University of Engineering & Technology Peshawar, Pakistan

CE301: Structure Analysis II

Module 04:

Analysis of S.I Pin Jointed Frames (Trusses) Using Flexibility method

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Topics to be Covered

- Introduction
- Prerequisites for using flexibility method for Trusses
- Indeterminacy of trusses
- Revision of Unit Load Method
- Flexibility method procedure for truss analysis
- Analysis of trusses Example 1
- Example 2
- Example 3
- Assignments

Flexibility Method for Trusses Analysis

❑ Introduction:

Trusses are analyzed with flexibility method due to

- To solve the problem in matrix notation, which is more systematic
- To compute reactions at all the supports.
- To compute internal resisting axial force in any member of the indeterminate Truss.

Flexibility Method for Trusses Analysis

The basic method for the analysis of indeterminate truss by force method is similar to the indeterminate beam analysis discussed in the previous lessons. Determine the degree of static indeterminacy of the structure. Identify the number of redundant reactions equal to the degree of indeterminacy. The redundants must be so selected that when the restraint corresponding to the redundants are removed, the resulting truss is statically determinate and stable. In case of E.I select support reactions as redundant actions in case of I.I selection member forces as redundant actions. E.I + I.I ⇒ select redundants from both support reactions & member forces.

Flexibility Method for Trusses Analysis

❑ **Prerequisites for Analysis with Flexibility method:**

It is necessary that students must have strong background of the following concepts before starting analysis with flexibility or any other matrix method.

- Enough concept of Matrix Algebra
- Must be able to find the Statical Indeterminacy of trusses
- Concept of methods for finding displacements (Unit load method for displacement)
- Must be able to analyze a determinate truss

Flexibility Method for Trusses Analysis

❑ Indeterminacy of Trusses:

The truss is said to be statically indeterminate when the total number of reactions and member axial forces exceed the total number of static equilibrium equations.

$$\mathrm{S.I} = (m + r) - 2j$$

where m, j and r are number of members, joints and unknown reaction components respectively.

Flexibility Method for Trusses Analysis

- **Types of Indeterminate trusses:**

  There are three different types of S.I trusses.

- Internally indeterminate trusses only ($I.I = m + 3 - 2j$)
- Externally indeterminate trusses only ($E.I = r – 3$)
- Internally & externally indeterminate trusses ($T.I= m + r - 2j$)

Flexibility Method for Trusses Analysis

- Internally indeterminate trusses:

A truss is said to be internally indeterminate if it has exactly three reaction components and more than (2j-3) members.

$$\mathrm{I.I} = (m + 3) - 2j$$

- Externally indeterminate trusses:

A planar truss is said to be externally indeterminate if the number of reactions exceeds the number of static equilibrium equations available.

$$\mathrm{E.I} = r - 3$$

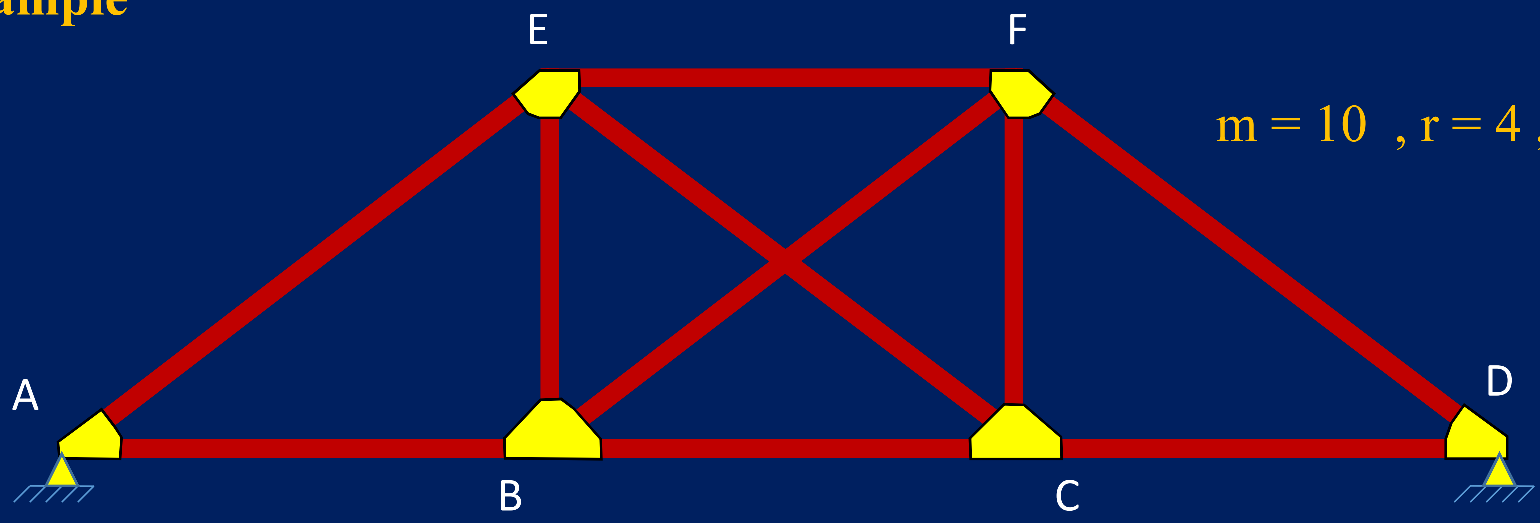
Flexibility Method for Trusses Analysis

- Internally & externally indeterminate trusses:

  A truss is both externally and internally indeterminate if it has more than three reaction components and also has more than (2j-3) members.

Flexibility Method for Trusses Analysis

- **Example**



m = 10 , r = 4 , j = 6

$$\mathrm{I.I} = (m+3) - 2j = 1 \circ$$

$$\mathrm{E.I} = r - 3 = 1 \circ$$

$$\mathrm{T.I} = (m+r) - 2j = 2 \circ$$

To analyze this truss we have to chose two redundant actions

One member force & one support reaction.

Flexibility Method for Trusses Analysis

❑ Unit Load Method for Displacement

- This Method is based on the principle of virtual work.
- Best suitable to find the slope & deflection of primary structure when subjected to external loads and unit loads.
- Displacement due to actual loads at any point is given by

$$\Delta_n = \sum \frac{P_i \, Un_i \, Li}{EA_i}$$

Note: When axial forces are predominant then use axial rigidity EA instead of EI

Flexibility Method for Trusses Analysis

- Displacement at any point due to a unit load at that point or any other point is given by (also called flexibility coefficient)

$$\delta_{nm} = \sum_{i=1}^{m} \frac{U_{ni}\, Um_i\, L_i}{EA_i} = f_{nm}$$

- Where P are the values of member forces in primary structure subjected to actual loads & U are the values of member forces in BDS subjected to unit loads.

Flexibility Method for Trusses Analysis

❑ Analysis Procedure

The following steps should be followed to analyze a statical indeterminate Truss using flexibility method.

1st find the degree of statical indeterminacy S.I (E.I & I.I).

- **Step # 01:** Identify redundants accordingly and obtain BDS

  Compute [DRS] values.

  No. of redundants = D.S.I

Assume # of redundants = 2
Just to understand the procedure

Note: when a truss is both externally & internally indeterminate then choose redundant actions from both member forces & support reactions.

DRS is initial support rotation or settlement corresponding to redundant locations

Flexibility Method for Trusses Analysis

- **Step # 02:** Analyze the BDS (primary structure) under the following loading conditions

i. Analyze the BDS when acted upon by the actual loads and compute the values of member forces P_i.

ii. Analyze the BDS when acted upon by the unit load at redundant location 1 & compute the values of member forces U_{1i}.

iii. Analyze the BDS when acted upon by the unit load at redundant location 2 & compute the values of member forces U_{2i}.

iv. And so on if redundants are more than 2.

Flexibility Method for Trusses Analysis

- **Step # 03:** Develop members forces table. Member forces table will consist of following information's for making the calculation work easy.

Members	Axial rigidity EA	Length	Forces in the released truss due to actual applied loading (P_i values)	Forces in the released truss due to unit load at redundant 1 (U_{1i} values)	Forces in the released truss due to unit load at redundant 2 (U_{2i} values)
AB	$(EA)_{AB}$	L_{AB}	P_{AB}	$(U_1)_{AB}$	$(U_2)_{AB}$
BC	$(EA)_{BC}$	L_{BC}	P_{BC}	$(U_1)_{BC}$	$(U_2)_{BC}$
And so on up to m members	$(EA)_m$	L_m	P_m	$(U_1)_m$	$(U_2)_m$

Flexibility Method for Trusses Analysis

- **Step # 04:** Find BDS Displacements due to external loads

  or Compute the values of DRL

$$\boldsymbol{DRL_1} = \sum_{i=1}^{m} \frac{\boldsymbol{P_i\, U_{1i}\, Li}}{\boldsymbol{EA_i}}$$

$$\boldsymbol{DRL_2} = \sum_{i=1}^{m} \frac{\boldsymbol{P_i\, U_{2i}\, Li}}{\boldsymbol{EA_i}}$$

Flexibility Method for Trusses Analysis

- **Step # 05:** Find BDS Displacements due to unit loads

  or Compute the values of flexibility coefficients

  & flexibility matrix.

$$f_{12} = f_{21} = \sum_{i=1}^{m} \frac{U_{1i} U_{2i} L_i}{EA_i}$$

$$f_{11} = \sum_{i=1}^{m} \frac{U_{1i} U_{1i} L_i}{EA_i} \qquad f_{22} = \sum_{i=1}^{m} \frac{U_{2i} U_{2i} L_i}{EA_i}$$

$$[f] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$$

Note: $f_{12} = f_{21}$

Flexibility Method for Trusses Analysis

Step # 06: Apply compatibility and principle of superposition at the location of the removed redundants and solve the matrices to compute the values of redundant actions AR. As we know

$$[DRS] = [DRL] + [f] \bullet [AR]$$

From this

$$[AR] = [f]^{-1} \bullet [DRS - DRL]$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

Flexibility Method for Trusses Analysis

Step # 07: Compute the member end actions

$$[AM] = [AML] + [AMR][AR]$$

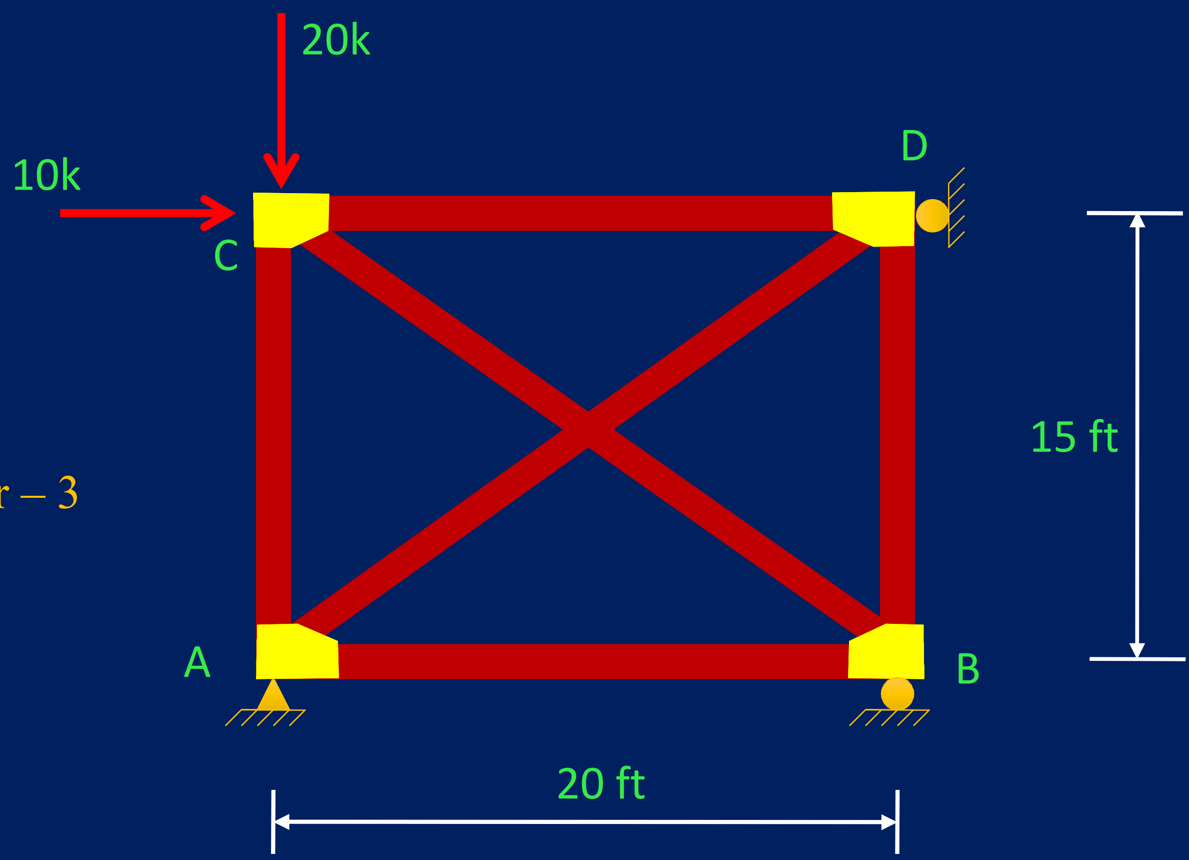
Note: Sign convention

Tension in member will be taken as +ive (away from the joints)

Compression in member will be taken as –ive (towards the joints)

Flexibility Method for Trusses Analysis

Problem 01: Analyze the given truss using flexibility method.



