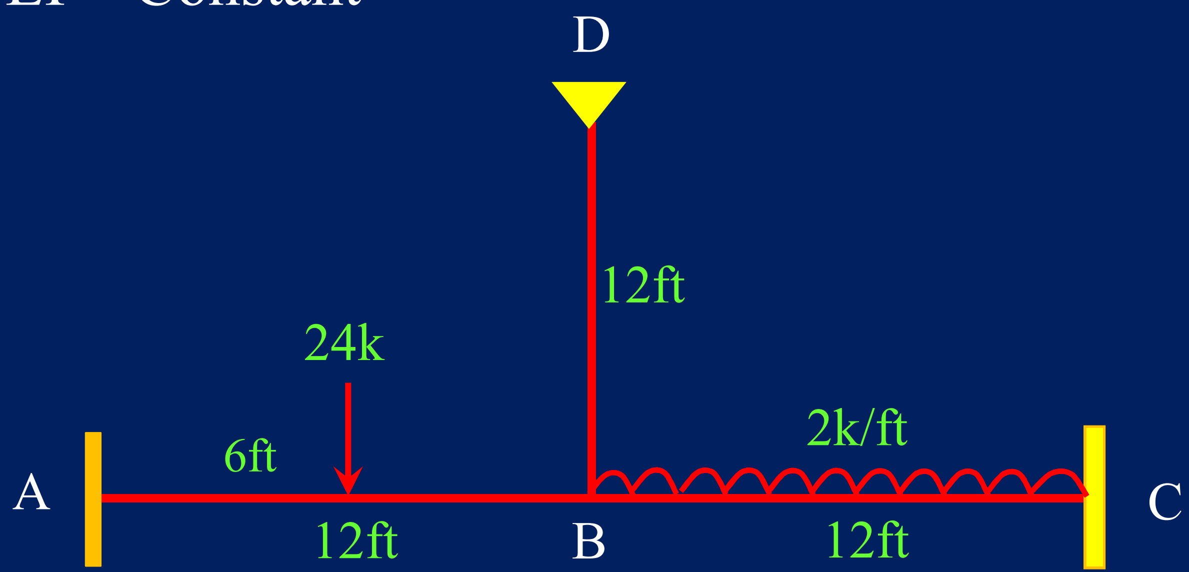
Stiffness Method for Frames Analysis

Assignment 04(a).

Q1. Find the member end actions and the draw shear force and bending moment diagram for the frames in problem 3 & 4

Q2. Analyze the frames given below using stiffness method.

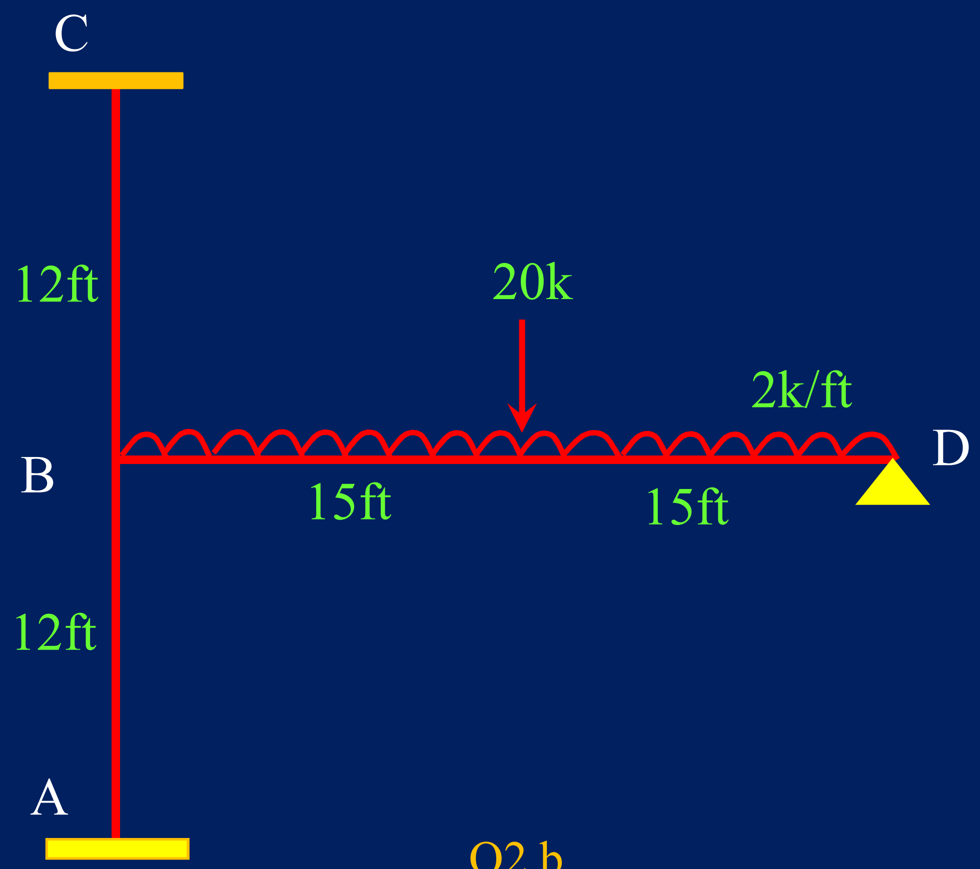
Take EI = Constant



Q2.a

Stiffness Method for Frames Analysis

Assignment 04(a).



Q2.b

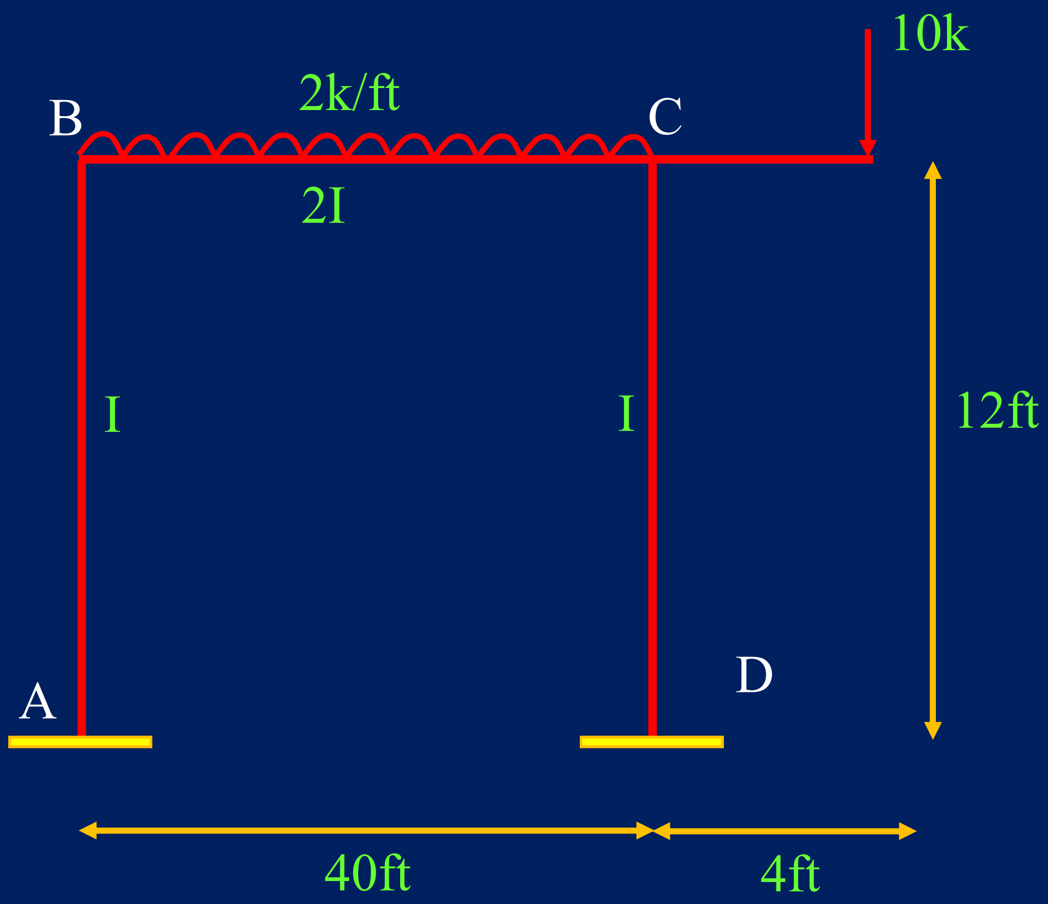
Stiffness Method for Frames Analysis

Part II

Analysis of Frames with side sway

Stiffness Method for Frames Analysis

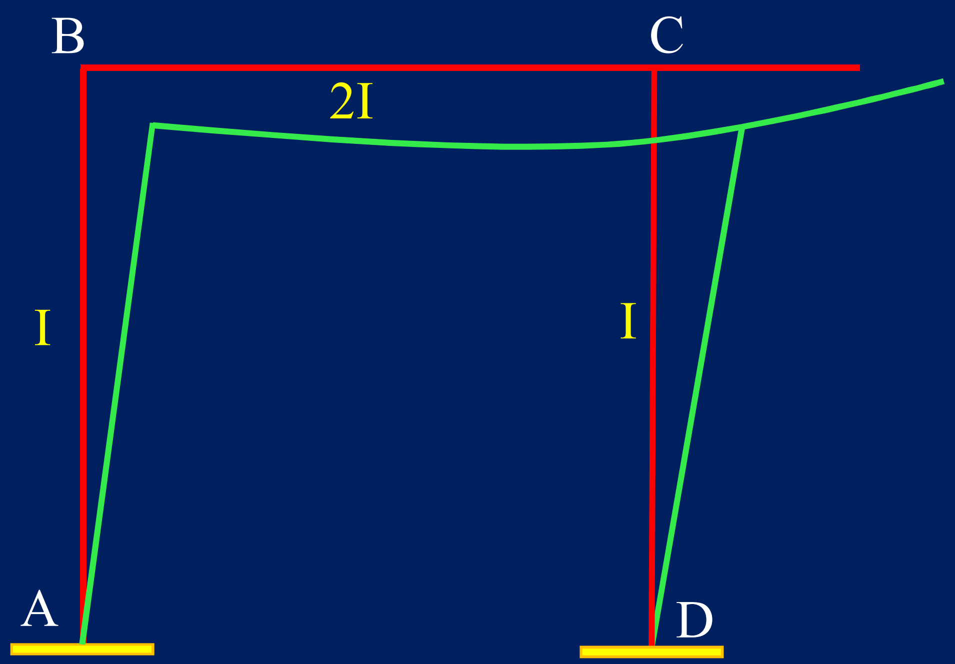
Problem 05: Analyze the given Frame using stiffness method.



Take EI = constant

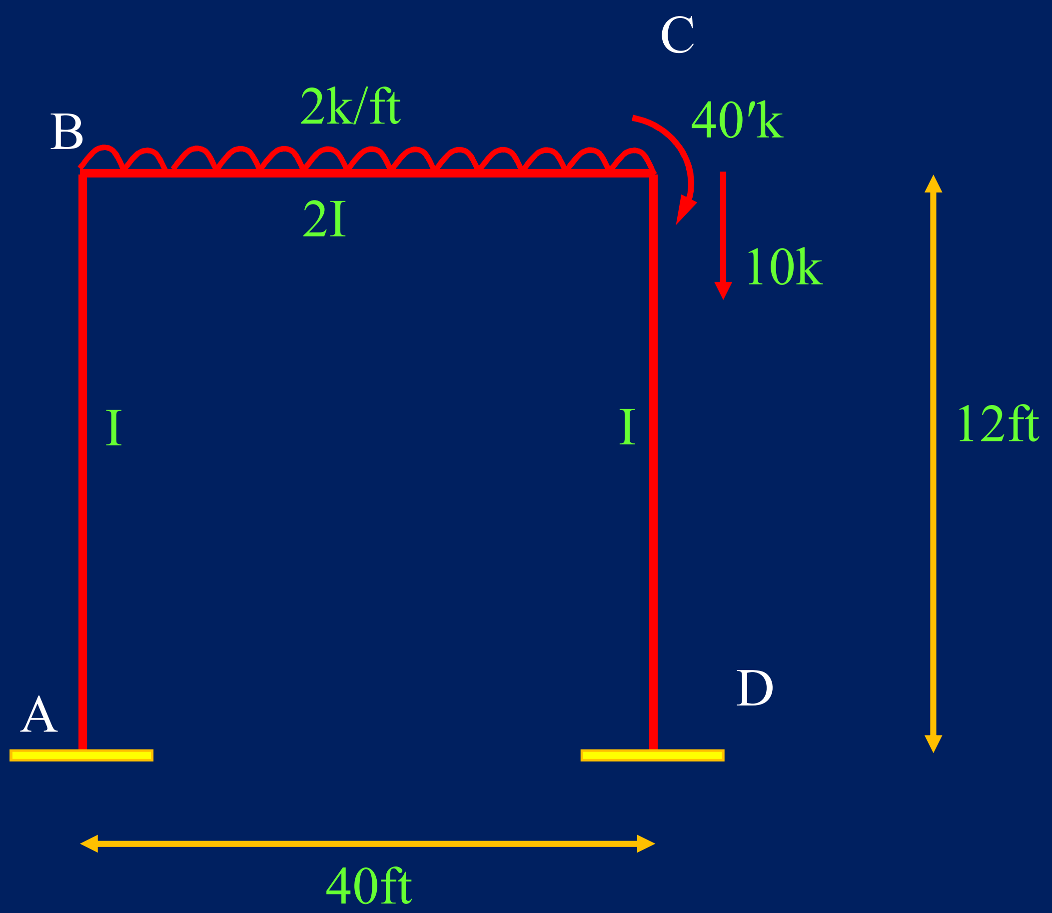
K.I = 3 degree (neglecting the axial effects)

Stiffness Method for Frames Analysis



Deflected shape

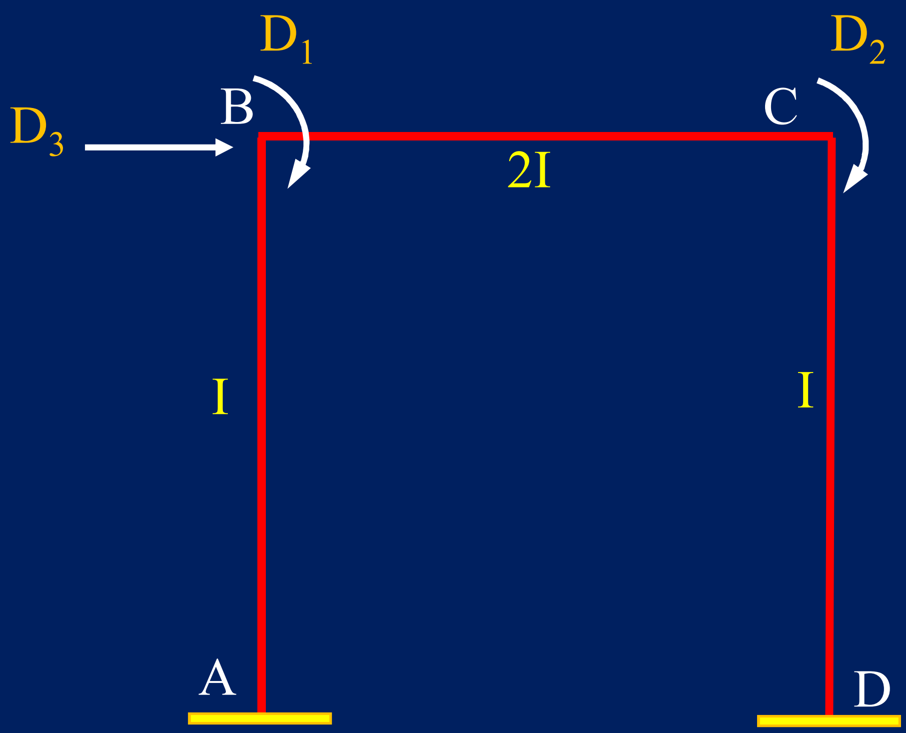
Stiffness Method for Frames Analysis



The effect of overhanging portion can be taken as moment and force at point C as shown above.

Stiffness Method for Frames Analysis

- **Step # 01:** Selection of redundant Joint displacements and assign coordinates at those locations. Also compute AD values.



