University of Engineering & Technology Peshawar, Pakistan

CE301: Structure Analysis II

Module 06:

Analysis of S.I Beams Using stiffness method

By:
Prof. Dr. Bashir Alam
Civil Engineering Department
UET , Peshawar

Topics to be Covered

- Introduction
- Prerequisites for using stiffness method
- Step wise procedure of stiffness method for beam analysis
- Analysis of beam Example 1
- Example 2
- Example 3
- Assignment 03 (b)

Stiffness Method for Beams Analysis

❑ **Introduction:**

Beams are analyzed with stiffness method due to

- To solve the problem in matrix notation, which is more systematic
- To compute reactions at all the supports.
- To compute internal resisting shear & bending moment at any section of the continuous beam.

Stiffness Method for Beams Analysis

❑ **Prerequisites for Analysis with stiffness method:**

It is necessary that students must have strong background of the following concepts before starting analysis with stiffness or any other matrix method.

- Enough concept of Matrix Algebra
- Must be able to find the Kinematic Indeterminacy
- Must know the formulas & concept of fixed end actions

Stiffness Method for Beams Analysis

❑ **Step wise Solution Procedure using Stiffness method method:**

The following steps must be followed while solving a structure using Stiffness method.

- **Step # 01:** Make the structure kinametically determinate, by restraining the joints i.e select the redundant joint displacement.

Stiffness Method for Beams Analysis

Step # 02: Apply the actual external loads on the BKDS (Basic kinametically determinate structure) and find the actions at the locations of redundant joints (compute fixed end actions) this will generate ADL matrix.

Step # 03: Apply the redundant joint displacement on the BKDS (To standardize the procedure, only a unit displacement is applied in the +ve direction) this will generate stiffness coefficient matrix.

Stiffness Method for Beams Analysis

Step # 04: Apply equilibrium condition at the location of the redundant joint displacement to write equilibrium equations and solve for unknown joint displacement.

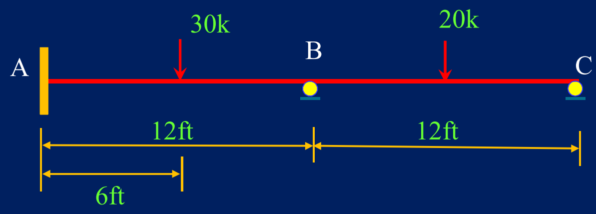
$$[AD] = [ADL] + [S] \bullet [D]$$

$$[D] = [S]^{-1} \bullet [AD - ADL]$$

Step # 05: Compute the member end actions .

Stiffness Method for Beams Analysis

Problem 01: Analyze the given beam using stiffness method.

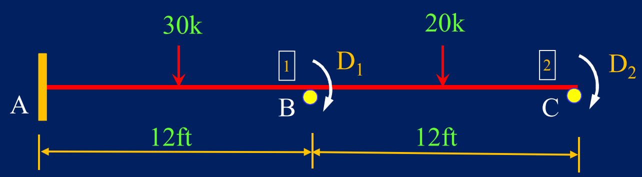


Take EI = constant

K.I = 2 degree (neglecting the axial effects)
So two redundant joint displacements should be chosen.

Stiffness Method for Beams Analysis

- **Step # 01:** Selection of redundant Joint displacements and assign coordinates at those locations. Also compute AD values.



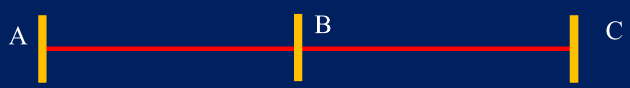
Rotation at B & C is taken as redundant joint displacement.

$$[D]_{2*1} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[AD]_{2*1} = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Stiffness Method for Beams Analysis

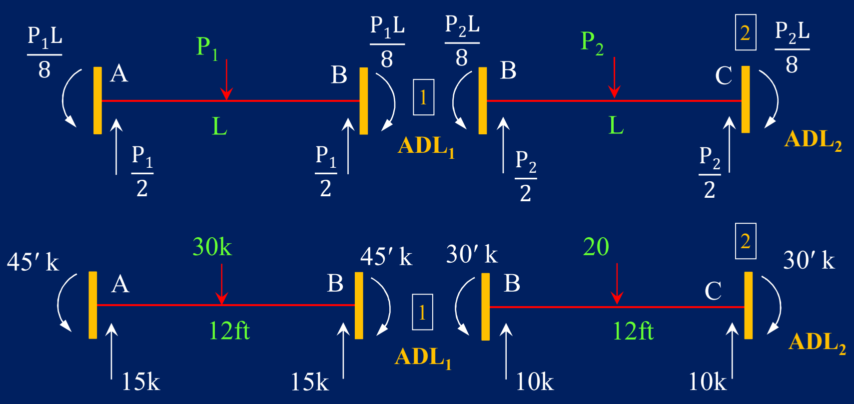
- Restrain all the degrees of freedom to get the restrained structure.



Basic kinematic determinate structure (BKDS) or restrained structure

Stiffness Method for Beams Analysis

- **Step # 02 :** Restrained structure acted upon by the actual loads. compute the values of actions in the restrained structure corresponding to the redundant locations. This will generate ADL matrix.(Fixed end actions)



Stiffness Method for Beams Analysis

$$ADL_1 = \; 45 - 30 = 15' \, k$$

$$ADL_2 = \; 30' \, k$$

$$[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 30 \end{bmatrix}$$

Note:
For rotation corresponding action is moment.

ADL_1 = moment in the restrained structure under the actual loads corresponding to redundant joint displacement 1.
ADL_2 = moment in the restrained structure under the actual loads corresponding to redundant joint displacement 2.

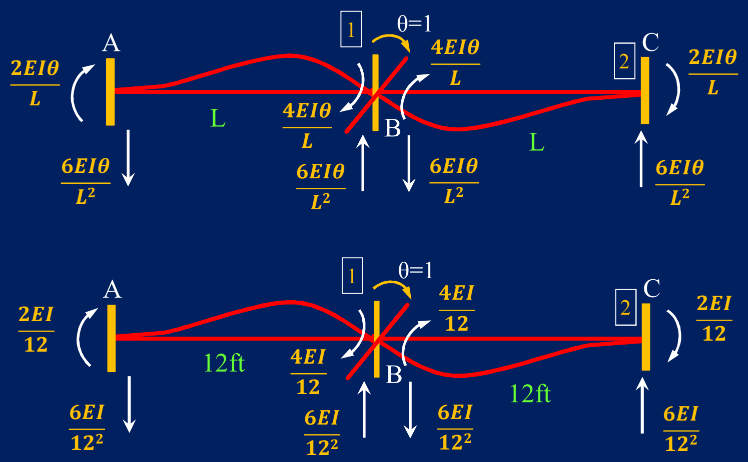
Stiffness Method for Beams Analysis

Step # 03 : Primary structure acted upon by a unit value of D & computation of stiffness coefficients " S" values in the BKDS corresponding to the redundant joint displacement locations.

- 1st a unit rotation is applied at location 1 & prevent it at 2 as shown. Compute the values of S_{11} and S_{21} .
- Then apply a unit rotation at the redundant displacement location 2 and prevented at 1 as shown. Compute the values of S_{12} & S_{22} .

Stiffness Method for Beams Analysis

i. 1st a unit rotation is applied at location 1 & prevent it at 2 (D_1=1 & D_2=0)as shown. Compute the values of S_{11} and S_{21} .



