DEVELOPMENT OF INTERACTION DIAGRAMS

DEVELOPMENT OF INTERACTION DIAGRAMS

Should an axial compressive load be applied to a short concrete member, it will be subjected to a uniform strain or shortening, as is shown in Figure 10.3(a). If a moment with zero axial load is applied to the same member, the result will be bending about the member's neutral axis such that strain is proportional to the distance from the neutral axis. This linear strain variation is shown in Figure 10.3(b). Should axial load and moment be applied at the same time, the resulting strain diagram will be a combination of two linear diagrams and will itself be linear, as illustrated in Figure 10.3(c). As a result of this linearity, we can assume certain numerical values of strain in one part of a column and determine strains at other locations by straight-line interpolation.

As the axial load applied to a column is changed, the moment that the column can resist will change. We will see how an interaction curve of nominal axial load and moment values can be developed for a particular column.

DEVELOPMENT OF INTERACTION DIAGRAMS

Assuming the concrete on the compression edge of the column will fail at a strain of 0.003, a strain can be assumed on the far edge of the column and the values of P_n and M_n can be computed by statics. Then holding the compression strain at 0.003 on the far edge, we can assume a series of different strains on the other edge and calculate P_n and M_n for each. Eventually a sufficient number of values will be obtained to plot an interaction curve such as the one shown later. The example illustrates the calculation of P_n and M_n for a column for one set of assumed strains.
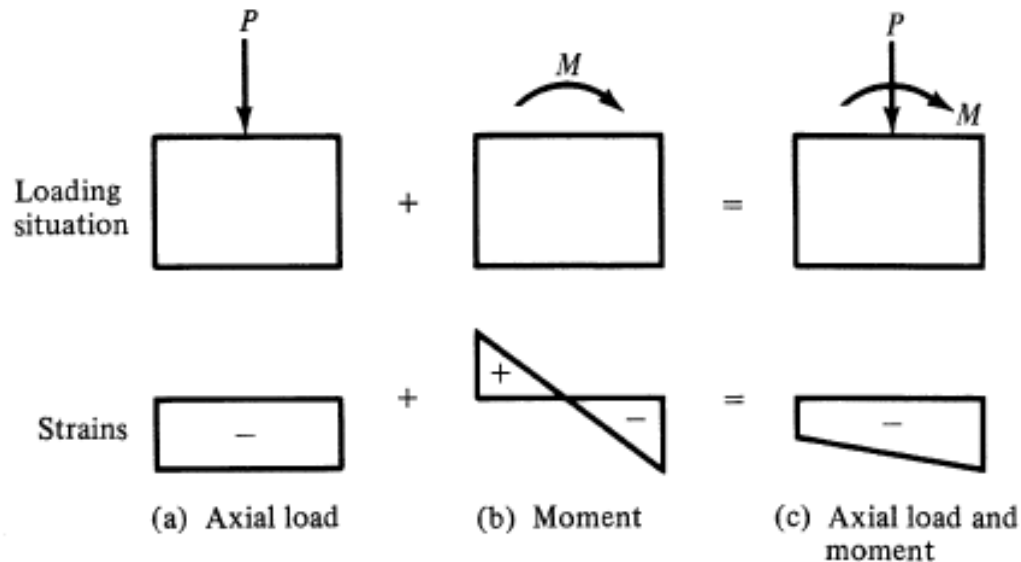

Figure 10.3 Column strains.

DEVELOPMENT OF INTERACTION DIAGRAMS

Example 2

It is assumed that the column of Figure 10.4 has a strain on its compression edge equal to -0.00300 and has a tensile strain of $+0.00200$ on its other edge. Determine the values of P_n and M_n that cause this strain distribution if $f_y = 60$ ksi and $f'_c = 4$ ksi.
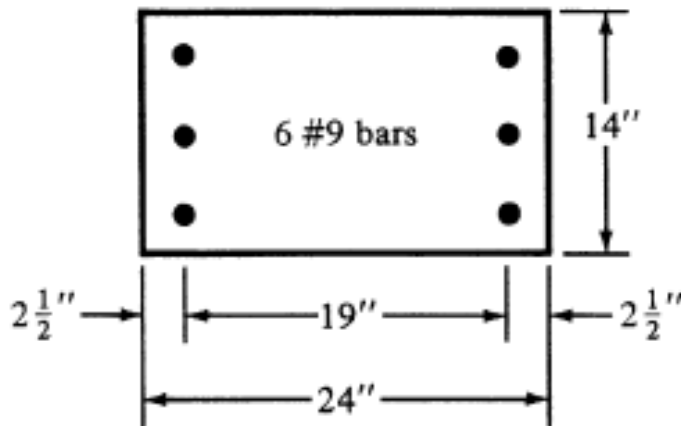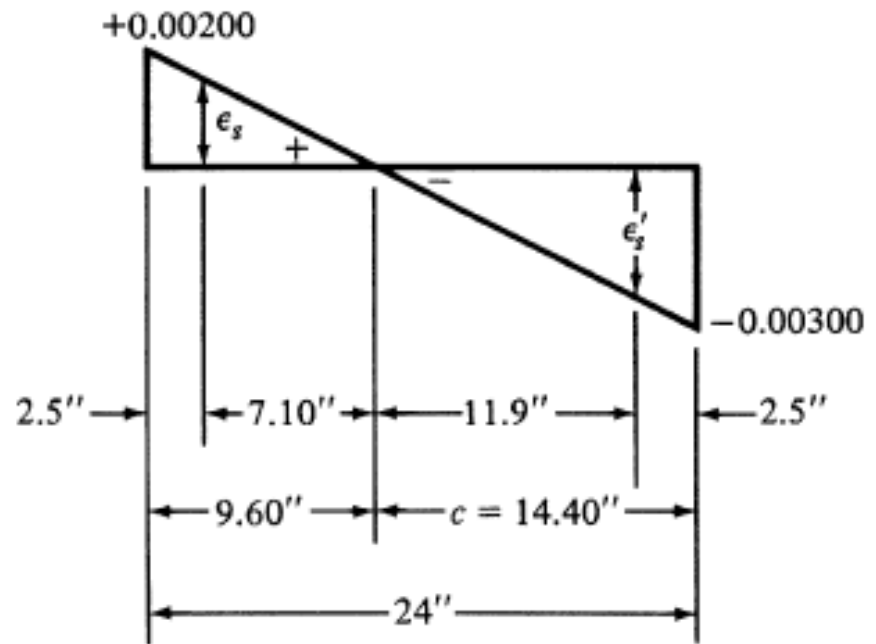
Solution:

Figure 10.4

Figure 10.5

DEVELOPMENT OF INTERACTION DIAGRAMS

Determine the values of c and of the steel strains ϵ_s', and ϵ_s by proportions with reference to the strain diagram shown in Figure 10.5.

$$c = \left(\frac{0.00300}{0.00300 + 0.00200} \right)(24) = 14.40 \text{ in.}$$

$$\epsilon_s' = \left(\frac{11.90}{14.40} \right)(0.00300) = 0.00248 > 0.00207 \quad \therefore \text{yields}$$

$$\epsilon_s = \left(\frac{7.10}{9.60} \right)(0.00200) = 0.00148$$

In the following calculations, C_c is the total compression in the concrete, C_s is the total compression in the compression steel, and T_s is the total tension in the tensile steel. Each of these values is computed below. The reader should note that C_s is reduced by $0.85f_cA_s$ to account for concrete displaced by the steel in compression.

DEVELOPMENT OF INTERACTION DIAGRAMS

$$a = (0.85)(14.40 \text{ in.}) = 12.24 \text{ in.}$$

$$C_c = (0.85)(12.24 \text{ in.})(14 \text{ in.})(4.0 \text{ ksi}) = -582.62 \text{ k}$$

$$C'_s = (60 \text{ ksi})(3.0 \text{ in.}^2) - (0.85)(3.0 \text{ in.}^2)(4.0 \text{ ksi}) = -169.8 \text{ k}$$

$$T_s = (0.00148)(29{,}000 \text{ ksi})(3.0 \text{ in.}^2) = +128.76 \text{ k}$$

By statics, *Pn* and *Mn* are determined with reference to Figure 10.6, where the values of C_c, C_s, and T_s are shown.

$$\Sigma V = 0$$

$$-P_n + 169.8 \text{ k} + 582.62 \text{ k} - 128.76 \text{ k} = 0$$

$$P_n = 623.7 \text{ k}$$

$$\phi P_n = (0.65)(623.7 \text{ k}) = 405.4 \text{ k}$$
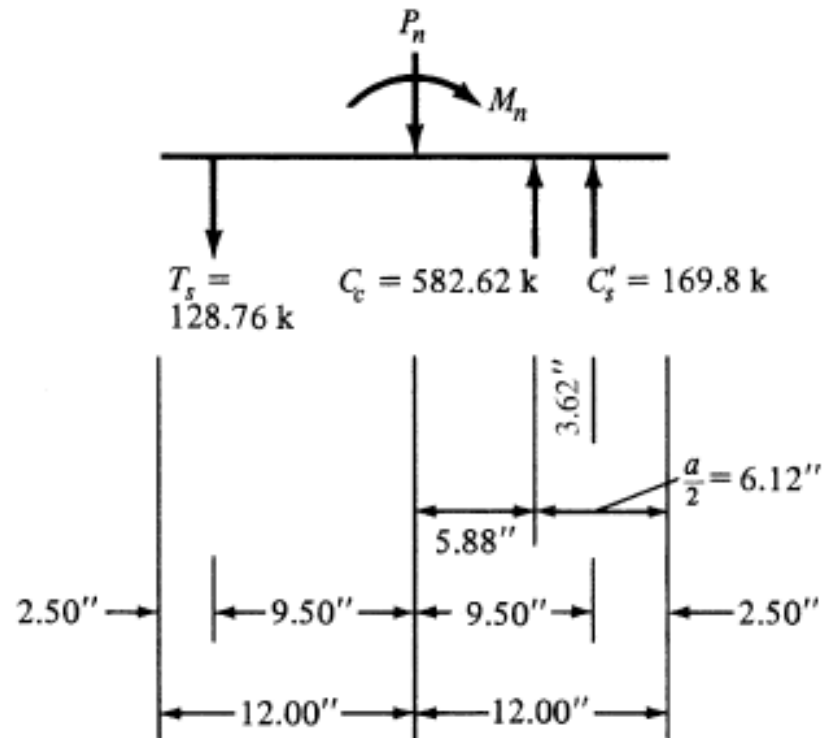
DEVELOPMENT OF INTERACTION DIAGRAMS

Figure 10.6

$\Sigma M = 0$ about Tensile Steel

$$(623.7 \text{ k})(9.50 \text{ in.}) + M_n - (582.62 \text{ k})(15.38 \text{ in.}) - (169.8 \text{ k})(19.00 \text{ in.}) = 0$$

$$M_n = 6261.3 \text{ in-k} = 521.8 \text{ ft-k}$$

$$\phi M_n = (0.65)(6261.3 \text{ in-k}) = 4069.8 \text{ in-k} = 339.2 \text{ ft-k}$$

DEVELOPMENT OF INTERACTION DIAGRAMS

In this manner, a series of P_n and M_n values are determined to correspond with a strain of -0.003 on the compression edge and varying strains on the far column edge. The resulting values are plotted on a curve as shown in Figure 10.8.

A few remarks are made here concerning the extreme points on this curve. One end of the curve will correspond to the case where P_n is at its maximum value and M_n is zero. For this case, P_n is determined as in Chapter 9 for the axially loaded column of Example 10.2.

$$P_n = 0.85f'_c(A_g - A_s) + A_s f_y$$
$$= (0.85)(4.0 \text{ ksi})(14 \text{ in.} \times 24 \text{ in.} - 6.00 \text{ in.}^2) + (6.00 \text{ in.}^2)(60 \text{ ksi})$$
$$= 1482 \text{ k}$$

On the other end of the curve, M_n is determined for the case where P_n is zero. This is the procedure used for a doubly reinforced member as previously described in Chapter 5. For the column of Example 10.2, M_n is equal to 297 ft-k.