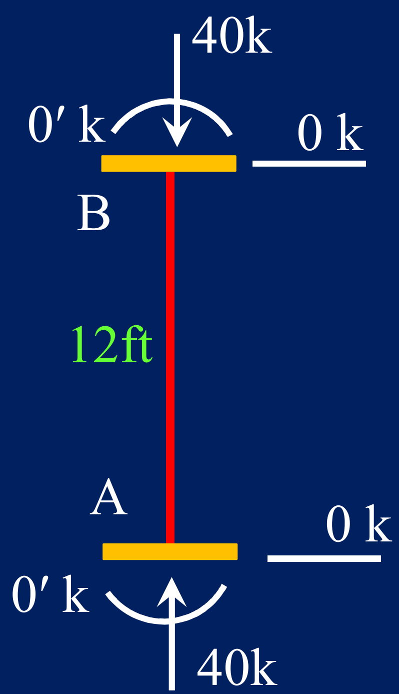
$$[D]_{3*1} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$$

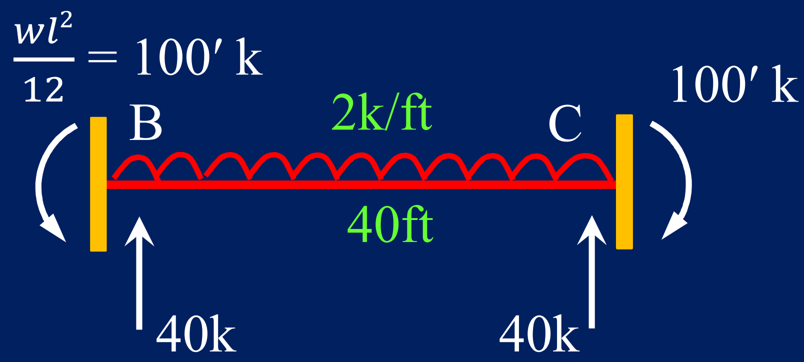
$$[AD]_{2*1} = \begin{bmatrix} AD_1 \\ AD_2 \\ AD_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 40'k \\ 0 \end{bmatrix}$$

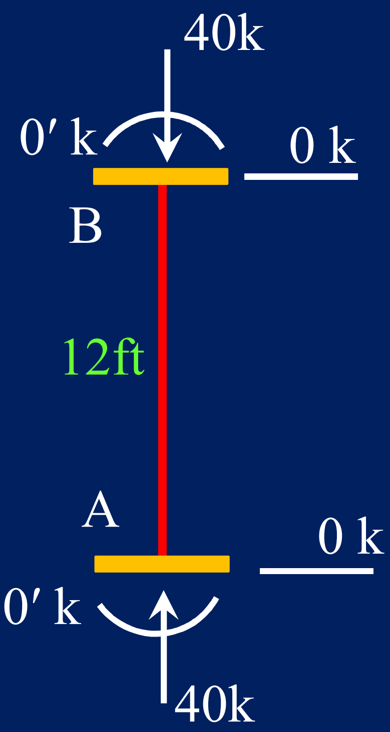
Rotation at B , C and Sway is taken as redundant joint displacement.

Stiffness Method for Frames Analysis

- **Step # 02 :** Restrained structure acted upon by the actual loads. compute the values of actions in the restrained structure corresponding to the redundant locations. This will generate ADL matrix.(Fixed end actions)







$$[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \\ ADL_3 \end{bmatrix} = \begin{bmatrix} -266.67 \\ 266.67 \\ 0 \end{bmatrix}$$

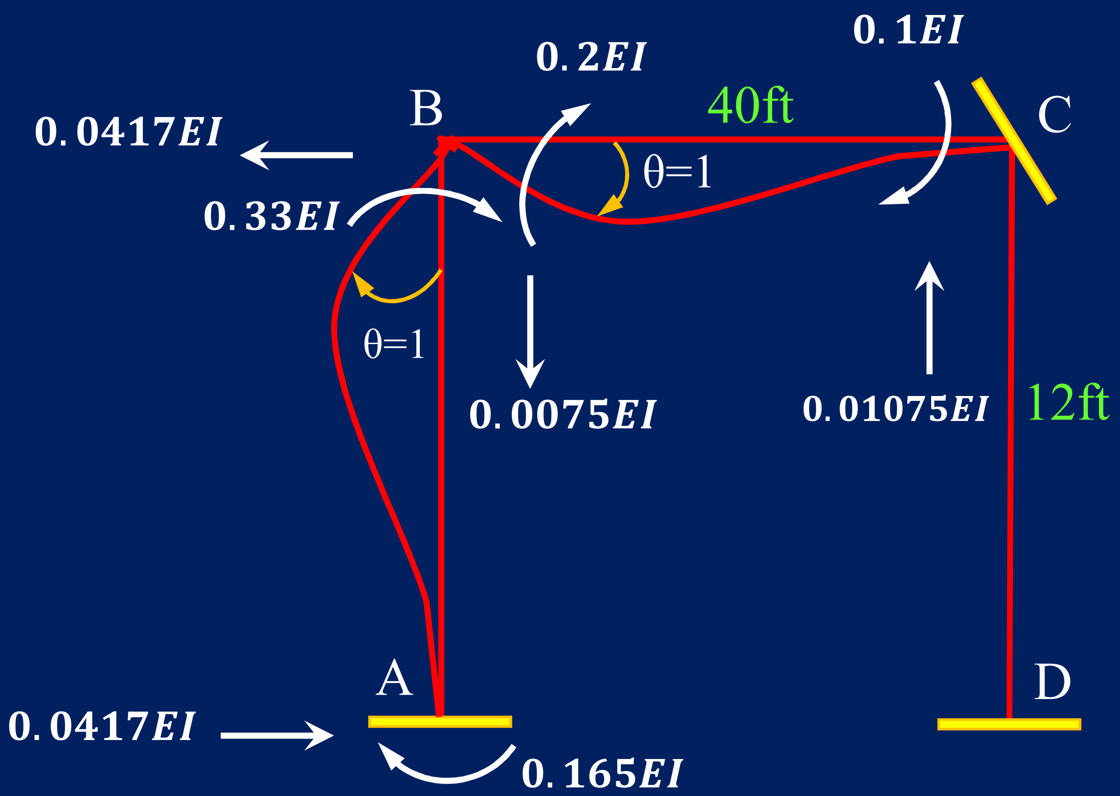
Stiffness Method for Frames Analysis

Step # 03 : Primary structure acted upon by a unit value of D & computation of stiffness coefficients " S" values in the BKDS corresponding to the redundant joint displacement locations.

- 1st a unit rotation is applied at location 1 & prevented at 2 & #as shown. Compute the values of S_{11} , S_{21} & S_{31}.
- Then a unit rotation is applied at redundant displacement location 2 and prevented at 1 & 3 as shown. Compute the values of S_{12} , S_{22} & S_{32} .
- Then a unit is applied at 3 and prevented at 1 & 2 as shown. Compute the values of S_{12} , S_{22} & S_{32} .

Stiffness Method for Frames Analysis

i. 1st a unit rotation is applied at location 1 & prevented at 2 & 3. (D_1=1 & $D_2 = D_3 = 0$)as shown. Compute the values of S_{11} , S_{21} & S_{31} .



Stiffness Method for Frames Analysis

Step # 03 (i): Contd…

$$S_{11} = 0.2EI + 0.33EI = 0.533EI$$

$$S_{21} = 0.1EI$$

$$S_{31} = -0.0417EI$$

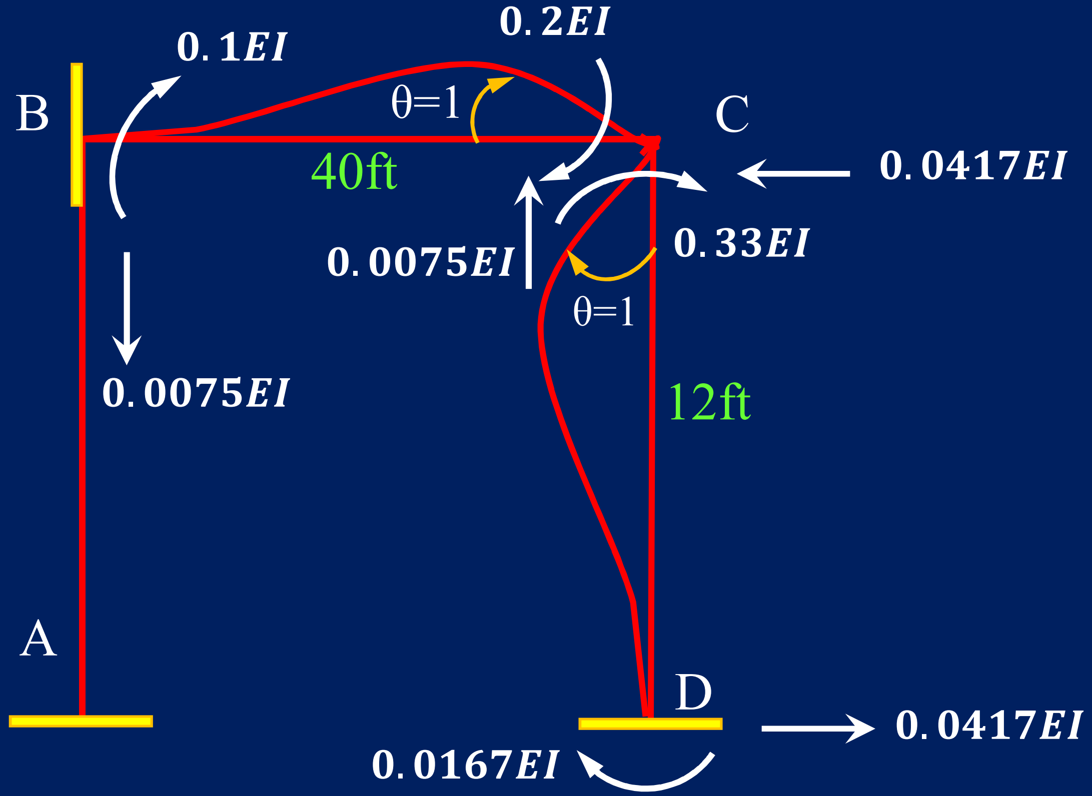
S_{11} = Action(sum of moments in this case) in the BKDS at redundant displacement location 1 due to unit rotation at that location.

S_{21} = Moment in the BDS at redundant displacement location 2 due to a unit rotation applied at location 1

S_{31} = Moment in the BDS at redundant displacement location 3 due to a unit rotation applied at location 1

Stiffness Method for Frames Analysis

ii. Now a unit rotation is applied at the redundant displacement location 2 and prevented at 1 & 3(D_2=1 & $D_1 = D_3 = 0$) as shown. Compute the values of S_{12} , S_{22} & S_{32} .



Stiffness Method for Frames Analysis

Step # 03 : Contd…

$$S_{12} = \ 0.1EI$$

$$S_{22} = 0.2EI + 0.333EI = 0.533EI$$

$$S_{32} = \ -0.0417EI$$

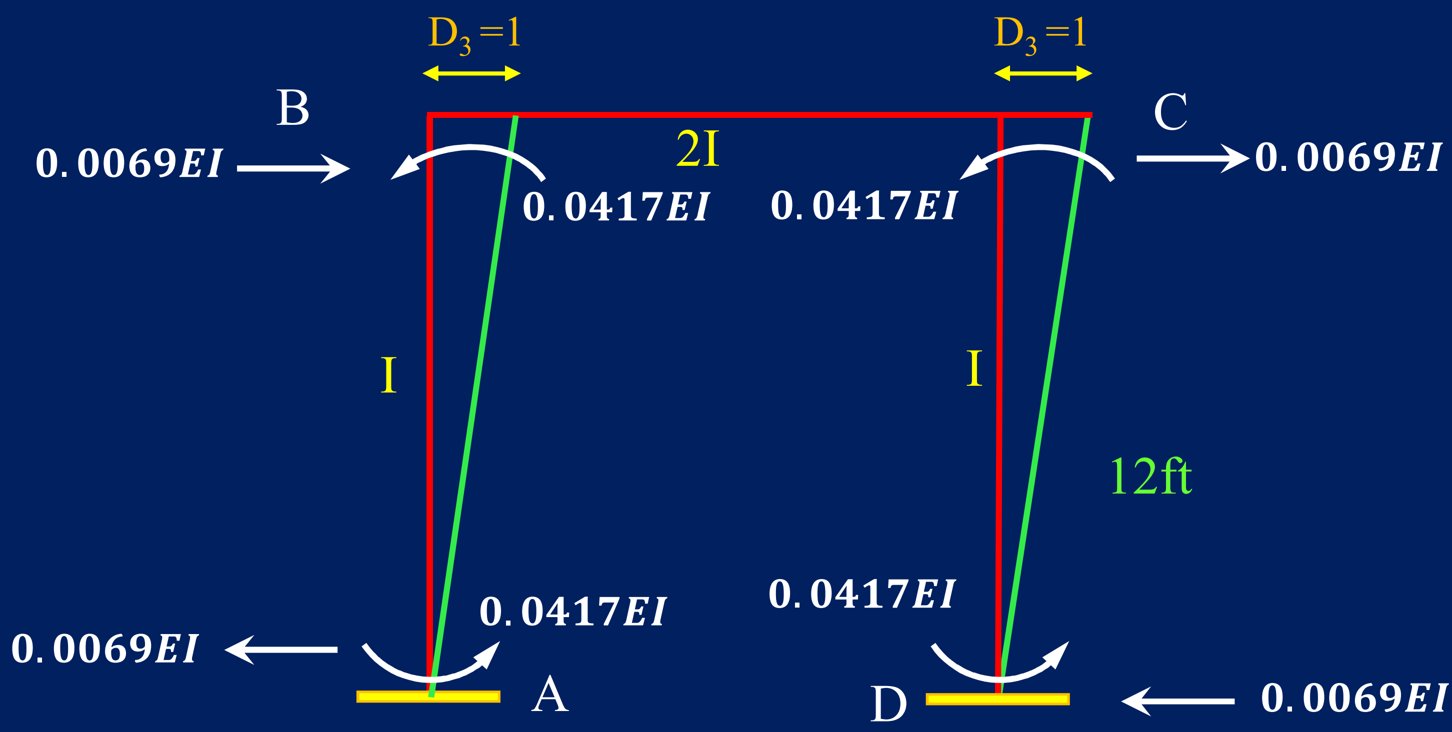
S_{12} = Moment in the BKDS at redundant displacement location 1 due to unit rotation applied at redundant displacement location 2.

S_{22} = Moment in the BKDS at redundant displacement location 2 due to a unit rotation applied at that location.

S_{23} = Moment in the BKDS at redundant displacement location 3 due to a unit rotation applied at redundant location 2.

Stiffness Method for Frames Analysis

iii. Now a unit rotation is applied at the redundant displacement location 2 and prevented at 1 (D_3=1 & D_1 = D_2 = 0) as shown. Compute the values of S_{13} , S_{23} & S_{33} .



Stiffness Method for Frames Analysis

Step # 03 : Contd…

$$S_{13} = \ -0.0417EI$$

$$S_{23} = \ -0.0417EI$$

$$S_{33} = 0.0069EI + 0.0069EI = 0.0138EI$$

S_{13} = Moment in the BKDS at redundant displacement location 1 due to unit rotation applied at redundant displacement location 2.

S_{23} = Moment in the BKDS at redundant displacement location 2 due to a unit rotation applied at that location.

S_{33} = Moment in the BKDS at redundant displacement location 3 due to a unit rotation applied at redundant location 2.

