DEVELOPMENT OF INTERACTION DIAGRAMS

The curve for P_n and M_n for a particular column may be extended into the range where P_n becomes a tensile load. We can proceed in exactly the same fashion as we did when P_n was compressive. A set of strains can be assumed, and the usual statics equations can be written and solved for P_n and M_n. Several different sets of strains were assumed for the column of Figure 10.4, and then the values of P_n and M_n were determined. The results were plotted at the bottom of Figure 10.8 and were connected with the dashed line which is labeled "tensile loads."

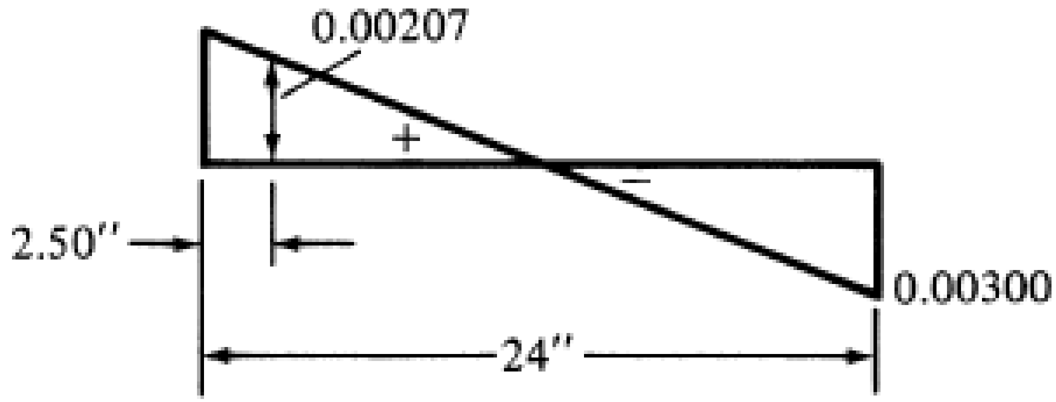


Figure 10.7

DEVELOPMENT OF INTERACTION DIAGRAMS