DEVELOPMENT OF INTERACTION DIAGRAMS

$P_n = 1482$ k, $M_n = 0$

Compression failure zone

$P_n = 623.7$ k, $M_n = 521.8$ ft-k

$P_{bn} = 504.4$ k, $M_{bn} = 559.7$ ft-k

Compressive loads

Tension failure zone

$P_n = 0$, $M_n = 297$ ft-k

Axial tension

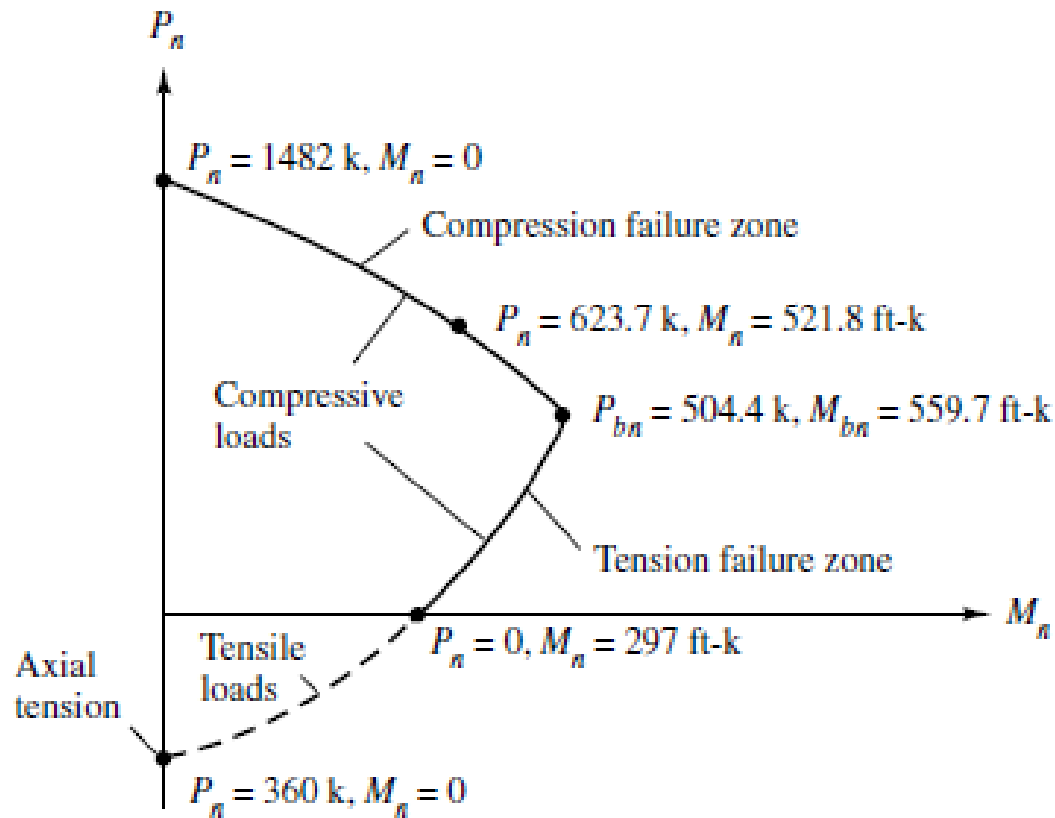Tensile loads

$P_n = 360$ k, $M_n = 0$

Figure 10.8 Interaction curve for the column of Figure 10.4. Notice these are nominal values.

DEVELOPMENT OF INTERACTION DIAGRAMS

A column reaches its ultimate capacity when the concrete reaches a compressive strain of 0.003. If the steel closest to the extreme tension side of the column reaches yield strain, or even more when the concrete reaches a strain of 0.003, the column is said to be tension controlled; otherwise, it is compression controlled. The transition point between these regions is the balance point. The term *balanced section* was used earlier in referring to a section whose compression concrete strain reached 0.003 at the same time as the tensile steel reached its yield strain at f_y/E_s. In a beam, this situation theoretically occurs when the steel percentage equaled ρ_b. A column can undergo a balanced failure no matter how much steel it has if it has the right combination of moment and axial load.

DEVELOPMENT OF INTERACTION DIAGRAMS

For columns, the definition of balanced loading is the same as it was for beams—that is, a column that has a strain of 0.003 on its compression side at the same time that its tensile steel on the other side has a strain of f_y/E_s. Although it is easily possible to prevent a balanced condition in beams by requiring that tensile steel strains be kept well above f_y/E_s, such is not the case for columns. Thus, for columns, it is not possible to prevent sudden compression failures or balanced failures. For every column, there is a balanced loading situation where an ultimate load, P_{bn}, placed at an eccentricity, e_b, will produce a moment, M_{bn}, at which time the balanced strains will be reached simultaneously.

At the balanced condition we have a strain of -0.003 on the compression edge of the column and a strain of $f_y/29 \times 10^3 = 60/29 \times 10^3 = 0.00207$ in the tensile steel. This information is shown in Figure 10.7. The same procedure used in Example 10.2 is used to find $P_n = 504.4$ k and $M_n = 559.7$ ft-k.

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

A slightly different approach is used in Example 4, where the average compression stress at ultimate load across the column cross section is assumed to equal some value—say, $0.5f_c$ to $0.6f_c$. This value is divided into P_n to determine the column area required. Cross-sectional dimensions are then selected, and the value of ρ_g is determined from the interaction curves. Again, if the percentage obtained seems unreasonable, the column size can be revised and a new steel percentage obtained.

In Examples 3 to 5, reinforcing bars are selected for three columns. The values of **K_n** = $P_n / f'_c A_g$ and **R_n** = $P_n e / f'_c A_g h$ are computed. The position of those values is located on the appropriate graph, and ρ_g is determined and multiplied by the gross area of the column in question to determine the reinforcing area needed.

DEVELOPMENT OF INTERACTION DIAGRAMS

The curve for P_n and M_n for a particular column may be extended into the range where P_n becomes a tensile load. We can proceed in exactly the same fashion as we did when P_n was compressive. A set of strains can be assumed, and the usual statics equations can be written and solved for P_n and M_n. Several different sets of strains were assumed for the column of Figure 10.4, and then the values of P_n and M_n were determined. The results were plotted at the bottom of Figure 10.8 and were connected with the dashed line which is labeled "tensile loads."
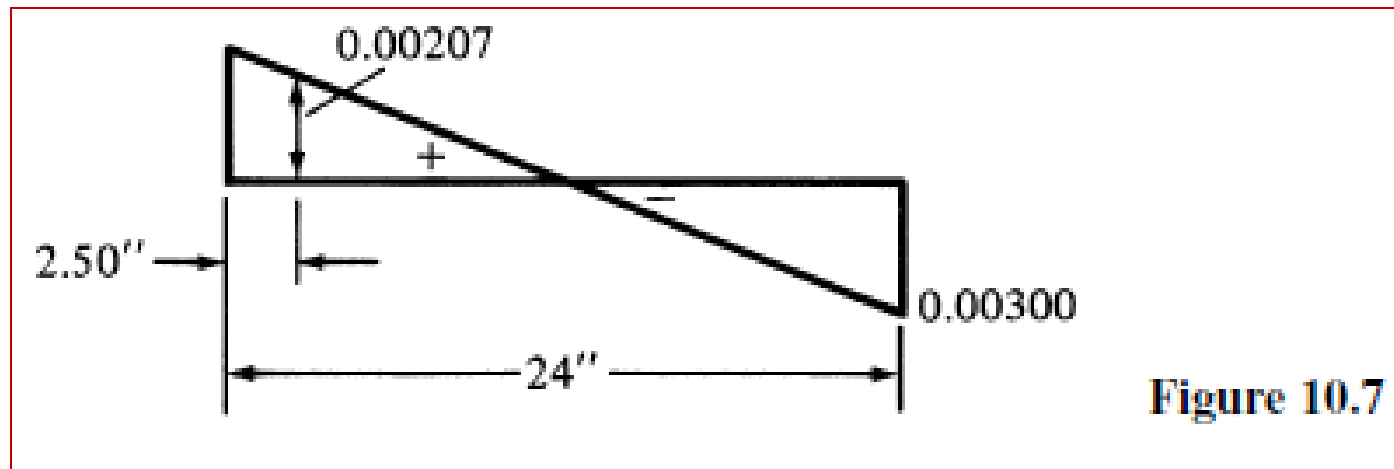
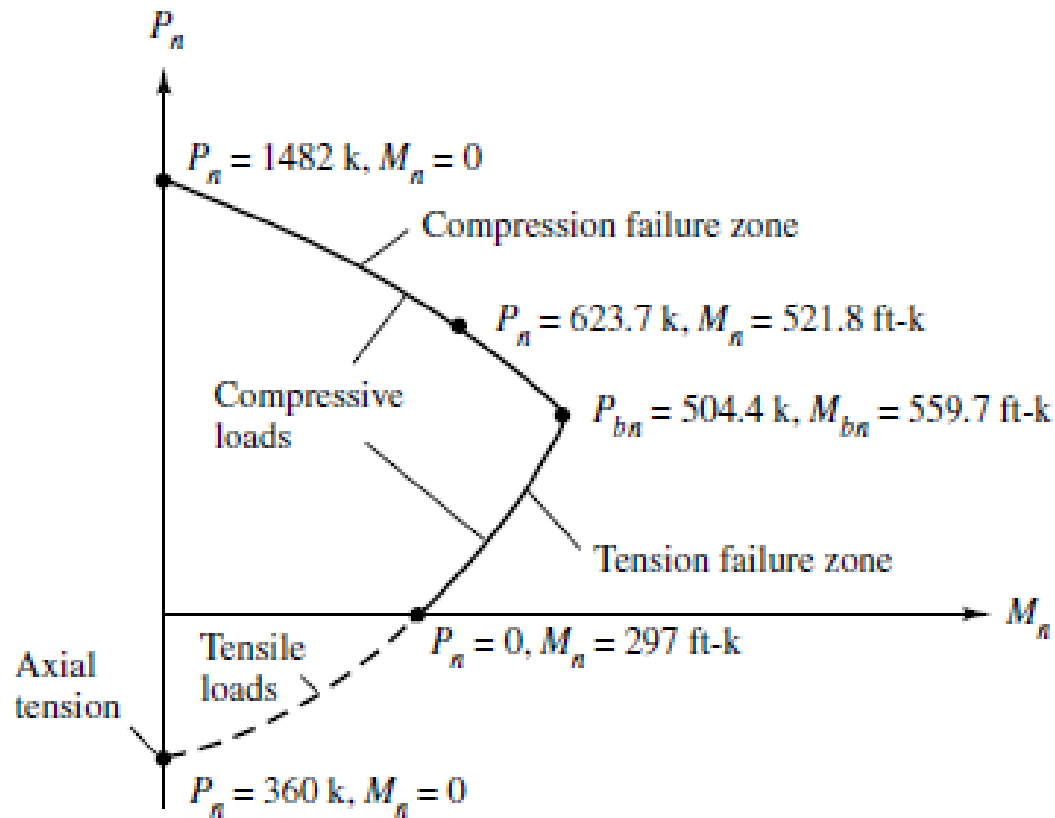Figure 10.7

DEVELOPMENT OF INTERACTION DIAGRAMS

Figure 10.8 Interaction curve for the column of Figure 10.4. Notice these are nominal values.

DEVELOPMENT OF INTERACTION DIAGRAMS

Because axial tension and bending are not very common for reinforced concrete columns, the tensile load part of the curves is not shown in subsequent figures in this chapter. You will note that the largest tensile value of P_n will occur when the moment is zero. For that situation, all of the column steel has yielded, and all of the concrete has cracked. Thus P_n will equal the total steel area A_s times the yield stress. For the column of Figure 10.4

$$P_n = A_s f_y = (6.0 \text{ in.}^2)(60 \text{ ksi}) = 360 \text{ k}$$

In this chapter P_n values were obtained only for rectangular tied columns. The same theory could be used for round columns, but the mathematics would be somewhat complicated because of the circular layout of the bars, and the calculations of distances would be rather tedious. Several approximate methods have been developed that greatly simplify the mathematics. Perhaps the best known of these is the one proposed by Charles Whitney, in which equivalent rectangular columns are used to replace the circular ones. This method gives results that correspond quite closely with test results.