Stiffness Method for Beams Analysis

Step # 03 (i): Contd…

$$S_{11} = \frac{4EI}{12} + \frac{4EI}{12} \qquad S_{21} = \frac{2EI}{12}$$

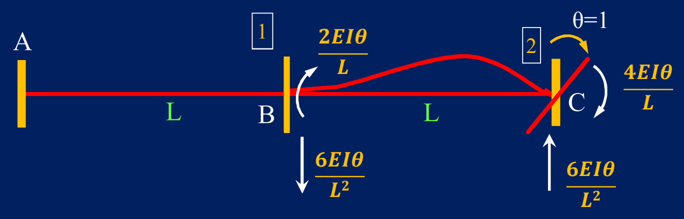
$$S_{11} = \frac{8\text{EI}}{12} \qquad S_{21} = \frac{2\text{EI}}{12}$$

S_{11} = Action(sum of moments in this case) in the BKDS at redundant displacement location 1 due to unit rotation at that location.

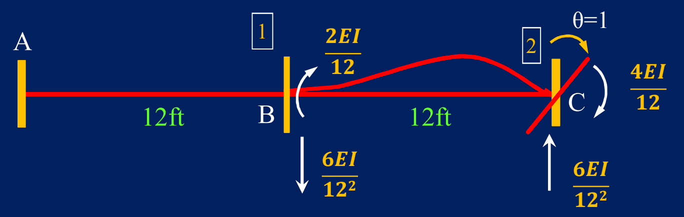
S_{21} = Moment in the BDS at redundant displacement location 2 due to a unit rotation applied at location 1

Stiffness Method for Beams Analysis

ii. Now a unit rotation is applied at the redundant displacement location 2 and prevented at 1 (D_2=1 & D_1=0) as shown. Compute the values of S_{12} & S_{22}.



where



Stiffness Method for Beams Analysis

Step # 03 : Contd…

$$S_{12} = \frac{2EI}{12} \qquad S_{21} = \frac{4EI}{12}$$

$$S_{11} = \frac{8EI}{12} \qquad S_{21} = \frac{4EI}{12}$$

S_{12} = Moment in the BKDS at redundant displacement location 1 due to unit rotation applied at redundant displacement location 2.

S_{22} = Moment in the BKDS at redundant displacement location 2 due to a unit rotation applied at that location.

Stiffness Method for Beams Analysis

Step # 03 : Contd…

$$S_{11} = \frac{8EI}{12} \qquad S_{12} = \frac{2EI}{12}$$

$$S_{21} = \frac{2EI}{12} \qquad S_{22} = \frac{4EI}{12}$$

$$[\,S\,] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$$[\,S\,] = EI \begin{bmatrix} \frac{8}{12} & \frac{2}{12} \\ \frac{2}{12} & \frac{4}{12} \end{bmatrix}$$

Stiffness coefficient matrix

Stiffness Method for Beams Analysis

Step # 04: Apply equilibrium condition at the location of the redundant joint displacement to write equilibrium equations and solve for unknown joint displacement.

$$AD_1 = ADL_1 + S_{11}D_1 + S_{12}D_2$$

$$AD_2 = ADL_2 + S_{21}D_1 + S_{22}D_2$$

$$\begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} + \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$[AD]_{2*1} = [ADL]_{2*1} + [S]_{2*2} \bullet [D]_{2*1}$$

$$[\boldsymbol{D}] = [\boldsymbol{S}]^{-1} \bullet [\boldsymbol{AD} - \boldsymbol{ADL}]$$

Stiffness Method for Beams Analysis

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix}$$

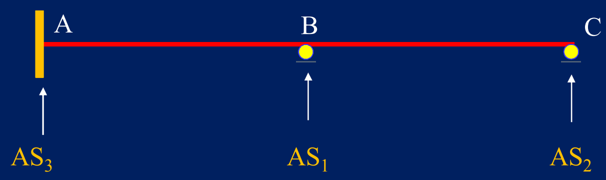
$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{12}{EI} \begin{bmatrix} 8 & 2 \\ 2 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 0 - 15 \\ 0 - 30 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -90 \end{bmatrix}$$

-ive sign shows that our assumed redundant joint displacement direction is wrong

Stiffness Method for Beams Analysis

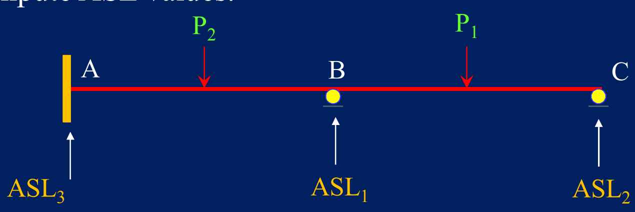
Step # 05(a): Before computing the member end action we can compute the support reactions directly using matrix approach .

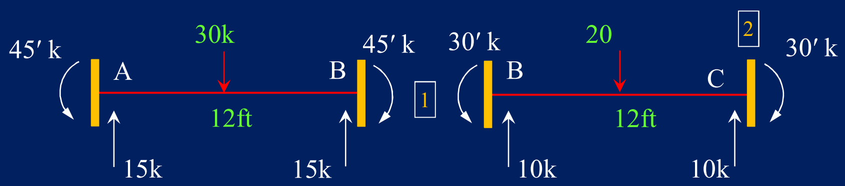


$$[AS] = [ASL] + [ASD][D]$$

Stiffness Method for Beams Analysis

i. Compute ASL values.



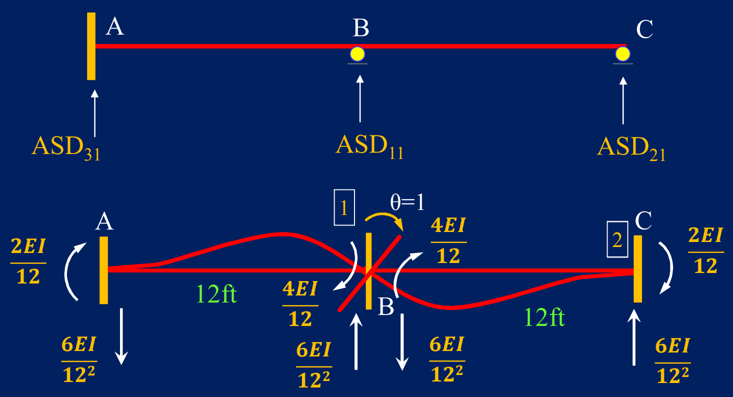


$$\begin{bmatrix} ASL_1 \\ ASL_2 \\ ASL_3 \end{bmatrix} = \begin{bmatrix} 25 \\ 10 \\ 15 \end{bmatrix}$$

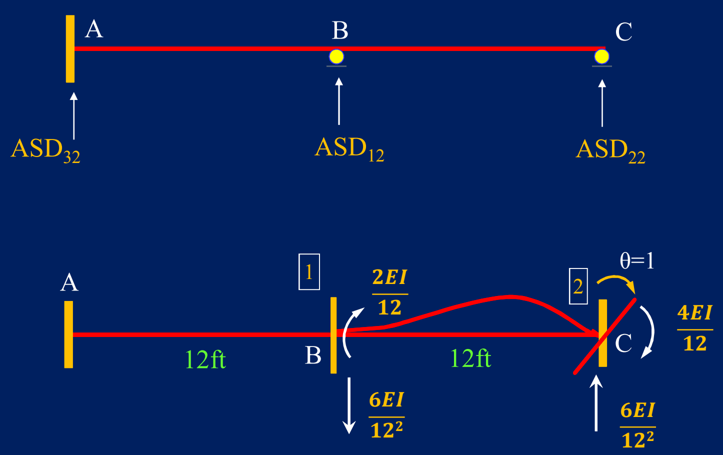
Stiffness Method for Beams Analysis

ii. Compute the ASD values.

- 1st apply a unit rotation at redundant location 1 and then at 2 in the restrained structure as shown below.