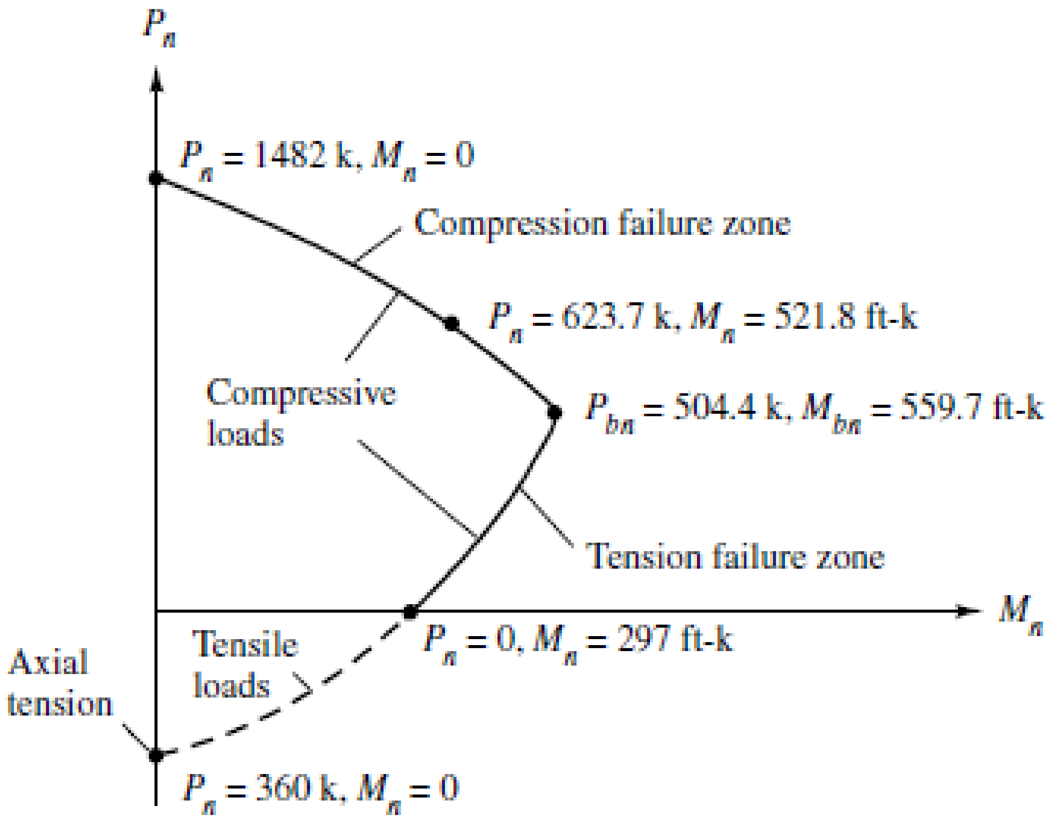
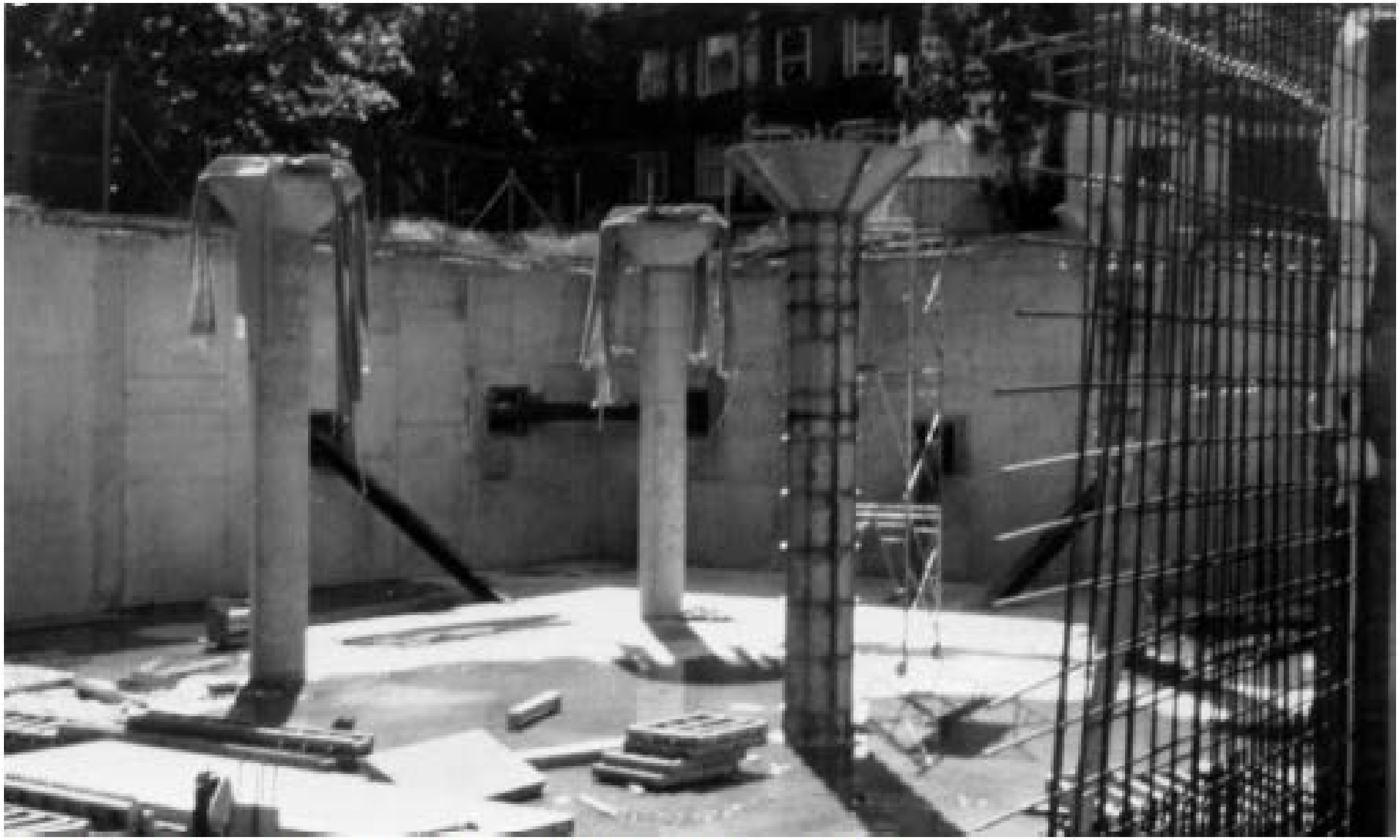


Figure 10.8 Interaction curve for the column of Figure 10.4. Notice these are nominal values.

DEVELOPMENT OF INTERACTION DIAGRAMS

Because axial tension and bending are not very common for reinforced concrete columns, the tensile load part of the curves is not shown in subsequent figures in this chapter. You will note that the largest tensile value of P_n will occur when the moment is zero. For that situation, all of the column steel has yielded, and all of the concrete has cracked. Thus P_n will equal the total steel area A_s times the yield stress. For the column of Figure 10.4



Round columns. (Courtesy of Economy Forms Corporation.)