Stiffness Method for Frames Analysis

Step # 03 : Contd…

$$S_{11} = 0.533EI \qquad S_{21} = 0.1EI \qquad S_{31} = -0.0417EI$$

$$S_{12} = 0.1EI \qquad S_{22} = 0.533EI \qquad S_{32} = -0.0417EI$$

$$S_{13} = -0.0417EI \qquad S_{23} = 0.0417EI \qquad S_{33} = 0.0138EI$$

$$[\,S\,] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} = EI \begin{bmatrix} 0.533 & 0.1 & -0.0417 \\ 0.1 & 0.533 & -0.0417 \\ -0.0417 & -0.0417 & 0.0138 \end{bmatrix}$$

Stiffness coefficient matrix

Stiffness Method for Frames Analysis

Step # 04: Apply equilibrium condition at the location of the redundant joint displacement to write equilibrium equations and solve for unknown joint displacement.

$$[AD]_{3\,*\,1} = [ADL]_{3\,*\,1} + [S]_{3\,*\,3} \bullet [D]_{3\,*\,1}$$

$$[\boldsymbol{D}] = [\boldsymbol{S}]^{-1} \bullet [\boldsymbol{AD} - \boldsymbol{ADL}]$$

Stiffness Method for Frames Analysis

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}^{-1} \begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \\ AD_2 - ADL_2 \end{bmatrix}$$

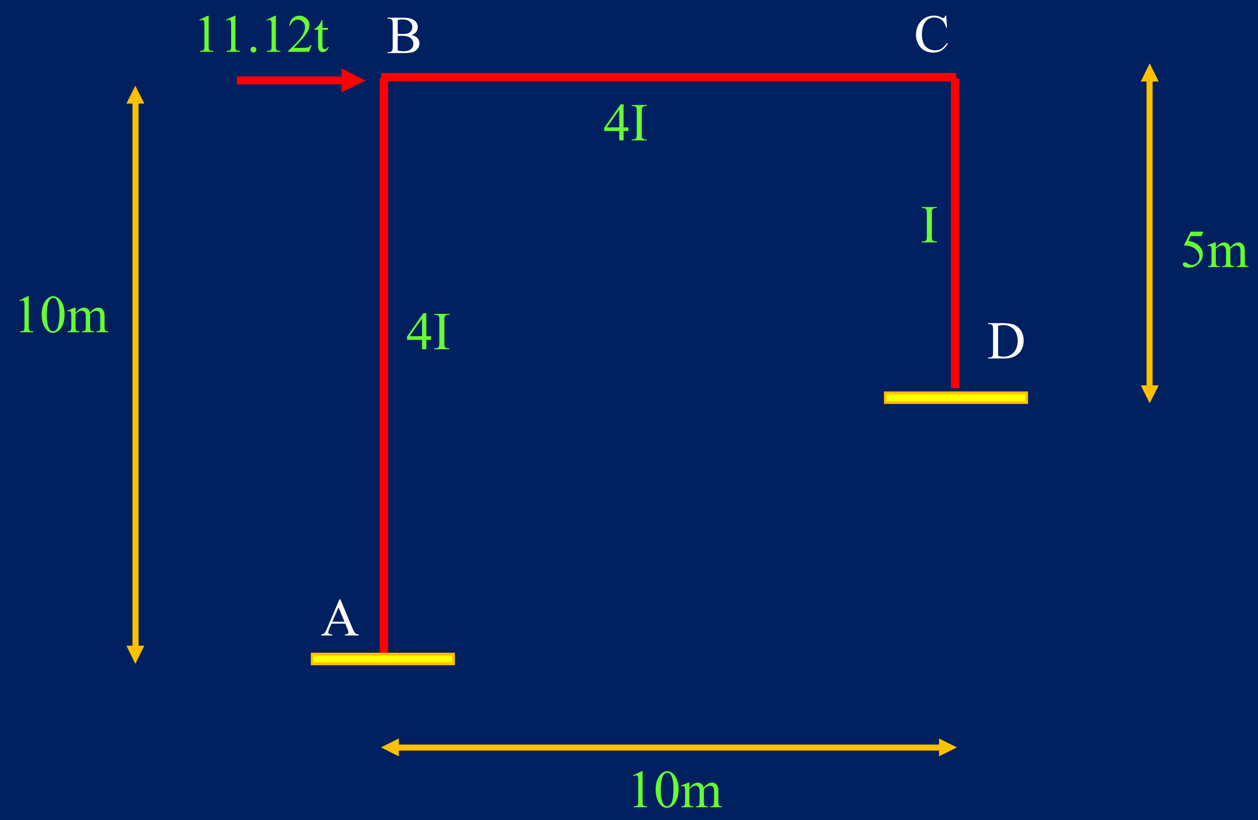
$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.533 & 0.1 & -0.0417 \\ 0.1 & 0.533 & -0.0417 \\ -0.0417 & -0.0417 & 0.0138 \end{bmatrix}^{-1} \begin{bmatrix} 0 - (-266) \\ 40 - 266.67 \\ 0 - 0 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 621.90 \\ -571.45 \\ 313.41 \end{bmatrix}$$

Step # 05: Compute the member end actions & then draw shear force and bending moment diagram. CLASS ACTIVITY

Stiffness Method for Frames Analysis

Problem 06: Analyze the given Frame using stiffness method if the yielding of support D to the right downwards in t-m units are $\frac{20}{EI}$ & $\frac{50}{EI}$ respectively.

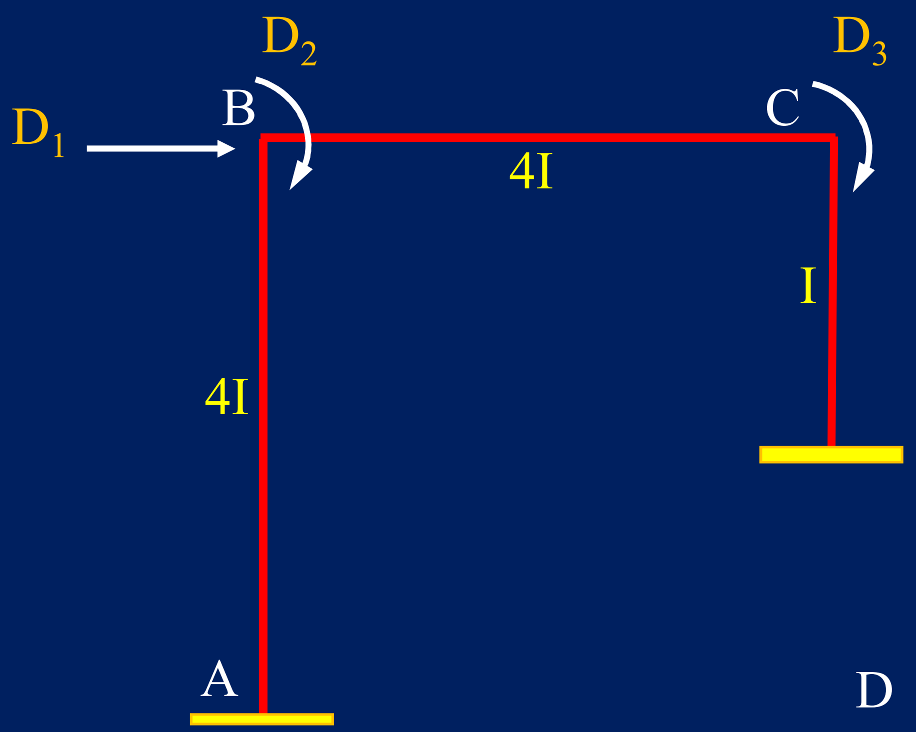


Take EI = con

K.I = 3 degree (neglecting the axial effects)

Stiffness Method for Frames Analysis

- **Step # 01:** Selection of redundant Joint displacements and assign coordinates at those locations. Also compute AD values.



$$[D]_{3*1} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$$

$$[AD]_{3*1} = \begin{bmatrix} AD_1 \\ AD_2 \\ AD_3 \end{bmatrix} = \begin{bmatrix} 11.12t \\ 0 \\ 0 \end{bmatrix}$$

Rotation at B , C and Sway is taken as redundant joint displacement.

Stiffness Method for Frames Analysis

- **Step # 02 :** Computation of ADL matrix.(Fixed end actions).

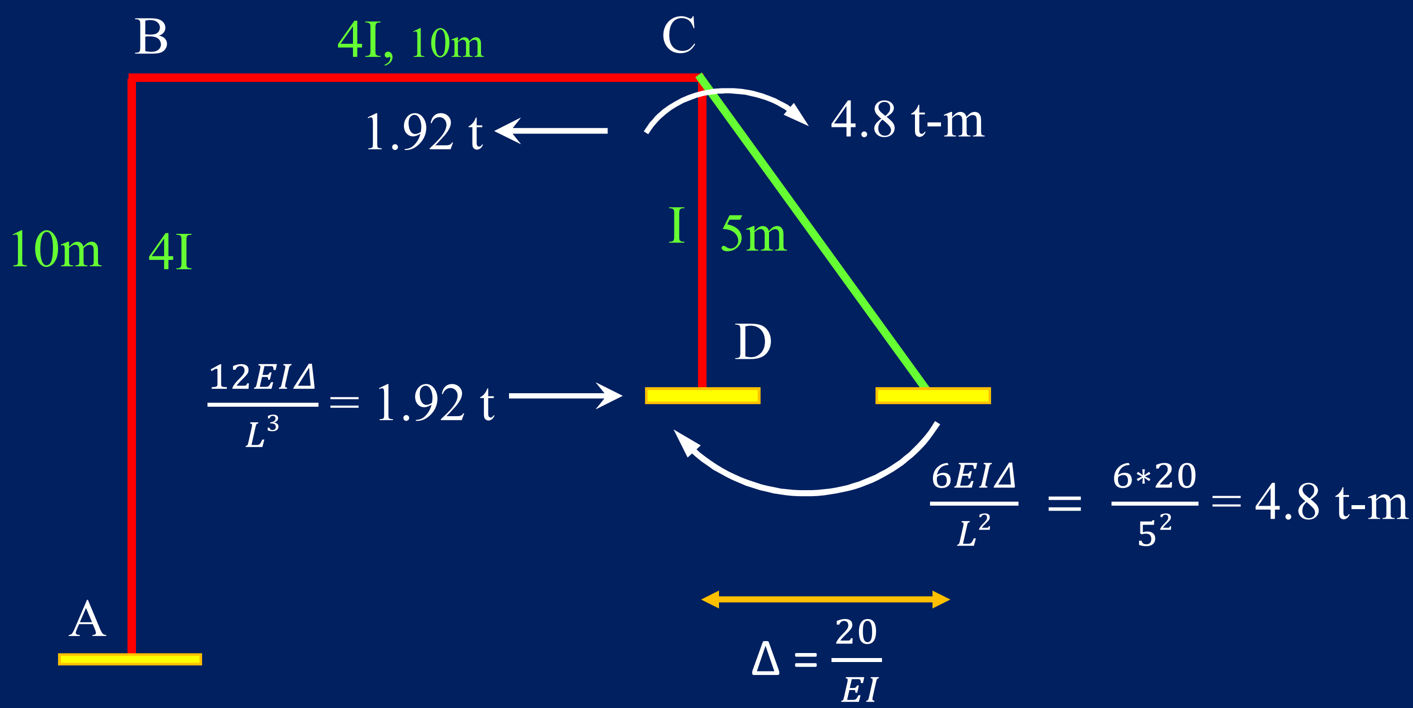
  As there is no external load acting on the frame that will cause fixed end actions so the ADL matrix will consist of restraining forces due to in direct loadings only.

1. Induce actions due to horizontal displacement of 20/EI (towards right) at support D.
2. Induce actions due to vertical displacement of 50/EI (downward) at support D.

Stiffness Method for Frames Analysis

- **Step # 02 :** Computation of ADL matrix.(Fixed end actions).

1. Induce actions due to horizontal displacement of 20/EI (towards right) at support D and get the values of ADL′.

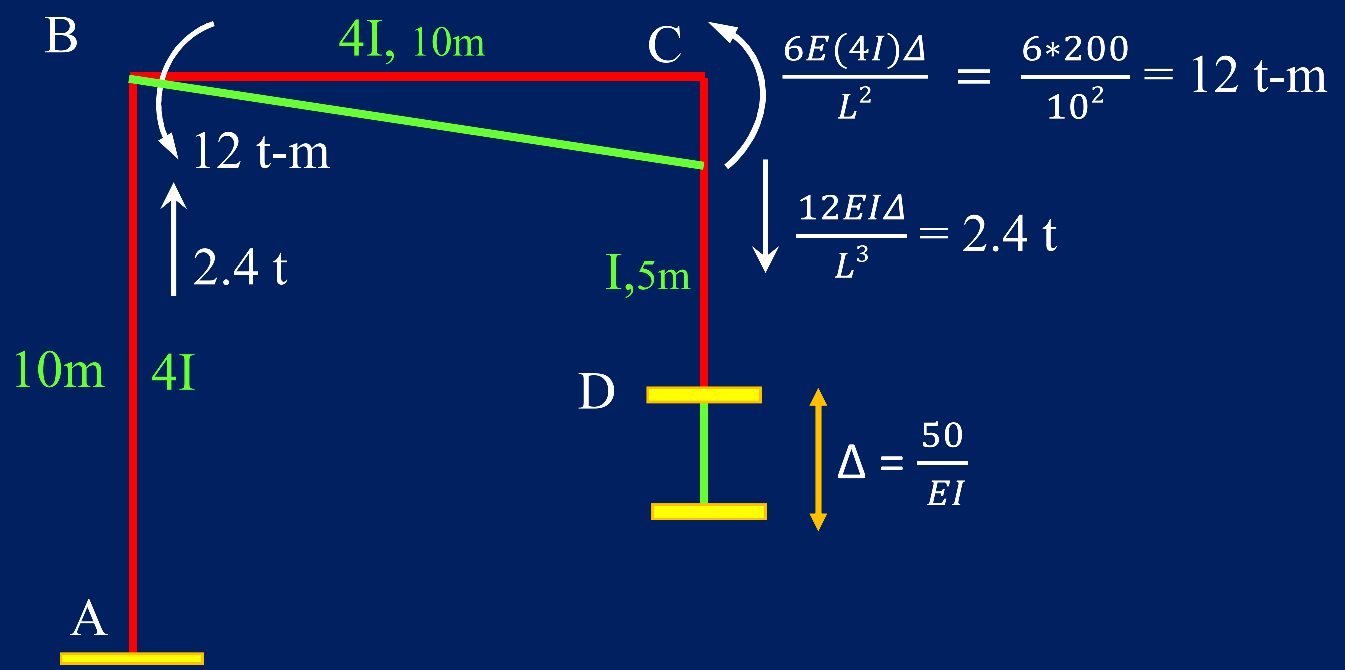


$ADL_1' = -1.92t$ $\quad ADL_2' = 0$ $\quad ADL_3' = 4.8t - m$

Stiffness Method for Frames Analysis

- **Step # 02 :** Computation of ADL matrix.(Fixed end actions).

1. Induce actions due to vertical displacement of 50/EI (downward) at support D and get the values of ADL″.



$ADL_1'' = 0t$ $\quad ADL_2'' = -12t - m$ $\quad ADL_3'' = -12t - m$

Stiffness Method for Frames Analysis

- **Step # 02 :** Computation of ADL matrix.(Fixed end actions).

$$ADL_1' = -1.92t \qquad ADL_2' = 0 \qquad ADL_3' = 4.8t - m$$

$$ADL_1'' = 0t \qquad ADL_2'' = -12t - m \qquad ADL_3'' = -12t - m$$

$$ADL_1 = ADL_1' + ADL_1'' = -1.92t$$

$$ADL_2 = ADL_2' + ADL_2'' = -12t - m$$

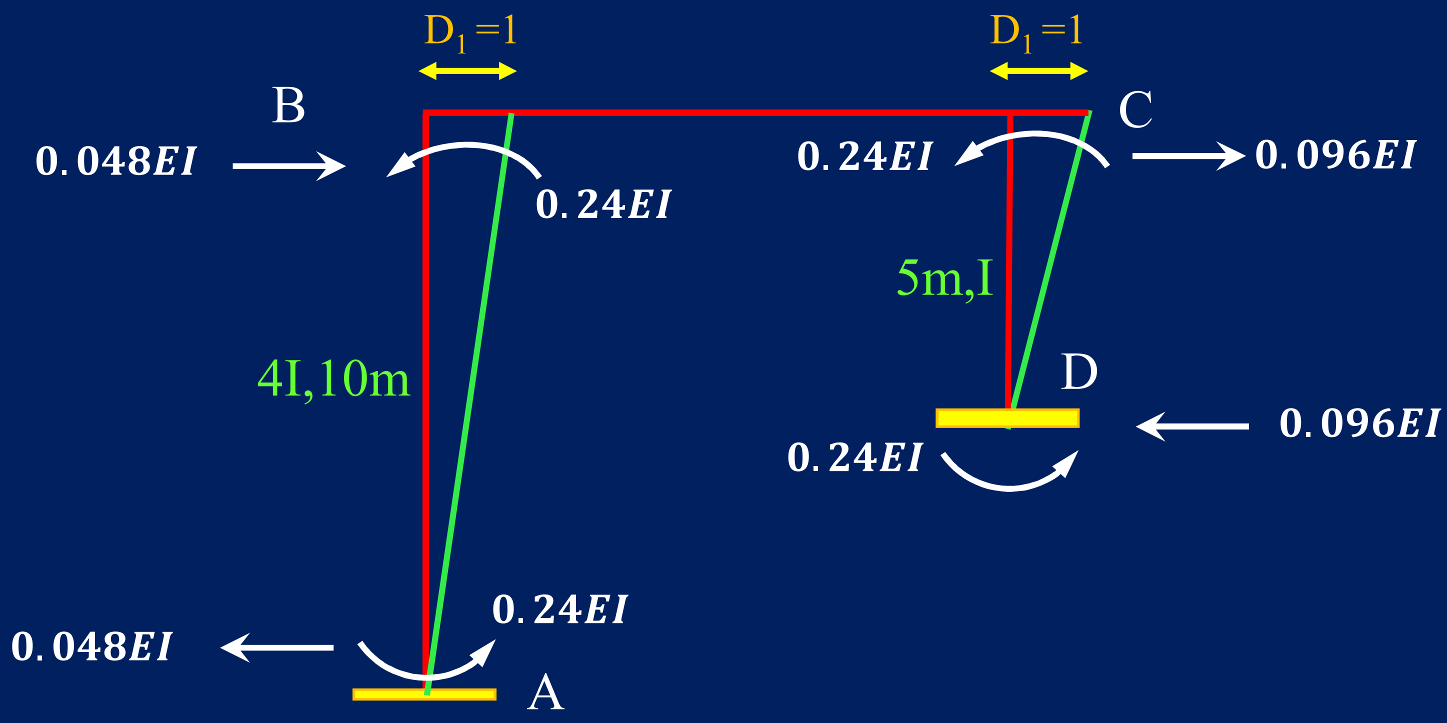
$$ADL_3 = ADL_3' + ADL_3'' = -7.2t - m$$

$$[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \\ ADL_3 \end{bmatrix} = \begin{bmatrix} -1.92 \\ -12 \\ -7.2 \end{bmatrix}$$

Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

i. When $D_1 = 1$ & $D_2 = D_3 = 0$ as shown.