Stiffness Method for Beams Analysis



Stiffness Method for Beams Analysis

So the ASD values are

$$[ASD]_{3*2} = \begin{bmatrix} ASD_{11} & ASD_{12} \\ ASD_{21} & ASD_{22} \\ ASD_{31} & ASD_{32} \end{bmatrix} = \text{EI}\begin{bmatrix} 0 & -0.041 \\ 0.041 & 0.041 \\ -0.5 & 0 \end{bmatrix}$$

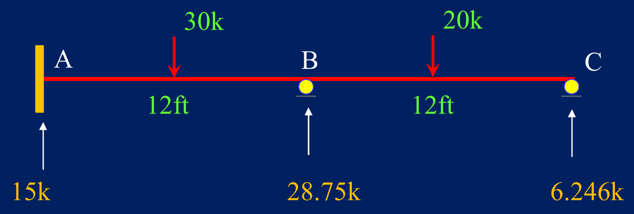
$$\begin{bmatrix} AS_1 \\ AS_2 \\ AS_3 \end{bmatrix} = \begin{bmatrix} ASL_1 \\ ASL_2 \\ ASL_3 \end{bmatrix} + \begin{bmatrix} ASD_{11} & ASD_{12} \\ ASD_{21} & ASD_{22} \\ ASD_{31} & ASD_{32} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

Stiffness Method for Beams Analysis

So the AS values are

$$\begin{bmatrix} AS_1 \\ AS_2 \\ AS_3 \end{bmatrix} = \begin{bmatrix} 25 \\ 10 \\ 15 \end{bmatrix} + \text{EI}\begin{bmatrix} 0 & -0.041 \\ 0.041 & 0.041 \\ -0.5 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -90 \end{bmatrix}$$

$$\begin{bmatrix} AS_1 \\ AS_2 \\ AS_3 \end{bmatrix} = \begin{bmatrix} 28.75 \\ 6.246 \\ 15 \end{bmatrix}$$

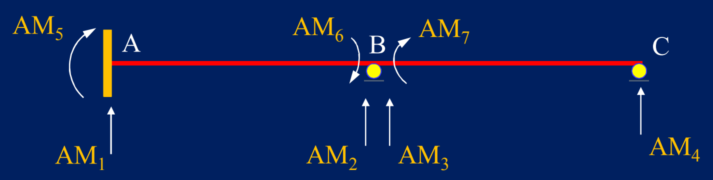


Structure with known support reactions

Stiffness Method for Beams Analysis

Step # 05(b): Compute the member end actions. As we know that

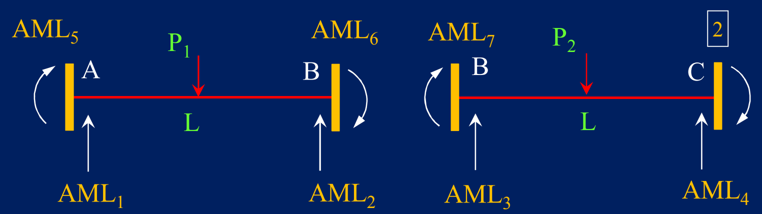
$$[AM] = [AML] + [AMR][AR]$$

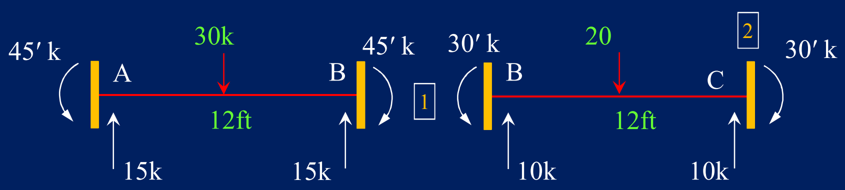


AM_n = is the member end action in the indeterminate structure at different location specified with a number "n". n shows the number of member end actions.

Stiffness Method for Beams Analysis

i. Compute AML values.





Stiffness Method for Beams Analysis

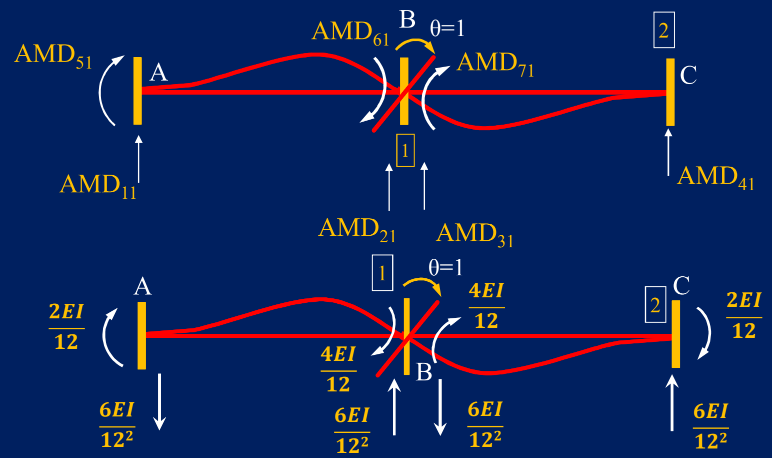
So AML values from the previous slide is.

$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \\ AML_7 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ 10 \\ 10 \\ -45 \\ 45 \\ -30 \end{bmatrix}$$

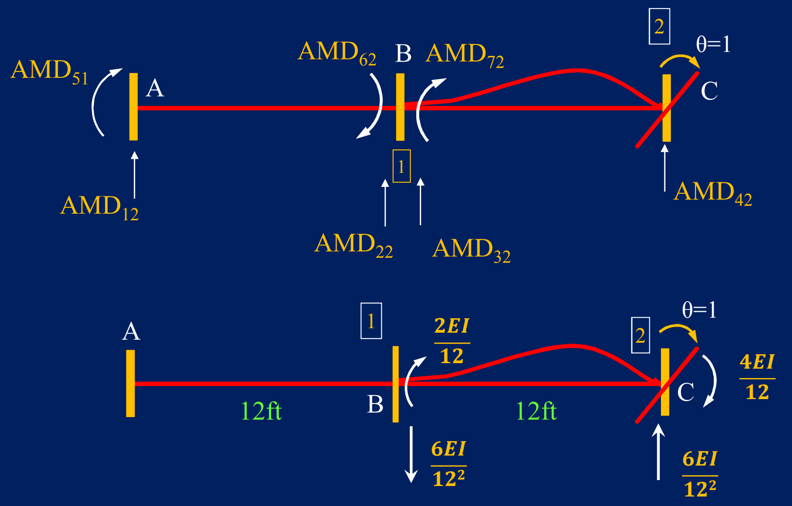
Stiffness Method for Beams Analysis

b) Compute the AMD values.

- 1st apply a unit rotation at redundant location 1 and then at 2 as shown below.



Stiffness Method for Beams Analysis



Stiffness Method for Beams Analysis

So the AMD values are

$$[AMD]_{7\,*\,2} = \begin{bmatrix} AMD_{11} & AMD_{12} \\ AMD_{21} & AMD_{22} \\ AMD_{31} & AMD_{32} \\ AMD_{41} & AMD_{42} \\ AMD_{51} & AMD_{52} \\ AMD_{61} & AMD_{62} \\ AMD_{71} & AMD_{72} \end{bmatrix} = \begin{bmatrix} -0.041 & 0 \\ 0.041 & 0 \\ -0.041 & -0.041 \\ 0.041 & 0.041 \\ 0.16 & 0 \\ 0.33 & 0 \\ 0.33 & 0.16 \end{bmatrix}$$