DEVELOPMENT OF INTERACTION DIAGRAMS

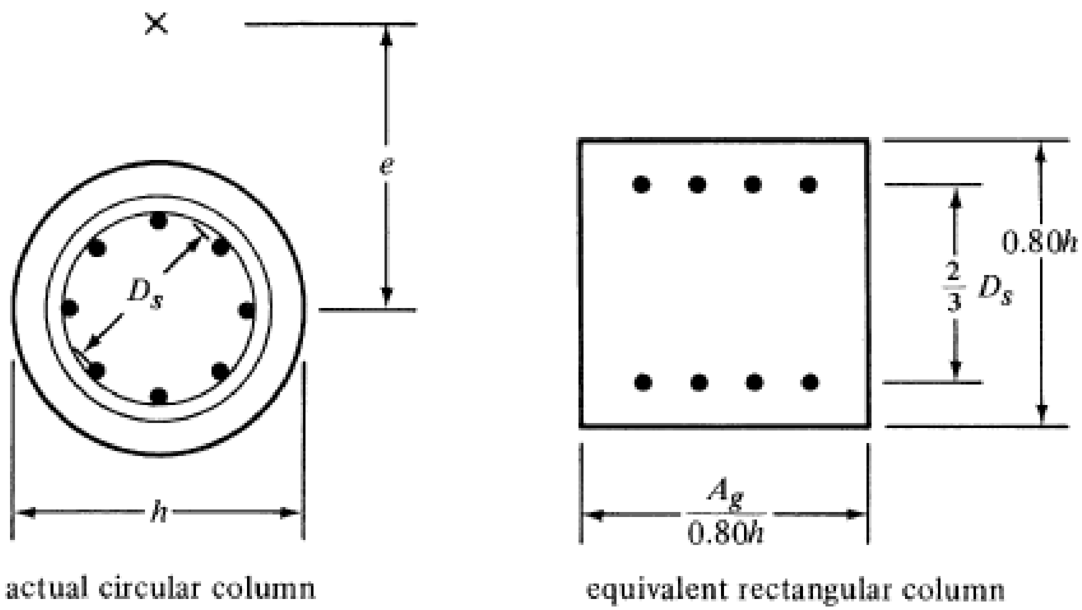


Figure 10.9 Replacing circular column with equivalent rectangular one.

$$P_n = A_s f_y = (6.0 \text{ in.}^2)(60 \text{ ksi}) = 360 \text{ k}$$

On some occasions, members subject to axial load and bending have unsymmetrical arrangements of reinforcing. Should this be the case, you must remember that eccentricity is correctly measured from the plastic centroid of the section.

DEVELOPMENT OF INTERACTION DIAGRAMS

In this chapter P_n values were obtained only for rectangular tied columns. The same theory could be used for round columns, but the mathematics would be somewhat complicated because of the circular layout of the bars, and the calculations of distances would be rather tedious. Several approximate methods have been developed that greatly simplify the mathematics. Perhaps the best known of these is the one proposed by Charles Whitney, in which equivalent rectangular columns are used to replace the circular ones. This method gives results that correspond quite closely with test results.

In Whitney's method, the area of the equivalent column is made equal to the area of the actual circular column, and its depth in the direction of bending is 0.80 times the outside diameter of the real column. One-half the steel is assumed to be placed on one side of the equivalent column and one-half on the other. The distance between these two areas of steel is assumed to equal two-thirds of the diameter (D_s) of a circle passing through the center of the bars in the real column. These values are illustrated in Figure 10.9. Once the equivalent column is established, the calculations for P_n and M_n are made as for rectangular columns.

USE OF INTERACTION DIAGRAMS

USE OF INTERACTION DIAGRAMS

We have seen that by statics the values of P_n and M_n for a given column with a certain set of strains can easily be determined. Preparing an interaction curve with a hand calculator for just one column, however, is quite tedious. Imagine the work involved in a design situation where various sizes, concrete strengths, and steel percentages need to be considered. Consequently, designers resort almost completely to computer programs, computer generated interaction diagrams, or tables for their column calculations. The remainder of this chapter is concerned primarily with computer-generated interaction diagrams such as the one in Figure 10.10. As we have seen, such a diagram is drawn for a column as the load changes from one of a pure axial nature through varying combinations of axial loads and moments and on to a pure bending situation.

USE OF INTERACTION DIAGRAMS

Interaction diagrams are obviously useful for studying the strengths of columns with varying proportions of loads and moments. Any combination of loading that falls inside the curve is satisfactory, whereas any combination falling outside the curve represents failure.

If a column is loaded to failure with an axial load only, the failure will occur at point *A* on the diagram (Figure 10.10). Moving out from point *A* on the curve, the axial load capacity decreases as the proportion of bending moment increases. At the very bottom of the curve, point *C* represents the bending strength of the member if it is subjected to moment only with no axial load present. In between the extreme points *A* and *C*, the column fails due to a combination of axial load and bending. Point *B* is called the *balanced point* and represents the balanced loading case, where theoretically a compression failure and tensile yielding occur simultaneously.