EQUIVALENT RECTANGULAR COLUMN

In Whitney's method, the area of the equivalent column is made equal to the area of the actual circular column, and its depth in the direction of bending is 0.80 times the outside diameter of the real column. One-half the steel is assumed to be placed on one side of the equivalent column and one-half on the other. The distance between these two areas of steel is assumed to equal two-thirds of the diameter (D_s) of a circle passing through the center of the bars in the real column. These values are illustrated in Figure 10.9. Once the equivalent column is established, the calculations for P_n and M_n are made as for rectangular columns.
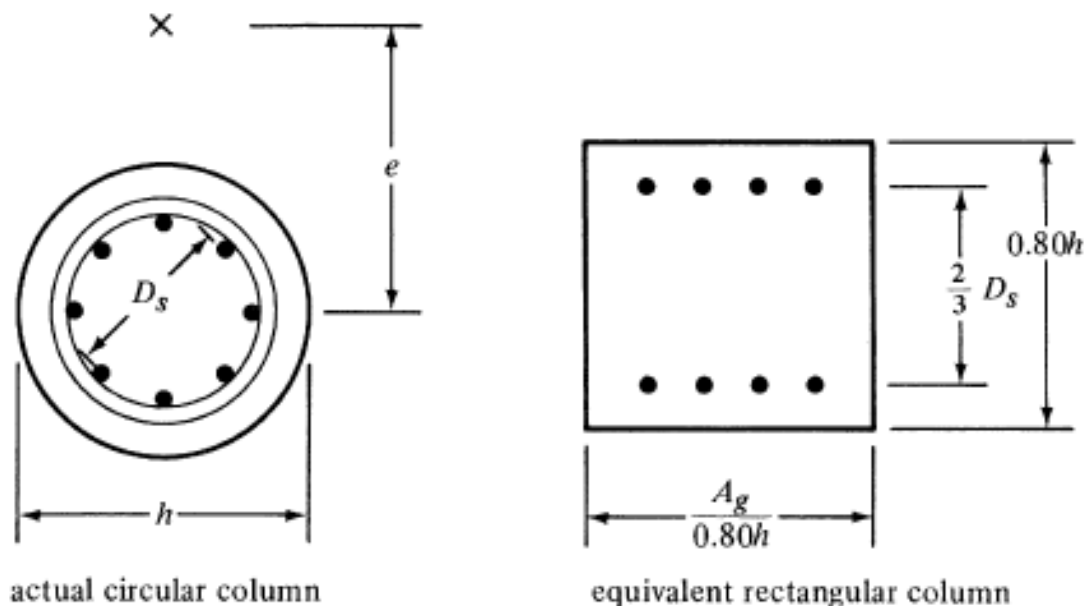

actual circular column   equivalent rectangular column

Figure 10.9 Replacing circular column with equivalent rectangular one.

EQUIVALENT RECTANGULAR COLUMN

On some occasions, members subject to axial load and bending have unsymmetrical arrangements of reinforcing. Should this be the case, you must remember that eccentricity is correctly measured from the plastic centroid of the section.

USE OF INTERACTION DIAGRAMS

USE OF INTERACTION DIAGRAMS

We have seen that by statics the values of P_n and M_n for a given column with a certain set of strains can easily be determined. Preparing an interaction curve with a hand calculator for just one column, however, is quite tedious. Imagine the work involved in a design situation where various sizes, concrete strengths, and steel percentages need to be considered. Consequently, designers resort almost completely to computer programs, computer generated interaction diagrams, or tables for their column calculations. As we have seen, such a diagram is drawn for a column as the load changes from one of a pure axial nature through varying combinations of axial loads and moments and on to a pure bending situation.

USE OF INTERACTION DIAGRAMS

Interaction diagrams are obviously useful for studying the strengths of columns with varying proportions of loads and moments. Any combination of loading that falls inside the curve is satisfactory, whereas any combination falling outside the curve represents failure.

If a column is loaded to failure with an axial load only, the failure will occur at point *A* on the diagram (Figure 10.10). Moving out from point *A* on the curve, the axial load capacity decreases as the proportion of bending moment increases. At the very bottom of the curve, point *C* represents the bending strength of the member if it is subjected to moment only with no axial load present. In between the extreme points *A* and *C*, the column fails due to a combination of axial load and bending. Point *B* is called the *balanced point* and represents the balanced loading case, where theoretically a compression failure and tensile yielding occur simultaneously.
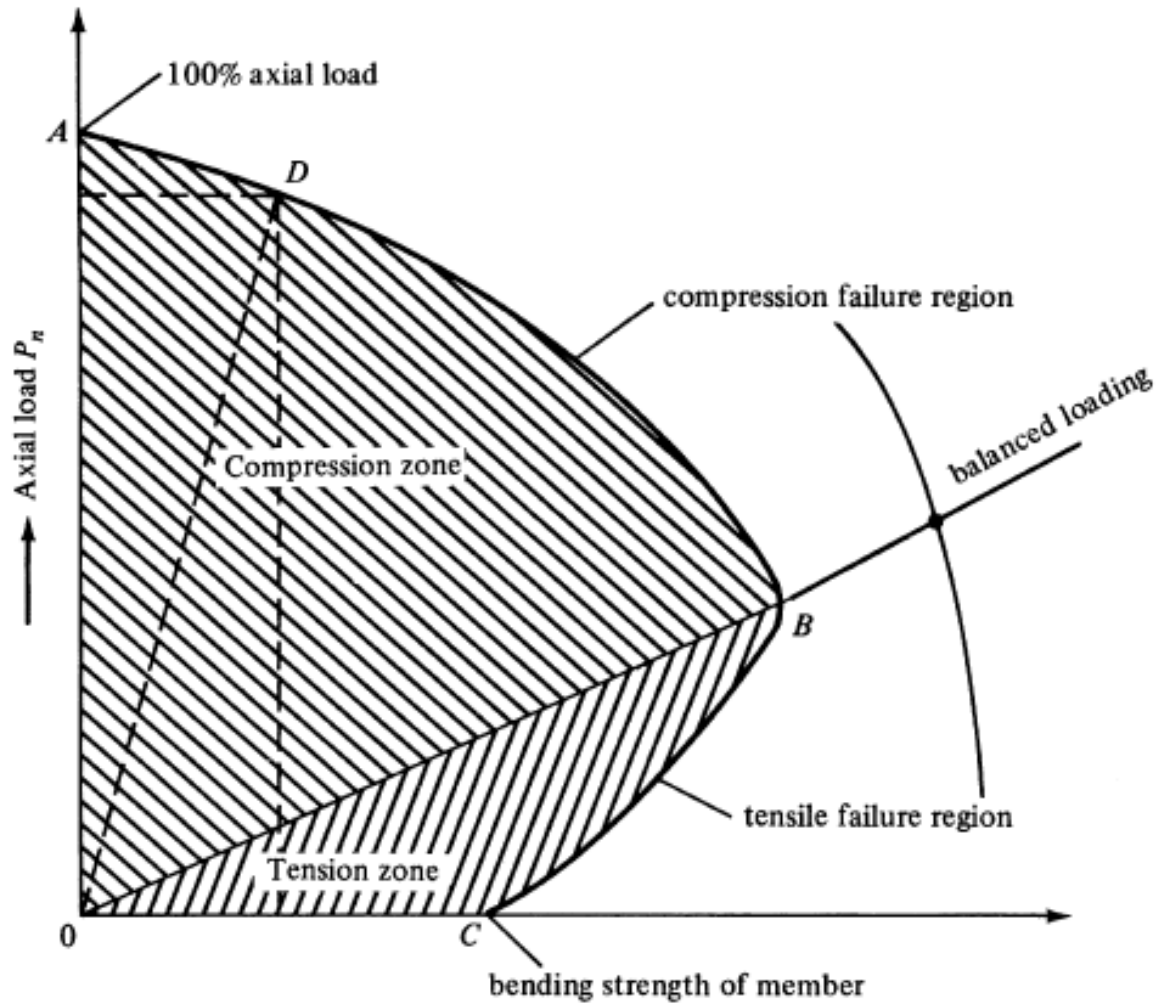
USE OF INTERACTION DIAGRAMS

Figure 10.10 Column interaction diagram.

USE OF INTERACTION DIAGRAMS

Refer to point *D* on the curve. The horizontal and vertical dashed lines to this point indicate a particular combination of axial load and moment at which the column will fail. Should a radial line be drawn from point 0 to the interaction curve at any point (as to *D* in this case), it will represent a constant eccentricity of load, that is, a constant ratio of moment to axial load.

Shape of the lower part of the curve from *B* to *C*, where bending predominates is of interest. From *A* to *B* on the curve the moment capacity of a section increases as the axial load decreases, but just the opposite occurs from *B* to *C*.

The part of the curve from *B* to *C* represents the range of tensile failures. Any axial compressive load in that range tends to reduce the stresses in the tensile bars, with the result that a larger moment can be resisted.

USE OF INTERACTION DIAGRAMS

In Figure 10.11 an interaction curve is drawn for the 14" by 24" column with six #9 bars considered in Section 10.3. If eight #9 bars had been used in the same dimension column, another curve could be generated as shown in the figure; if ten #9 bars were used, still another curve would result. The shape of the new diagrams would be the same as for the six #9 curve, but the values of P_n and M_n would be larger.
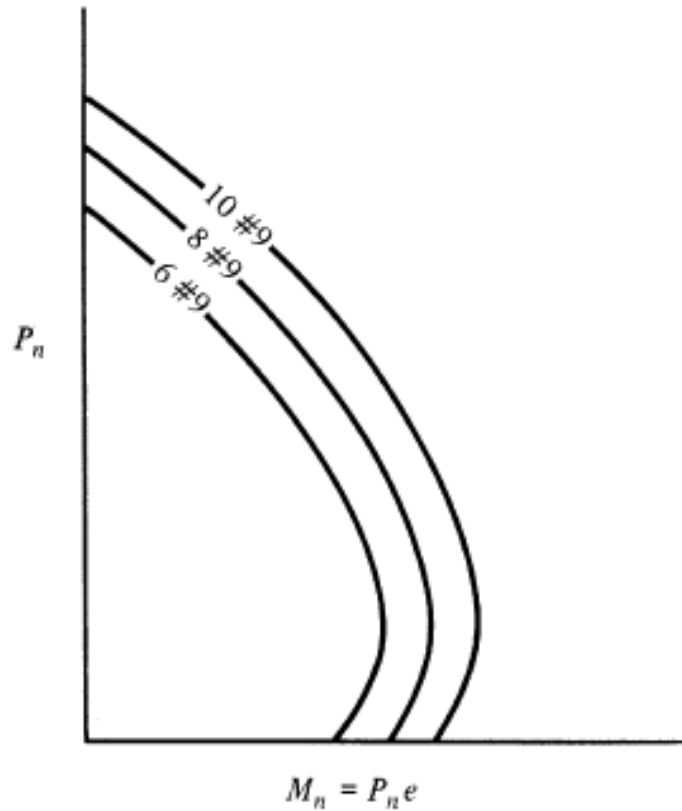
USE OF INTERACTION DIAGRAMS

Figure 10.11 Interaction curves for a rectangular column with different sets of reinforcing bars.