Stiffness Method for Beams Analysis

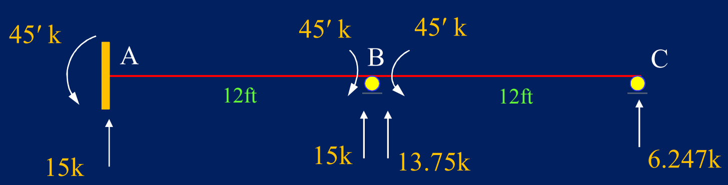
Now member end actions will be computed as given below

$$[AM] = [AML] + [AMR][AR]$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \\ AM_7 \end{bmatrix} = \begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \\ AML_7 \end{bmatrix} + \begin{bmatrix} AMD_{11} & AMD_{12} \\ AMD_{21} & AMD_{22} \\ AMD_{31} & AMD_{32} \\ AMD_{41} & AMD_{42} \\ AMD_{51} & AMD_{52} \\ AMD_{61} & AMD_{62} \\ AMD_{71} & AMD_{72} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

Stiffness Method for Beams Analysis

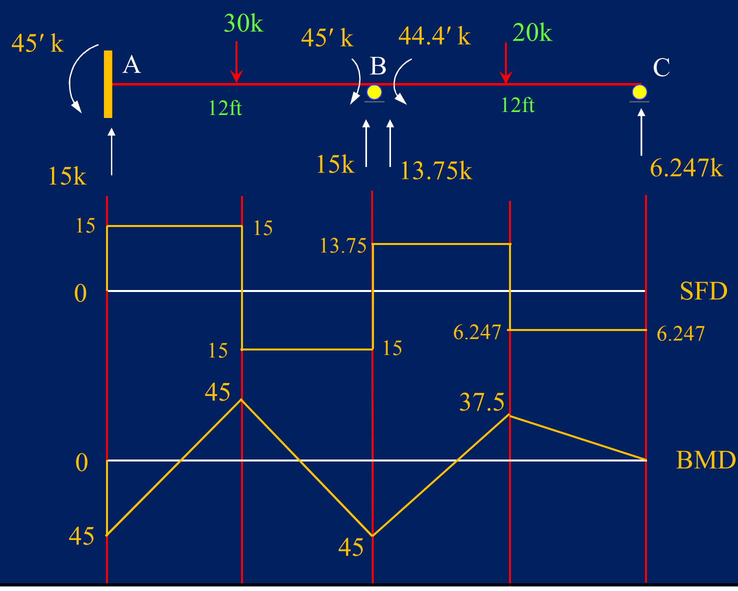
$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \\ AM_7 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ 10 \\ 10 \\ -45 \\ 45 \\ -30 \end{bmatrix} + \begin{bmatrix} -0.041 & 0 \\ 0.041 & 0 \\ -0.041 & -0.041 \\ 0.041 & 0.041 \\ 0.16 & 0 \\ 0.33 & 0 \\ 0.33 & 0.16 \end{bmatrix} \begin{bmatrix} 0 \\ -90 \end{bmatrix} = \begin{bmatrix} 15\ \text{k} \\ 15\ \text{k} \\ 13.75\ \text{k} \\ 6.247\ \text{k} \\ -45'\ \text{k} \\ 45'\ \text{k} \\ -45'\ \text{k} \end{bmatrix}$$



Complete analyzed structure. Shear force and bending moment diagrams are given on next slide.

Stiffness Method for Beams Analysis

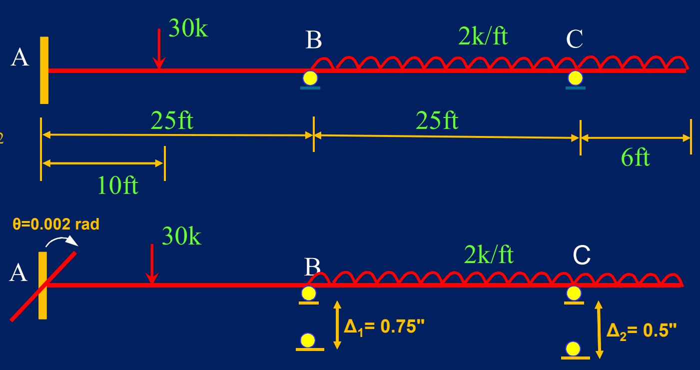
Now Shear force and Bending moment diagram



Flexibility Method for Beams Analysis

Problem 02: Analyze the given beam using stiffness method, if support A rotates by 0.002 rad clockwise, support B settles down by 0.75 in & support C settles down by 0.5 in.

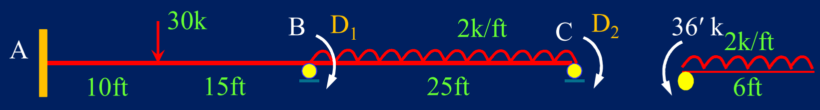
Take
E = 30000 ksi
I = 800 in^4
EI = 166666.6 k-ft^2



K.I = 2 degree
So two redundant joint displacements should be chosen.(ignoring the axial effects).

Stiffness Method for Beams Analysis

- **Step # 01:** Selection of redundant Joint displacements and assign coordinates at those locations. Also compute AD values.



The effect of overhanging part is taken as AD_2

Rotation at B & C is taken as redundant joint displacement.

$$[D]_{2*1} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[AD]_{2*1} = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 36 \end{bmatrix}$$

Stiffness Method for Beams Analysis

- Restrain all the degrees of freedom to get the restrained structure.

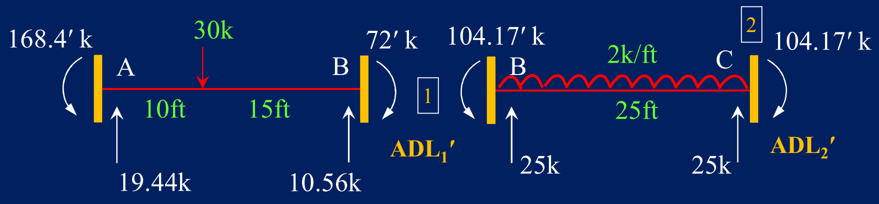
Basic kinematic determinate structure (BKDS) or restrained structure

Stiffness Method for Beams Analysis

- **Step # 02 : Compute ADL matrix.**

i. Due to direct loadings (ADL′)

Note:
For Fixed end actions see formulas tables in module 5.

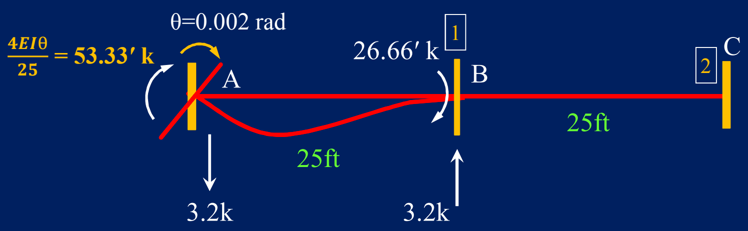


$$[ADL'] = \begin{bmatrix} ADL_1' \\ ADL_2' \end{bmatrix} = \begin{bmatrix} -32.17 \\ 104.17 \end{bmatrix}$$

Stiffness Method for Beams Analysis

- **Step # 02 : Compute ADL matrix.**

ii. Due to indirect loadings i.e due to 0.002 rad C.W rotation at A (ADL″)

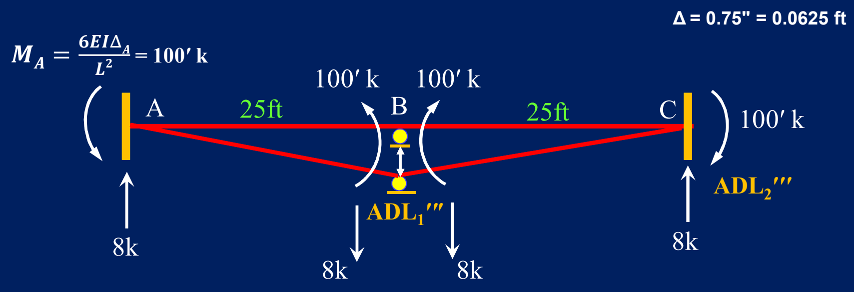


$$[ADL''] = \begin{bmatrix} ADL_1'' \\ ADL_2'' \end{bmatrix} = \begin{bmatrix} 26.66 \\ 0 \end{bmatrix}$$