USE OF INTERACTION DIAGRAMS

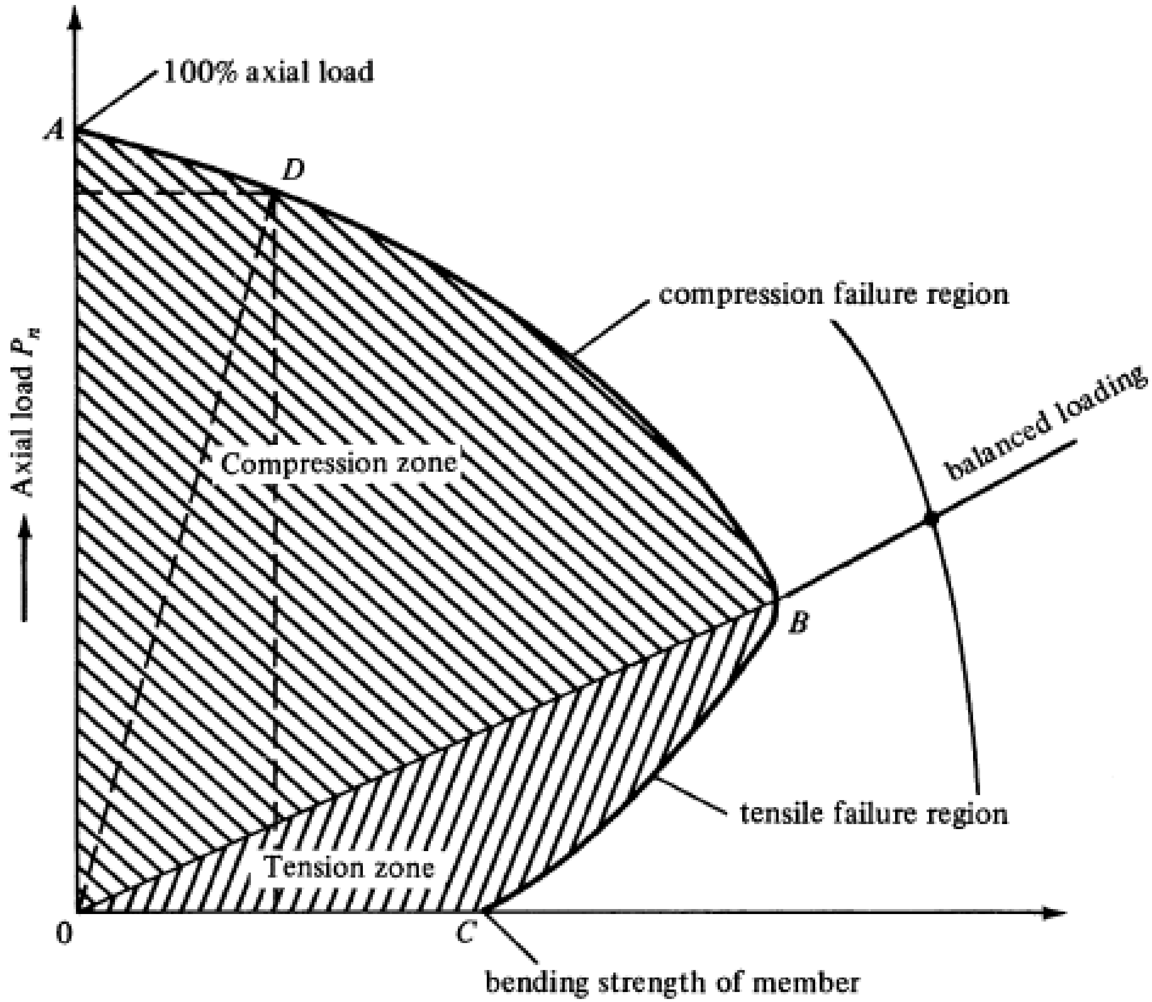


Figure 10.10 Column interaction diagram.

USE OF INTERACTION DIAGRAMS

Refer to point *D* on the curve. The horizontal and vertical dashed lines to this point indicate a particular combination of axial load and moment at which the column will fail. Should a radial line be drawn from point 0 to the interaction curve at any point (as to *D* in this case), it will represent a constant eccentricity of load, that is, a constant ratio of moment to axial load.

You may be somewhat puzzled by the shape of the lower part of the curve from *B* to *C*, where bending predominates. From *A* to *B* on the curve the moment capacity of a section increases as the axial load decreases, but just the opposite occurs from *B* to *C*. A little thought on this point, however, shows that the result is quite logical after all. The part of the curve from *B* to *C* represents the range of tensile failures. Any axial compressive load in that range tends to reduce the stresses in the tensile bars, with the result that a larger moment can be resisted.

USE OF INTERACTION DIAGRAMS

In Figure 10.11 an interaction curve is drawn for the 14” by 24” column with six #9 bars considered in Section 10.3. If eight #9 bars had been used in the same dimension column, another curve could be generated as shown in the figure; if ten #9 bars were used, still another curve would result. The shape of the new diagrams would be the same as for the six #9 curve, but the values of P_n and M_n would be larger.

CODE MODIFICATIONS OF COLUMN INTERACTION DIAGRAMS

CODE MODIFICATIONS OF COLUMN INTERACTION DIAGRAMS

If interaction curves for P_n values were prepared, they would be of the types shown in Figures 10.10 and 10.11. To use such curves to obtain design values, they would have to have three modifications made to them as specified in the Code. These modifications are as follows:

(a). The Code 9.3.2 specifies strength reduction or ϕ factors (0.65 for tied columns and 0.70 for spiral columns) that must be multiplied by P_n values. If a P_n curve for a particular column were multiplied by ϕ, the result would be a curve something like the ones shown in Figure 10.12.

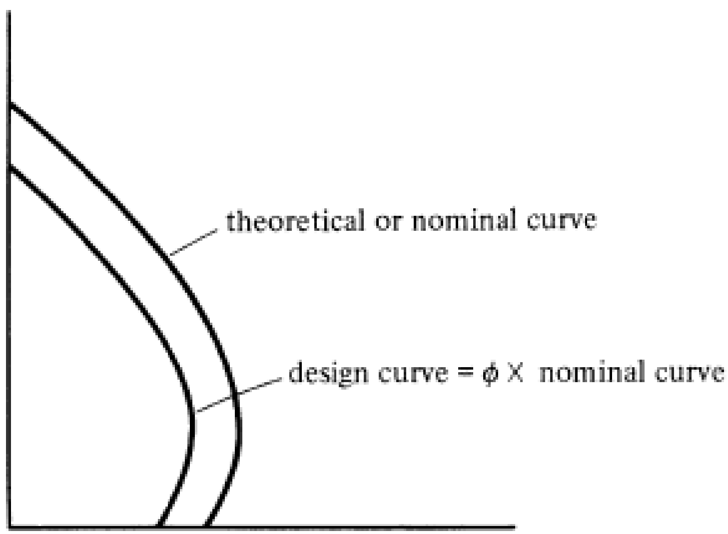


Figure 10.12 Curves for P_n and ϕP_n for a single column.