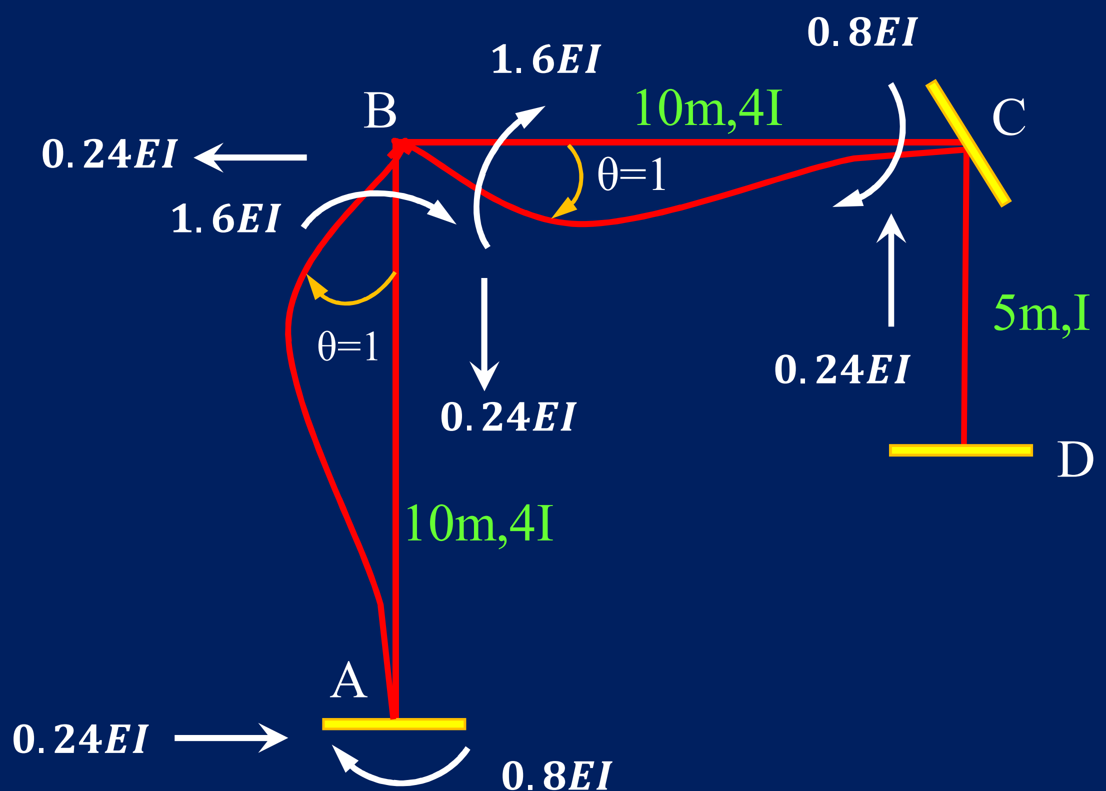
$$S_{11} = (0.48 + 0.096)EI = 0.144EI$$

$$S_{21} = -0.24EI \qquad S_{31} = -0.24EI$$

Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

ii. When $D_2 = 1$ & $D_1 = D_3 = 0$ as shown.

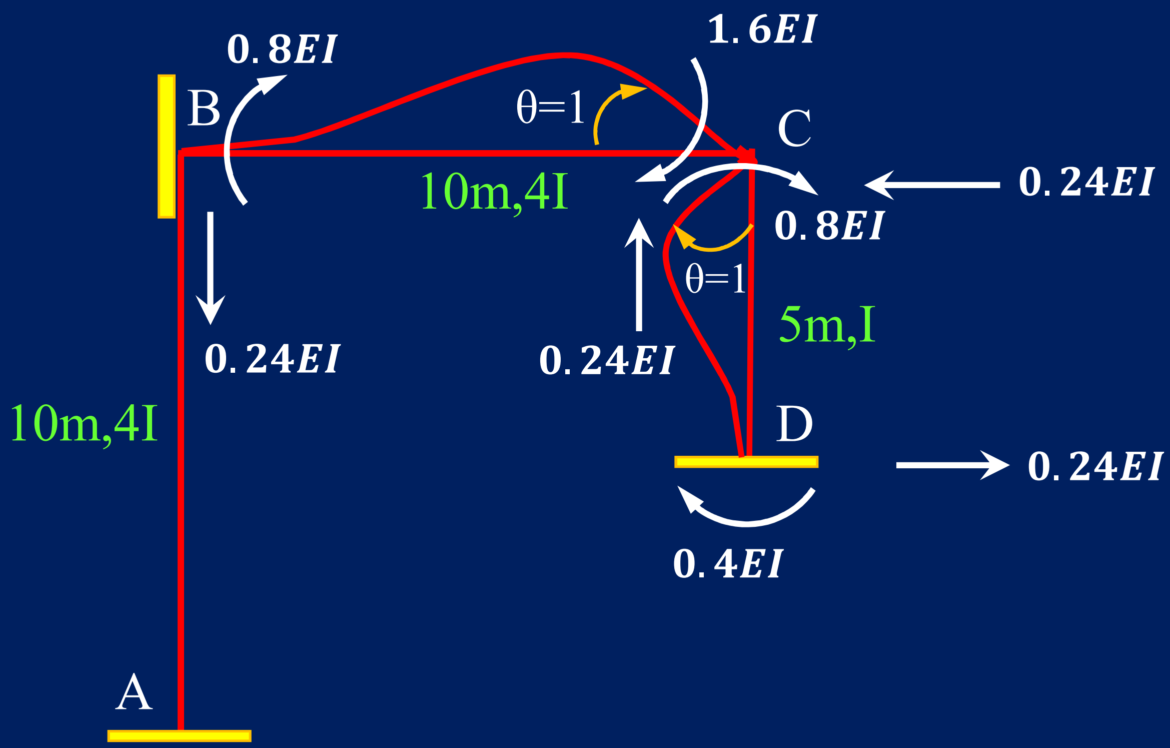


$S_{12} = \ -0.24EI$ $\qquad S_{22} = \ 1.6EI + 1.6EI = 3.2EI$ $\qquad S_{32} = 0.8EI$

Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

iii. When $D_3 = 1$ & $D_1 = D_2 = 0$ as shown.



$S_{13} = -0.24EI$ $\quad$ $S_{23} = 0.8EI$ $\quad$ $S_{32} = 1.6EI + 0.8EI = 2.4EI$

Stiffness Method for Frames Analysis

Step # 03 : Contd…

$$S_{11} = 0.144EI \qquad S_{21} = -0.24EI \qquad S_{31} = -0.24EI$$

$$S_{12} = -0.24EI \qquad S_{22} = 3.2EI \qquad S_{32} = 0.8EI$$

$$S_{13} = -0.24EI \qquad S_{23} = 0.8EI \qquad S_{33} = 2.4EI$$

$$[\,S\,] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} = EI \begin{bmatrix} 0.144 & -0.24 & -0.24 \\ -0.24 & 3.20 & 0.80 \\ -0.24 & 0.80 & 2.40 \end{bmatrix}$$

Stiffness coefficient matrix

Stiffness Method for Frames Analysis

Step # 04: Apply equilibrium condition at the location of the redundant joint displacement to write equilibrium equations and solve for unknown joint displacement.

$$[AD]_{3\,*\,1} = [ADL]_{3\,*\,1} \quad + \quad [S]_{3\,*\,3} \bullet [D]_{3\,*\,1}$$

$$[\boldsymbol{D}] \;=\; [\boldsymbol{S}]^{-1} \bullet \; [\boldsymbol{AD} - \boldsymbol{ADL}]$$

Stiffness Method for Frames Analysis

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}^{-1} \begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \\ AD_2 - ADL_2 \end{bmatrix}$$

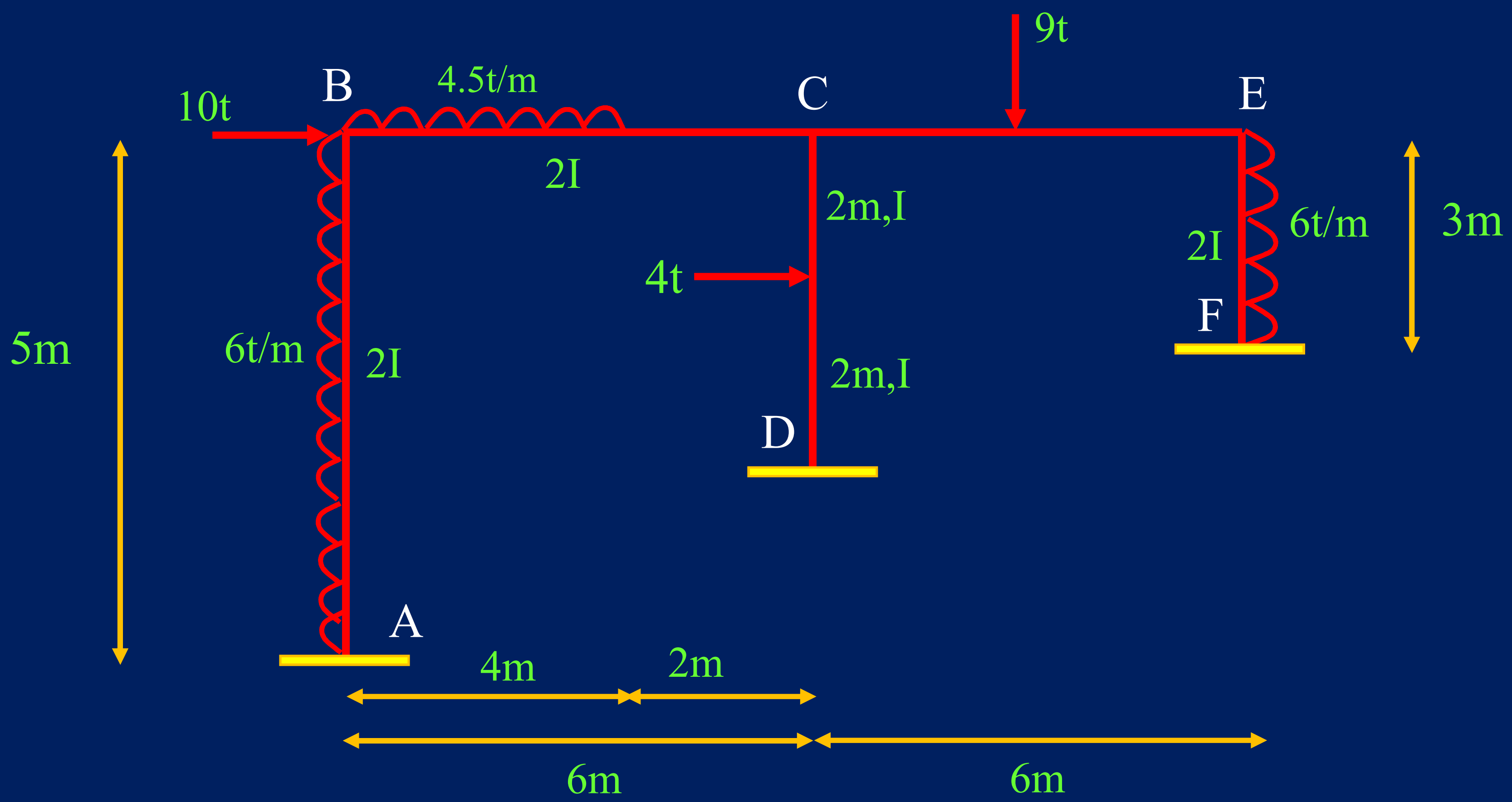
$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.144 & -0.24 & -0.24 \\ -0.24 & 3.20 & 0.80 \\ -0.24 & 0.80 & 2.40 \end{bmatrix}^{-1} \begin{bmatrix} 11.12 - (-1.92) \\ 0 - (-12) \\ 0 - (-7.2) \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 128.369 \\ 10.276 \\ 12.408 \end{bmatrix}$$

Step # 05: Compute the member end actions & then draw shear force and bending moment diagram. HOME WORK

Stiffness Method for Frames Analysis

Problem 07: Analyze the given Frame using stiffness method.

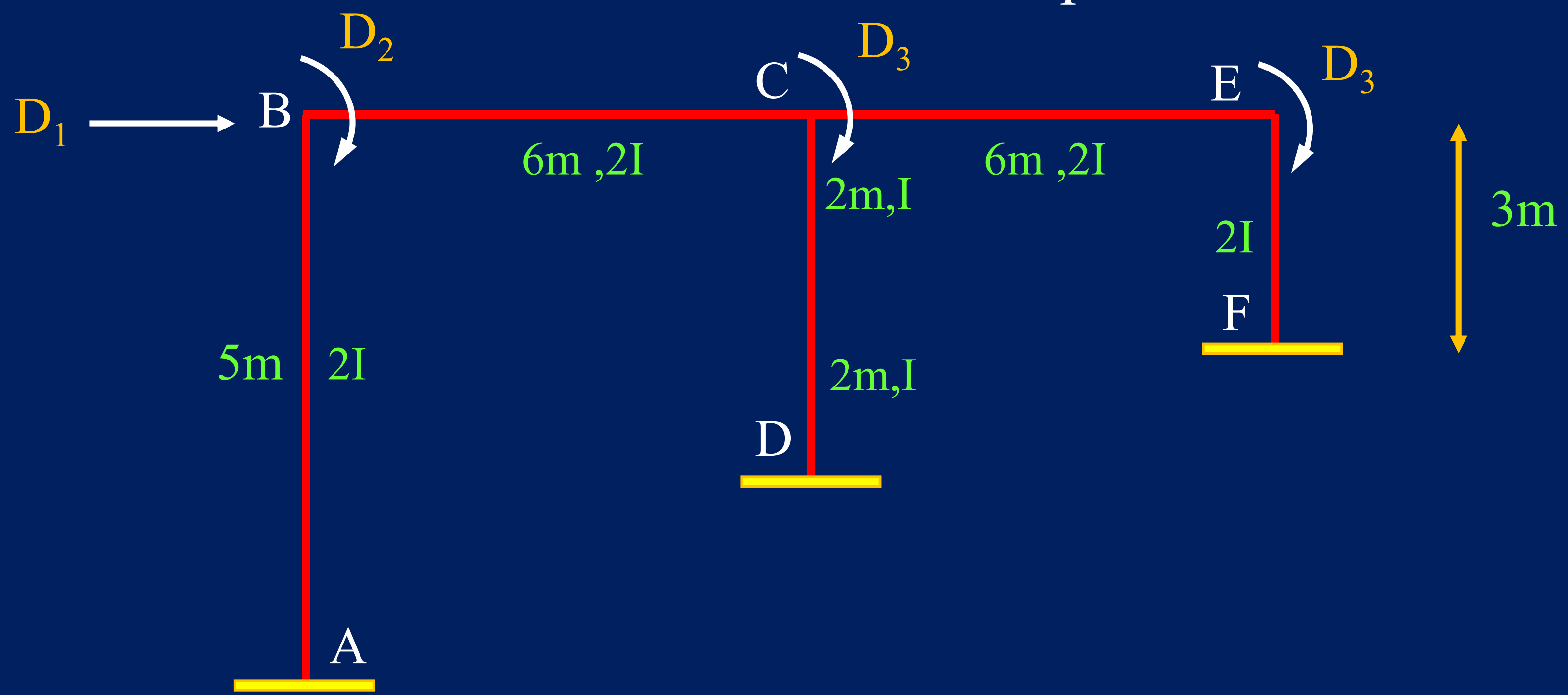


Take EI = constant

K.I = 4 degree (neglecting the axial effects)

Stiffness Method for Frames Analysis

- **Step # 01:** Selection of redundant Joint displacements and assign coordinates at those locations. Also compute AD values.