Stiffness Method for Beams Analysis

- **Step # 02 : Compute ADL matrix.**

iii. Due to indirect loadings i.e due to settlement at B (ADL‴)

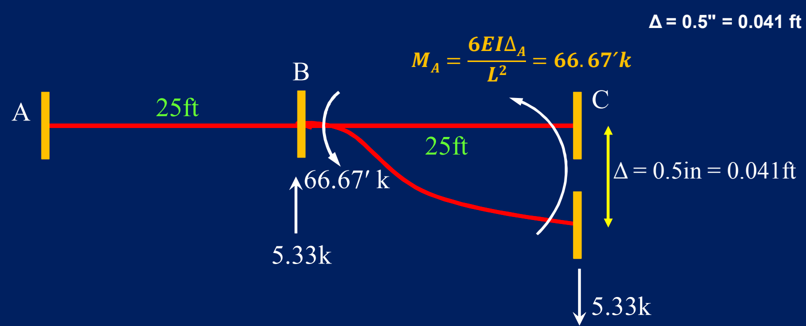


$$[ADL'''] = \begin{bmatrix} ADL_1''' \\ ADL_2''' \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \end{bmatrix}$$

Stiffness Method for Beams Analysis

- **Step # 02 : Compute ADL matrix.**

iv. Due to indirect loadings i.e due to settlement at B (ADL⁗)



$$[ADL''''] = \begin{bmatrix} ADL_1'''' \\ ADL_2'''' \end{bmatrix} = \begin{bmatrix} -66.67 \\ -66.67 \end{bmatrix}$$

Stiffness Method for Beams Analysis

$$ADL_1 = ADL_1' + ADL_1'' + ADL_1''' + ADL_1''''$$

$$ADL_1 = -32.17 + 26.67 + 0 - 67.67 = -72.17$$

$$ADL_2 = ADL_2' + ADL_2'' + ADL_2''' + ADL_2''''$$

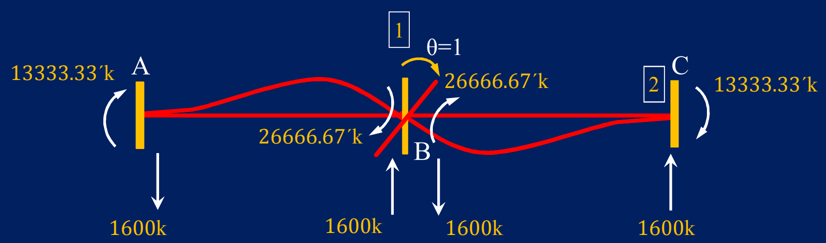
$$ADL_2 = 104.17 + 0 + 100 - 67.67 = 137.50$$

$$[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} = \begin{bmatrix} -72.17 \\ 137.50 \end{bmatrix}$$

Stiffness Method for Beams Analysis

Step # 03 : Computation of stiffness coefficients matrix

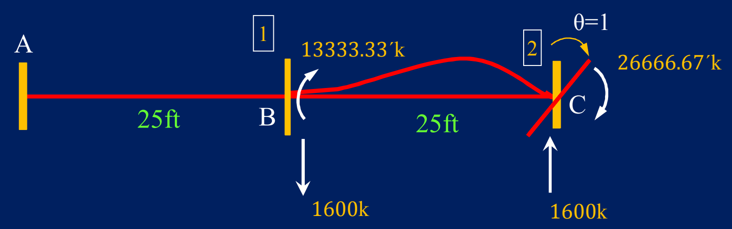
- 1st a unit rotation is applied at location 1 & prevent it at 2 (D_1= 1 & D_2=0) as shown. Compute the values of S_{11} and S_{21} .



$$S_{11} = 53333.33 \qquad S_{21} = 13333.33$$

Stiffness Method for Beams Analysis

ii. Now a unit rotation is applied at the redundant displacement location 2 and prevented at 1 ($D_1 = 0$ & $D_2 = 1$) as shown. Compute the values of S_{12} & S_{22}.



$S_{12} = 13333.33 \quad S_{22} = 26666.62$

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \Rightarrow [S] = \begin{bmatrix} 53333.33 & 13333.33 \\ 13333.33 & 26666.62 \end{bmatrix}$$

Stiffness Method for Beams Analysis

Step # 04: Apply equilibrium condition at the location of the redundant joint displacement to write equilibrium equations and solve for unknown joint displacement.

$$\begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} + \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix}$$

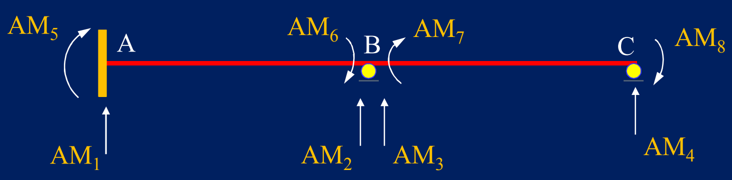
$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 53333.33 & 13333.33 \\ 13333.33 & 26666.62 \end{bmatrix}^{-1} \begin{bmatrix} 0 - (-72.17) \\ 36 - (137.50) \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.0026 \\ 0.0051 \end{bmatrix}$$

Stiffness Method for Beams Analysis

Step # 05(b): Compute the member end actions. As we know that

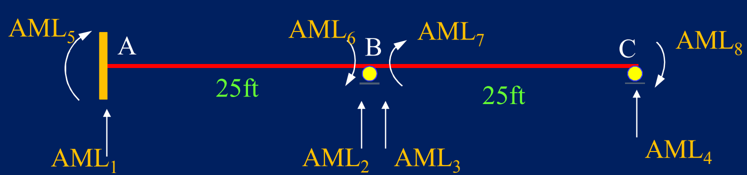
$$[AM] = [AML] + [AMR][AR]$$



AM_n = is the member end action in the indeterminate structure at different location specified with a number "n". n shows the number of member end actions.

Stiffness Method for Beams Analysis

i. Compute AML values.



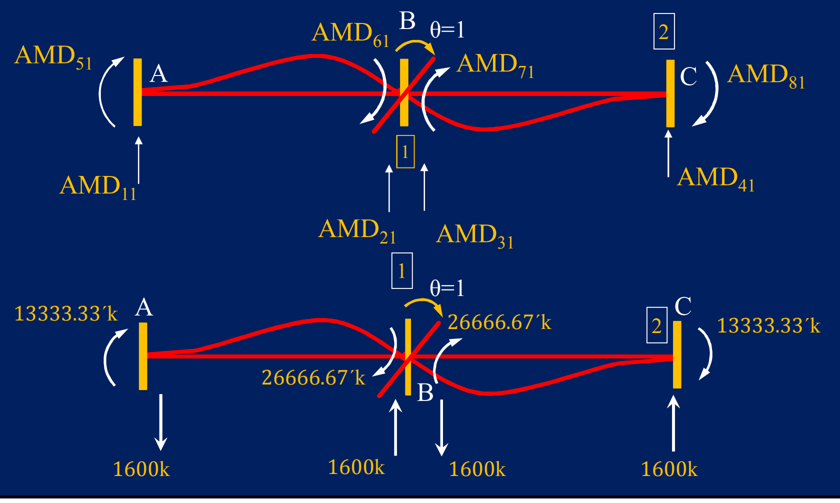
So AML values from the previous slides is.