CODE MODIFICATIONS OF COLUMN INTERACTION DIAGRAMS

(b). The second modification also refers to ϕ factors. The Code specifies values of 0.65 and 0.70 for tied and spiral columns, respectively. Should a column have quite a large moment and a very small axial load so that it falls on the lower part of the curve between points *B* and *C* (see Figure 10.10), the use of these small ϕ values may be a little unreasonable. For instance, for a member in pure bending(point *C* on the same curve) the specified ϕ is 0.90, but if the same member has a very small axial load added, ϕ would immediately fall to 0.65 or 0.70. Therefore, the Code (9.3.2.2) states that when members subject to axial load and bending have net tensile strains (*t*) between the limits for compression-controlled and tensile-controlled sections, they fall in the transition zone for ϕ. In this zone it is permissible to increase ϕ linearly from 0.65 or 0.70 to 0.90 as ε_t increases from the compression-controlled limit to 0.005. In this regard, the Figure R9.3.2 of the Code is again referred where the transition zone and the variation of ϕ values are clearly shown.

CODE MODIFICATIONS OF COLUMN INTERACTION DIAGRAMS

(c). As described in Chapter 9, maximum permissible column loads were specified for columns no matter how small their *e* values. As a result, the upper part of each design interaction curve is shown as a horizontal line representing the appropriate value of

$$P_u = \phi P_{n\,\max} \text{ for tied columns} = 0.80\phi[0.85f'_c(A_g - A_{st}) + f_y A_{st}]$$

(ACI Equation 10-2)

$$P_u = \phi P_{n\,\max} \text{ for spiral columns} = 0.85\phi[0.85f'_c(A_g - A_{st}) + f_y A_{st}]$$

(ACI Equation 10-1)

These formulas were developed to be approximately equivalent to loads applied with eccentricities of 0.10*h* for tied columns and 0.05*h* for spiral columns. Each of the three modifications described here is indicated on the design curve of Figure 10.13. In Figure 10.13, the solid curved line represents *Pu* and *Mu*, whereas the dashed curved line is *Pn* and *Mn*. The difference between the two curves is the φ factor. The two curves would have the same shape if the φ factor did not vary. Above the radial line labeled “balanced case,” φ = 0.65 (0.75 for spirals). Below the other radial line, labeled “strain of 0.005,” φ = 0.9. It varies between the two values in between, and the *Pu* versus *Mu* curve assumes a different shape.