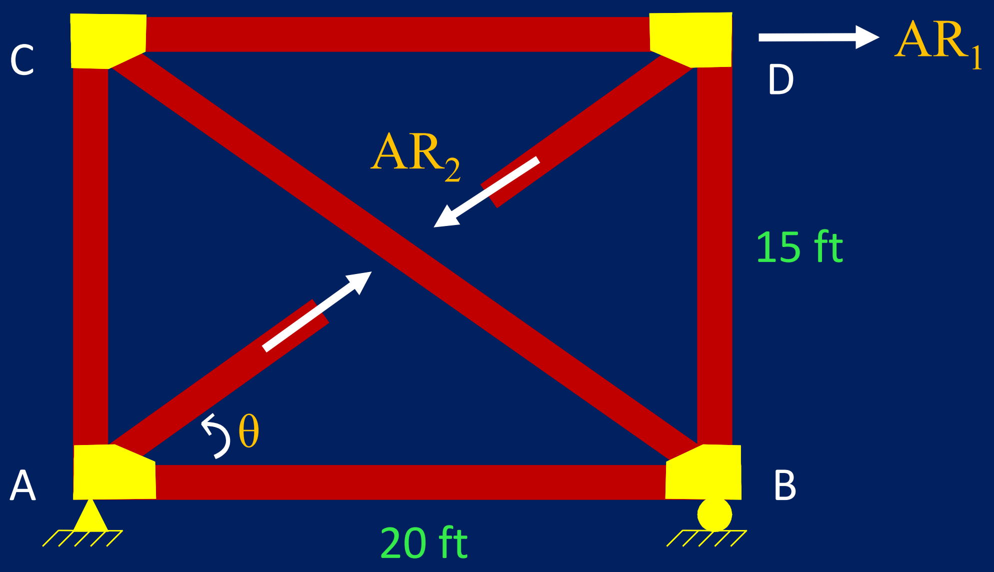
Take EA = constant

External indeterminacy = r – 3
E.I = 4 – 3 = 1°
I.I = 6 + 3 – 2(4) = 1°
T.I = 2 degrees

S.I = 2 degrees
So two redundant actions should be chosen.

Flexibility Method for Trusses Analysis

- **Step # 01:** Identify the redundants and obtain BDS also compute [DRS] values.



$$[AR] = \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[DRS] = \begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

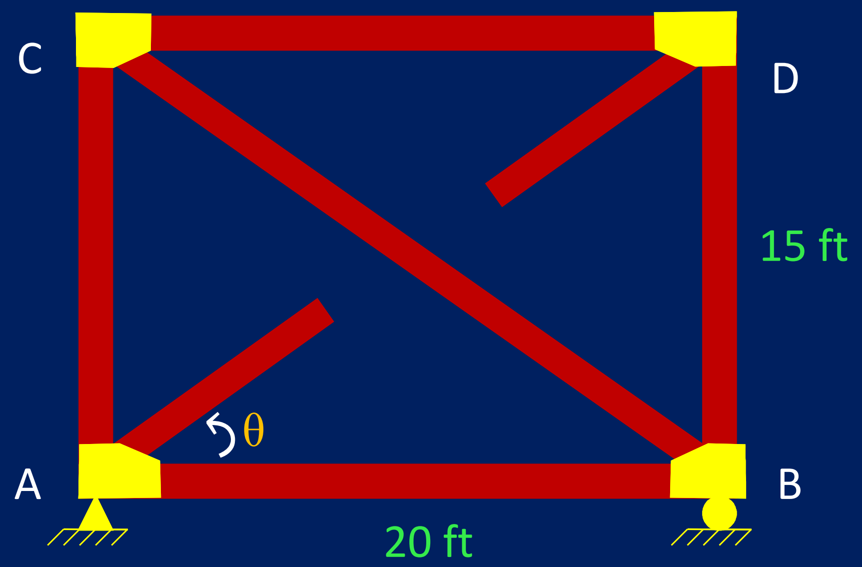
$$\tan^{-1}\frac{15}{20} = 36.86°$$

$$\sin\theta = 0.6$$

$$\cos\theta = 0.8$$

Horizontal reaction at D and member force in member AD are chosen as redundant actions

Flexibility Method for Trusses Analysis



Basic determinate structure (BDS) or Primary structure or Released structure

Flexibility Method for Trusses Analysis

- **Step # 02:** Analyze the BDS (primary structure) under the following loading conditions

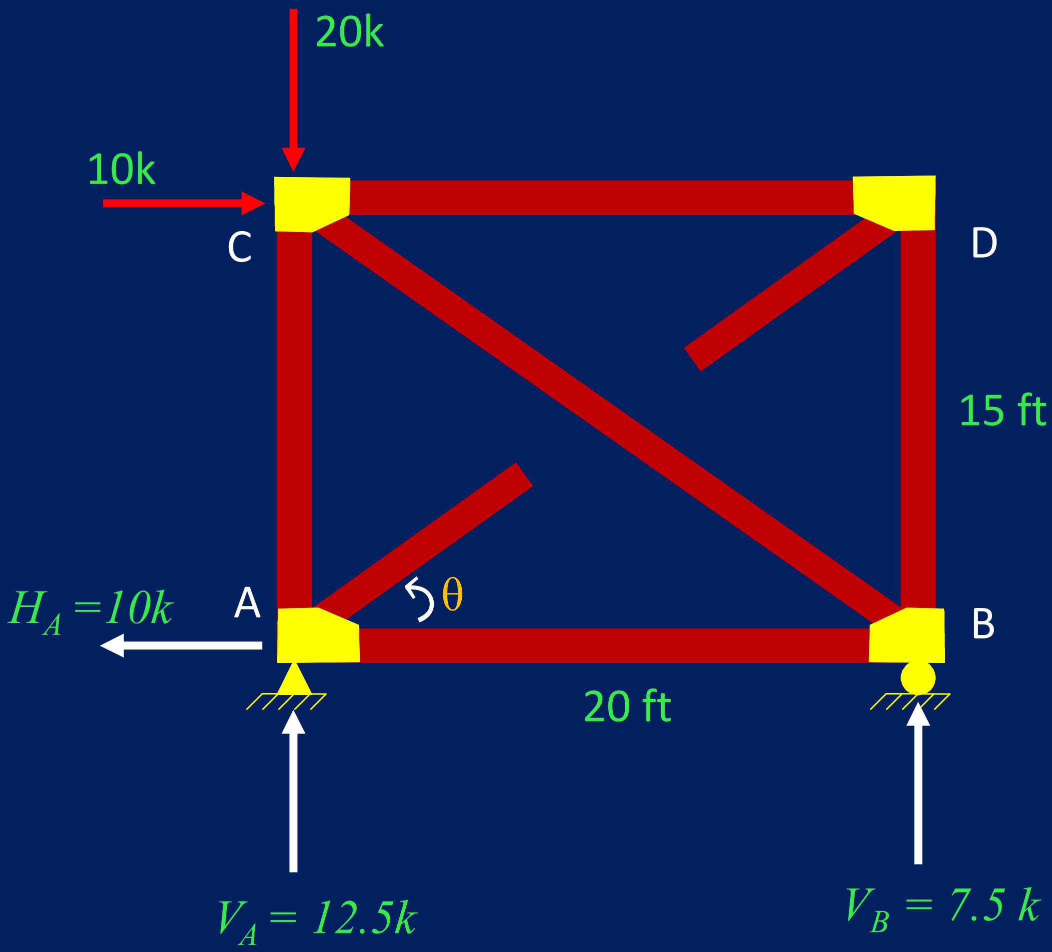
i. Analyze the BDS when acted upon by the actual loads and compute the values of member forces P_i.

ii. Analyze the BDS when acted upon by the unit load at redundant location 1 & compute the values of member forces U_{1i}.

iii. Analyze the BDS when acted upon by the unit load at redundant location 2 & compute the values of member forces U_{2i}.

Flexibility Method for Trusses Analysis

i. BDS acted upon by the actual loads



$$\sum M_A = 0$$

$$10 * 15 - V_B * 20 = 0$$
$$V_B = 7.5\ k$$

$$\sum Fy = 0$$

$$V_A = 20 - 7.5$$

$$V_A = 12.5\ k$$

Now

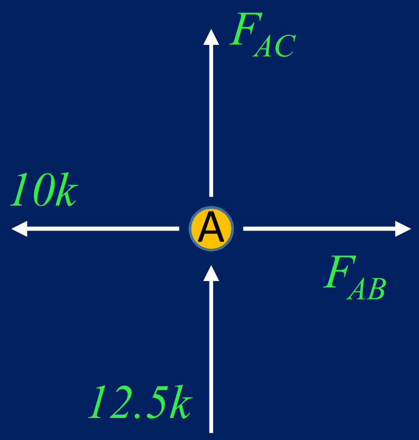
$$\sum Fx = 0$$

$$H_A = 10$$

Flexibility Method for Trusses Analysis

Now find all the member forces using method of joints.