$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \\ AML_7 \\ AML_8 \end{bmatrix} = \begin{bmatrix} 24.24 \\ 5.76 \\ 22.34 \\ 27.64 \\ -154.67 \\ -1.33 \\ -70.81 \\ 137.50 \end{bmatrix}$$

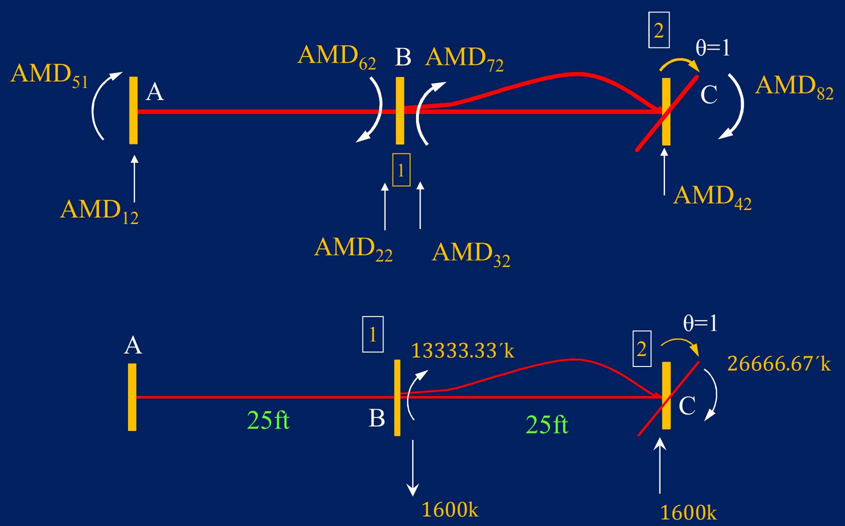
Stiffness Method for Beams Analysis

b) Compute the AMD values.

- 1st apply a unit rotation at redundant location 1 and then at 2 as shown below.



Stiffness Method for Beams Analysis



Stiffness Method for Beams Analysis

So the AMD values are

$$[AMD]_{7*2} = \begin{bmatrix} AMD_{11} & AMD_{12} \\ AMD_{21} & AMD_{22} \\ AMD_{31} & AMD_{32} \\ AMD_{41} & AMD_{42} \\ AMD_{51} & AMD_{52} \\ AMD_{61} & AMD_{62} \\ AMD_{71} & AMD_{72} \\ AMD_{81} & AMD_{82} \end{bmatrix} = \begin{bmatrix} -1600 & 0 \\ 1600 & 0 \\ -1600 & -1600 \\ 1600 & 1600 \\ 13333.33 & 0 \\ 26666.67 & 0 \\ 26666.67 & 13333.33 \\ 13333.33 & 26666.67 \end{bmatrix}$$

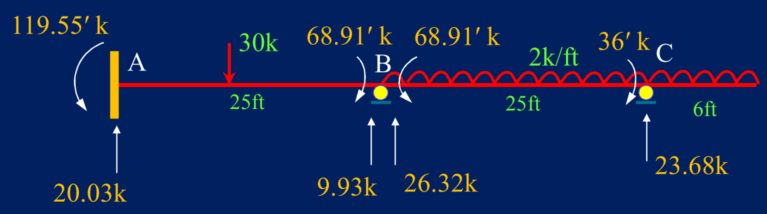
Stiffness Method for Beams Analysis

Now member end actions will be computed as given below

$$[AM]= [AML] + [AMR][AR]$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \\ AM_7 \\ AM_8 \end{bmatrix} = \begin{bmatrix} 24.24 \\ 5.76 \\ 22.34 \\ 27.64 \\ -154.67 \\ -1.33 \\ -70.81 \\ 137.50 \end{bmatrix} + \begin{bmatrix} -1600 & 0 \\ 1600 & 0 \\ -1600 & -1600 \\ 1600 & 1600 \\ 13333.33 & 0 \\ 26666.67 & 0 \\ 26666.67 & 13333.33 \\ 13333.33 & 26666.67 \end{bmatrix} \begin{bmatrix} 0.0026 \\ 0.0051 \end{bmatrix} = \begin{bmatrix} 20.03\text{ k} \\ 9.93\text{ k} \\ 26.32\text{ k} \\ 23.68\text{ k} \\ -119.55'\text{k} \\ 68.91'\text{ k} \\ -68.91'\text{ k} \\ 36'\text{ k} \end{bmatrix}$$

Stiffness Method for Beams Analysis



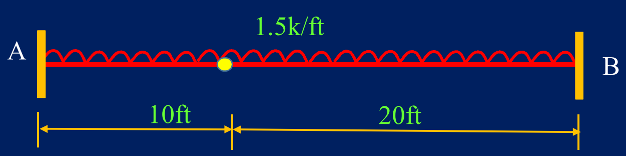
Complete analyzed structure.

Class activity: Draw Shear force and bending moment diagrams.

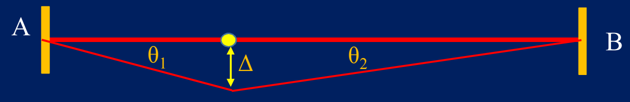
Stiffness Method for Beams Analysis

Problem 03: Analyze the given beam using stiffness method.

Take
EI = constant



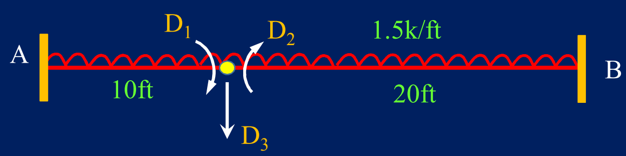
- As the I.H is not at the middle of the beam so $\theta_1 \neq \theta_2$ so θ_1 , θ_2 and Δ need to be restrained.



K.I = 3 degree
So three redundant joint displacements should be chosen.(ignoring the axial effects).

Stiffness Method for Beams Analysis

- **Step # 01:** Selection of redundant Joint displacements and assign coordinates at those locations. Also compute AD values.



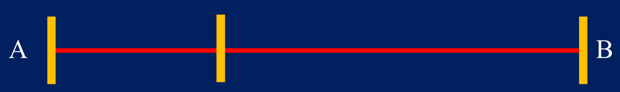
Two Rotations & one translation at I.H is taken as redundant joint displacement as shown.

$$[D]_{3*1} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$$

$$[AD]_{3*1} = \begin{bmatrix} AD_1 \\ AD_2 \\ AD_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Stiffness Method for Beams Analysis

- Restrain all the degrees of freedom to get the restrained structure.

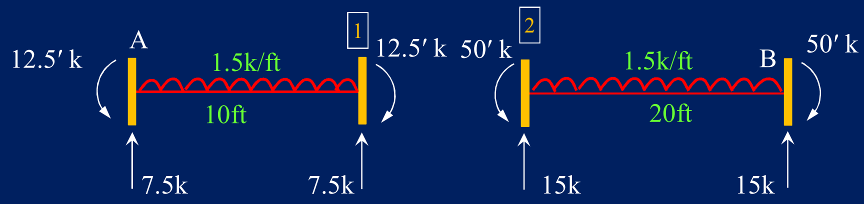


Basic kinematic determinate structure (BKDS) or restrained structure

Stiffness Method for Beams Analysis

- **Step # 02 : Compute ADL matrix.**

Note:
For Fixed end actions see formulas tables in module 5.



$$[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \\ ADL_3 \end{bmatrix} = \begin{bmatrix} 12.5 \\ -50 \\ -22.5 \end{bmatrix}$$

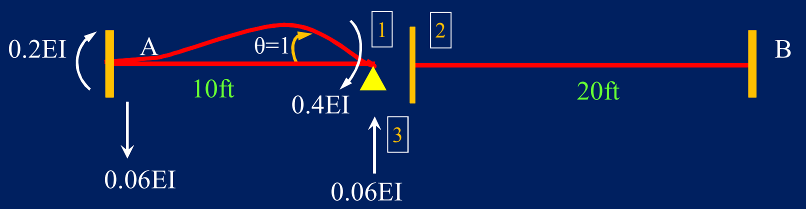
Stiffness Method for Beams Analysis

Step # 03 : Computation of stiffness coefficients matrix

- 1st a unit rotation is applied at location 1 & prevent it at 2 & 3.