CODE MODIFICATIONS OF COLUMN INTERACTION DIAGRAMS

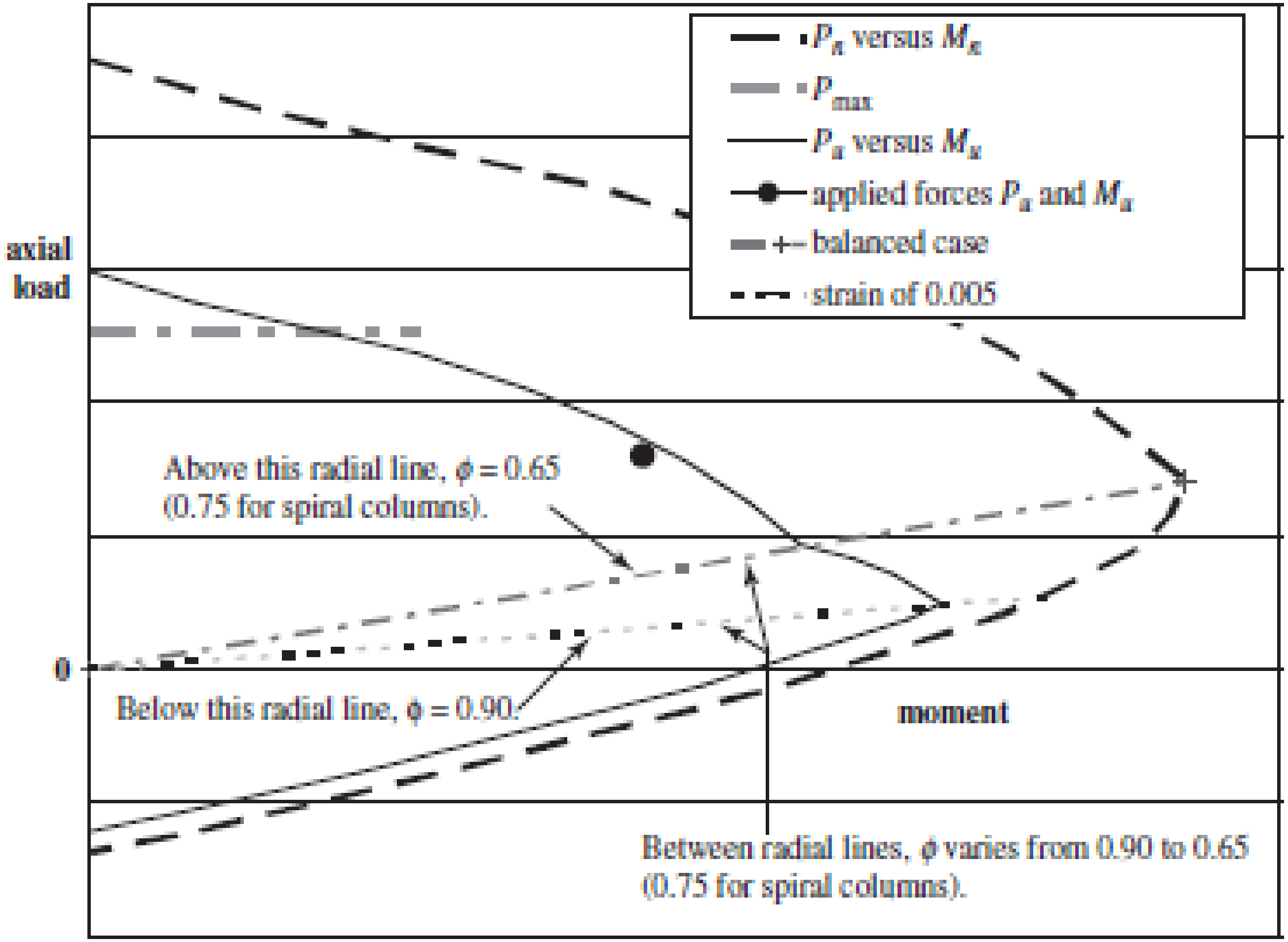


FIGURE 10.13 A column interaction curve adjusted for the three modifications described in this section (10.5).

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

If individual column interaction diagrams were prepared as described in the preceding sections, it would be necessary to have a diagram for each different column cross section, for each different set of concrete and steel grades, and for each different bar arrangement. The result would be an astronomical number of diagrams. The number can be tremendously reduced, however, if the diagrams are plotted with ordinates of $K_n = P_n/f'_c A_g$ (instead of P_n) and with abscissas of $R_n = P_n e/f'_c A_g h$ (instead of M_n). The resulting normalized interaction diagrams can be used for cross sections with widely varying dimensions. The ACI has prepared normalized interaction curves in this manner for the different cross section and bar arrangement situations shown in Figure 10.14 and for different grades of steel and concrete.

Two of the ACI diagrams are given in Figures 10.15 and 10.16, while Appendix A (Graphs A.2–A.13) presents several other ones for the situations given in parts (a), (b), and (d) of Figure 10.14. *Notice that these ACI diagrams do not include the three modifications described in the last section.*

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

The ACI column interaction diagrams are used in Examples 10.3 to 10.7 to design or analyze columns for different situations. In order to correctly use these diagrams, it is necessary to compute the value of γ (gamma), which is equal to the distance from the center of the bars on one side of the column to the center of the bars on the other side of the column divided by h, the depth of the column (both values being taken in the direction of bending). Usually the value of γ obtained falls in between a pair of curves, and interpolation of the curve readings will have to be made.