CODE MODIFICATIONS OF COLUMN INTERACTION DIAGRAMS

CODE MODIFICATIONS OF COLUMN INTERACTION DIAGRAMS

If interaction curves for P_n values were prepared, they would be of the types shown in Figures 10.10 and 10.11. To use such curves to obtain design values, they would have to have three modifications made to them as specified in the Code. These modifications are as follows:

(a).   The Code 9.3.2 specifies strength reduction or ϕ factors (0.65 for tied columns and 0.70 for spiral columns) that must be multiplied by P_n values. If a P_n curve for a particular column were multiplied by ϕ, the result would be a curve something like the ones shown in Figure 10.12.
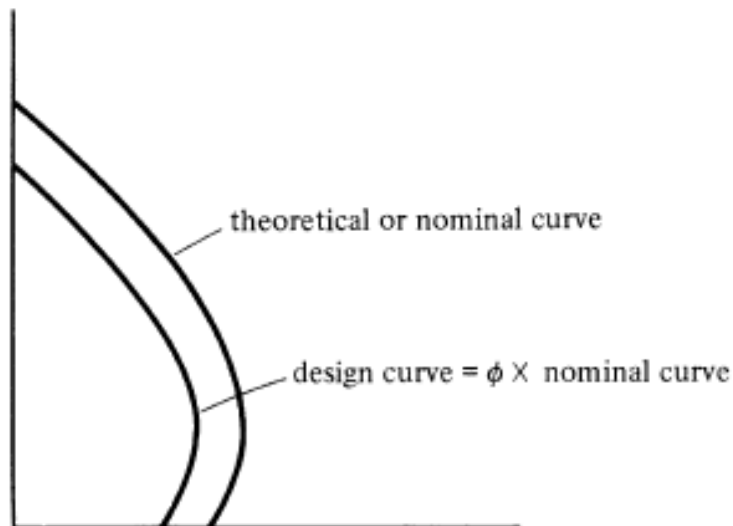
theoretical or nominal curve

design curve = $\phi \times$ nominal curve

Figure 10.12 Curves for P_n and ϕP_n for a single column.

CODE MODIFICATIONS OF COLUMN INTERACTION DIAGRAMS

(b). The second modification also refers to φ factors. The Code specifies values of 0.65 and 0.70 for tied and spiral columns, respectively. Should a column have quite a large moment and a very small axial load so that it falls on the lower part of the curve between points *B* and *C* (see Figure 10.10), the use of these small φ values may be a little unreasonable. For instance, for a member in pure bending(point *C* on the same curve) the specified φ is 0.90, but if the same member has a very small axial load added, φ would immediately fall to 0.65 or 0.70. Therefore, the Code (9.3.2.2) states that when members subject to axial load and bending have net tensile strains (*t*) between the limits for compression-controlled and tensile-controlled sections, they fall in the transition zone for φ. In this zone it is permissible to increase φ linearly from 0.65 or 0.70 to 0.90 as ε_t increases from the compression-controlled limit to 0.005. In this regard, the Figure R9.3.2 of the Code may also be referred.

CODE MODIFICATIONS OF COLUMN INTERACTION DIAGRAMS

(c). As described in Chapter 9, maximum permissible column loads were specified for columns no matter how small their *e* values. As a result, the upper part of each design interaction curve is shown as a horizontal line representing the appropriate value of *Pu.*

$$P_u = \phi P_{n\,max} \text{ for tied columns} = 0.80\phi[0.85f'_c(A_g - A_{st}) + f_y A_{st}]$$
$$\text{(ACI Equation 10-2)}$$
$$P_u = \phi P_{n\,max} \text{ for spiral columns} = 0.85\phi[0.85f'_c(A_g - A_{st}) + f_y A_{st}]$$
$$\text{(ACI Equation 10-1)}$$

These formulas were developed to be approximately equivalent to loads applied with eccentricities of 0.10*h* for tied columns and 0.05*h* for spiral columns. Each of the three modifications described here is indicated on the design curve of Figure 10.13. In Figure 10.13, the solid curved line represents *Pu* and *Mu*, whereas the dashed curved line is *Pn* and *Mn*. The difference between the two curves is the *φ* factor. The two curves would have the same shape if the *φ* factor did not vary. Above the radial line labeled "balanced case," *φ* = 0.65 (0.75 for spirals). Below the other radial line, labeled "strain of 0.005," *φ* = 0.9. It varies between the two values in between, and the *Pu* versus *Mu* curve assumes a different shape.
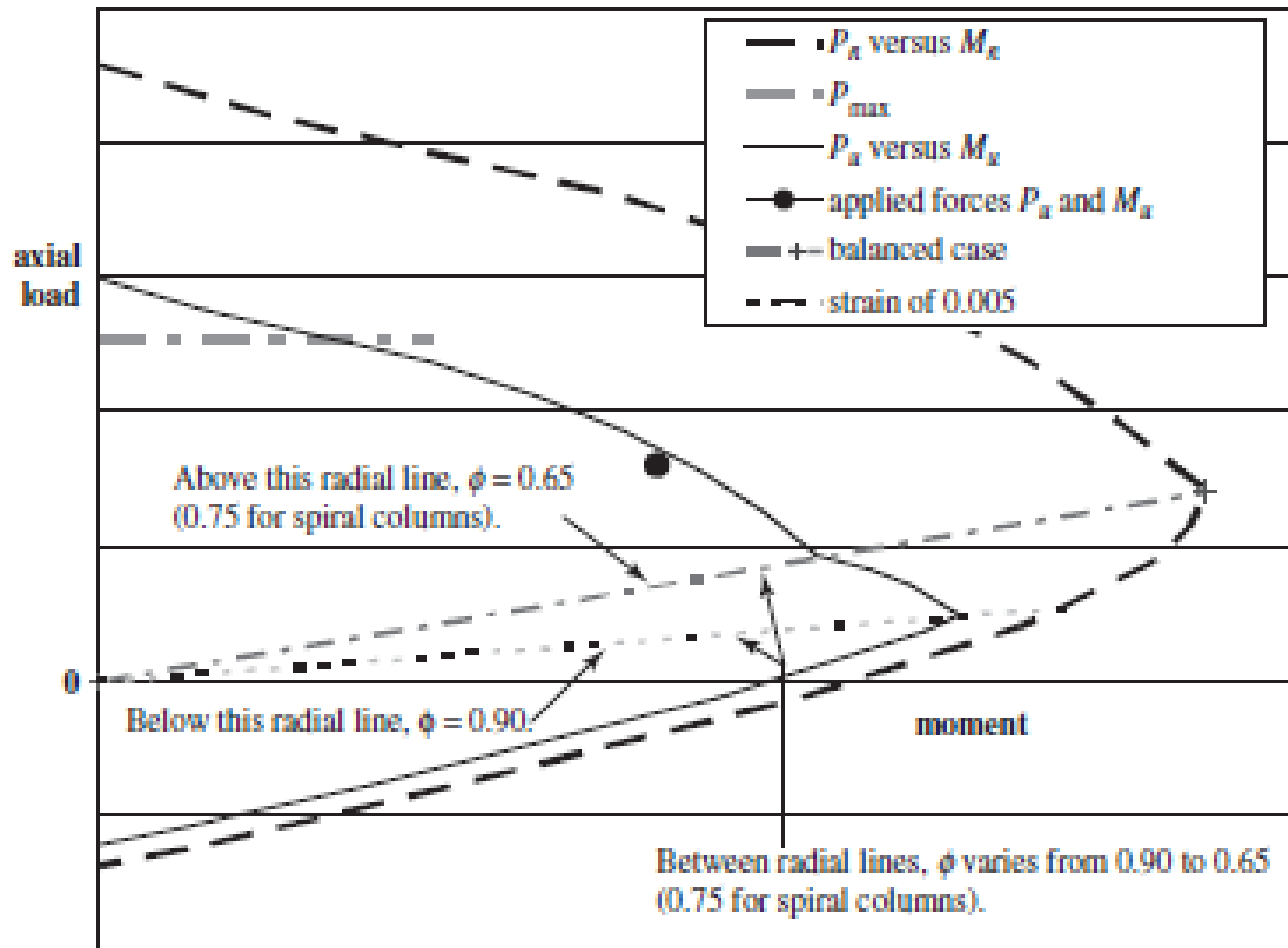
FIGURE 10.13 A column interaction curve adjusted for the three modifications described in this section (10.5).

Legend (within figure):
- P_a versus M_g
- P_{max}
- P_a versus M_g
- applied forces P_a and M_a
- balanced case
- strain of 0.005

axial load

Above this radial line, $\phi = 0.65$ (0.75 for spiral columns).

Below this radial line, $\phi = 0.90$.

0

moment

Between radial lines, ϕ varies from 0.90 to 0.65 (0.75 for spiral columns).

CODE MODIFICATIONS OF COLUMN INTERACTION DIAGRAMS

Cut off because of maximum ϕP_n formulas (c)

ϕP_n

ϕP_n (a)

$0.10 f_c' A_g$
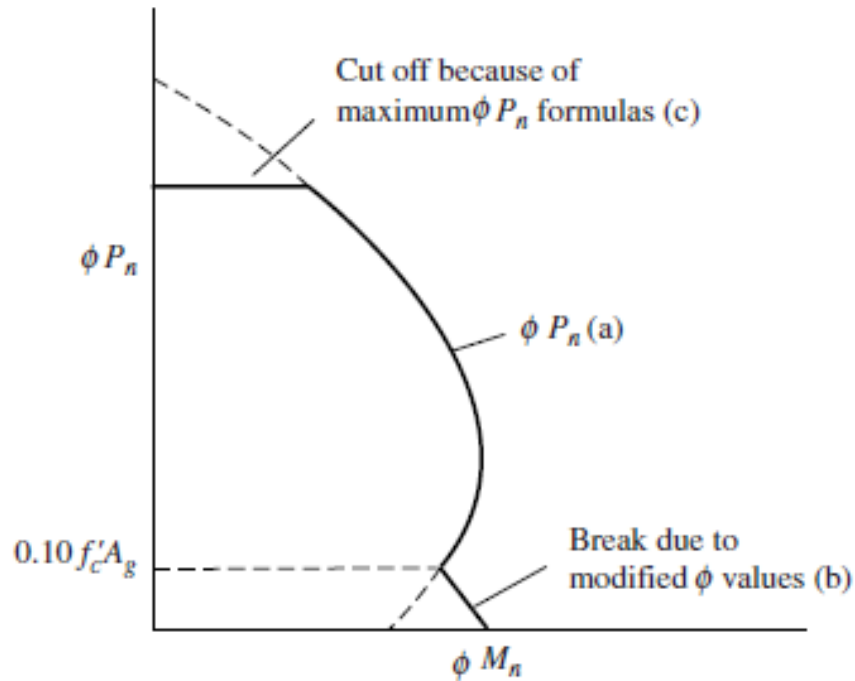
Break due to modified ϕ values (b)

ϕM_n

Figure 10.13a A column interaction curve adjusted for the three modifications described in this section (10.5).

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED
COLUMNS USING INTERACTION DIAGRAMS

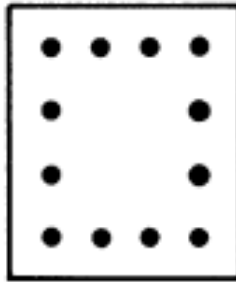## DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

If individual column interaction diagrams were prepared as described in the preceding sections, it would be necessary to have a diagram for each different column cross section, for each different set of concrete and steel grades, and for each different bar arrangement. The result would be an astronomical number of diagrams. The number can be tremendously reduced, however, if the diagrams are plotted with ordinates of $K_n = P_n/f'_c A_g$ (instead of P_n) and with abscissas of $R_n = P_n e/f'_c A_g h$ (instead of M_n). The resulting normalized interaction diagrams can be used for cross sections with widely varying dimensions. The ACI has prepared normalized interaction curves in this manner for the different cross section and bar arrangement situations shown in Figure 10.14 and for different grades of steel and concrete.