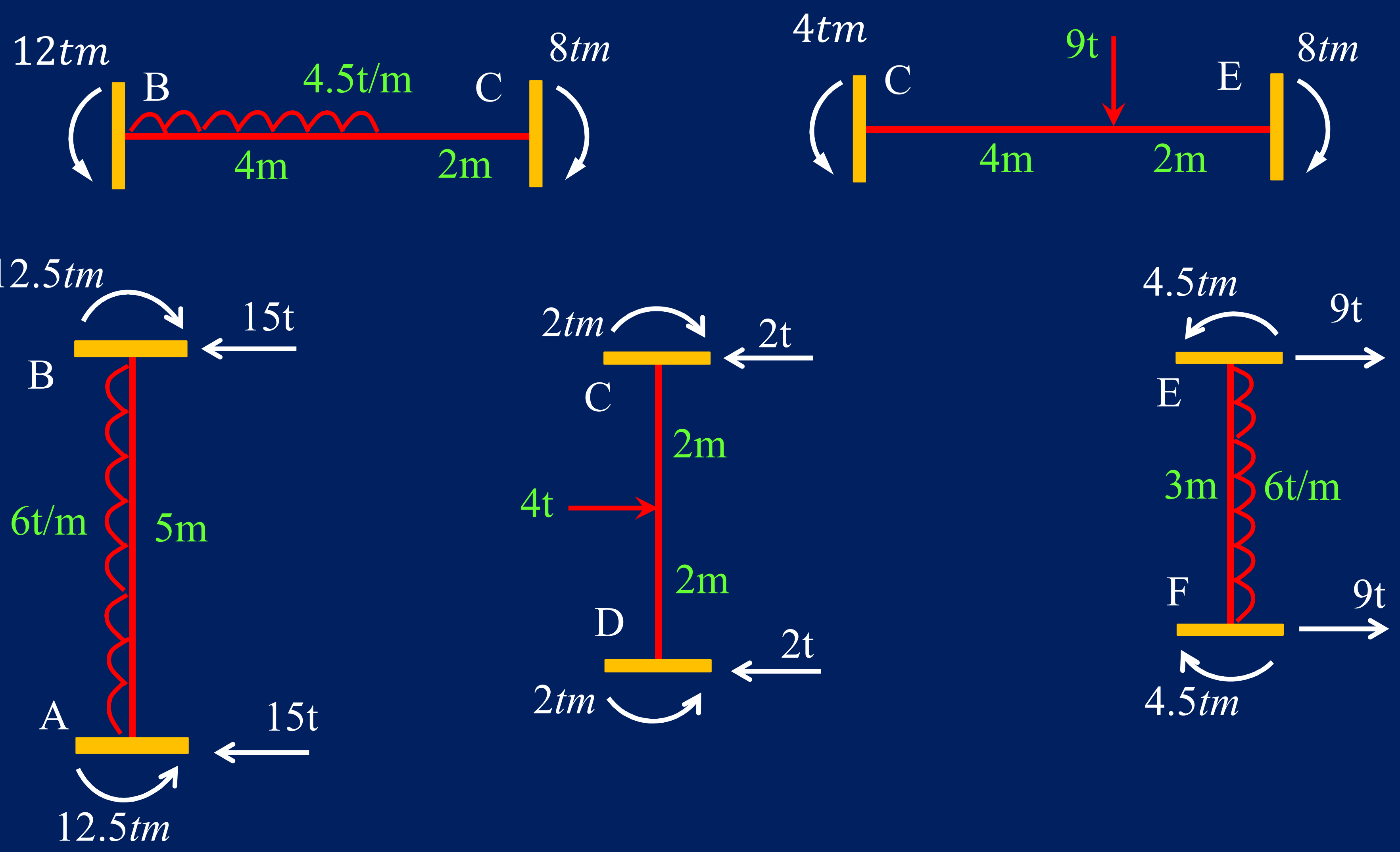


$$[D]_{4*1} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \end{bmatrix}$$

$$[AD]_{4*1} = \begin{bmatrix} AD_1 \\ AD_2 \\ AD_3 \\ AD_4 \end{bmatrix} = \begin{bmatrix} 10t \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Stiffness Method for Frames Analysis

- **Step # 02 :** Computation of ADL matrix.(Fixed end actions).



Stiffness Method for Frames Analysis

- **Step # 02 :** Computation of ADL matrix.(Fixed end actions).

$$ADL_1 = -15 - 2 + 9 = -8.0t$$

$$ADL_2 = 12.5 - 12 = -0.5t - m$$

$$ADL_3 = 8 + 2 + 4 = 6.0t - m$$

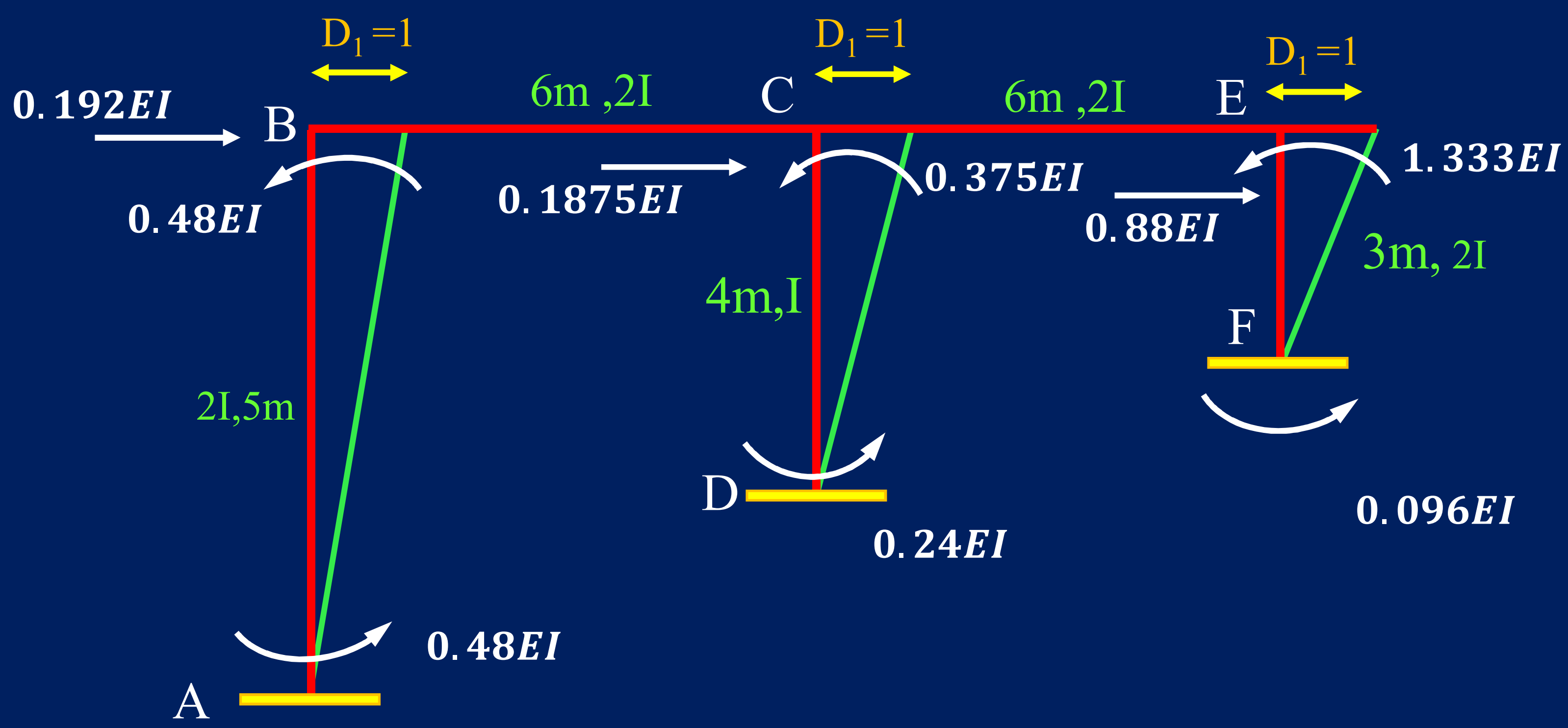
$$ADL_4 = 8 - 4.5 = 3.5t - m$$

$$[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \\ ADL_3 \\ ADL_4 \end{bmatrix} = \begin{bmatrix} -266.67 \\ 266.67 \\ 0 \\ 3.5 \end{bmatrix}$$

Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

i. When $D_1 = 1$ & $D_2 = D_3 = D_4 = 0$ as shown.



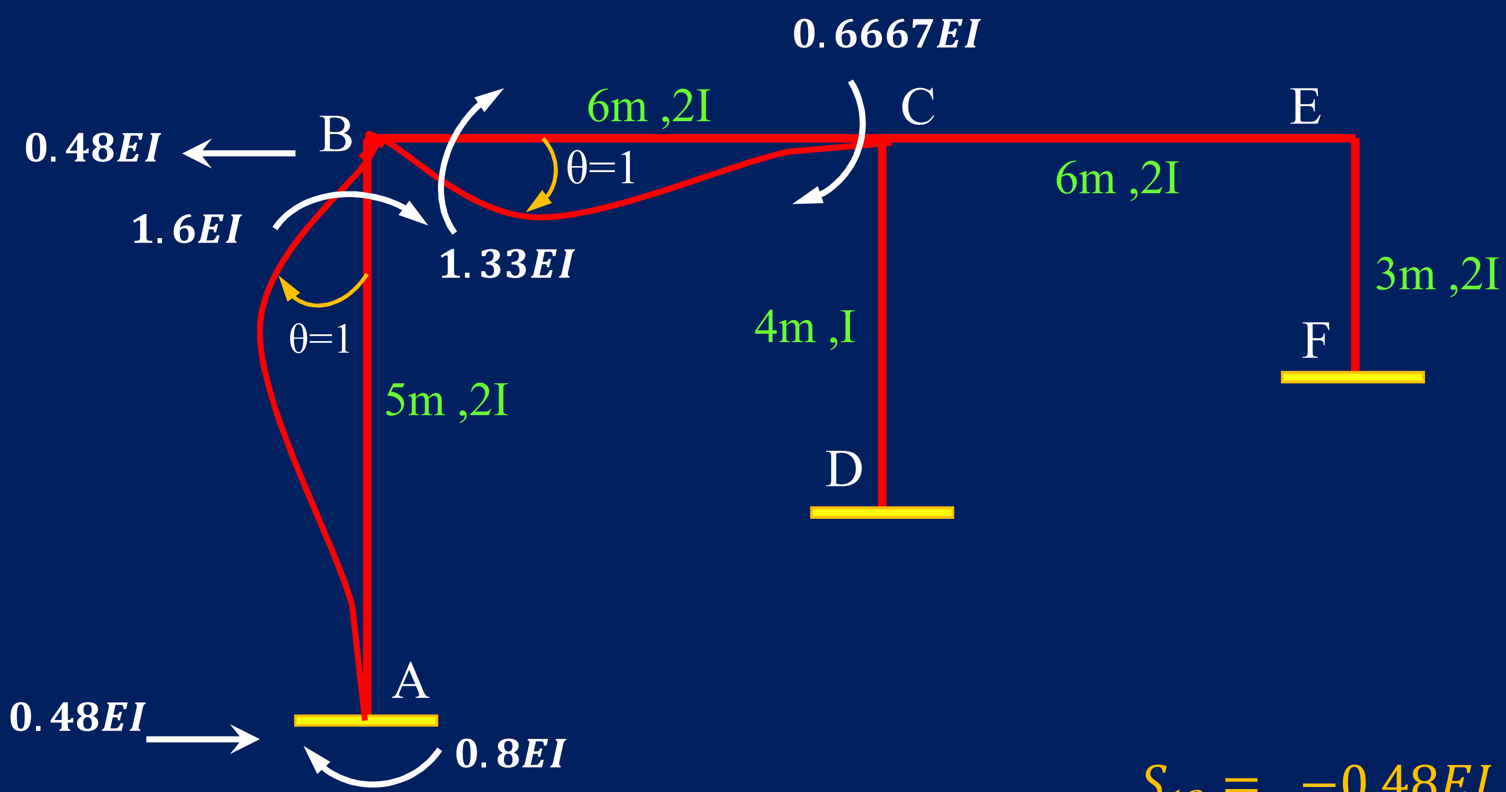
$$S_{11} = (0.192 + 0.1875 + 0.88)EI = 1.2683EI$$

$$S_{21} = -0.48EI \qquad S_{31} = -0.375EI \qquad S_{41} = -1.333EI$$

Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

ii. When $D_2 = 1$ & $D_1 = D_3 = D_4 = 0$ as shown.



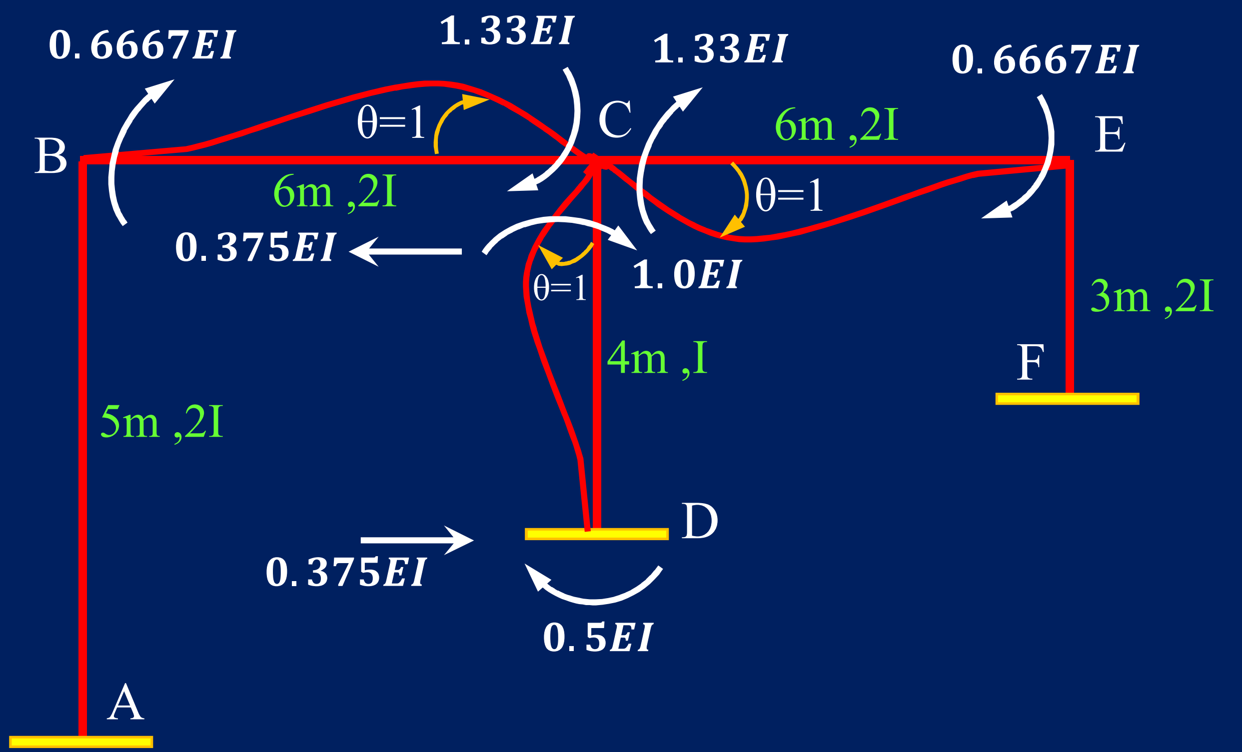
$S_{12} = \; -0.48EI$

$S_{22} = \; 1.6EI + 1.33EI = 2.933EI$ $\quad S_{32} = 0.6667EI$ $\quad S_{42} = 0$

Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

iii. When $D_3 = 1$ & $D_1 = D_2 = D_4 = 0$ as shown.