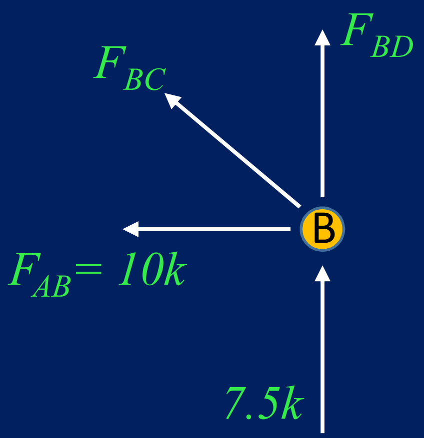
Joint A:



$$\sum Fy = 0 \quad \Rightarrow \quad F_{AC} = -12.5\ k$$

$$\sum Fx = 0 \quad \Rightarrow \quad F_{AB} = 10\ k$$

Joint B:



$$\sum Fx = 0$$

$$FBC \cos 36.86 = -10$$

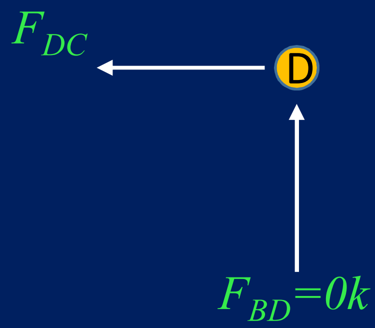
$$F_{BC} = -12.5\ k$$

$$\sum Fy = 0$$

$$F_{BD} = -FBC \sin 36.86 - 7.5$$

$$F_{BD} = 0k$$

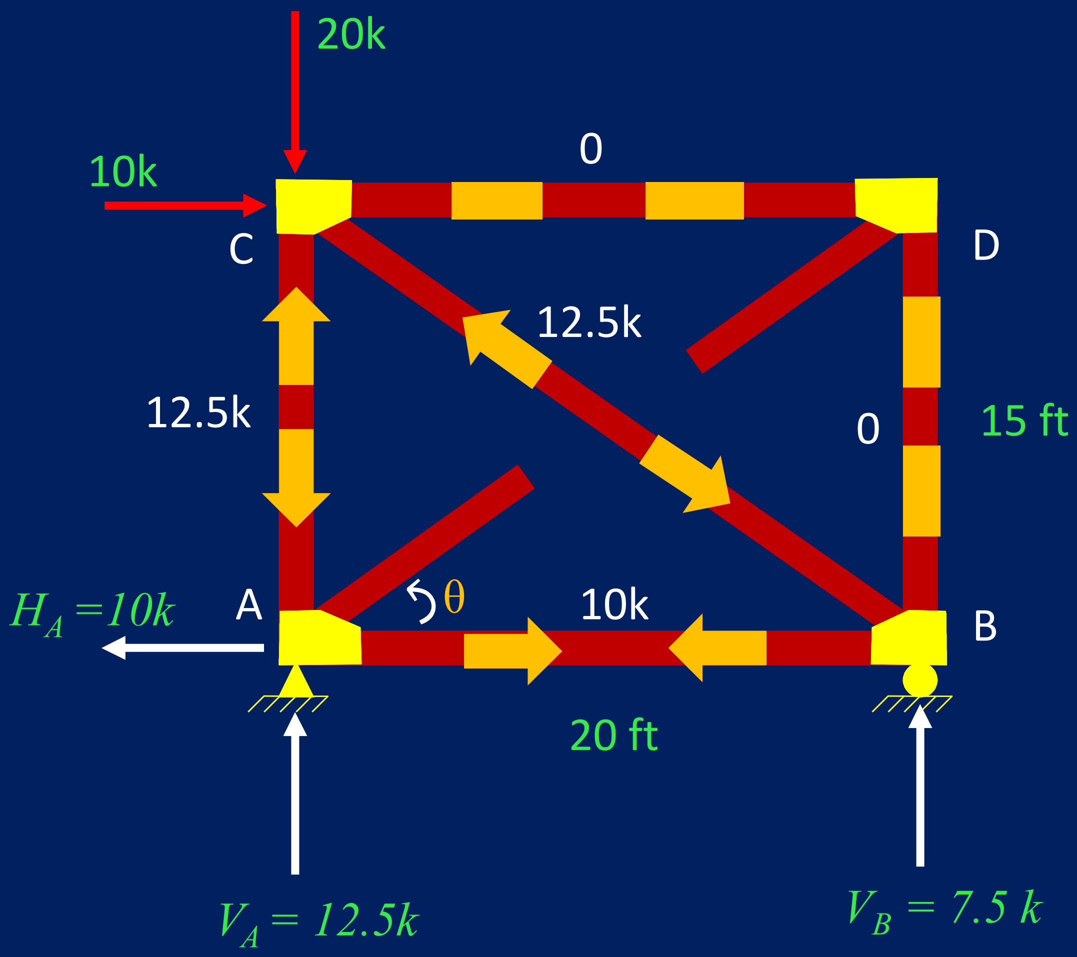
Joint D:



$$\sum Fx = 0$$

$$\Rightarrow F_{DC} = 0\ k$$

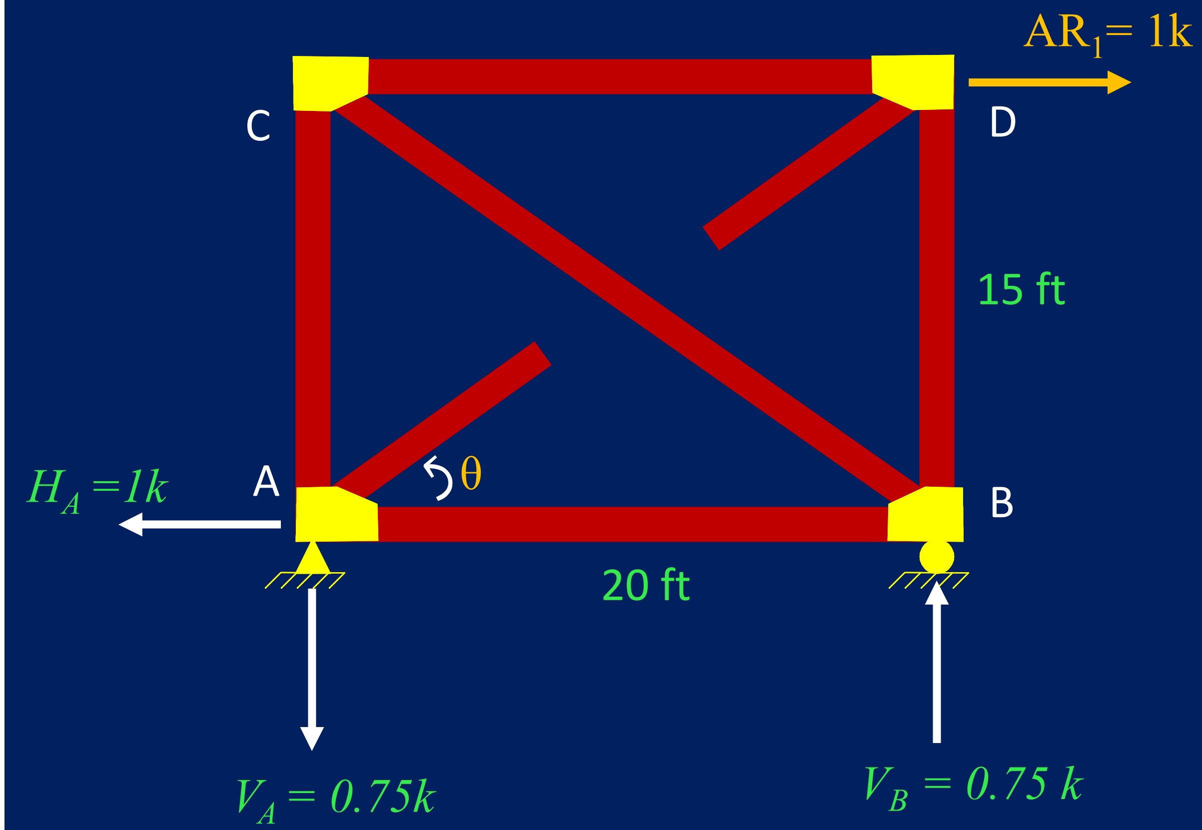
Flexibility Method for Trusses Analysis



P_i -Values

Flexibility Method for Trusses Analysis

ii. BDS acted upon by the Unit load at redundant location 1.



$$\sum M_A = 0$$

$$1 * 15 = VB * 20$$

$$V_B = 0.75\ k$$

$$\sum Fy = 0$$

$$V_A = -0.75\ k$$

Now

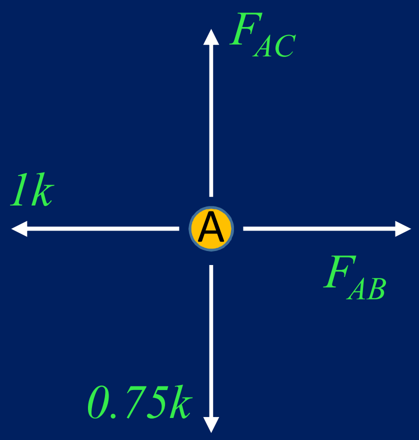
$$\sum Fx = 0$$

$$H_A = 1k$$

Flexibility Method for Trusses Analysis

Now find all the member forces using method of joints.

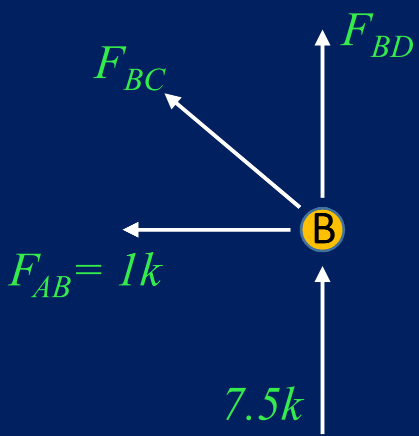
Joint A:



$$\sum Fy = 0 \;\Rightarrow\; F_{AC} = 0.75\;k$$

$$\sum Fx = 0 \;\Rightarrow\; F_{AB} = 1\;k$$

Joint B:



$$\sum Fx = 0$$

$$FBC \cos 36.86 = -1$$

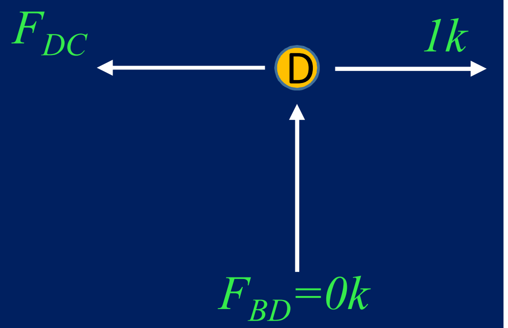
$$F_{BC} = -1.25\;k$$

$$\sum Fy = 0$$

$$F_{BD} = -FBC \sin 36.86 - 0.75$$

$$F_{BD} = 0k$$

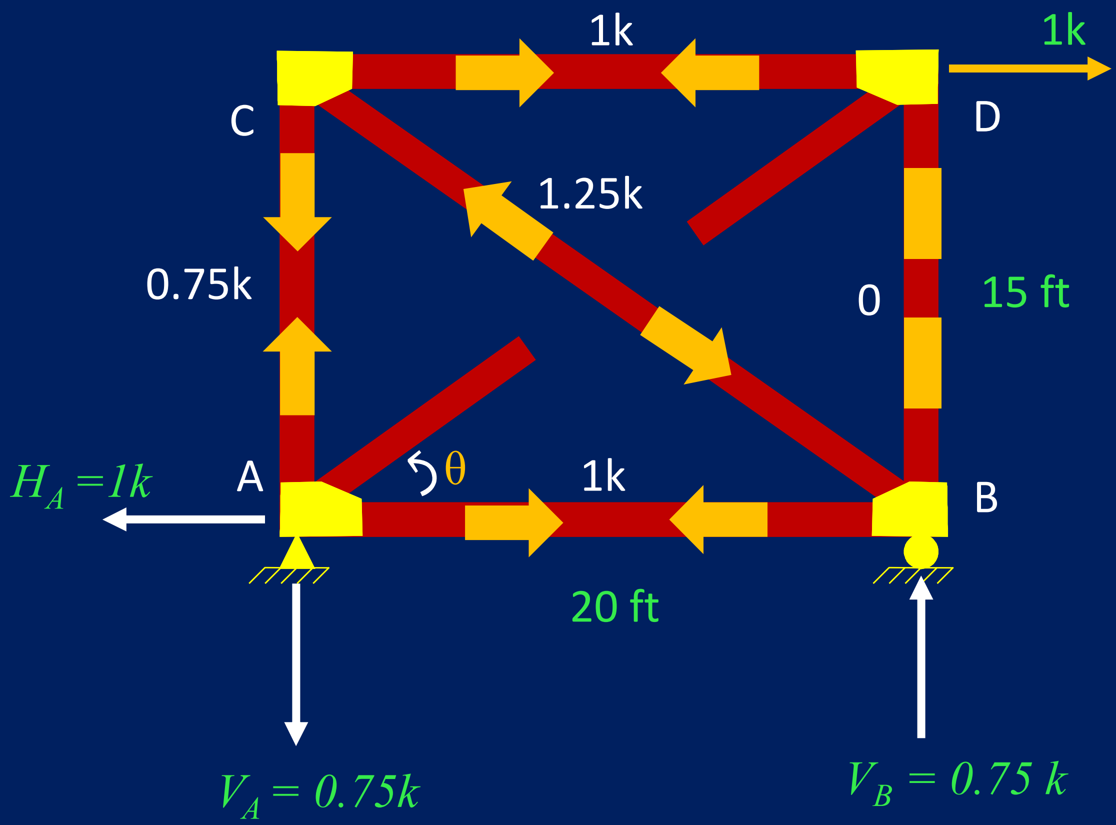
Joint D:



$$\sum Fx = 0$$

$$\Rightarrow F_{DC} = 1\;k$$

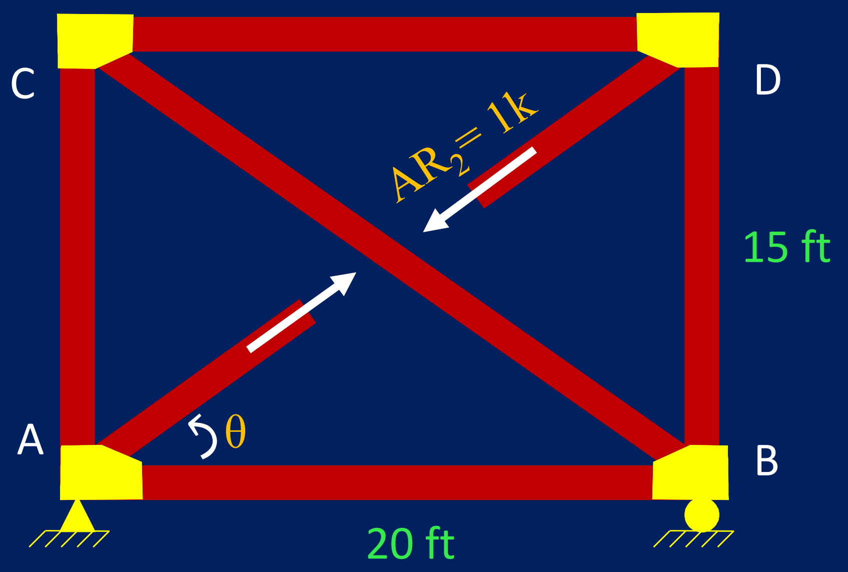
Flexibility Method for Trusses Analysis



U_{1i} - Values

Flexibility Method for Trusses Analysis

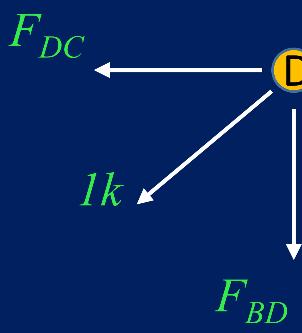
iii. BDS acted upon by the Unit load at redundant location 2.



Flexibility Method for Trusses Analysis

Now find all the member forces using method of joints.

Joint D:



$$\sum Fx = 0$$

$$\Rightarrow \quad F_{DC} = -1 \cos 36.86$$

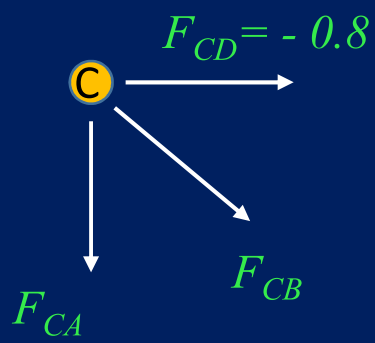
$$\Rightarrow \quad F_{DC} = -0.8\ k$$

$$\sum Fy = 0$$

$$\Rightarrow \quad F_{BD} = -1 \sin 36.86$$

$$F_{BD} = -0.6\ k$$

Joint C:



$$\sum Fx = 0$$

$$\Rightarrow \quad F_{CB} = -FCD / \cos 36.86$$

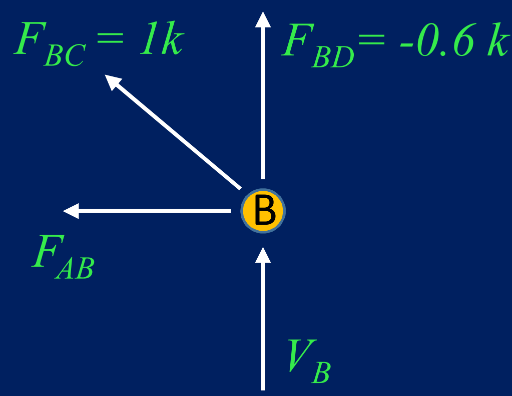
$$\Rightarrow \quad F_{CB} = 1.0\ k$$

$$\sum Fy = 0$$

$$\Rightarrow \quad F_{CA} = -FCB \sin 36.86$$

$$F_{CA} = -0.6\ k$$

Joint B:



$$\sum Fx = 0$$

$$FAB = -1 \cos 36.86$$

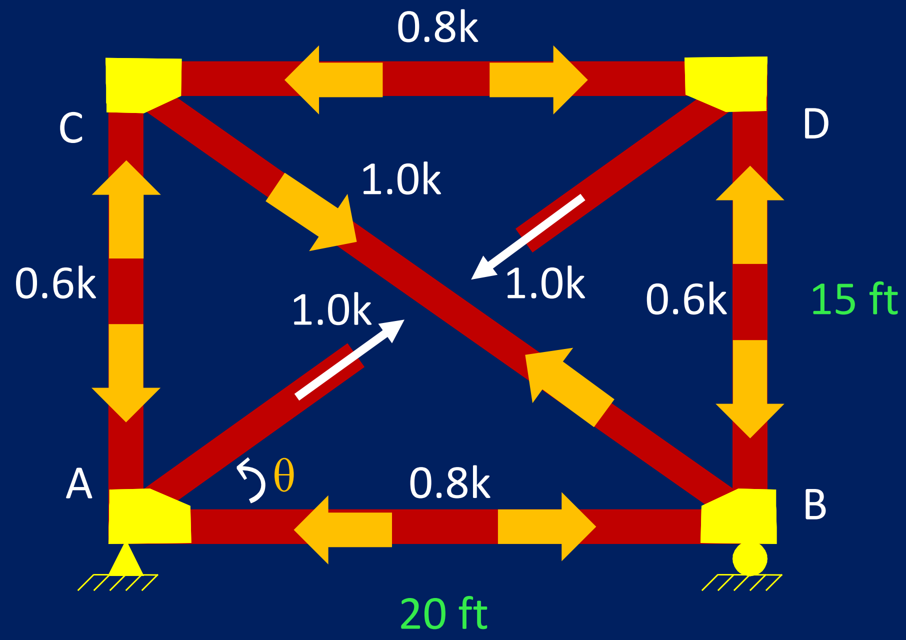
$$F_{AB} = -0.8\ k$$

$$\sum Fy = 0$$

$$V_B - 0.6 + 0.6 = 0$$

$$V_B = 0k$$

Flexibility Method for Trusses Analysis



U_{2i} - Values

Flexibility Method for Trusses Analysis

- **Step # 03:** Develop Member forces table.

Member	**AE**	**Length, L_i (ft)**	**P_i values**	**U_{1i} Values**	**U_{2i} values**
AB	Constant	20	10	1	-0.8
BC		25	-12.5	-1.25	1
CD		20	0	1	-0.8
AD		25	0	0	1
AC		15	-12.5	0.75	-0.6
BD		15	0	0	-0.6

Flexibility Method for Trusses Analysis

- **Step # 04:** Find BDS Displacements due to external loads or Compute the values of DRL matrix.

$$\boldsymbol{DRL_1 = \sum_{i=1}^{m} \frac{P_i\, U_{1i}\, Li}{EA_i}}$$

$$DRL_1 = \frac{1}{EA}\begin{bmatrix}(20 * 10 * 1) + (25 * -12.5 * -1.25) + 0 \\ + 0 + (15 * -12.5 * 0.75) + 0\end{bmatrix}$$

$$DRL_1 = \frac{450}{EA}$$

Flexibility Method for Trusses Analysis

$$\boldsymbol{DRL_2 = \sum_{i=1}^{m} \frac{P_i\, U_{2i}\, Li}{EA_i}}$$

$$DRL_2 = \quad \frac{1}{EA}\begin{bmatrix} (20 * 10 * -0.8) + (25 * -12.5 * 1) + 0 \\ + 0 + (15 * 0.75 * 0.75) + 0 \end{bmatrix}$$

$$DRL_2 = \frac{-360}{EA}$$

$$DRL_1 = \frac{450}{EA} \qquad DRL_2 = \frac{-360}{EA}$$

$$[DRL] = \begin{bmatrix} DRL_1 \\ DRL_2 \end{bmatrix} = \frac{1}{EA}\begin{bmatrix} 450 \\ -360 \end{bmatrix}$$

Flexibility Method for Trusses Analysis

- **Step # 05:** Find BDS Displacements due to unit loads

  or Compute the values of flexibility coefficients

  & flexibility matrix.

$$f_{11} = \sum_{i=1}^{m} \frac{U_{1i} U_{1i} L_i}{EA_i}$$

$$f_{11} = \frac{1}{EA}\begin{bmatrix}(20 * 1 * 1) + (25 * -1.25 * -1.25) + (20 * 1 * 1) \\ + 0 + (15 * -12.5 * -0.6) + 0\end{bmatrix}$$

$$f_{11} = \frac{87.5}{EA}$$

Flexibility Method for Trusses Analysis

$$f_{12} = f_{21} = \sum_{i=1}^{m} \frac{U_{1i} U_{2i} L_i}{EA_i}$$

$$f_{12} = f_{21} = \frac{1}{EA}\left[\begin{array}{c}(20 * 1 * -0.8) + (25 * -1.25 * 1) + (20 * 1 * -0.8) \\ + 0 + (15 * 0.75 * -0.6) + 0\end{array}\right]$$

$$f_{12} = f_{21} = \frac{-70}{EA}$$

$$f_{22} = \sum_{i=1}^{m} \frac{U_{2i} U_{2i} L_i}{EA_i}$$

$$f_{22} = \frac{1}{EA}\left[\begin{array}{c}(20 * -0.8 * -0.8) + (25 * 1 * 1) + (20 * -0.8 * -0.8) \\ + (25 * 1 * 1) + (15 * -0.6 * -0.6) + (15 * -0.6 * -0.6)\end{array}\right]$$

$$f_{22} = \frac{86.40}{EA}$$

Flexibility Method for Trusses Analysis

$$f_{11} = \frac{87.5}{EA} \qquad f_{22} = \frac{86.40}{EA}$$

$$f_{12} = f_{21} = \frac{-70}{EA}$$

$$[\,f\,] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$$

$$[\,f\,] = \frac{1}{EA}\begin{bmatrix} 87.5 & -70 \\ -70 & 86.40 \end{bmatrix}$$

Flexibility Method for Trusses Analysis

Step # 06: Apply compatibility and principle of superposition at the location of the removed redundants and solve the matrices to compute the values of redundant actions AR. As we know

$$[DRS] = [DRL] + [f] \bullet [AR]$$

From this

$$[AR] = [f]^{-1} \bullet [DRS - DRL]$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