Two of the ACI diagrams are given in Figures 10.15 and 10.16, while Appendix A (Graphs A.2–A.13) presents several other ones for the situations given in parts (a), (b), and (d) of Figure 10.14. *Notice that these ACI diagrams do not include the three modifications described just now.*
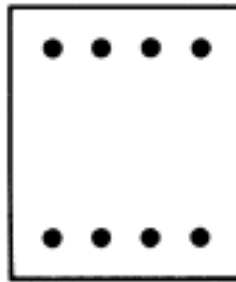
DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

The ACI column interaction diagrams are used in Examples 10.3 to 10.7 to design or analyze columns for different situations. In order to correctly use these diagrams, it is necessary to compute the value of γ (gamma), which is equal to the distance from the center of the bars on one side of the column to the center of the bars on the other side of the column divided by h, the depth of the column (both values being taken in the direction of bending). Usually the value of γ obtained falls in between a pair of curves, and interpolation of the curve readings will have to be made.
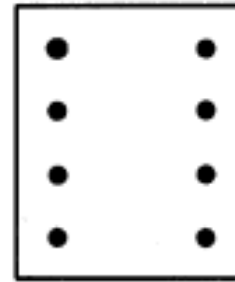
DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS
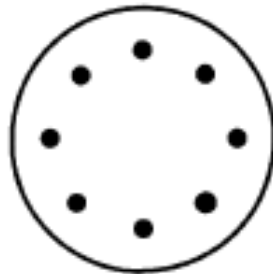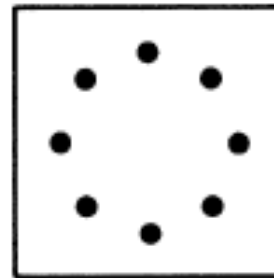

tied column
with bars on
all 4 faces
(a)

tied column
with bars
on 2 end
faces
(b)

tied column
with bars
on 2 lateral
faces
(c)

round
spiral
column
(d)

square
spiral
column
(e)

Figure 10.14

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS
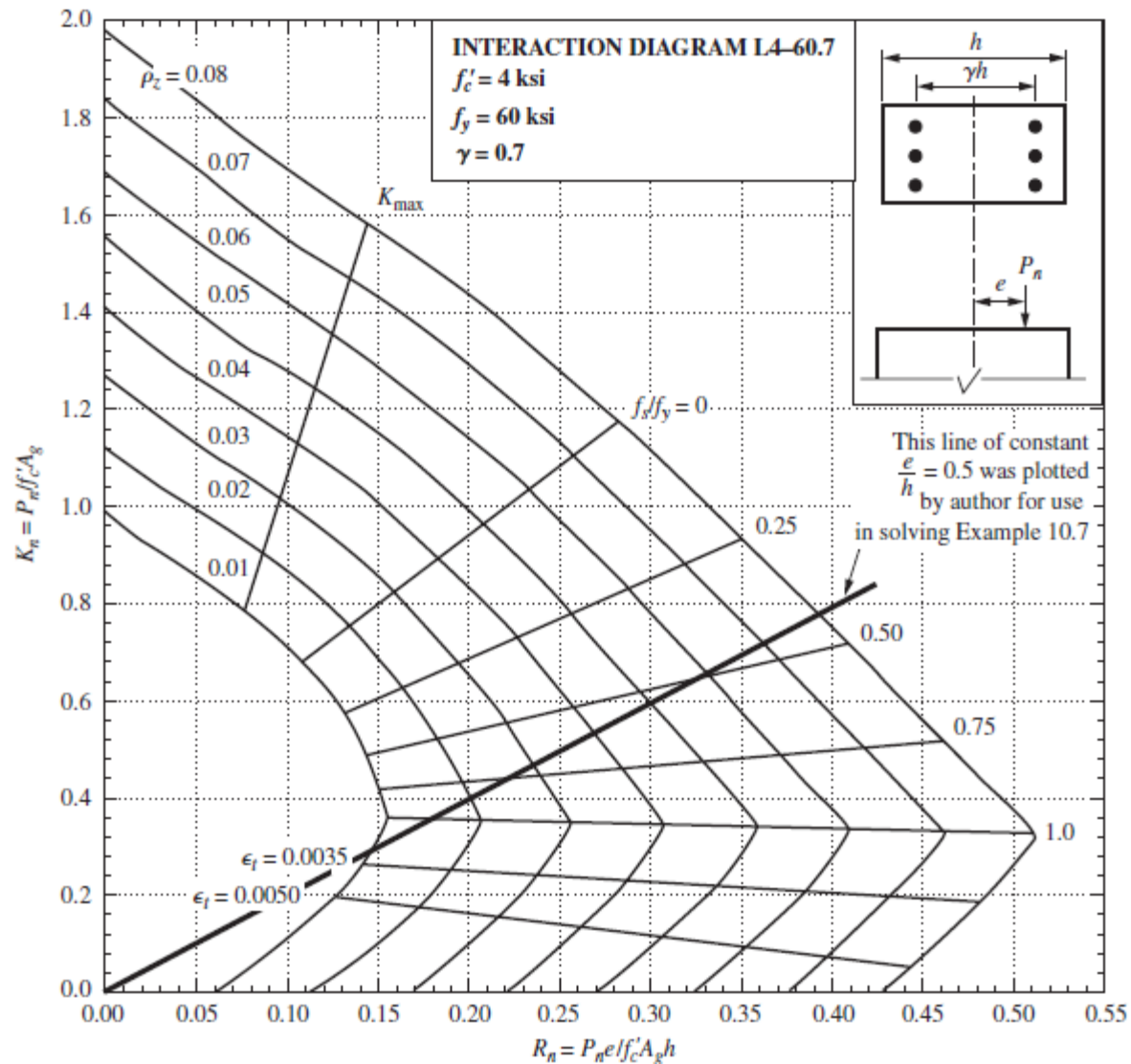

Figure 10.15 ACI rectangular column interaction diagrams when bars are placed on two faces only. (Permission of American Concrete Institute.)

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

Caution

Be sure that the column picture at the upper right of the interaction curve being used agrees with the column being considered. In other words, are there bars on two faces of the column or on all four faces? If the wrong curves are selected, the answers may be quite incorrect.

Although several methods are available for selecting column sizes, a trial-and-error method is about as good as any. With this procedure the designer estimates what he or she thinks is a reasonable column size and then determines the steel percentage required for that column size from the interaction diagram. If it is felt that the ρ_g determined is unreasonably large or small, another column size can be selected and the new required ρ_g selected from the diagrams, and so on. In this regard, the selection of columns for which ρ_g is greater than 4 or 5% results in congestion of the steel, particularly at splices, and consequent difficulties in getting the concrete down into the forms.

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS
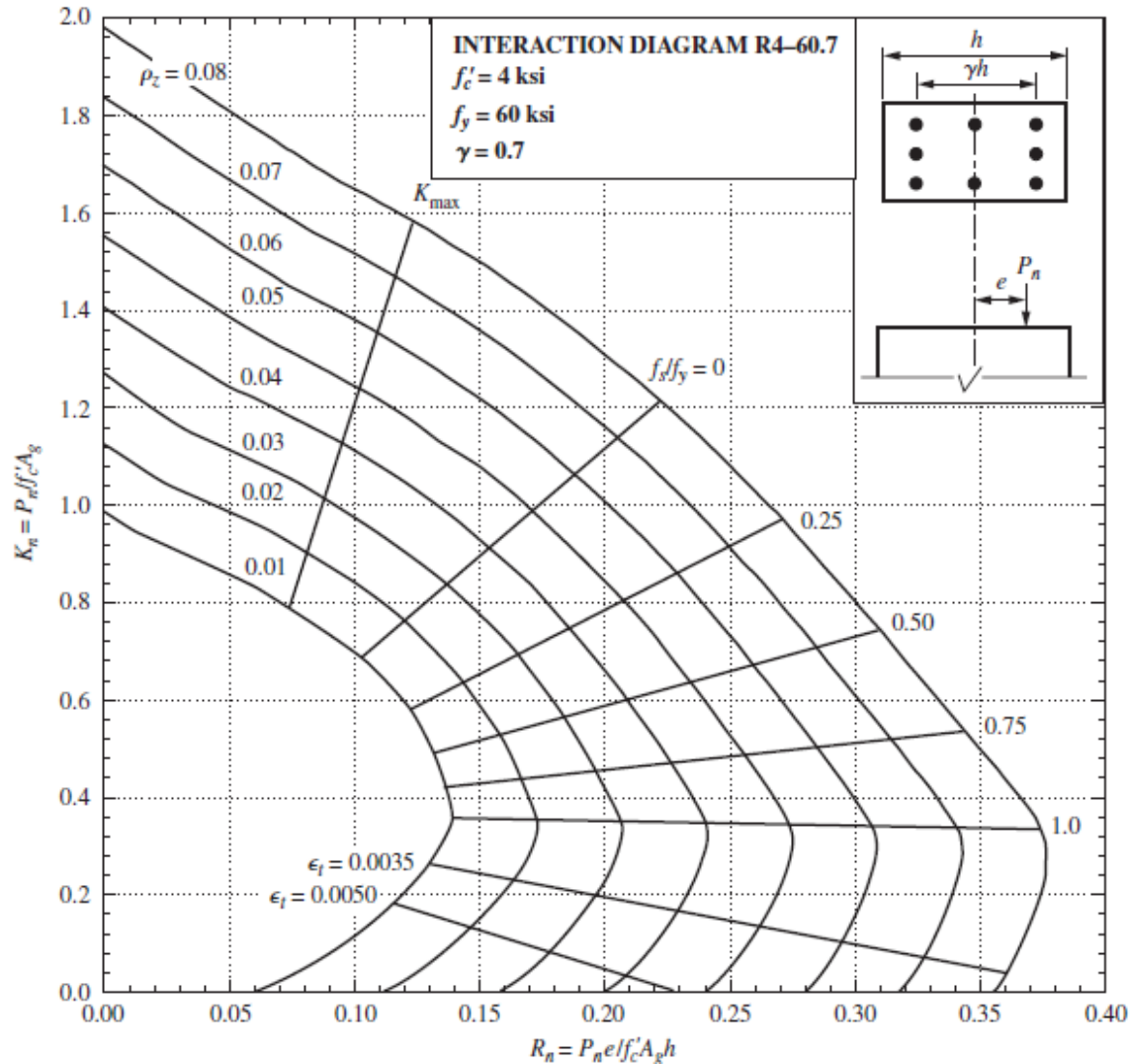

Figure 10.16 ACI rectangular column interaction diagram when bars are placed along all four faces. (Permission of American Concrete Institute.)

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

Example 3

The short 14×20-in. tied column of Figure 10.17 is to be used to support the following loads and moments: $P_D = 125$ k, $P_L = 140$ k, $M_D = 75$ ft-k and $M_L = 90$ ft-k. If $f'_c = 4000$ psi and $f_y = 60,000$ psi, select reinforcing bars to be placed in its end faces only using appropriate ACI column interaction diagrams.
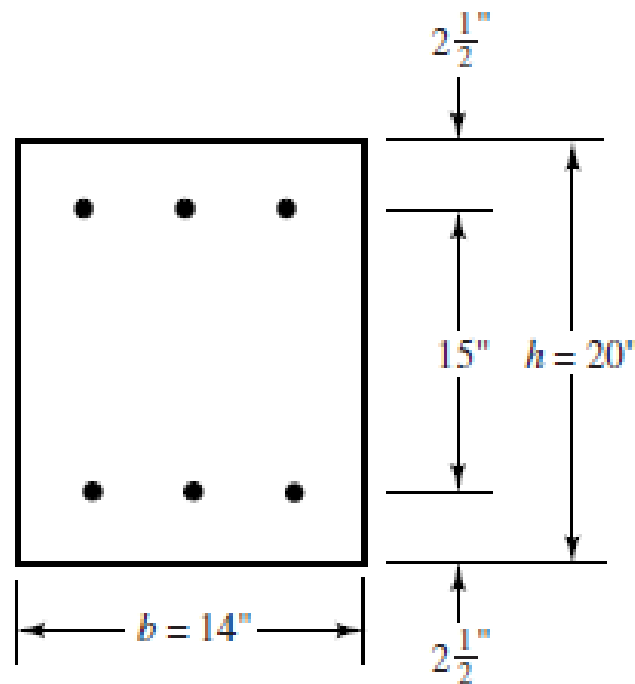

Figure 10.17

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

Solution:

$$P_u = (1.2)(125 \text{ k}) + (1.6)(140 \text{ k}) = 374 \text{ k}$$

$$P_n = \frac{374 \text{ k}}{0.65} = 575.4 \text{ k}$$

$$M_u = (1.2)(75 \text{ ft-k}) + (1.6)(90 \text{ ft-k}) = 234 \text{ ft-k}$$

$$M_n = \frac{234 \text{ ft-k}}{0.65} = 360 \text{ ft-k}$$

$$e = \frac{(12 \text{ in/ft})(360 \text{ ft-k})}{575.4 \text{ k}} = 7.51 \text{ in.}$$

$$\gamma = \frac{15 \text{ in.}}{20 \text{ in.}} = 0.75$$

Compute values of K_n and R_n

$$K_n = \frac{P_n}{f'_c A_g} = \frac{575.4 \text{ k}}{(4 \text{ ksi})(14 \text{ in.} \times 20 \text{ in.})} = 0.513$$

$$R_n = \frac{P_n e}{f'_c A_g h} = \frac{(575.4 \text{ k})(7.51 \text{ in.})}{(4 \text{ ksi})(14 \text{ in.} \times 20 \text{ in.})(20 \text{ in.})} = 0.193$$

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

The value of γ falls between γ values for Graphs 3 and 4 of Appendix A. Therefore, interpolating between the two as follows:

γ	0.70	0.75	0.80
ρ_g	0.0220	0.0202	0.0185

$$A_s = \rho_g bh = (0.0202)(14 \text{ in.})(20 \text{ in.}) = 5.66 \text{ in.}^2$$

Use 6 #9 bars = 6.00 in.^{2}

Notes

(a) Note that $\phi = 0.65$ as initially assumed since the graphs used show $\frac{f_s}{f_y}$ is <1.0 and thus $\epsilon_t < 0.002$.