  (D_1= 1 , D_2=0 & D3=0) as shown.

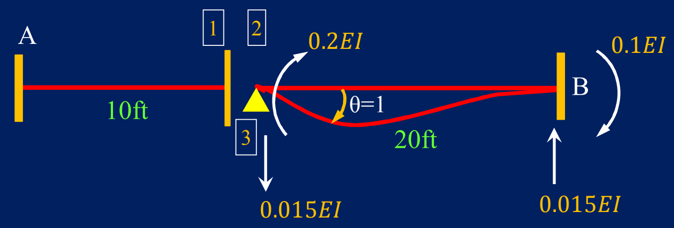
  Compute the values of S_{11} , S_{21} & S_{31}.



$S_{11} = 0.4EI$ $S_{21} = 0$ $S_{31} = -0.06EI$

Stiffness Method for Beams Analysis

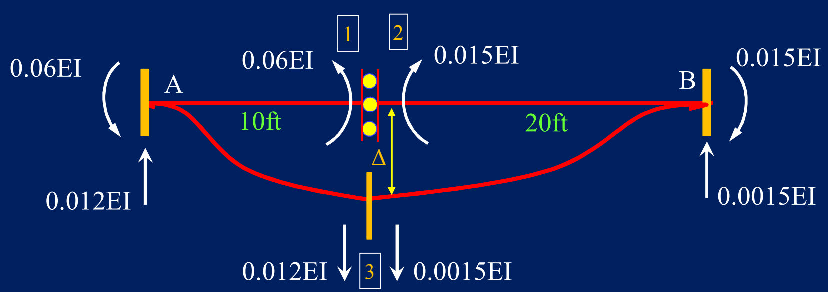
ii. Now a unit rotation is applied at the redundant displacement location 2 and prevented at 1 & .3 ($D_2 = 1$ & $D_1 = D_3 = 0$) as shown. Compute the values of S_{12} , S_{22} & S_{32}



$S_{12} = 0$ $\quad S_{22} = 0.2EI$ $\quad S_{32} = 0.015EI$

Stiffness Method for Beams Analysis

ii. Now a unit rotation is applied at the redundant displacement location 2 and prevented at 1 & 3 ($D_3 = 1$ & $D_1 = D_2 = 0$) as shown. Compute the values of S_{13} , S_{23} & S_{33}



$S_{13} = -0.06EI$ $\quad S_{23} = 0.015EI$ $\quad S_{32} = 0.0135EI$

Stiffness Method for Beams Analysis

So Stiffness coefficient matrix will be

$$S_{11} = 0.4EI \qquad S_{21} = 0 \qquad S_{31} = -0.06EI$$

$$S_{12} = 0 \qquad S_{22} = 0.2EI \qquad S_{32} = 0.015EI$$

$$S_{13} = -0.06EI \qquad S_{23} = 0.015EI \qquad S_{32} = 0.0135EI$$

$$[\,S\,] = \begin{bmatrix} 0.4 & 0 & -0.06 \\ 0 & 0.2 & 0.015 \\ -0.06 & 0.015 & 0.0135 \end{bmatrix}$$

Stiffness Method for Beams Analysis

Step # 04: Apply equilibrium condition at the location of the redundant joint displacement to write equilibrium equations and solve for unknown joint displacement.

$$[AD]_{3\,*\,1} = [ADL]_{3\,*\,1} + [S]_{3\,*\,3} \bullet [D]_{3\,*\,1}$$

$$[\boldsymbol{D}] = [\boldsymbol{S}]^{-1} \bullet [\boldsymbol{AD} - \boldsymbol{ADL}]$$

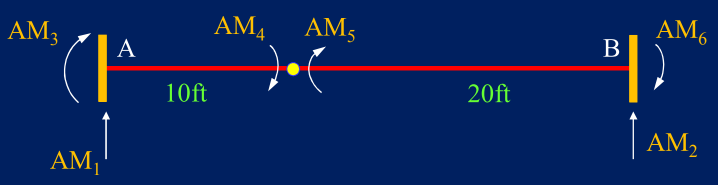
$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 0.4 & 0 & -0.06 \\ 0 & 0.2 & 0.015 \\ -0.06 & 0.015 & 0.0135 \end{bmatrix}^{-1} \begin{bmatrix} 0-(12.5) \\ 0-(-50) \\ 0-(-22.5) \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} 718.75 \\ -125 \\ 5000 \end{bmatrix} \frac{1}{EI}$$

Stiffness Method for Beams Analysis

Step # 05(b): Compute the member end actions. As we know that

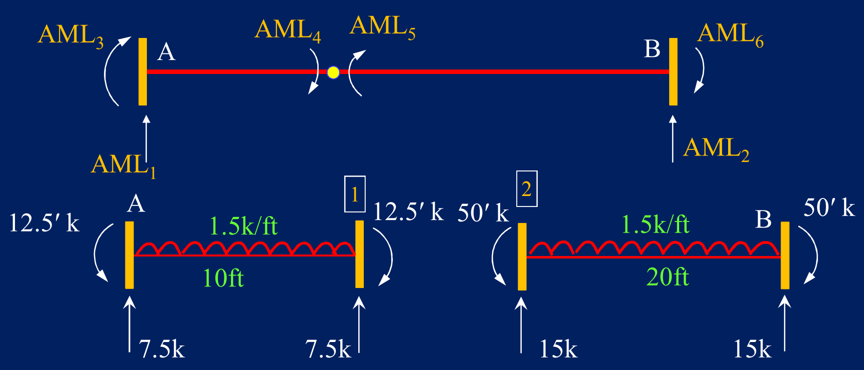
$$[AM] = [AML] + [AMR][AR]$$



AM_n = is the member end action in the indeterminate structure at different location specified with a number "n". n shows the number of member end actions.

Stiffness Method for Beams Analysis

i. Compute AML values.

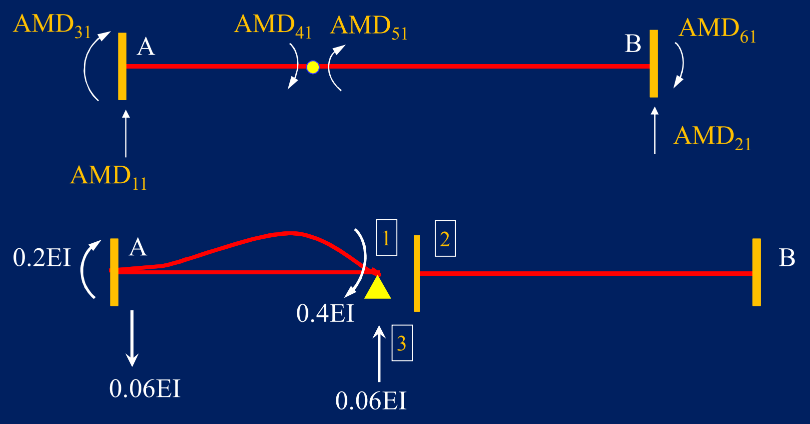


$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} = \begin{bmatrix} 7.5 \\ 15 \\ -12.5 \\ 12.5 \\ -50 \\ 50 \end{bmatrix}$$

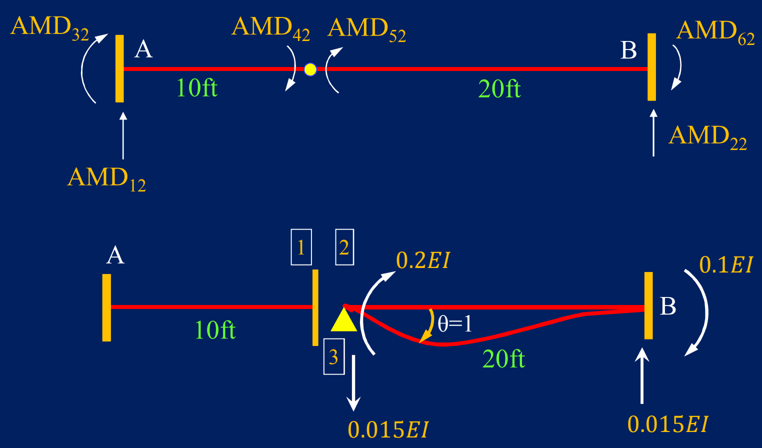
Stiffness Method for Beams Analysis

b) Compute the AMD values.