Flexibility Method for Trusses Analysis

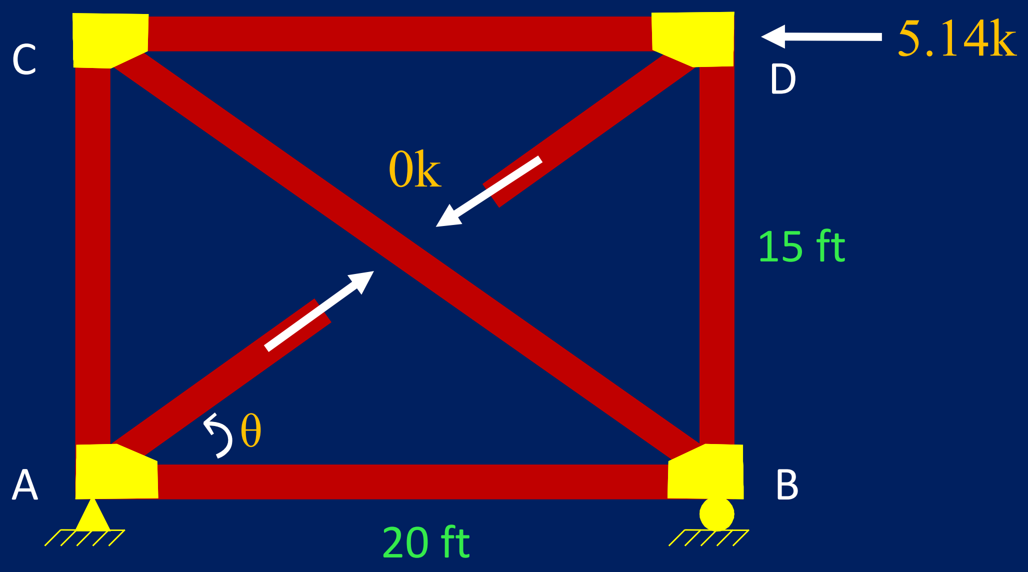
$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = EA \begin{bmatrix} 87.5 & -70 \\ -70 & 86.40 \end{bmatrix}^{-1} \begin{bmatrix} 0 - (450) \\ 0 - (-360) \end{bmatrix} \times \frac{1}{EA}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} -5.14 \\ 0 \end{bmatrix}$$

-ive sign shows that our assumed redundant action direction is wrong

Flexibility Method for Trusses Analysis

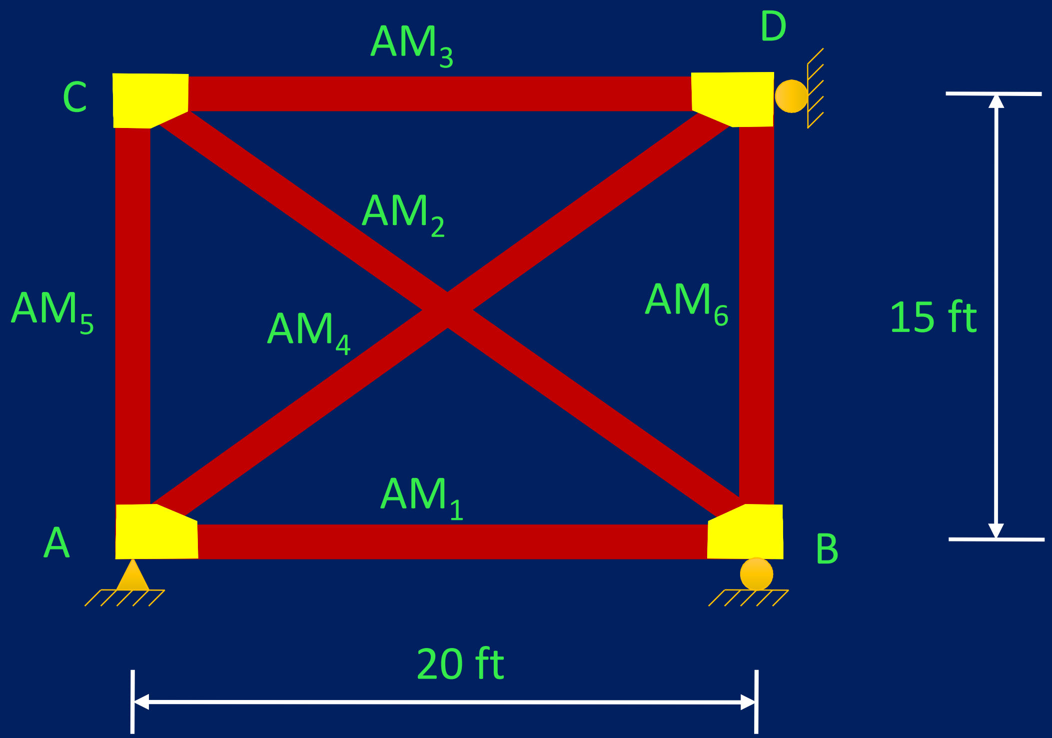


Final determinate structure

Flexibility Method for Trusses Analysis

Step # 07: Compute the member end actions. As we know that

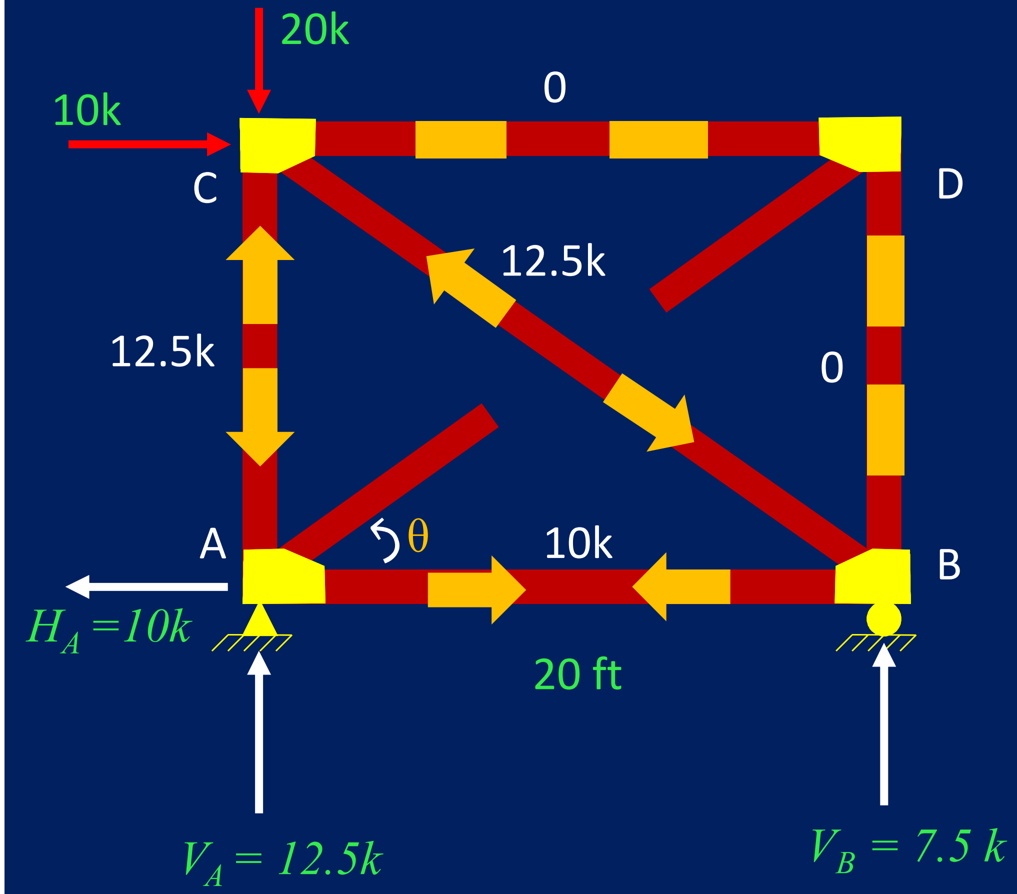
$$[AM] = [AML] + [AMR][AR]$$

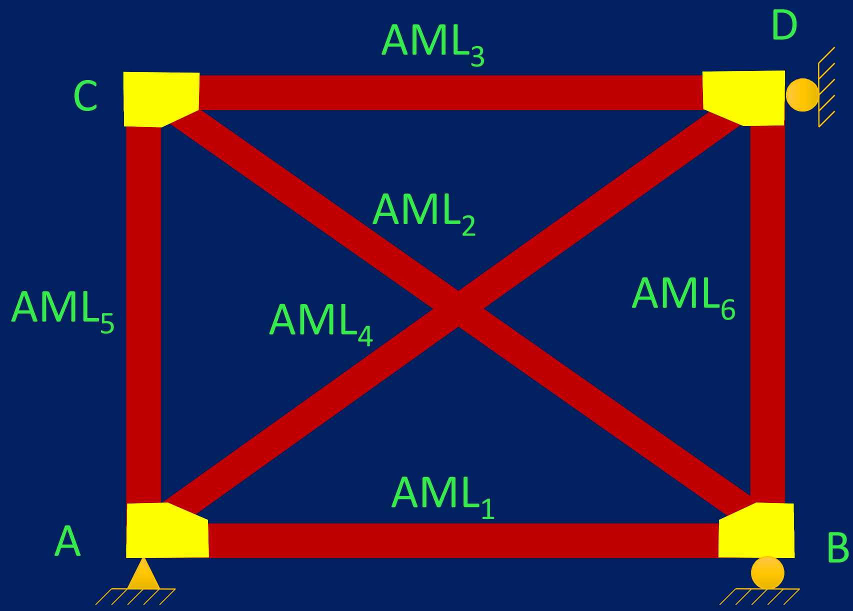


$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix}$$

Flexibility Method for Trusses Analysis

a) Compute AML values.