(b) Code requirements must be checked as in Example 9.1. (See Figure 10.25 to understand.)

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

Example 4

Design a short square column for the following conditions: P_u = 600 k, M_u = 80 ft-k, f_c = 4000 psi, and f_y = 60,000 psi. Place the bars uniformly around all four faces of the column.

Solution:

Assume the column will have an average compression stress = about $0.6f'_c$ = 2400 psi = 2.4 ksi.

$$A_{g\ required} = \frac{600\ k}{2.4\ ksi} = 250\ in.^2$$

Try a 16-in. × 16-in. column (A_g = 256 in.^{2}) with the bar arrangement shown in Figure 10.18.

$$e = \frac{M_u}{P_u} = \frac{(12\ in/ft)\,(80\ ft\text{-}k)}{600\ k} = 1.60\ in.$$

$$P_n = \frac{P_u}{\phi} = \frac{600\ k}{0.65} = 923.1\ k$$

$$K_n = \frac{P_n}{f'_c A_g} = \frac{923.1\ k}{(4\ ksi)\,(16\ in.\ \times\ 16\ in.)} = 0.901$$

$$R_n = \frac{P_n e}{f'_c A_g h} = \frac{(923.1\ k)\,(1.6\ in.)}{(4\ ksi)\,(16\ in.\ \times\ 16\ in.)\,(16\ in.)} = 0.0901$$

$$\gamma = \frac{11\ in.}{16\ in.} = 0.6875$$

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

Interpolating between values given in Graphs 6 and 7 of Appendix A.

γ	0.600	0.6875	0.700
ρ_g	0.025	0.023	0.022

$A_s = (0.023)(16 \text{ in.})(16 \text{ in.}) = 5.89 \text{ in.}^2$
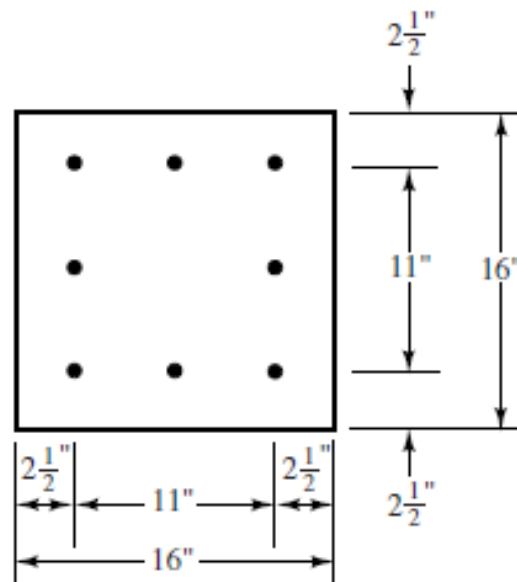
Use 8 #8 bars $= 6.28 \text{ in.}^2$

Figure 10.18

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

Notes

(a) Note that $\phi = 0.65$ as initially assumed since the graphs used show $f_s/f_y < 1.0$ and, thus, $\epsilon_t < 0.002$.

(b) Code requirements must be checked as in Example 9.1. (See Figure 10.25.)

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

Example 5

Using the ACI column interaction graphs, select reinforcing for the short round spiral column shown in Figure 10.19 if $f'_c = 4000$ psi, $f_y = 60,000$ psi, $P_u = 500$ k, and $M_u = 225$ ft-k.

Solution:

$$e = \frac{(12 \text{ in/ft})(225 \text{ ft-k})}{500 \text{ k}} = 5.40 \text{ in.}$$

$$P_n = \frac{P_u}{\phi} = \frac{500 \text{ k}}{0.75} = 666.7 \text{ k}$$

$$K_n = \frac{P_n}{f'_c A_g} = \frac{666.7 \text{ k}}{(4 \text{ ksi})(314 \text{ in.}^2)} = 0.531$$

$$R_n = \frac{P_n e}{f'_c A_g h} = \frac{(666.7 \text{ k})(5.40 \text{ in.})}{(4 \text{ ksi})(314 \text{ in.}^2)(20 \text{ in.})} = 0.143$$

$$\gamma = \frac{15 \text{ in.}}{20 \text{ in.}} = 0.75$$

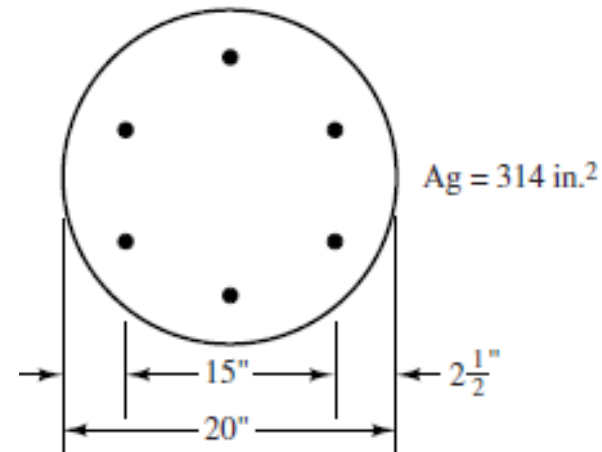$A_g = 314 \text{ in.}^2$

15" $2\frac{1}{2}"$

20"

Figure 10.19

By interpolation between Graphs 11 and 12 of Appendix A, ρ_g is found to equal 0.0235 and $f_s/f_y < 1.0$.

$$\rho A_g = (0.0235)(314 \text{ in.}^2) = 7.38 \text{ in.}^2$$

Use 8 #9 bars $= 8.00 \text{ in.}^2$

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

In Example 6 it is desired to select a 14-in. wide column with approximately 2% steel. This is done by trying different column depths and then determining the steel percentage required in each case.

Example 6

Design a 14-in.-wide rectangular short tied column with bars only in the two end faces for $P_u = 500$ k, $M_u = 250$ ft-k, $f'_c = 4000$ psi, and $f_y = 60,000$ psi. Select a column with approximately 2% steel.

Solution:

$$e = \frac{M_u}{P_u} = \frac{(12 \text{ in/ft})(250 \text{ ft-k})}{500 \text{ k}} = 6.00 \text{ in.}$$

$$P_n = \frac{P_u}{\phi} = \frac{500 \text{ k}}{0.65} = 769.2 \text{ k}$$

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

Example 6

Solution:

Trying several column sizes and determining reinforcing.

Trial sizes (in.)	14 × 20	14 × 22	14 × 24
$K_n = \dfrac{P_n}{f'_c A_g}$	0.687	0.624	0.572
$R_n = \dfrac{P_n e}{f'_c A_g h}$	0.206	0.170	0.143
$\gamma = \dfrac{h - 2 \times 2.50}{h}$	0.750	0.773	0.792
ρ_g by interpolation	0.0315	0.020	0.011

Use 14-in. × 22-in. column

$$A_g = (0.020)(14 \text{ in.} \times 22 \text{ in.}) = 6.16 \text{ in.}^2$$

Use 8 #8 bars $= 6.28$ in.^{2}

Same notes as for Examples 10.3 and 10.4.

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

One more illustration of the use of the ACI interaction diagrams is presented with Example 7. In this example, the nominal column load Pn at a given eccentricity which a column can support is determined.

With reference to the ACI interaction curves, the reader should carefully note that the value of R_n (which is $P_n e / f'_c A_g h$) for a particular column, equals e/h times the value of K_n ($P_n / f'_c A_g$)for that column. This fact needs to be understood when the user desires to determine the nominal load that a column can support at a given eccentricity.
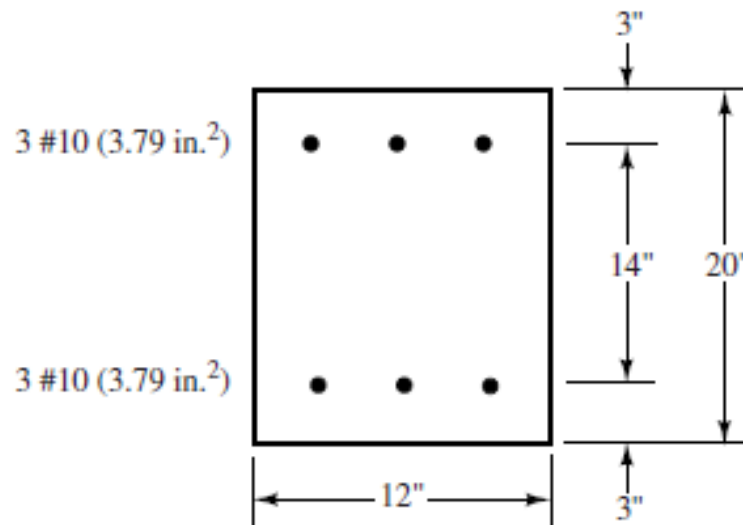

Figure 10.20

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

In Example 7 the nominal load that the short column of Figure 10.20 can support at an eccentricity of 10 in. with respect to the x axis is determined. If we plot on the interaction diagram the intersection point of K_n and R_n for a particular column and draw a straight line from that point to the lower left corner or origin of the figure, we will have a line with a constant e/h. For the column of Example 6 $e/h = 10/20 = 0.5$. Therefore a line is plotted from the origin through a set of assumed values for K_n and R_n in the proportion of 10/20 to each other. In this case, K_n was set equal to 0.8 and $R_n = 0.5 \times 0.8 = 0.4$. Next a line was drawn from that intersection point to the origin of the diagram as shown in Figure 10.16. Finally, the intersection of this line with ρ_g (0.0316 in this example) was determined, and the value of K_n or R_n was read. This latter value enables us to compute P_n.

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

Example 7

Using the appropriate interaction curves, determine the value of P_n for the short tied column shown in Figure 10.20 if $e_x = 10$ in. Assume $f'_c = 4000$ psi and $f_y = 60,000$ psi.

Solution:

$$\frac{e}{h} = \frac{10 \text{ in.}}{20 \text{ in.}} = 0.500$$

$$\rho_g = \frac{(2)(3.79 \text{ in.}^2)}{(12 \text{ in.})(20 \text{ in.})} = 0.0316$$

$$\gamma = \frac{14 \text{ in.}}{20 \text{ in.}} = 0.700$$

Plotting a straight line through the origin and the intersection of assumed values of K_n and R_n (say, 0.8 and 0.4, respectively).
For ρ_g of 0.0316, we read the value of $R_n = 0.24$:

$$R_n = \frac{P_n e}{f'_c A_g h} = 0.24$$

$$P_n = \frac{(0.24)(4 \text{ ksi})(12 \text{ in.} \times 20 \text{ in.})(20 \text{ in.})}{10 \text{ in.}} = 460.8 \text{ k}$$

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

When the usual column is subjected to axial load and moment, it seems reasonable to assume initially that $\phi = 0.65$ for tied columns and 0.75 for spiral columns. It is to be remembered, however, that under certain conditions these φ values may be increased, as discussed.