$S_{13} = -0.375EI$

$S_{23} = 0.6667EI$

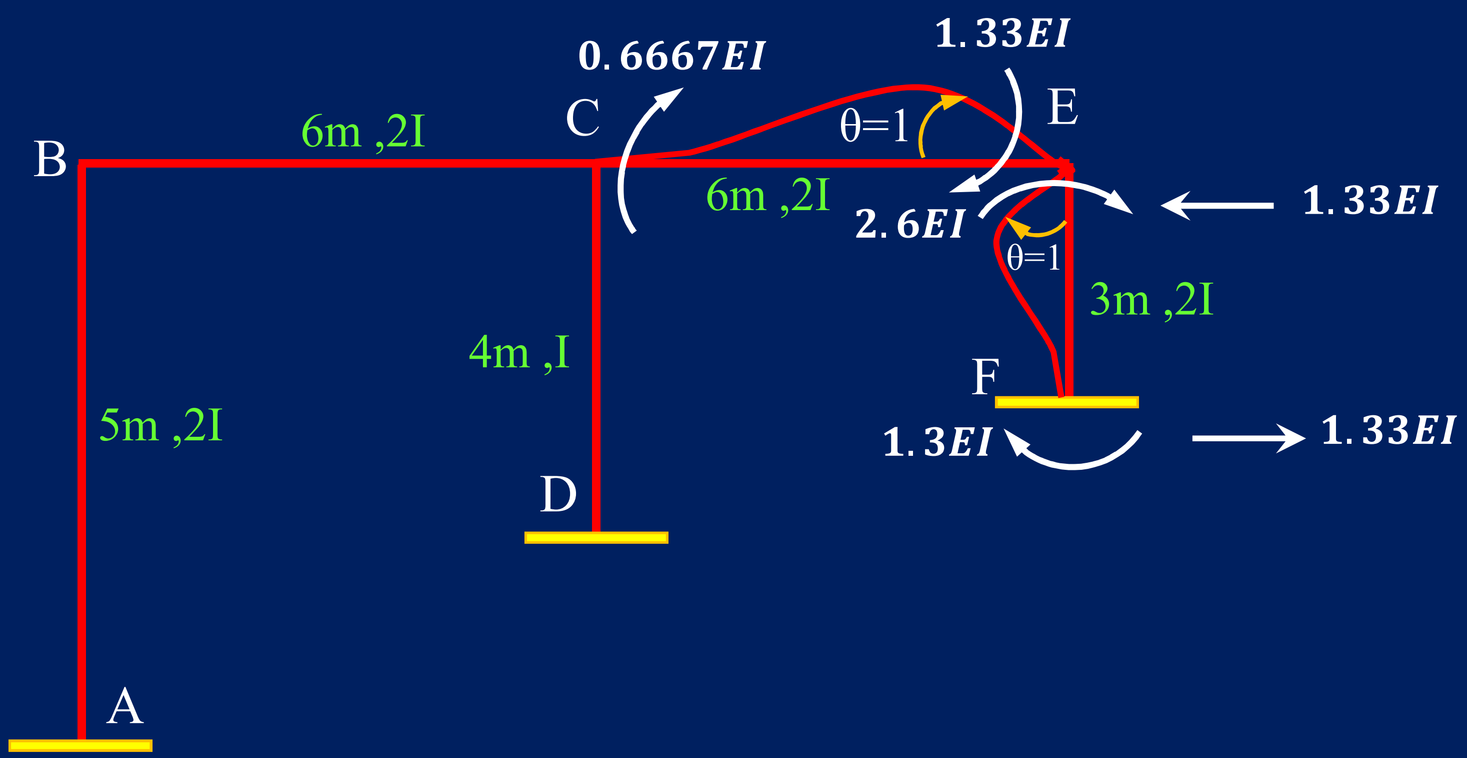
$S_{32} = 1.33EI + 1.33EI + 1EI = 3.6667EI$

$S_{43} = 0.6667EI$

Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

iv. When $D_4 = 1$ & $D_1 = D_2 = D_3 = 0$ as shown.



$S_{14} = -1.333EI$ $\quad$ $S_{24} = 0EI$

$S_{34} = 0.6667EI$ $\quad$ $S_{43} = 1.333EI + 2.666EI = 4.0EI$

Stiffness Method for Frames Analysis

Step # 03 : Contd…

$S_{11} = 1.2683EI$ $S_{21} = -0.48EI$ $S_{31} = -0.375EI$ $S_{41} = -1.333EI$

$S_{12} = -0.48EI$ $S_{22} = 2.933EI$ $S_{32} = 0.6667EI$ $S_{42} = 0$

$S_{13} = -0.375EI$ $S_{23} = 0.6667EI$ $S_{33} = 3.6667EI$ $S_{43} = 0.6667EI$

$S_{14} = -1.333EI$ $S_{24} = 0$ $S_{34} = 0.6667EI$ $S_{44} = 4.00EI$

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} = EI \begin{bmatrix} 1.2683 & -0.48 & -0.375 & -1.333 \\ -0.48 & 2.933 & 0.6667 & 0 \\ -0.375 & 0.6667 & 3.6667 & 0.6667 \\ -1.333 & 0 & 0.6667 & 4.00 \end{bmatrix}$$

Stiffness Method for Frames Analysis

Step # 04: Apply equilibrium condition at the location of the redundant joint displacement to write equilibrium equations and solve for unknown joint displacement.

$$[AD]_{3\,*\,1} = [ADL]_{3\,*\,1} + [S]_{3\,*\,3} \bullet [D]_{3\,*\,1}$$

$$[\boldsymbol{D}] = [\boldsymbol{S}]^{-1} \bullet [\boldsymbol{AD} - \boldsymbol{ADL}]$$

Stiffness Method for Frames Analysis

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}^{-1} \begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \\ AD_3 - ADL_3 \\ AD_4 - ADL_4 \end{bmatrix}$$

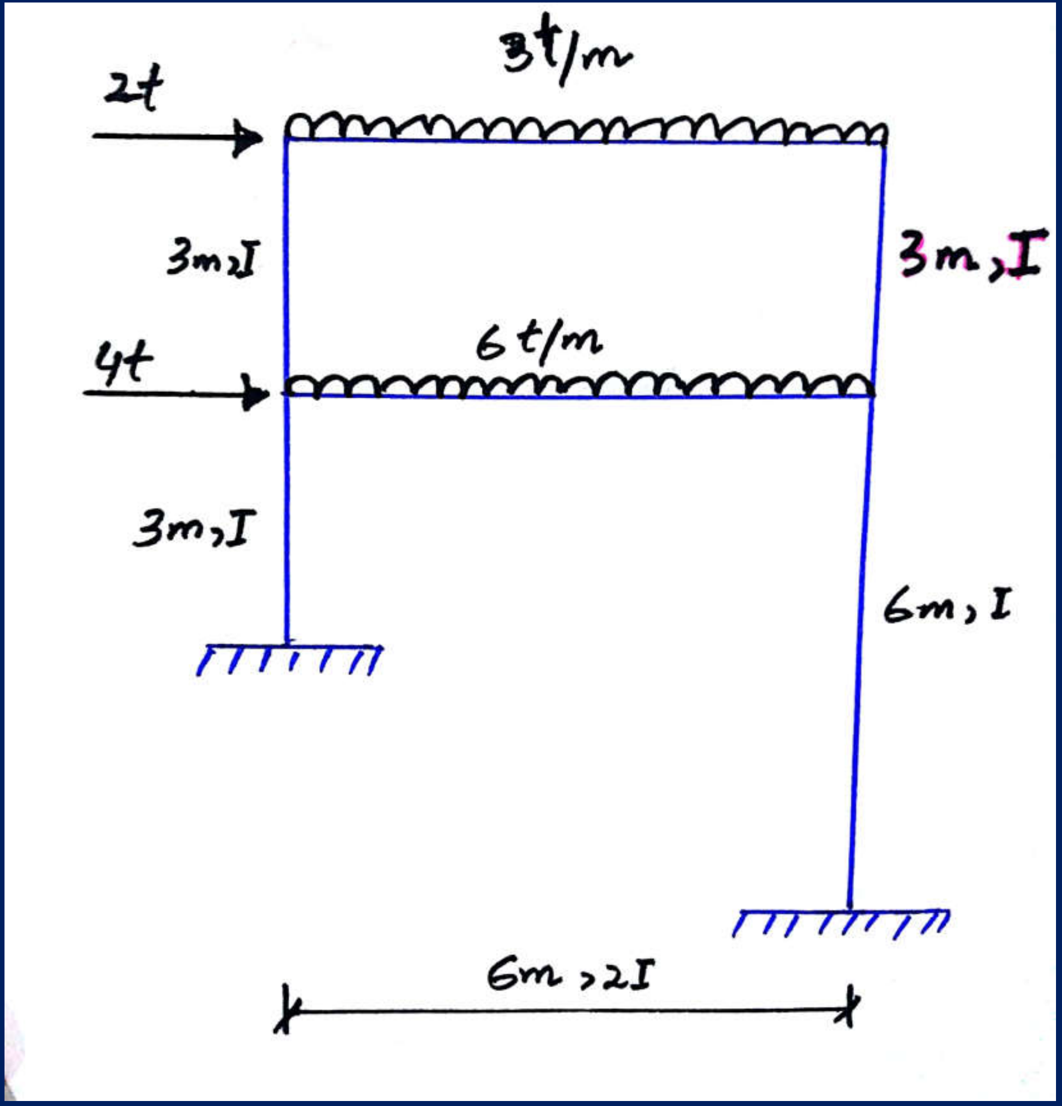
$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 1.2683 & -0.48 & -0.375 & -1.333 \\ -0.48 & 2.933 & 0.6667 & 0 \\ -0.375 & 0.6667 & 3.6667 & 0.6667 \\ -1.333 & 0 & 0.6667 & 4.00 \end{bmatrix}^{-1} \begin{bmatrix} 10 - (-8.0) \\ 0 - (0.5) \\ 0 - (6.0) \\ 0 - 3.5 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 22.40 \\ 3.786 \\ -1.271 \\ 6.802 \end{bmatrix}$$

Step # 05: Compute the member end actions & then draw shear force and bending moment diagram. Class WORK

Stiffness Method for Frames Analysis

Problem 08: Analyze the given Frame using stiffness method.

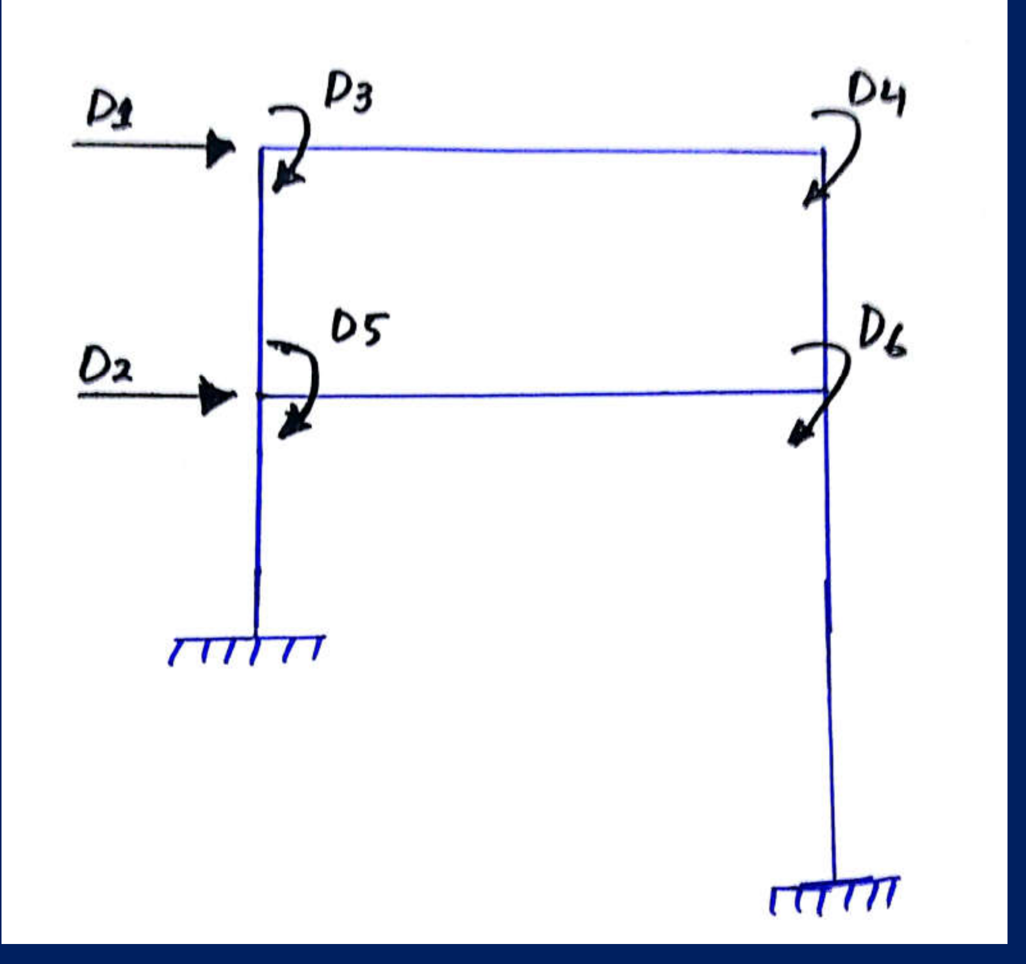


K.I = 6 degree

Take EI = constant

Stiffness Method for Frames Analysis

- **Step # 01:** Selection of redundant Joint displacements and assign coordinates at those locations. Also compute AD values.

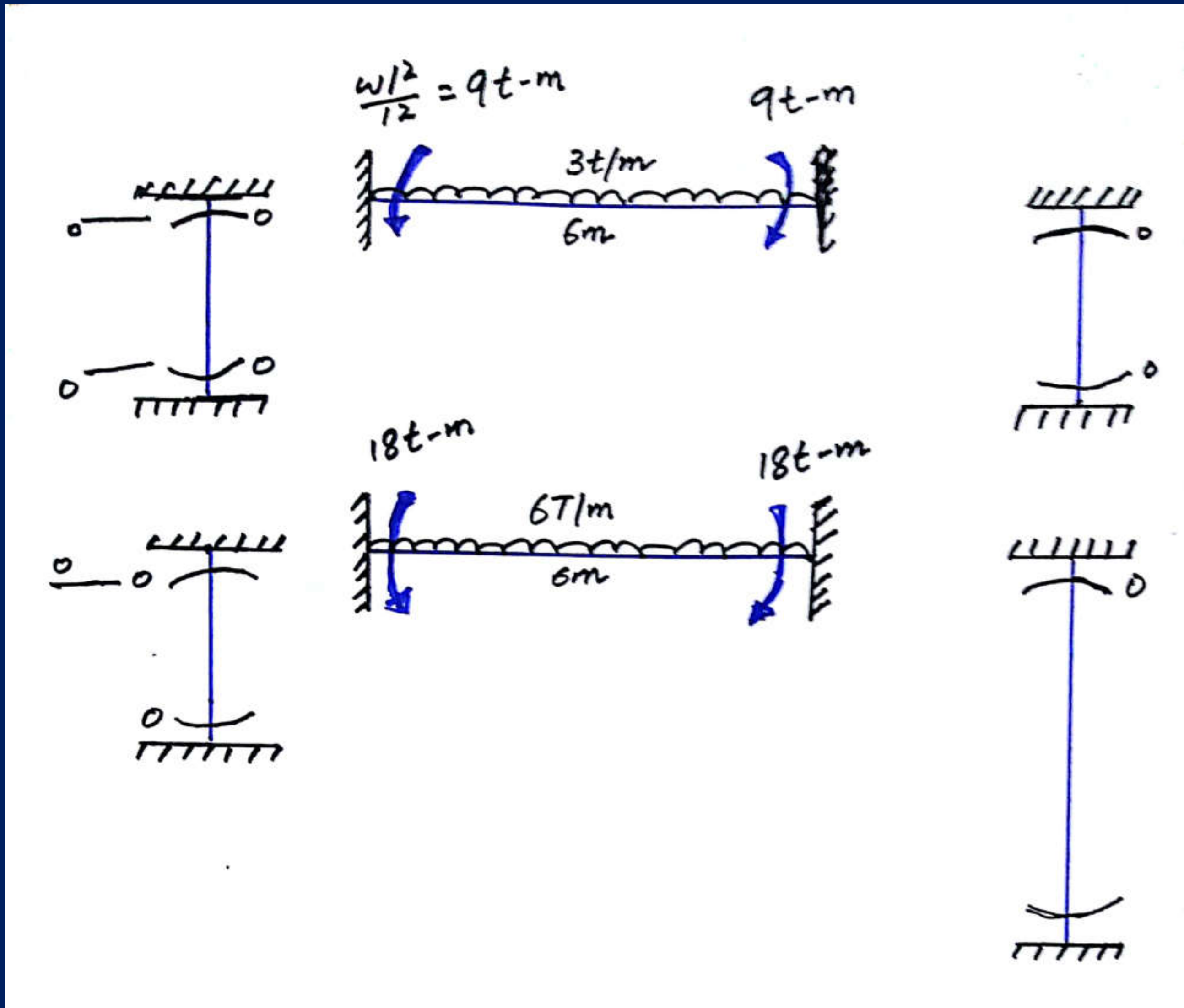


$$[AD]_{6*1} = \begin{bmatrix} AD_1 \\ AD_2 \\ AD_3 \\ AD_4 \\ AD_5 \\ AD_6 \end{bmatrix} = \begin{bmatrix} 2t \\ 4t \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[D]_{6*1} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix} = [?]$$

Stiffness Method for Frames Analysis

- **Step # 02 :** Computation of ADL matrix.(Fixed end actions).