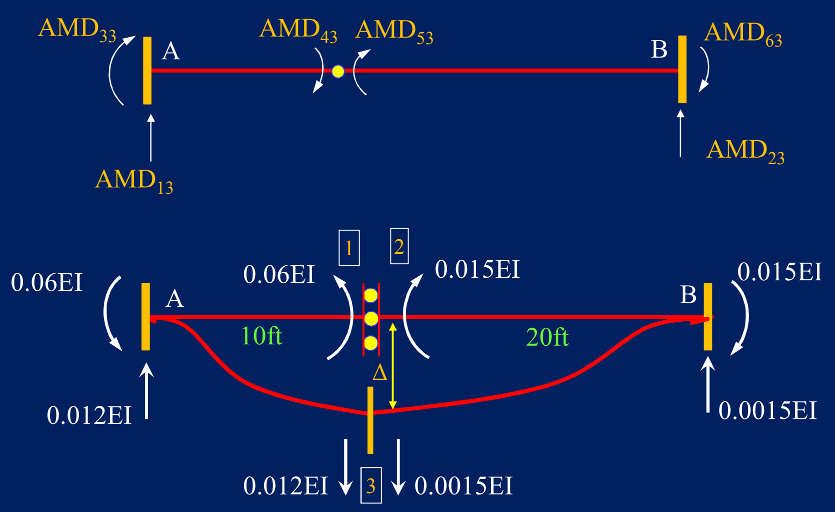
- 1st apply a unit rotation at redundant location 1 then at 2 then a unit translation at 3 as shown below.



Stiffness Method for Beams Analysis



Stiffness Method for Beams Analysis



Stiffness Method for Beams Analysis

So the AMD values are

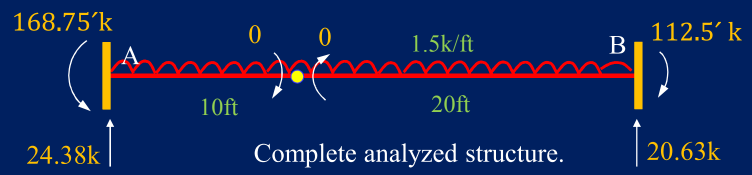
$$[AMD]=\begin{bmatrix} AMD_{11} & AMD_{12} & AMD_{13} \\ AMD_{21} & AMD_{22} & AMD_{23} \\ AMD_{31} & AMD_{32} & AMD_{33} \\ AMD_{41} & AMD_{42} & AMD_{43} \\ AMD_{51} & AMD_{52} & AMD_{53} \\ AMD_{61} & AMD_{62} & AMD_{63} \end{bmatrix}=\begin{bmatrix} -0.06 & 0 & 0.012 \\ 0 & 0.015 & 0.0015 \\ 0.2 & 0 & -0.06 \\ 0.4 & 1600 & -0.06 \\ 0 & 0.2 & 0.015 \\ 0 & 0.1 & 0.015 \end{bmatrix}$$

Stiffness Method for Beams Analysis

Now member end actions will be computed as given below

$$[AM] = [AML] + [AMR][AR]$$

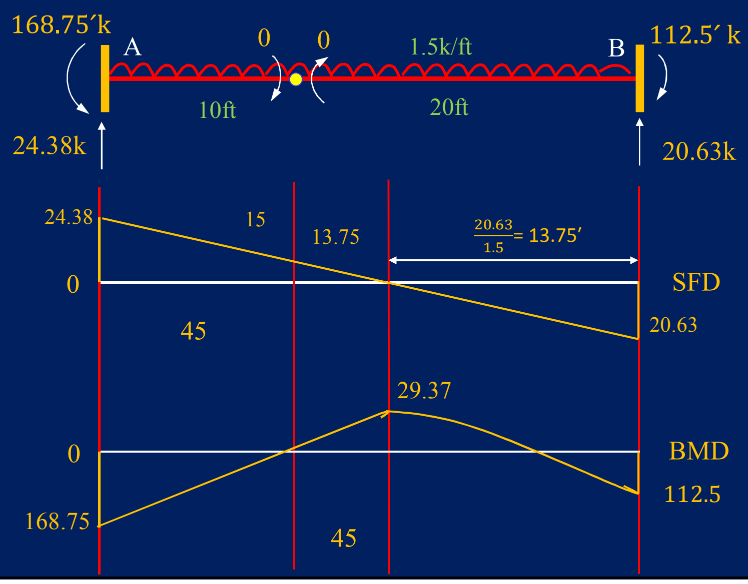
$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} 7.5 \\ 15 \\ -12.5 \\ 12.5 \\ -50 \\ 50 \end{bmatrix} + \begin{bmatrix} -0.06 & 0 & 0.012 \\ 0 & 0.015 & 0.0015 \\ 0.2 & 0 & -0.06 \\ 0.4 & 1600 & -0.06 \\ 0 & 0.2 & 0.015 \\ 0 & 0.1 & 0.015 \end{bmatrix} \begin{bmatrix} 718.75 \\ -125 \\ 5000 \end{bmatrix} = \begin{bmatrix} 24.38k \\ 20.63\ k \\ -168.75'k \\ 0 \\ 0 \\ 112.5'\ k \end{bmatrix}$$



Complete analyzed structure.

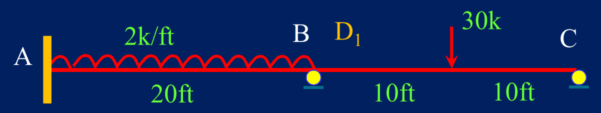
Stiffness Method for Beams Analysis

Now Shear force and Bending moment diagram

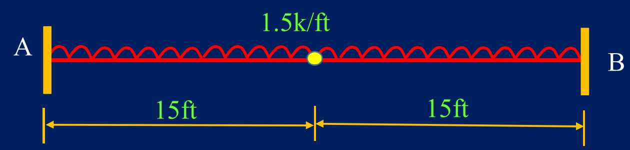


Stiffness Method for Beams Analysis

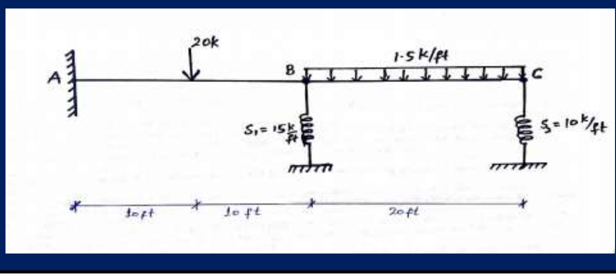
Assignment # 03(b) : Analyze the given beams using stiffness method.



Take
EI = constant



Take
EI = constant



Take
E = 30000 ksi
I = 800 in^4
EI = 166666.6 k-ft^2

References

- Structural Analysis by R. C. Hibbeler
- Matrix structural analysis by William Mc Guire
- Matrix analysis of frame structures by William Weaver
- Online Civil Engineering blogs