SHEAR IN COLUMNS

SHEAR IN COLUMNS

The shearing forces in interior columns are usually quite small and normally do not control the design. However, the shearing forces in exterior columns can be large, particularly in columns bent in double curvature. Section 11.3.1.2 of the ACI Code provides the following equations for determining the shearing force that can be carried by the concrete for a member subjected simultaneously to axial compression and shearing forces.

$$V_c = 2\left(1 + \frac{N_u}{2000A_g}\right)\sqrt{f'_c}\,b_w d \qquad \text{(ACI Equation 11-4)}$$

In SI units this equation is:

$$V_c = \left(1 + \frac{N_u}{14A_g}\right)\left(\frac{\sqrt{f'_c}}{6}\right)b_w d$$

SHEAR IN COLUMNS

In these equations, N_u is the factored axial force acting simultaneously with the factored shearing force, V_u, that is applied to the member. The value of N_u/A_g is the average factored axial stress in the column and is expressed in units of psi. Should V_u be greater than $\phi V_c/2$, it will be necessary to calculate required tie spacing using the stirrup spacing procedures described in Chapter 8. The results will be closer tie spacing than required by the usual column rules.

Sections 11.3.3 and 11.5.6.3 of the ACI Code specify the method for calculating the contribution of the concrete to the total shear strength of circular columns and for calculating the contribution of shear reinforcement for cases where circular hoops, ties, or spirals are present. According to the Commentary of the Code in their Section 11.3.3, the entire cross section in circular columns is effective in resisting shearing forces. The shear area, $b_w d$, in ACI Equation 11-4 then would be equal to the gross area of the column. However, to provide for compatibility with other calculations requiring an effective depth, ACI requires that the shear area be computed as an equivalent rectangular area in which:

SHEAR IN COLUMNS

$$b_w = D$$
$$d = 0.8D$$

In these equations, D is the gross diameter of the column. If the constant modifying D in the effective depth equation were equal to /4, which is equal to 0.7854, the effective rectangular area would be equal to the gross area of the circular column. As such, the area of the column is overestimated by a little less than 2% when using the equivalent area prescribed by ACI.

BIAXIAL BENDING

BIAXIAL BENDING

Many columns are subjected to biaxial bending, that is, bending about both axes. Corner columns in buildings where beams and girders frame into the columns from both directions are the most common cases, but there are others, such as where columns are cast monolithically as part of frames in both directions or where columns are supporting heavy spandrel beams. Bridge piers are almost always subject to biaxial bending.

Circular columns have polar symmetry and thus the same ultimate capacity in all directions. The design process is the same, therefore, regardless of the directions of the moments. If there is bending about both the *x* and *y* axes, the biaxial moment can be computed by combining the two moments or their eccentricities as follows:

$$M_u = \sqrt{(M_{ux})^2 + (M_{uy})^2}$$

or

$$e = \sqrt{(e_x)^2 + (e_y)^2}$$

BIAXIAL BENDING

For shapes other than circular ones, it is necessary to consider the three-dimensional interaction effects. Whenever possible, it is desirable to make columns subject to biaxial bending circular in shape. Should it be necessary to use square or rectangular columns for such cases, the reinforcing should be placed uniformly around the perimeters.

You might quite logically think that you could determine P_n for a biaxially loaded column by using static equations, as was done in Example 2. Such a procedure will lead to the correct answer, but the mathematics involved is so complicated due to the shape of the compression side of the column that the method is not a practical one. Nevertheless, a few comments are made about this type of solution, and reference is made to Figure 10.21.
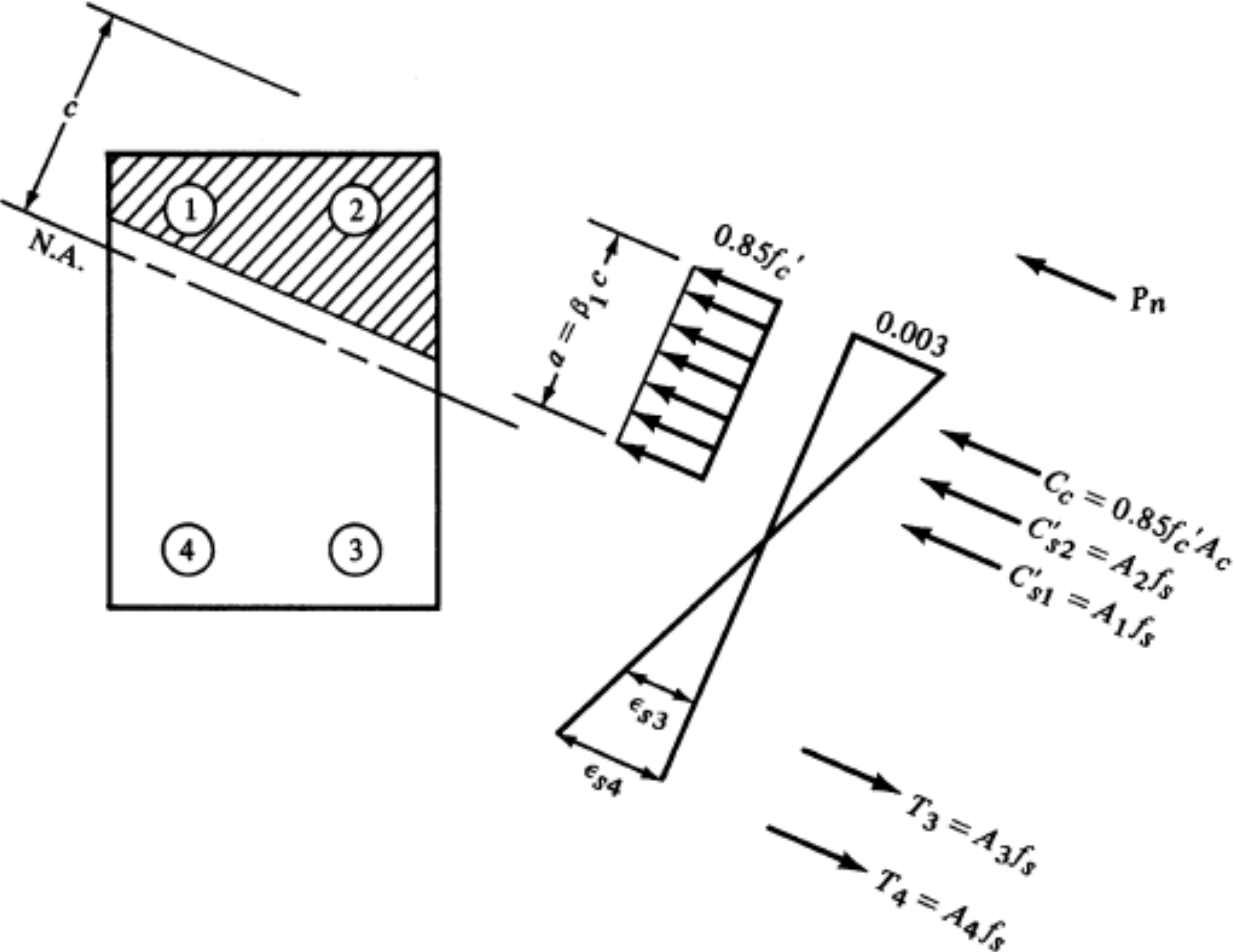
BIAXIAL BENDING

Figure 10.21

BIAXIAL BENDING

An assumed location is selected for the neutral axis, and the appropriate strain triangles are drawn as shown in the figure 10.21. The usual equations are written with $C_c = 0.85 f'_c$ times the shaded area *Ac* and with each bar having a force equal to its cross-sectional area times its stress. The solution of the equation yields the load that would establish that neutral axis—but the designer usually starts with certain loads and eccentricities and does not know the neutral axis location. Furthermore, the neutral axis is probably not even perpendicular to the resultant *e* =

$$\sqrt{(e_x)^2 + (e_y)^2}.$$

For column shapes other than circular ones, it is desirable to consider three-dimensional interaction curves such as the one shown in Figure 10.22. In this figure the curve labeled M_{nxo} represents the interaction curve if bending occurs about the *x* axis only, while the one labeled M_{nyo} is the one if bending occurs about the *y* axis only.

BIAXIAL BENDING

In this figure, for a constant P_n, the hatched plane shown represents the contour of M_n for bending about any axis.

Today, the analysis of columns subject to biaxial bending is primarily done with computers. One of the approximate methods that is useful in analysis and that can be handled with pocket calculators includes the use of the so-called reciprocal interaction equation, which was developed by Professor Boris Bresler of the University of California at Berkeley. This equation, which is shown in Section R10.3.6 of the ACI Commentary, follows:

$$\frac{1}{P_{ni}} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_o}$$

where

P_{ni} = the nominal axial load capacity of the section when the load is placed at a given eccentricity along both axes.

P_{nx} = the nominal axial load capacity of the section when the load is placed at an eccentricity e_x.

P_{ny} = the nominal axial load capacity of the section when the load is placed at an eccentricity e_y.

P_o = the nominal axial load capacity of the section when the load is placed with a zero eccentricity. It is usually taken as $0.85 f'_c A_g + f_y A_s$.
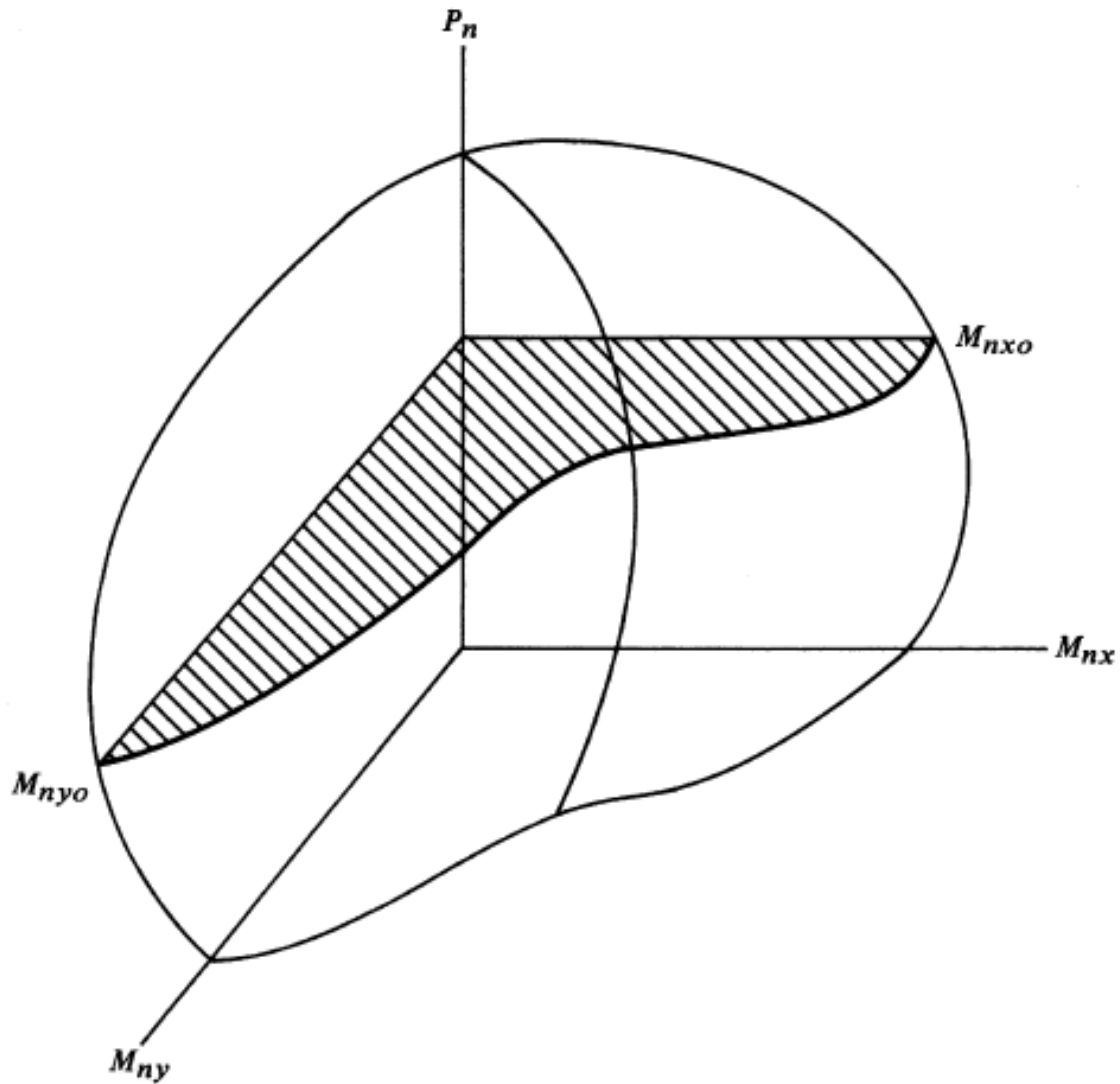
BIAXIAL BENDING