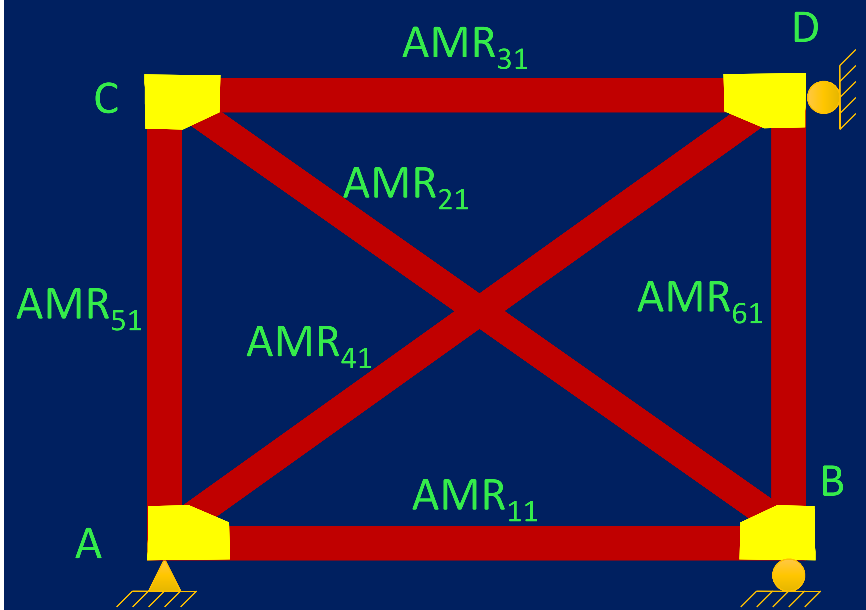


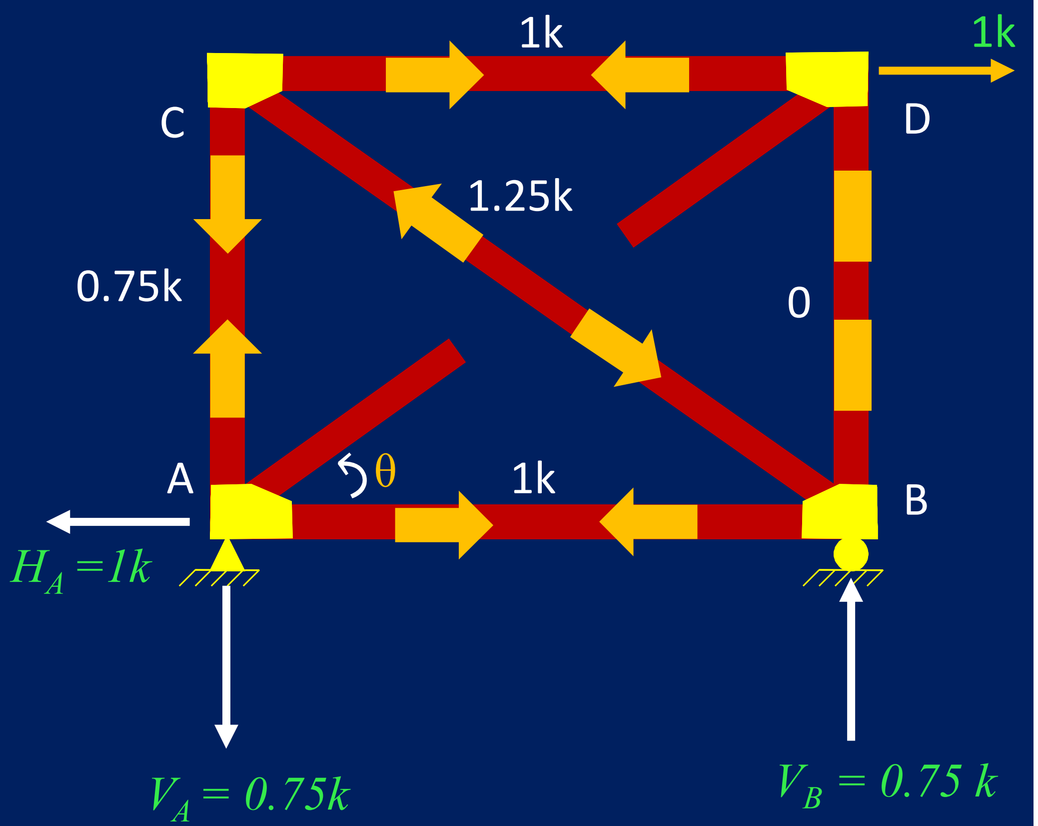


$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} = \begin{bmatrix} 10 \\ -12.5 \\ 0 \\ 0 \\ -12.5 \\ 0 \end{bmatrix}$$

Flexibility Method for Trusses Analysis

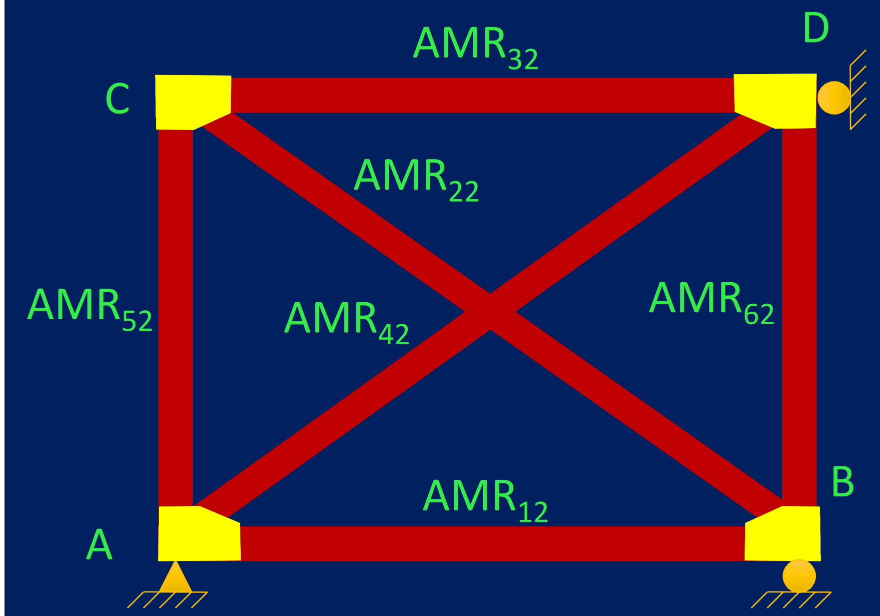
b) Compute the AMR values: when unit is applied at redundant location 1.

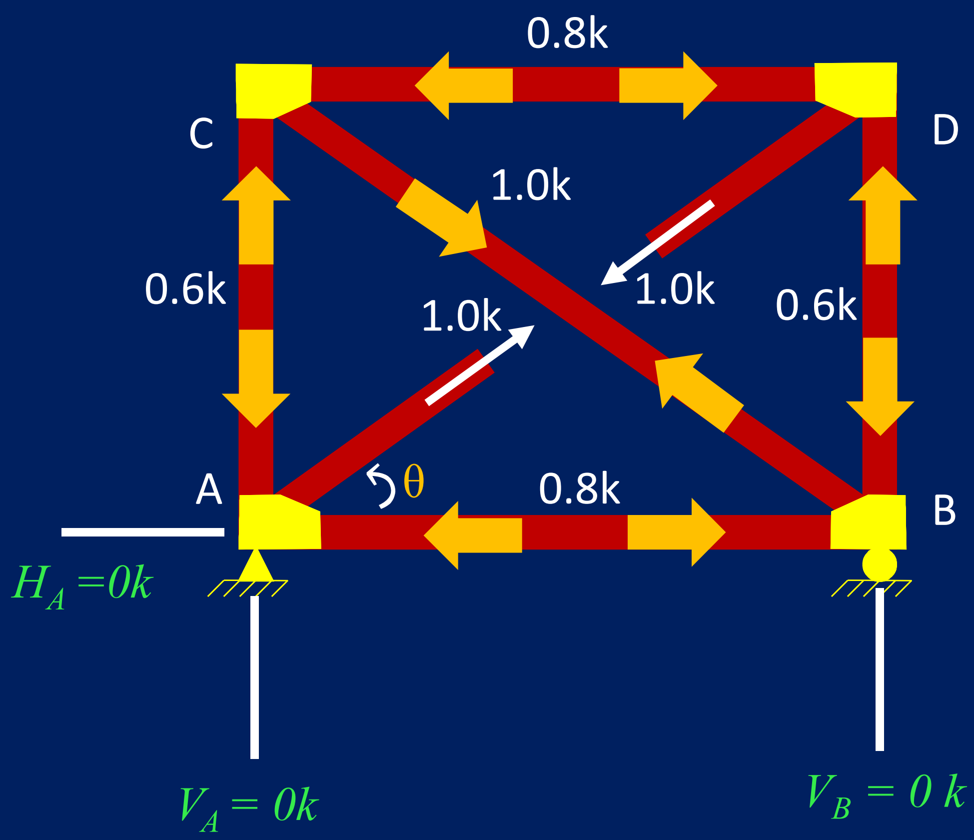




Flexibility Method for Trusses Analysis

b) Compute the AMR values: when unit is applied at redundant location 2.





Flexibility Method for Trusses Analysis

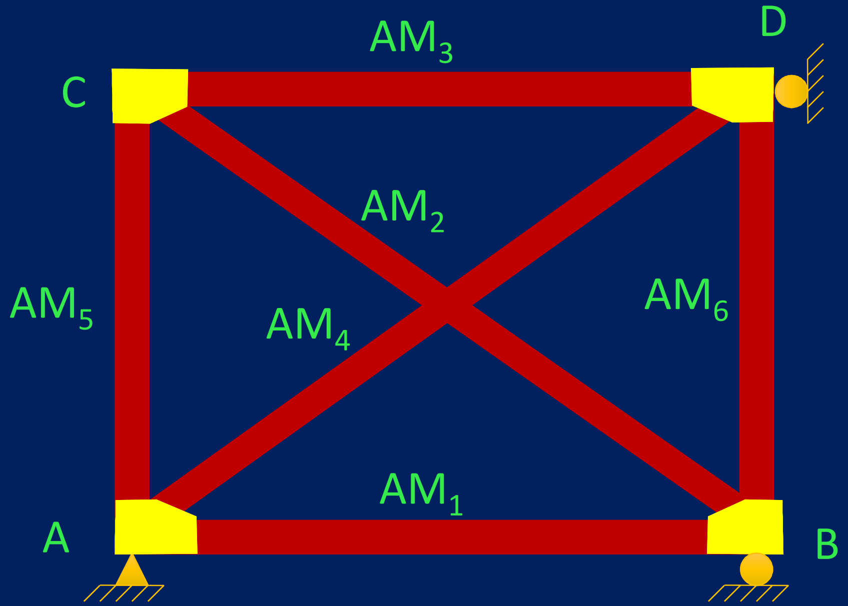
$$[AMR]_{6*2} = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix} = \begin{bmatrix} 1 & -0.8 \\ -1.25 & 1 \\ 1 & -0.8 \\ 0 & 1 \\ 0.75 & -0.6 \\ 1 & -0.6 \end{bmatrix}$$

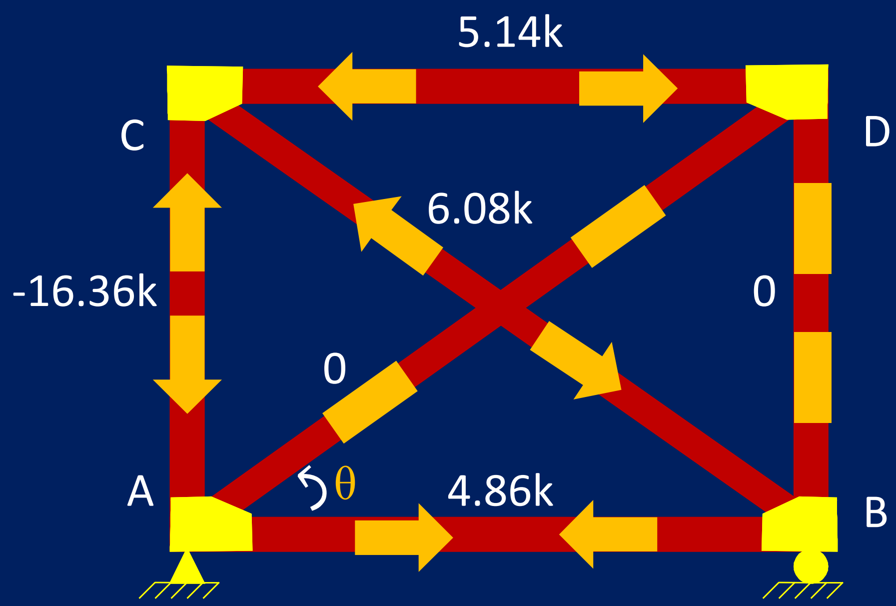
Now member end actions will be computed as given below

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} + \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix} \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix}$$

Flexibility Method for Trusses Analysis

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} 10 \\ -12.5 \\ 0 \\ 0 \\ -12.5 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & -0.8 \\ -1.25 & 1 \\ 1 & -0.8 \\ 0 & 1 \\ 0.75 & -0.6 \\ 1 & -0.6 \end{bmatrix} \begin{bmatrix} -5.14 \\ 0 \end{bmatrix} = \begin{bmatrix} 4.86 \\ -6.08 \\ -5.14 \\ 0 \\ -16.36 \\ 0 \end{bmatrix}$$