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

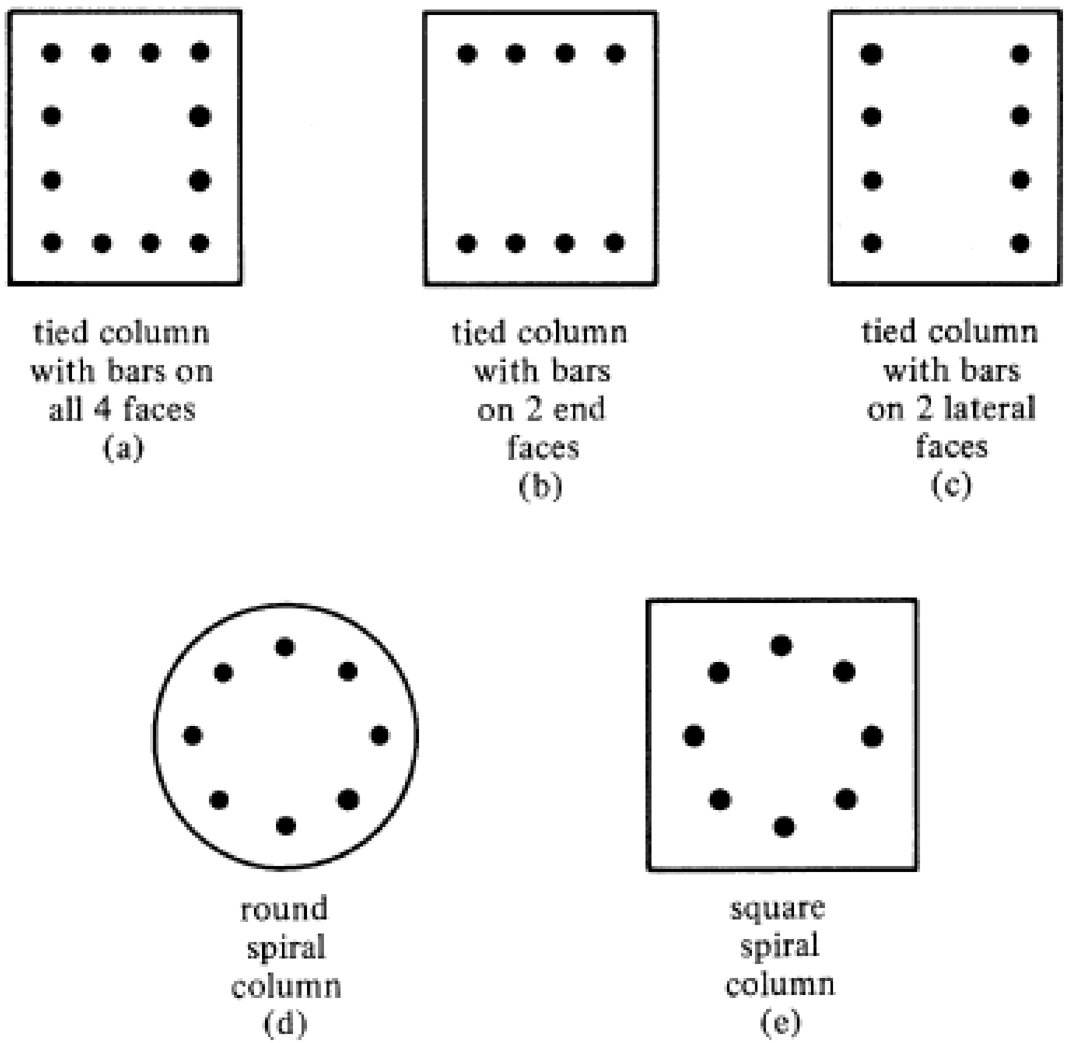


Figure 10.14

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

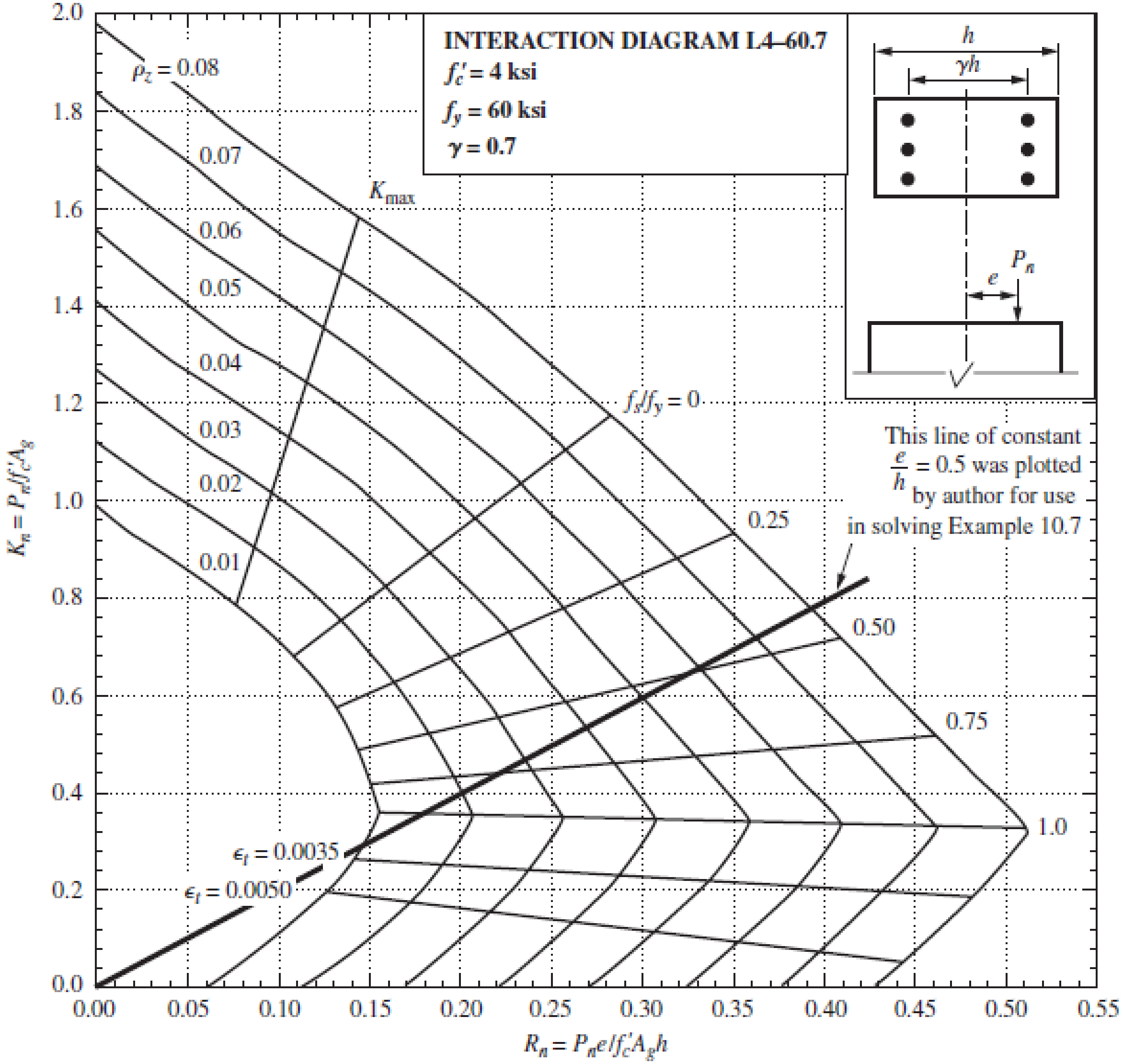


Figure 10.15 ACI rectangular column interaction diagrams when bars are placed on two faces only. (Permission of American Concrete Institute.)