Figure 10.22

BIAXIAL BENDING

The Bresler equation works rather well as long as P_{ni} is at least as large as $0.10Po$. Should P_{ni} be less than $0.10Po$, it is satisfactory to neglect the axial force completely and design the section as a member subject to biaxial bending only. This procedure is a little on the conservative side. For this lower part of the interaction curve, it will be remembered that a little axial load increases the moment capacity of the section. The Bresler equation does not apply to axial tension loads. Professor Bresler found that the ultimate loads predicted by his equation for the conditions described do not vary from test results by more than 10%.

Example 10.8 illustrates the use of the reciprocal theorem for the analysis of a column subjected to biaxial bending. The procedure for calculating P_{nx} and P_{ny} is the same as the one used for the prior examples of this chapter.

BIAXIAL BENDING

Example 8

Determine the design capacity P_{ni} of the short tied column shown in Figure 10.23, which is subjected to biaxial bending. $f'_c = 4000$ psi, $f_y = 60,000$ psi, $e_x = 16$ in., and $e_y = 8$ in.

Solution:

For Bending about x-Axis

$$\gamma = \frac{20 \text{ in.}}{25 \text{ in.}} = 0.80$$

$$\rho_g = \frac{8.00 \text{ in.}^2}{(15 \text{ in.})(25 \text{ in.})} = 0.0213$$

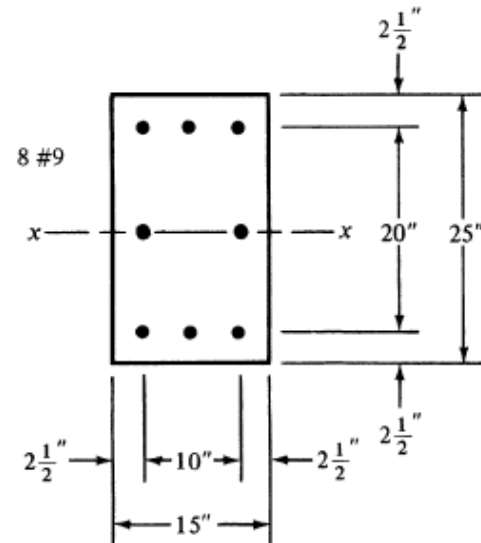$$\frac{e}{h} = \frac{16 \text{ in.}}{25 \text{ in.}} = 0.64$$

Figure 10.23

Drawing Line of Constant $e/h = 0.64$ in Graph 8 of Appendix A and Estimating R_n Corresponding to $\rho_g = 0.0213$, Read $R_n \cong 0.185$

$$R_n = \frac{P_{nx}e}{f'_c A_g h} = 0.185$$

$$P_{nx} = \frac{(4 \text{ ksi})(15 \text{ in.} \times 25 \text{ in.})(25 \text{ in.})(0.185)}{16 \text{ in.}} = 434 \text{ k}$$

BIAXIAL BENDING

Example 8

Solution:

For Bending about y-Axis

$$\gamma = \frac{10 \text{ in.}}{15 \text{ in.}} = 0.667$$

$$\rho_g = \frac{8.00 \text{ in.}^2}{(15 \text{ in.})(25 \text{ in.})} = 0.0213$$

$$\frac{e}{h} = \frac{8 \text{ in.}}{15 \text{ in.}} = 0.533$$

Drawing Radial Lines of Constant e/h (0.533) in Graphs 6 and 7 of Appendix A and Interpolating between Them for $\gamma = 0.667$

$$R_n = \frac{P_{ny}e}{f_c' A_g h} = 0.163$$

$$P_{ny} = \frac{(4 \text{ ksi})(15 \text{ in.} \times 25 \text{ in.})(15 \text{ in.})(0.163)}{8 \text{ in.}} = 458 \text{ k}$$

Determining Axial Load Capacity of Section

$$P_o = (0.85)(4.0 \text{ ksi})(15 \text{ in.} \times 25 \text{ in.}) + (8.00 \text{ in.}^2)(60 \text{ ksi}) = 1755 \text{ k}$$

BIAXIAL BENDING

Example 8

Solution:

Using the Bresler Expression to Determine P_{ni}

$$\frac{1}{P_{ni}} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_o}$$

$$\frac{1}{P_{ni}} = \frac{1}{434\ k} + \frac{1}{458\ k} - \frac{1}{1755\ k}$$

Multiplying through by 1755 k

$$\frac{1755\ k}{P_{ni}} = 4.044 + 3.832 - 1$$

$$P_{ni} = \underline{\underline{255.3\ k}}$$

BIAXIAL BENDING

If the moments in the weak direction (y axis here) are rather small compared to bending in the strong direction (x axis), it is rather common to neglect the smaller moment. This practice is probably reasonable as long as e_y is less than about 20% of e_x, since the Bresler expression will show little reduction for P_{ni}. For the example just solved, an e_y equal to 50% of ex caused the axial load capacity to be reduced by approximately 40%.

Example 10.9 illustrates the design of a column subject to biaxial bending. The Bresler expression, which is of little use in the proportioning of such members, is used to check the capacities of the sections selected by some other procedure. Exact theoretical designs of columns subject to biaxial bending are very complicated and, as a result, are seldom handled with pocket calculators. They are proportioned either by approximate methods or with computer programs.

Design of Biaxially Loaded Columns

Design of Biaxially Loaded Columns

During the past few decades, several approximate methods have been introduced for the design of columns with biaxial moments. For instance, quite a few design charts are available with which satisfactory designs may be made. The problems are reduced to very simple calculations in which coefficients are taken from the charts and used to magnify the moments about a single axis. Designs are then made with the regular uniaxial design charts.

Another approximate procedure that works fairly well for design office calculations is used for Example 10.9. If this simple method is applied to square columns, the values of both M_{nx} and M_{ny} are assumed to act about both the x-axis and the y-axis (i.e., $M_x = M_y = M_{nx} + M_{ny}$). The steel is selected about one of the axes and is spread around the column, and the Bresler expression is used to check the ultimate load capacity of the eccentrically loaded column.

Design of Biaxially Loaded Columns

Should a rectangular section be used where the y axis is the weaker direction, it would seem logical to calculate $M_y = M_{nx} + M_{ny}$ and to use that moment to select the steel required about the y axis and spread the computed steel area over the whole column cross section. Although such a procedure will produce safe designs, the resulting columns may be rather uneconomical because they will often be much too strong about the strong axis. A fairly satisfactory approximation is to calculate $My = Mnx + Mny$ and multiply it by b/h, and with that moment design the column about the weaker axis.

Example 9 illustrates the design of a short square column subject to biaxial bending. The approximate method described in the last two paragraphs is used, and the Bresler expression is used for checking the results. If this had been a long column, it would have been necessary to magnify the design moments for slenderness effects, regardless of the design method used.

Design of Biaxially Loaded Columns

Example 9

Select the reinforcing needed for the short square tied column shown in Figure 10.24 for the following: $P_D = 100\,k$, $P_L = 200\,k$, $M_{DX} = 50\,ft\text{-}k$, $M_{LX} = 110\,ft\text{-}k$, $M_{DY} = 40\,ft\text{-}k$, $M_{LY} = 90\,ft\text{-}k$, $f'_c = 4000\,psi$, and $f_y = 60,000\,psi$.

Solution:

Computing Design Values

$$P_u = (1.2)(100\,k) + (1.6)(200\,k) = 440\,k$$

$$\frac{P_u}{f'_c A_g} = \frac{440\,k}{(4\,ksi)(484\,in.^2)} = 0.227$$

Assume $\phi = 0.65$

$$P_n = \frac{440\,k}{0.65} = 677\,k$$

$$M_{ux} = (1.2)(50\,ft\text{-}k) + (1.6)(110\,ft\text{-}k) = 236\,ft\text{-}k$$

$$M_{nx} = \frac{236\,ft\text{-}k}{0.65} = 363\,ft\text{-}k$$

$$M_{uy} = (1.2)(40\,ft\text{-}k) + (1.6)(90\,ft\text{-}k) = 192\,ft\text{-}k$$

$$M_{ny} = \frac{192\,ft\text{-}k}{0.65} = 295\,ft\text{-}k$$

Design of Biaxially Loaded Columns

Example 9

Solution:

As a result of biaxial bending, the design moment about the x or y axis is assumed to equal $M_{nx} + M_{ny} = 363 + 295 = 658$ ft-k.

Determining Steel Required

Figure 10.24

$$e_x = e_y = \frac{(12 \text{ in/ft})(658 \text{ ft-k})}{677 \text{ k}} = 11.66 \text{ in.}$$

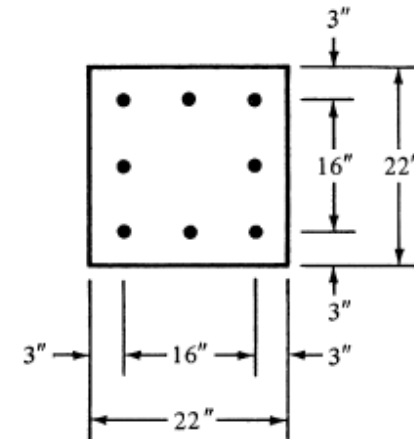$$\gamma = \frac{16 \text{ in.}}{22 \text{ in.}} = 0.727$$

By interpolation from interaction diagrams with bars on all four faces,

$$\rho_g = 0.0235$$

$$A_s = (0.0235)(22 \text{ in.})(22 \text{ in.}) = 11.37 \text{ in.}^2$$

Use 8 #11 $= 12.50$ in.^{2}

Design of Biaxially Loaded Columns

    A review of the column with the Bresler expression gives a P_{ni} = 804 k > 677 k, which is satisfactory. Should the reader go through the Bresler equation here, he or she must remember to calculate the correct e_x and e_y values for use with the interaction diagrams. For instance,

$$e_x = \frac{(12 \text{ in/ft}) (363 \text{ ft-k})}{677 \text{ k}} = 6.43 \text{ in.}$$

When a beam is subjected to biaxial bending, the following approximate interaction equation may be used for design purposes:

$$\frac{M_x}{M_{ux}} + \frac{M_y}{M_{uy}} \leq 1.0$$

Design of Biaxially Loaded Columns

In this expression M_x and M_y are the design moments, M_{ux} is the design moment capacity of the section if bending occurs about the x axis only, and M_{uy} is the design moment capacity if bending occurs about the y axis only. This same expression may be satisfactorily used for axially loaded members if the design axial load is about 15% or less of the axial load capacity of the section. For a detailed discussion of this subject, the reader is referred to the *Handbook of Concrete Engineering*.

Numerous other methods are available for the design of biaxially loaded columns. One method that is particularly useful to the design profession is the PCA Load Contour Method, which is also recommended in the ACI *Design Handbook*