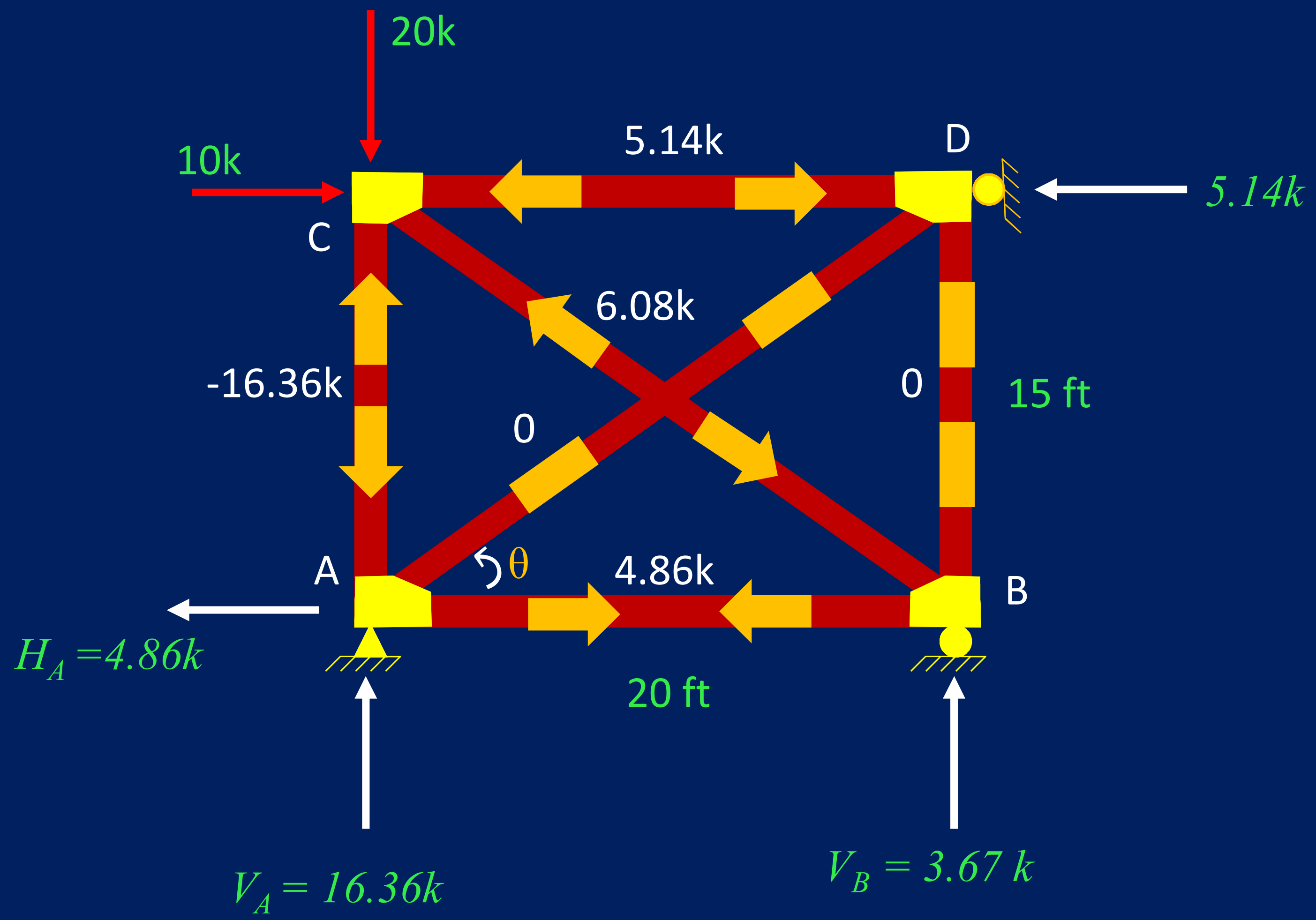




Truss with all member forces are known

Flexibility Method for Trusses Analysis

To find the support reaction simply apply equilibrium equations and we will get

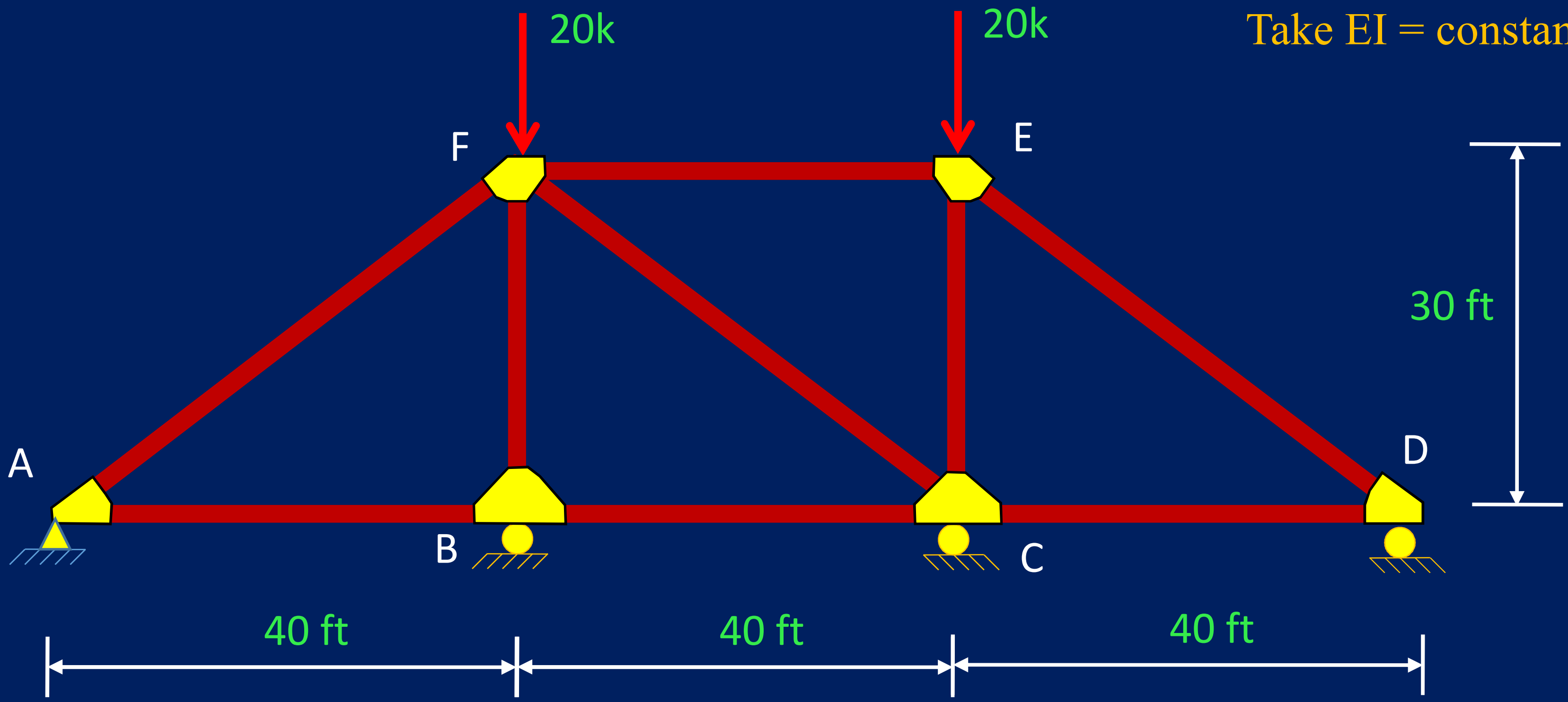


Completely analyzed structure

Flexibility Method for Trusses Analysis

Problem 02: Analyze the given truss using flexibility method.

Take EI = constant



E.I = 5 – 3 = 2°

I.I = 9 + 3 – 2(6) = 0°

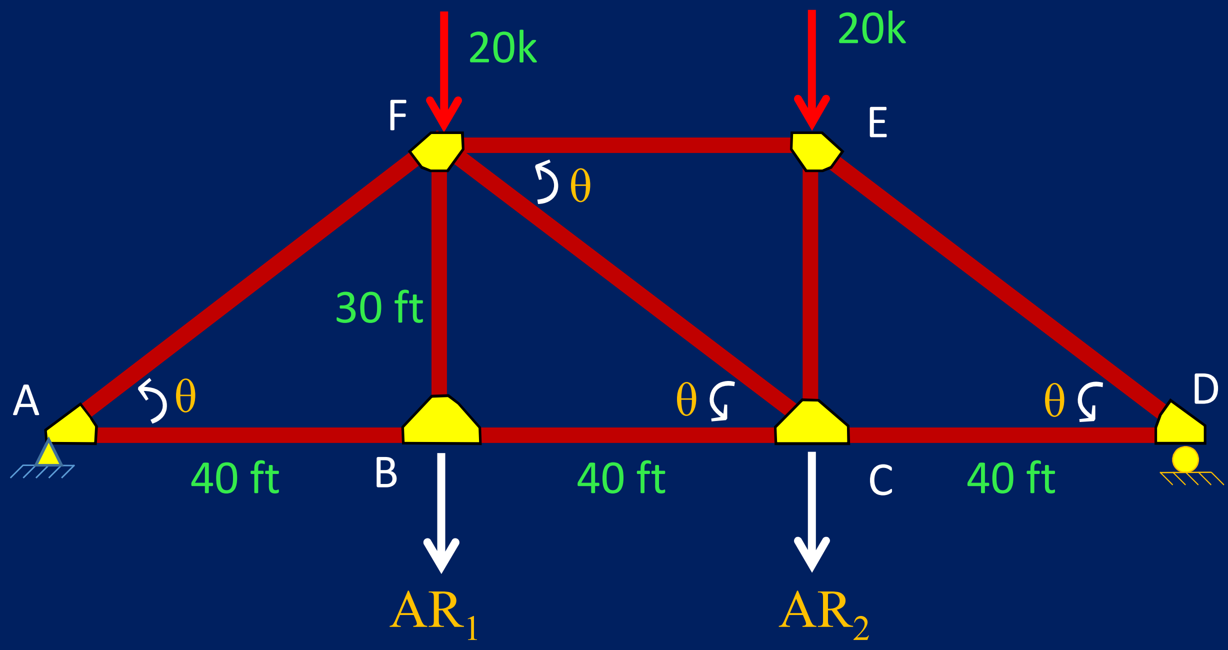
T.I = 2 degrees

S.I = 2 degrees

So two redundant actions should be chosen.

Flexibility Method for Trusses Analysis

- **Step # 01:** Identify the redundants and obtain BDS also compute [DRS] values.



$$[AR] = \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[DRS] = \begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

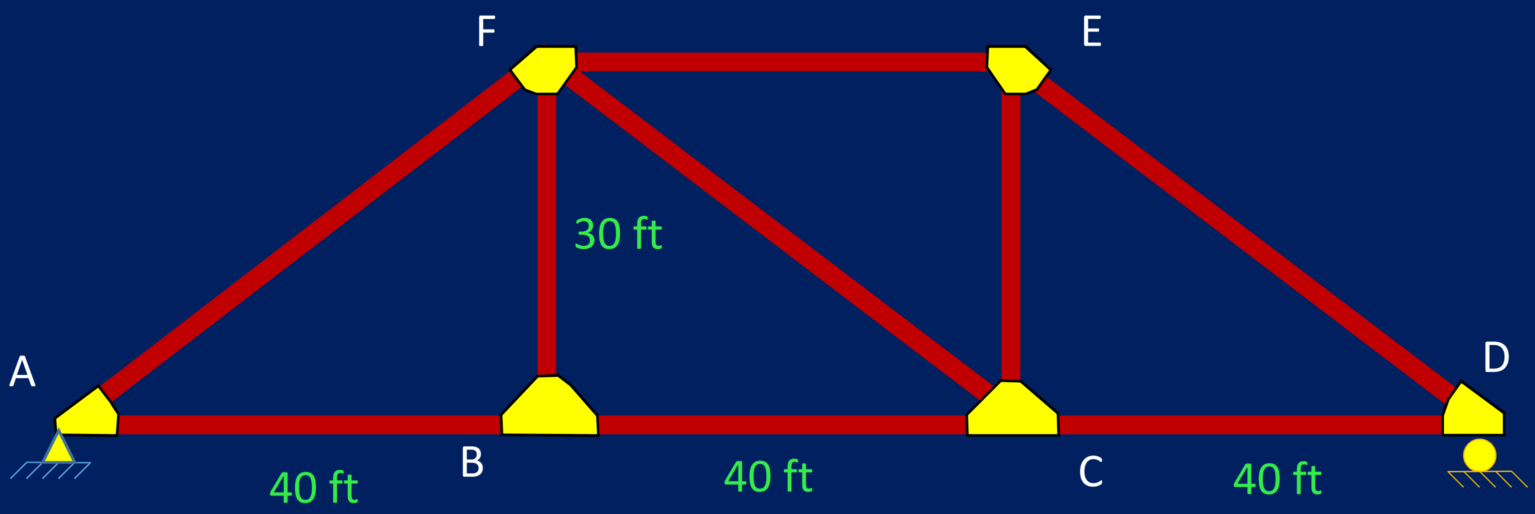
$$\tan^{-1}\frac{30}{40} = 36.86°$$

$$\sin\theta = 0.6$$

$$\cos\theta = 0.8$$

Vertical reaction at B & C are chosen as redundant actions

Flexibility Method for Trusses Analysis



Basic determinate structure (BDS) or Primary structure or Released structure

Flexibility Method for Trusses Analysis

- **Step # 02:** Analyze the BDS (primary structure) under the following loading conditions