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

Caution

Be sure that the column picture at the upper right of the interaction curve being used agrees with the column being considered. In other words, are there bars on two faces of the column or on all four faces? If the wrong curves are selected, the answers may be quite incorrect.

Although several methods are available for selecting column sizes, a trial-and-error method is about as good as any. With this procedure the designer estimates what he or she thinks is a reasonable column size and then determines the steel percentage required for that column size from the interaction diagram. If it is felt that the ρ_g determined is unreasonably large or small, another column size can be selected and the new required ρ_g selected from the diagrams, and so on. In this regard, the selection of columns for which ρ_g is greater than 4 or 5% results in congestion of the steel, particularly at splices, and consequent difficulties in getting the concrete down into the forms.

DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

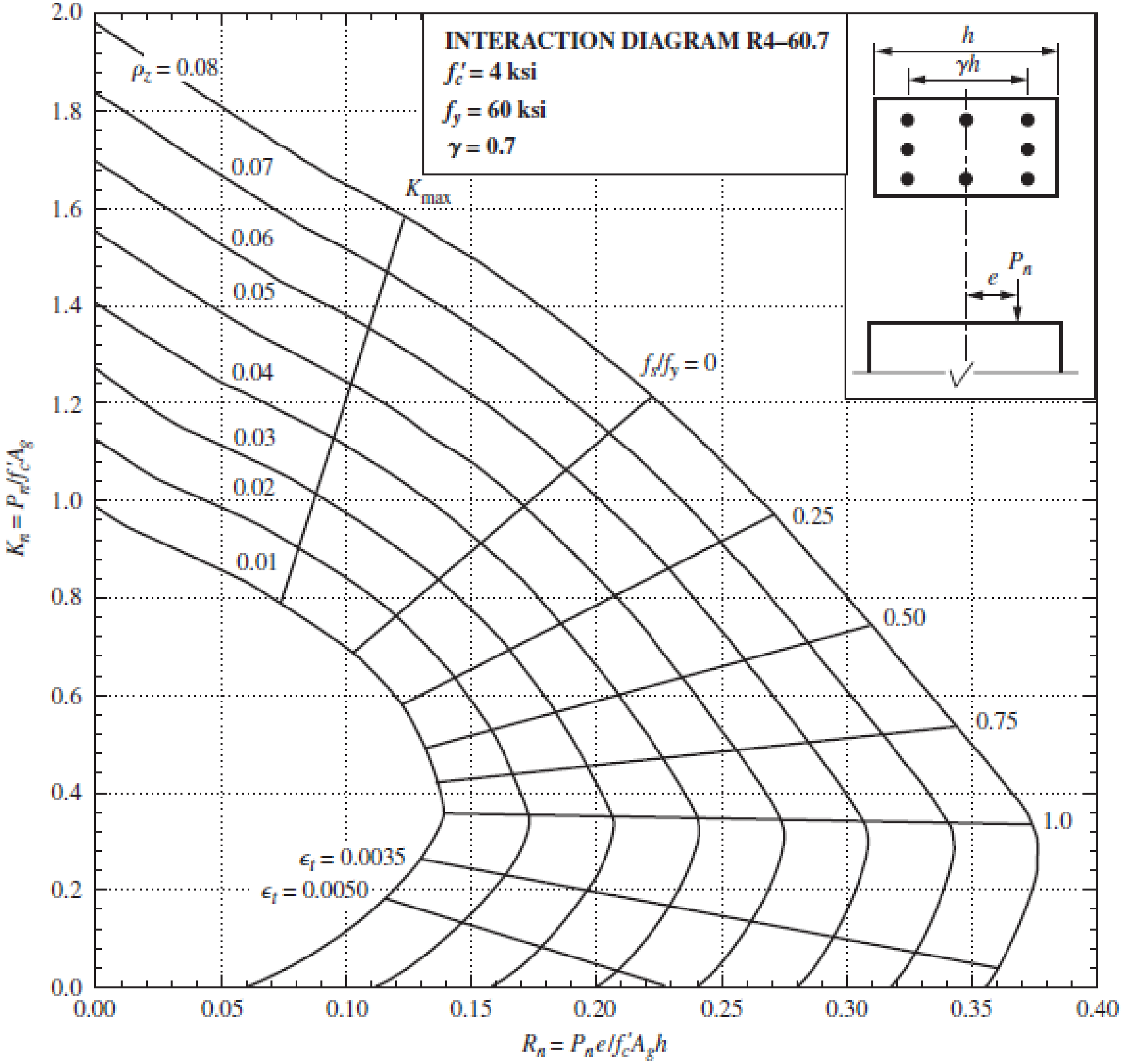


Figure 10.16 ACI rectangular column interaction diagram when bars are placed along all four faces. (Permission of American Concrete Institute.)