Stiffness Method for Frames Analysis

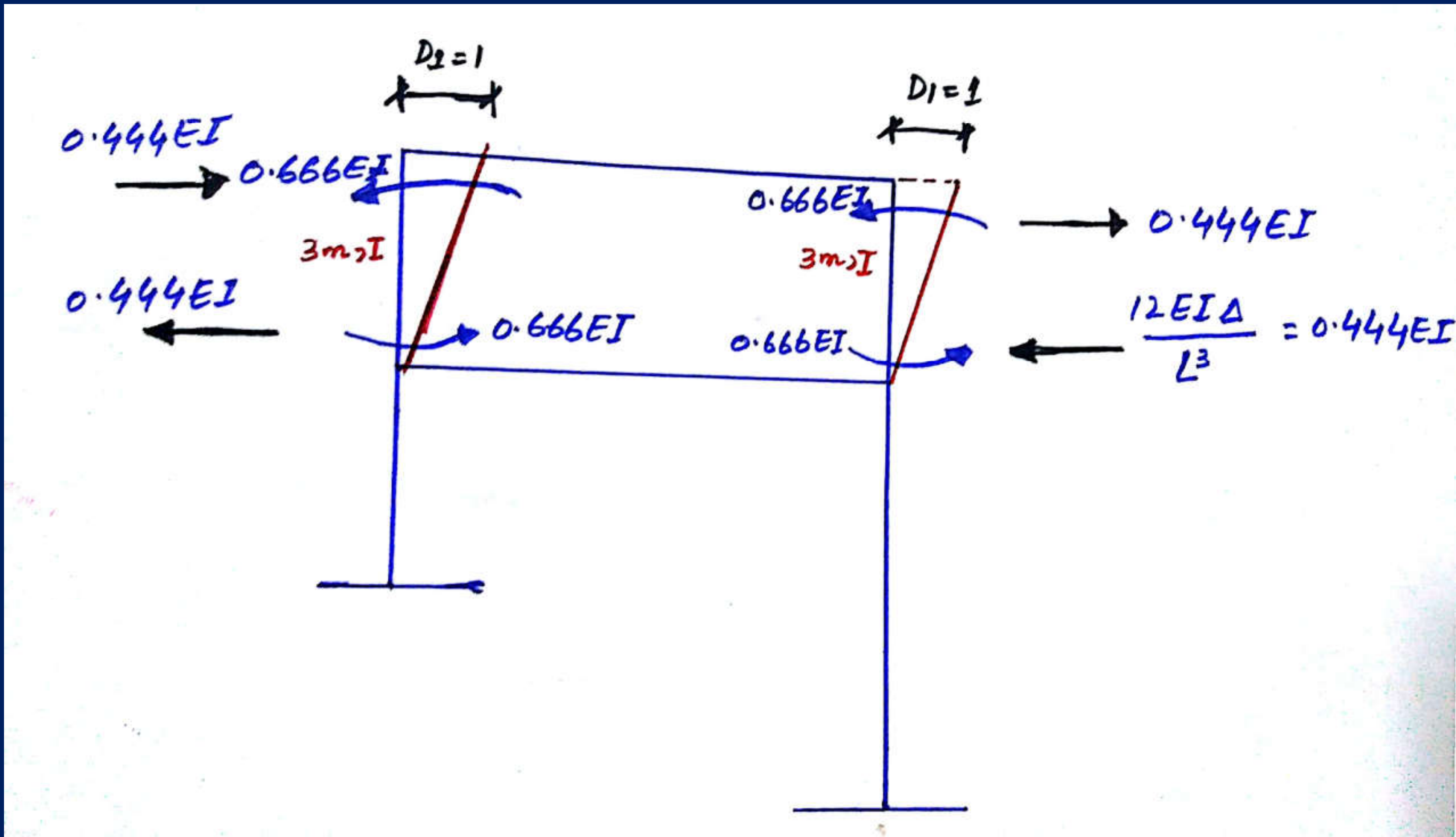
- **Step # 02 :** Computation of ADL matrix.(Fixed end actions).

$$[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \\ ADL_3 \\ ADL_4 \\ ADL_5 \\ ADL_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -9 \\ 9 \\ -18 \\ 18 \end{bmatrix}$$

Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

i. When $D_1 = 1$ & $D_2 = D_3 = D_4 = D_5 = D_6 = 0$ as shown.



Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

i. When $D_1 = 1$ & $D_2 = D_3 = D_4 = D_5 = D_6 = 0$ as shown.

$$S_{11} = (0.444 + 0.444)EI = 0.888EI$$

$$S_{21} = -0.888EI$$

$$S_{31} = -0.666EI$$

$$S_{41} = -0.666EI$$

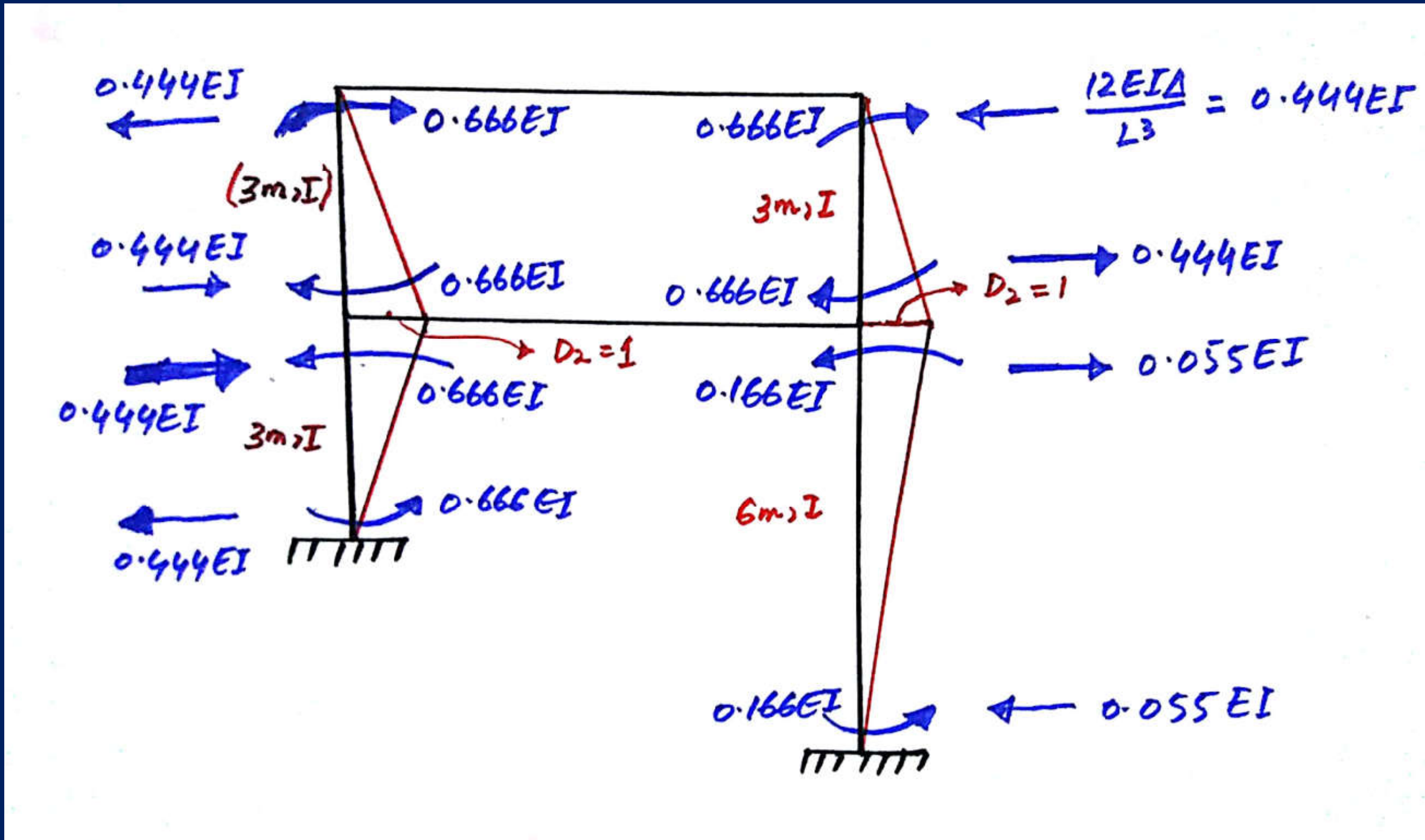
$$S_{51} = -0.666EI$$

$$S_{61} = -0.666EI$$

Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

ii. When $D_2 = 1$ & $D_1 = D_3 = D_4 = D_5 = D_6 = 0$ as shown.



Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

ii. When $D_2 = 1$ & $D_1 = D_3 = D_4 = D_5 = D_6 = 0$ as shown.

$$S_{12} = -0.888EI$$

$$S_{22} = 1.387EI$$

$$S_{32} = 0.666EI$$

$$S_{42} = 0.666EI$$

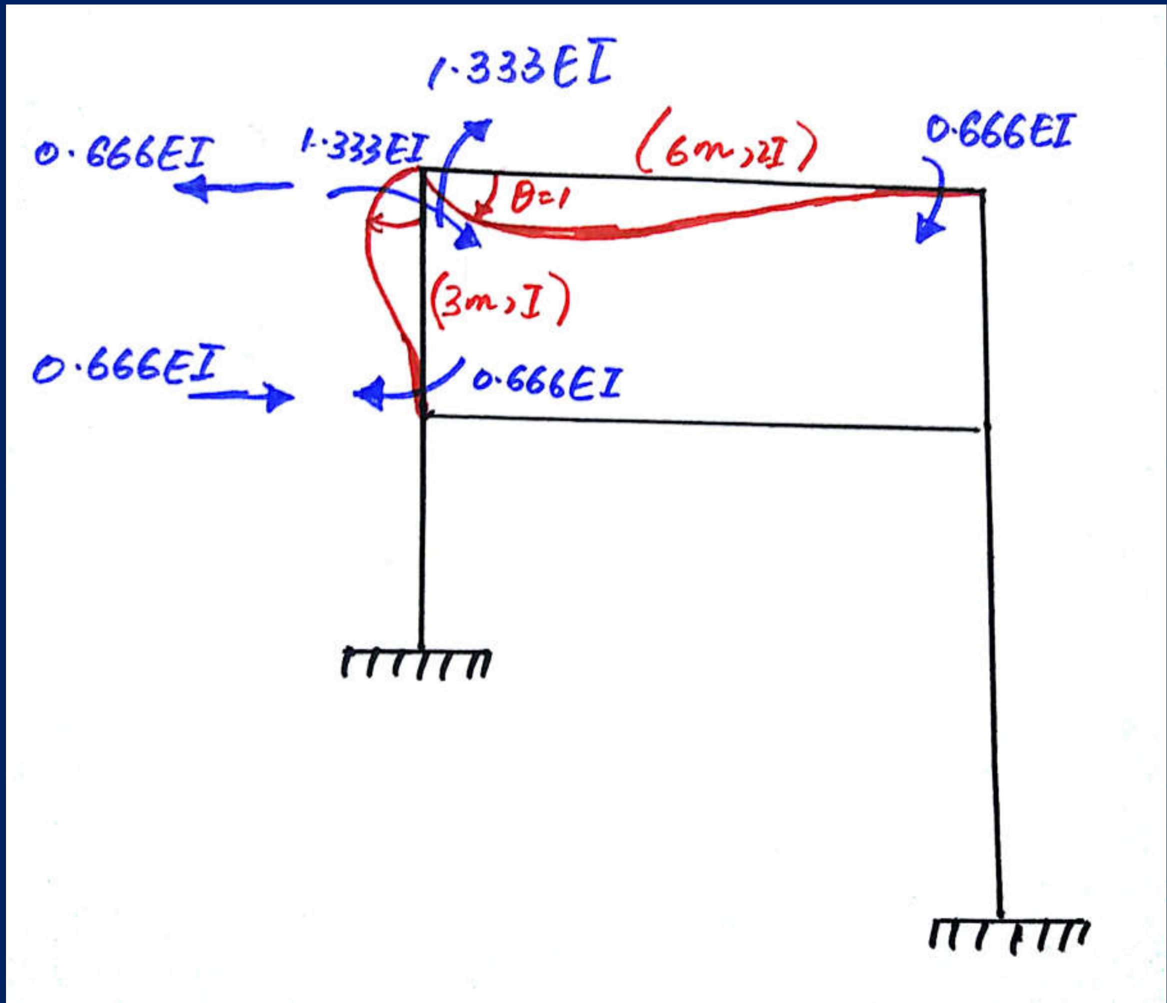
$$S_{52} = 0\ EI$$

$$S_{62} = 0.5EI$$

Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

iii. When $D_3 = 1$ & $D_1 = D_2 = D_4 = D_5 = D_6 = 0$ as shown.



Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

iii. When $D_3 = 1$ & $D_1 = D_2 = D_4 = D_5 = D_6 = 0$ as shown.

$$S_{13} = -0.666EI$$

$$S_{23} = 0.666EI$$

$$S_{33} = 2.666EI$$

$$S_{43} = 0.666EI$$

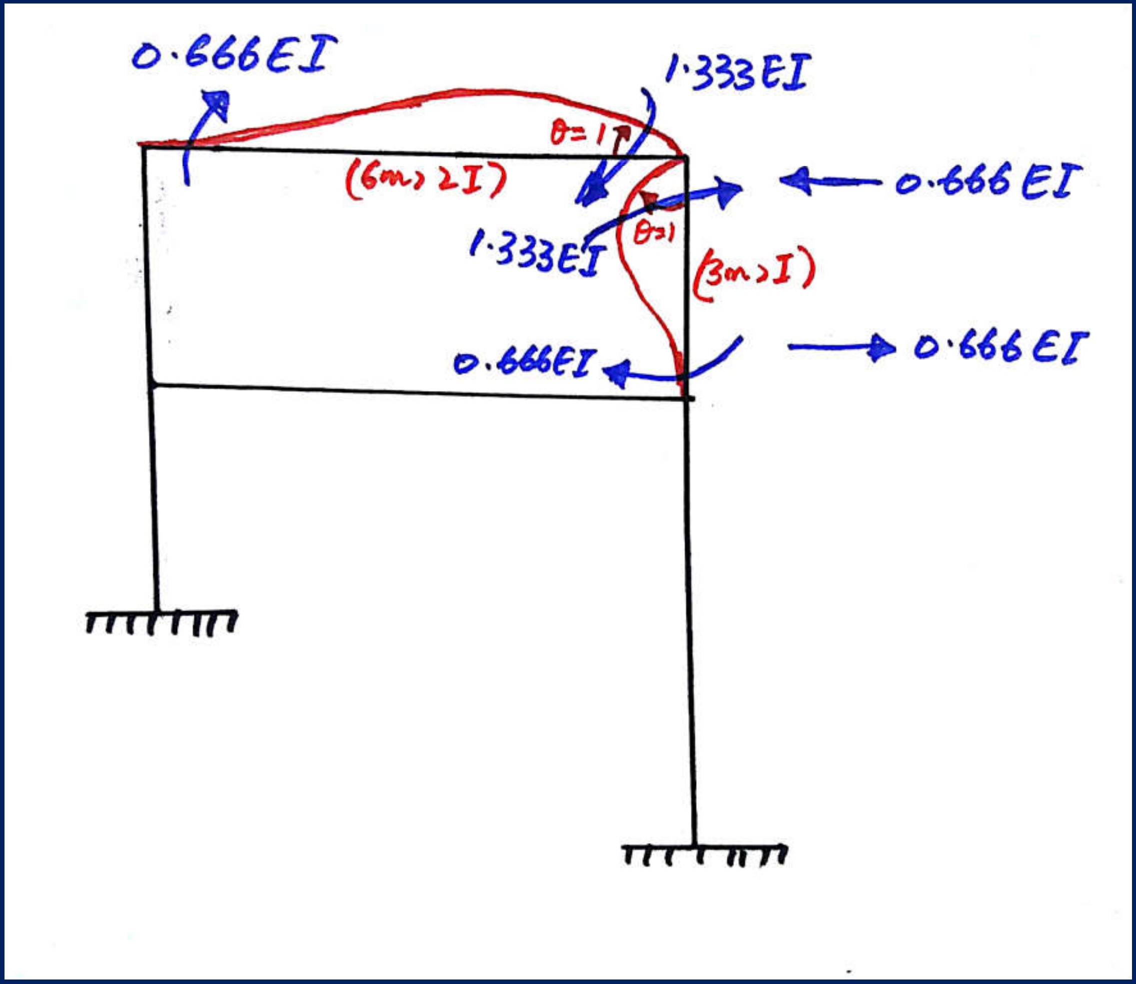
$$S_{53} = 0.666\ EI$$

$$S_{63} = 0\ EI$$

Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

iv. When $D_4 = 1$ & $D_1 = D_2 = D_3 = D_5 = D_6 = 0$ as shown.



Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

iv. When $D_4 = 1$ & $D_1 = D_2 = D_3 = D_5 = D_6 = 0$ as shown.

$$S_{14} = -0.666EI$$

$$S_{24} = 0.666EI$$

$$S_{34} = 0.666EI$$

$$S_{44} = 2.666EI$$

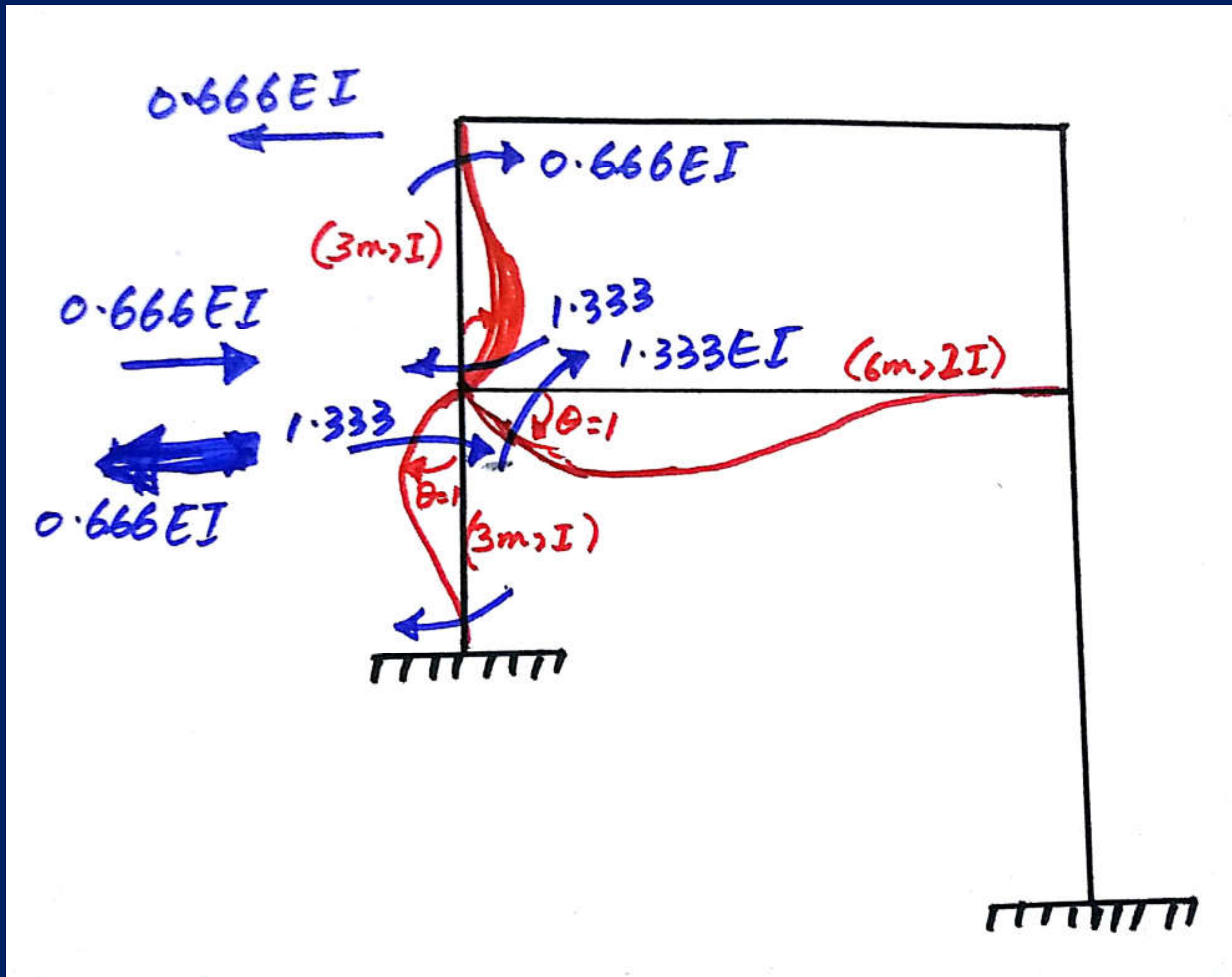
$$S_{54} = 0EI$$

$$S_{64} = 0.666\ EI$$

Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

v. When $D_5 = 1$ & $D_1 = D_2 = D_3 = D_4 = D_6 = 0$ as shown.



Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

v. When $D_5 = 1$ & $D_1 = D_2 = D_3 = D_4 = D_6 = 0$ as shown.

$$S_{15} = \ -0.666EI$$

$$S_{25} = 0EI$$

$$S_{35} = 0.666EI$$

$$S_{45} = 0EI$$

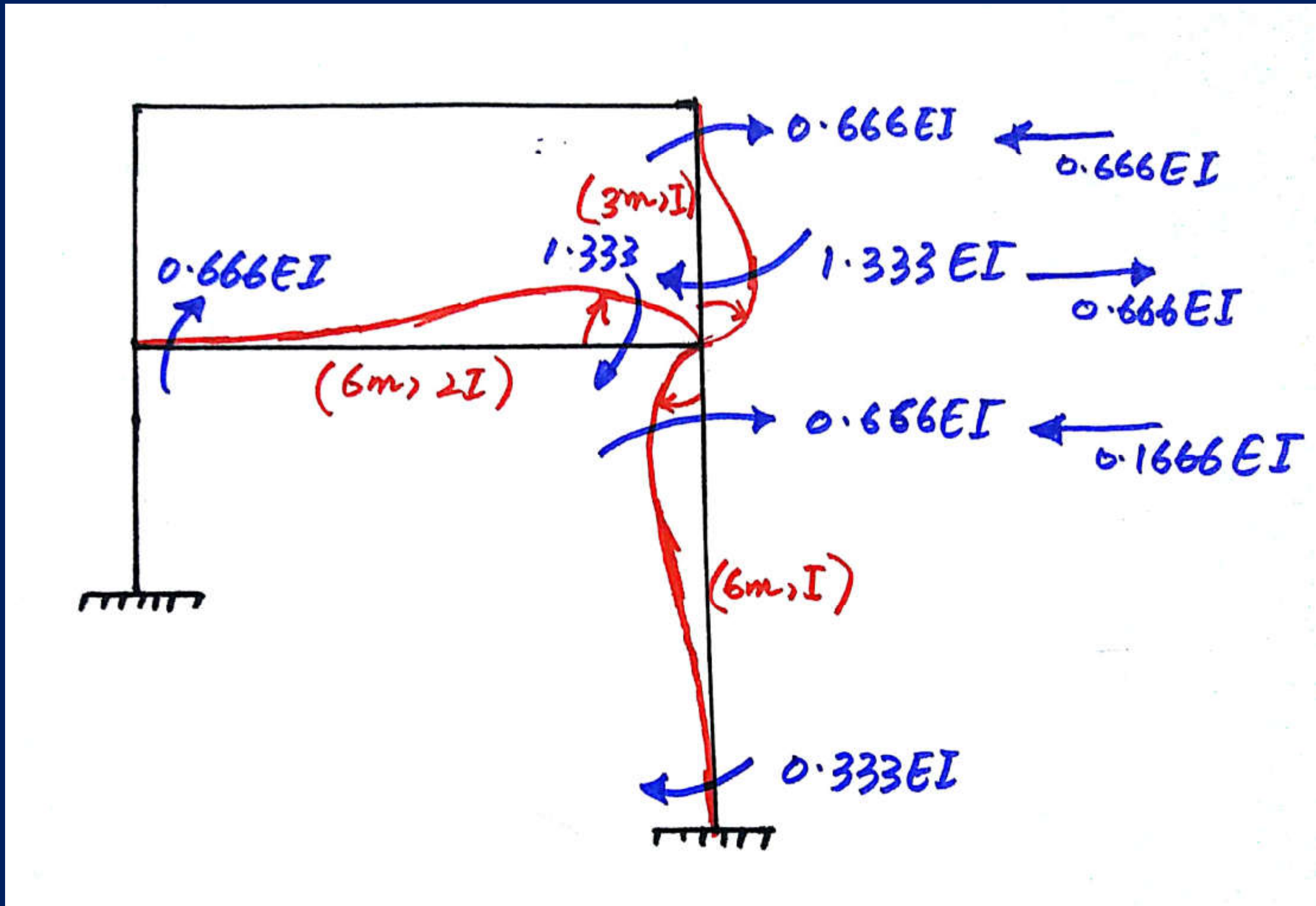
$$S_{55} = 4.0EI$$

$$S_{65} = 0.666\ EI$$

Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

vi. When $D_6 = 1$ & $D_1 = D_2 = D_3 = D_4 = D_5 = 0$ as shown.



Stiffness Method for Frames Analysis

Step # 03 : Computation of stiffness coefficient matrix

vi. When $D_6 = 1$ & $D_1 = D_2 = D_3 = D_4 = D_5 = 0$ as shown.

$$S_{16} = -0.666EI$$

$$S_{26} = 0.5EI$$

$$S_{36} = 0EI$$

$$S_{46} = 0.666EI$$

$$S_{56} = 0.666EI$$

$$S_{66} = 3.332\ EI$$

Stiffness Method for Frames Analysis

Step # 03 : Contd…

$$[\,S\,] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{15} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{35} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix}$$

$$= EI \begin{bmatrix} 0.888 & -0.888 & -0.666 & -0.666 & -0.666 & -0.666 \\ -0.888 & 1.387 & 0.666 & 0.666 & 0 & 0.5 \\ -0.666 & 0.666 & 2.666 & 0.666 & 0.666 & 0 \\ -0.666 & 0.666 & 0.666 & 2.666 & 0 & 0.666 \\ -0.666 & 0 & 0.666 & 0 & 4.0 & 0.666 \\ -0.666 & 0.5 & 0 & 0.666 & 0.666 & 3.332 \end{bmatrix}$$

Stiffness Method for Frames Analysis

Step # 04: Apply equilibrium condition at the location of the redundant joint displacement to write equilibrium equations and solve for unknown joint displacement.

$$[AD]_{3\,*\,1} = [ADL]_{3\,*\,1} + [S]_{3\,*\,3} \bullet [D]_{3\,*\,1}$$

$$[\boldsymbol{D}] = [\boldsymbol{S}]^{-1} \bullet [\boldsymbol{AD} - \boldsymbol{ADL}]$$

Stiffness Method for Frames Analysis

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.888 & -0.888 & -0.666 & -0.666 & -0.666 & -0.666 \\ -0.888 & 1.387 & 0.666 & 0.666 & 0 & 0.5 \\ -0.666 & 0.666 & 2.666 & 0.666 & 0.666 & 0 \\ -0.666 & 0.666 & 0.666 & 2.666 & 0 & 0.666 \\ -0.666 & 0 & 0.666 & 0 & 4.0 & 0.666 \\ -0.666 & 0.5 & 0 & 0.666 & 0.666 & 3.332 \end{bmatrix}^{-1} \begin{bmatrix} 2t - 0 \\ 4t - 0 \\ 0 + 9 \\ 0 - 9 \\ 0 + 18 \\ 0 - 18 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 6.6186 \\ 2.8619 \\ 3.3155 \\ 1.9350 \\ 5.9258 \\ -5.3151 \end{bmatrix}$$

Step # 05: Compute the member end actions & then draw shear force and bending moment diagram.

Stiffness Method for Frames Analysis

Assignment 04(b).

Q1. Find the member end actions and the draw shear force and bending moment diagram for the frames in problem 5,6,7 & 8.

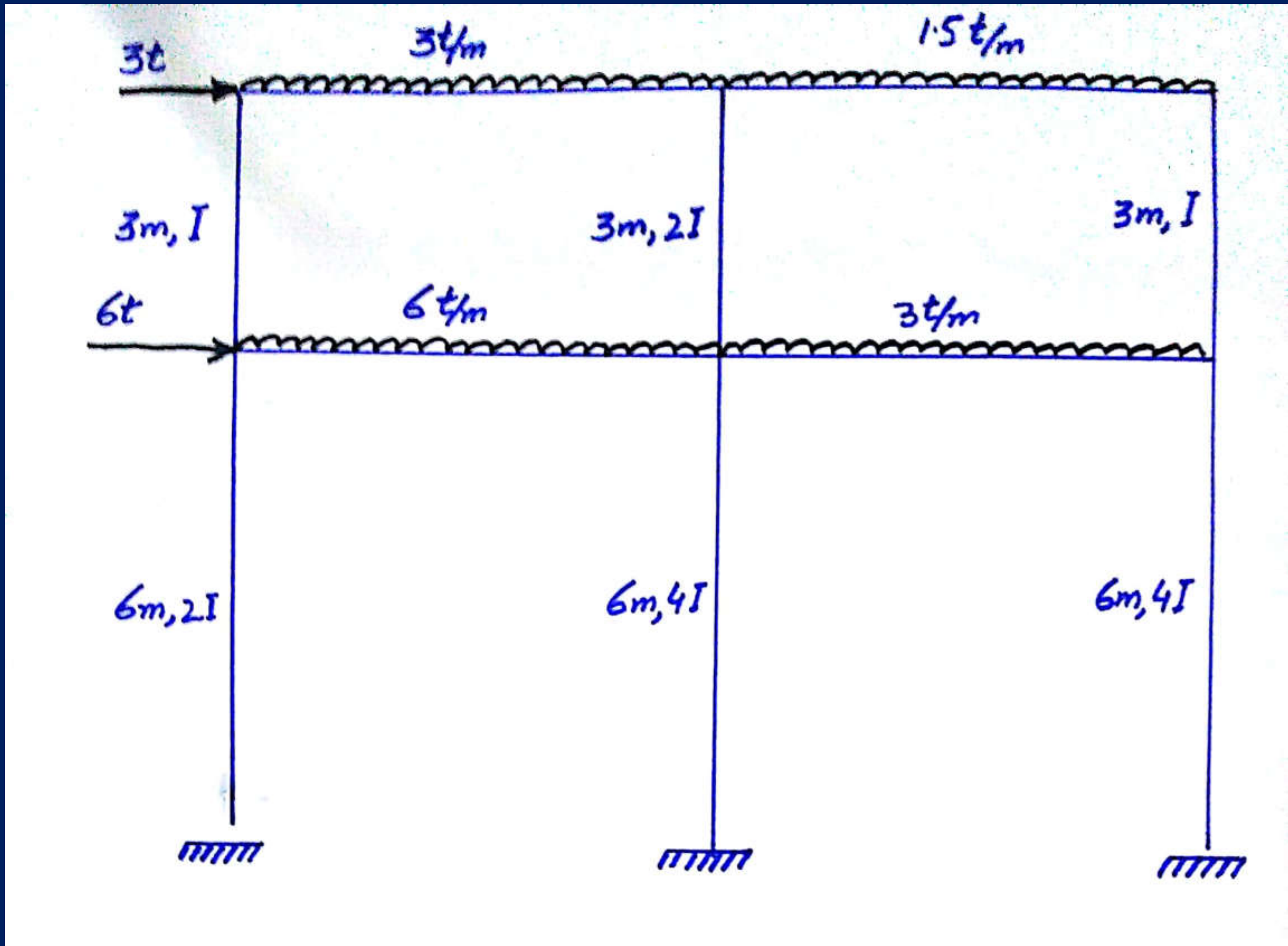
Q2. Develop stiffness matrix for the frame shown in fig. on next slide.

Note: submit assignment 4(a & b) both in next class.

Stiffness Method for Frames Analysis

Assignment 04(b).

Q2.



References

- Structural Analysis by R. C. Hibbeler
- Matrix structural analysis by William Mc Guire
- Matrix analysis of frame structures by William Weaver
- Online Civil Engineering blogs