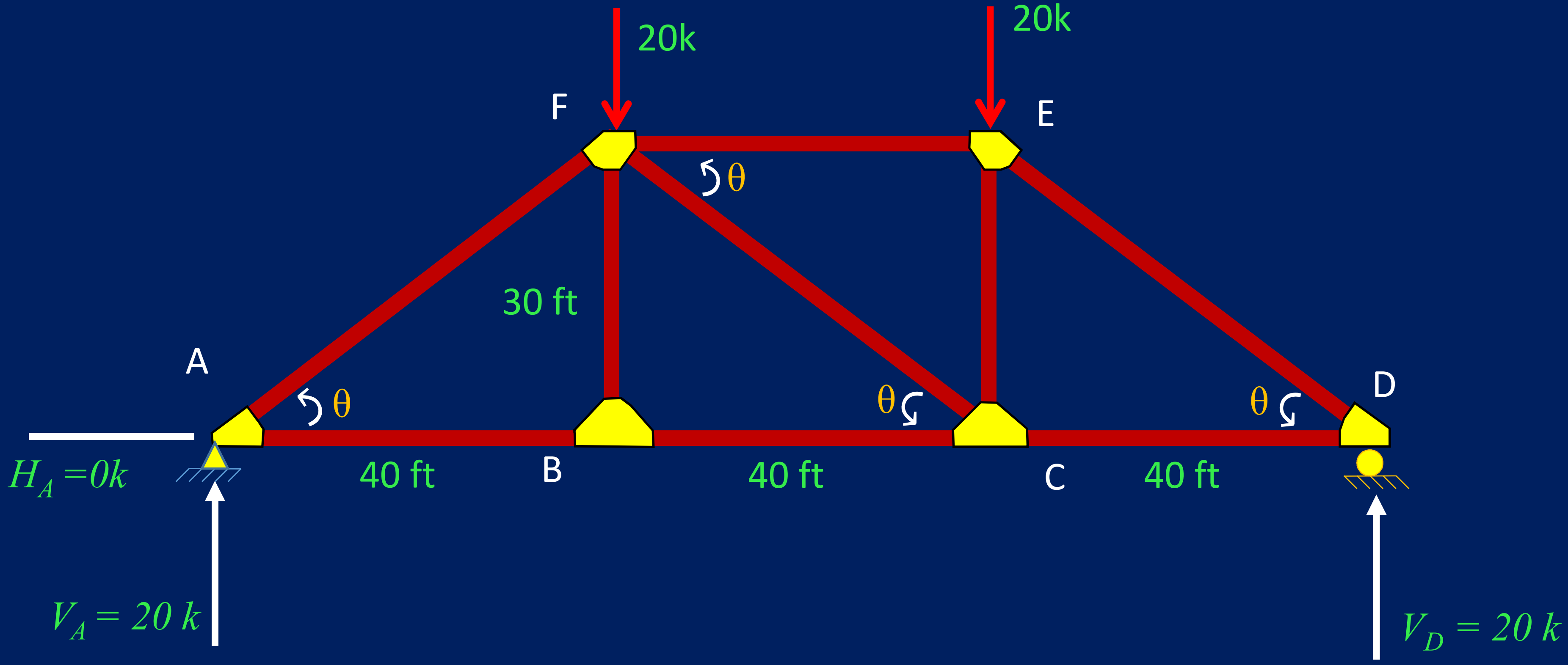
i. Analyze the BDS when acted upon by the actual loads and compute the values of member forces P_i.

ii. Analyze the BDS when acted upon by the unit load at redundant location 1 & compute the values of member forces U_{1i}.

iii. Analyze the BDS when acted upon by the unit load at redundant location 2 & compute the values of member forces U_{2i}.

Flexibility Method for Trusses Analysis

i. BDS acted upon by the actual loads



$$\sum M_A = 0$$

$$20 * 40 + 20 * 80 - V_D * 120 = 0$$
$$V_D = 20\ k$$

$$\sum Fy = 0$$

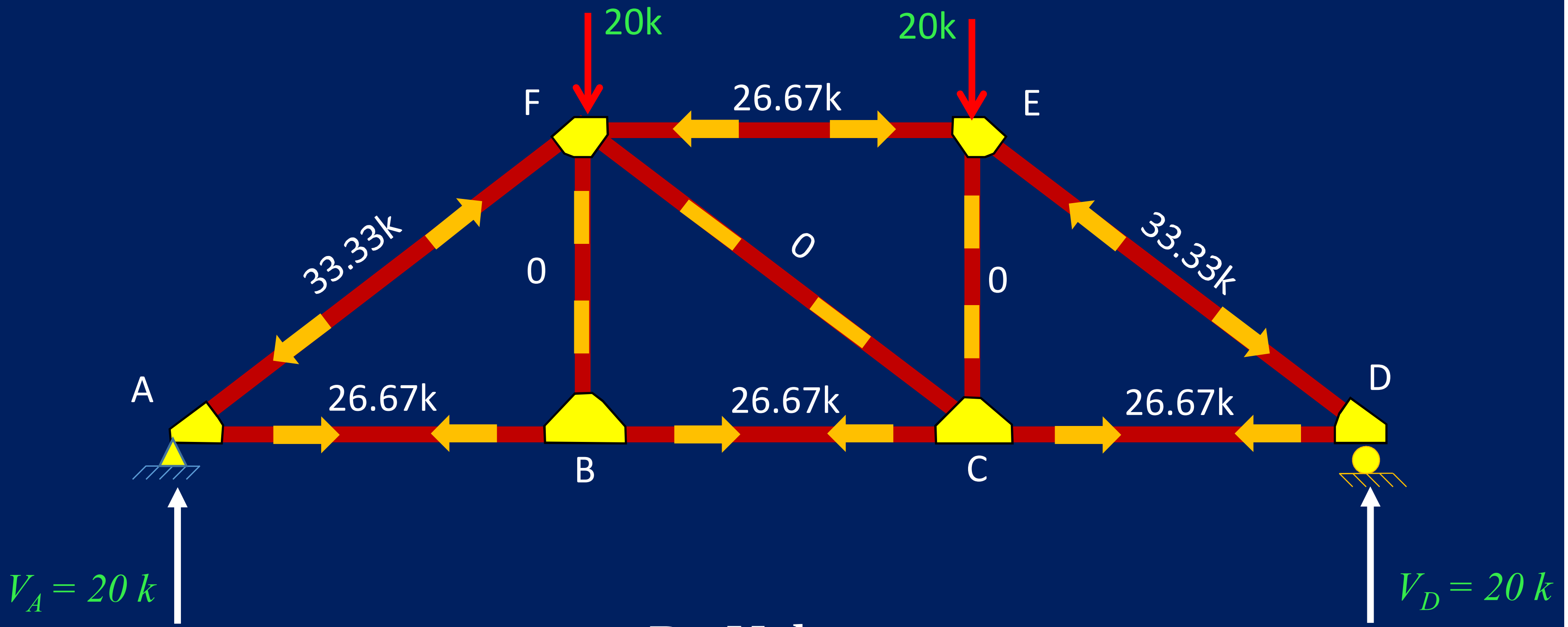
$$VA = 40 - 20$$
$$V_A = 20\ k$$

$$\sum Fx = 0$$

$$H_A = 0k$$

Flexibility Method for Trusses Analysis

Now find all the member forces using method of joints & we will get

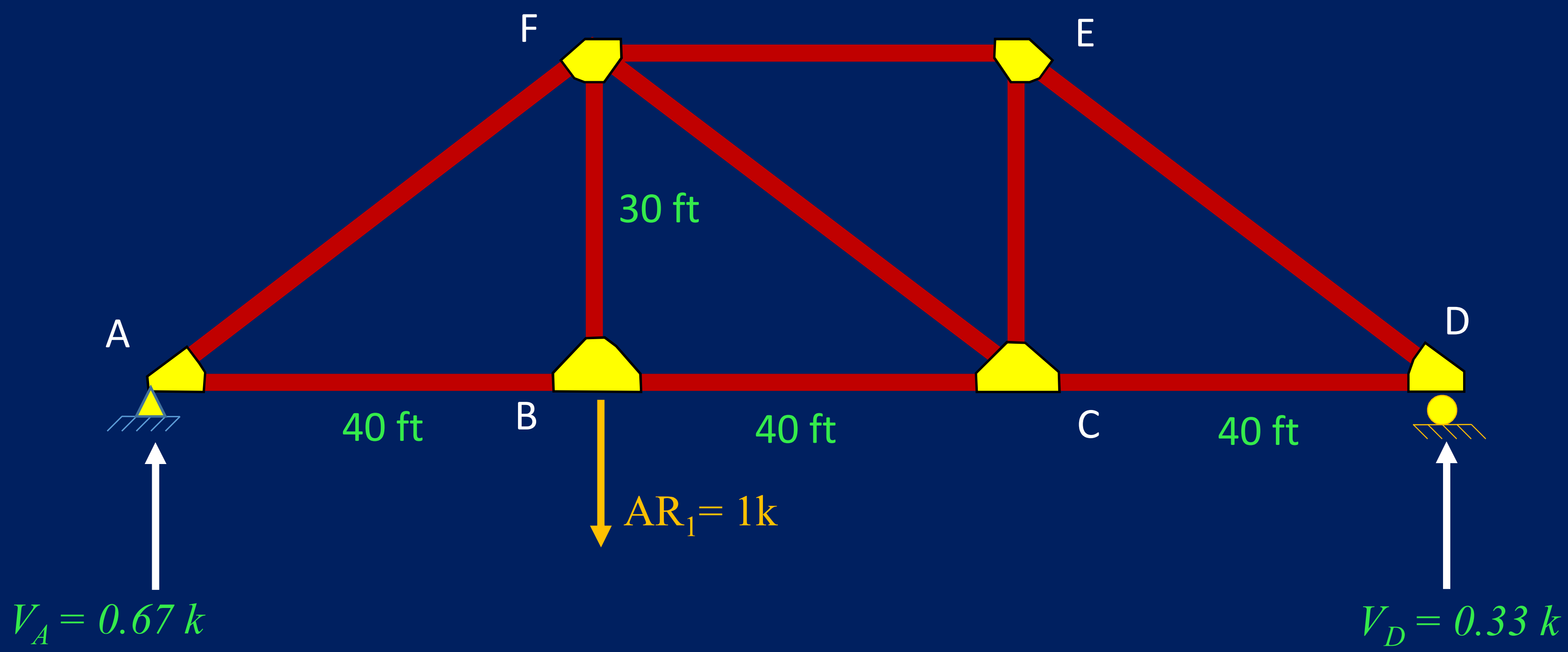


P_i -Values

Class Activity: Verify the member forces using method of joints

Flexibility Method for Trusses Analysis

ii. BDS acted upon by the Unit load at redundant location 1.



$$\sum M_A = 0$$

$$1 * 20 = VD * 120$$

$$V_D = 0.33\ k$$

$$\sum Fy = 0$$

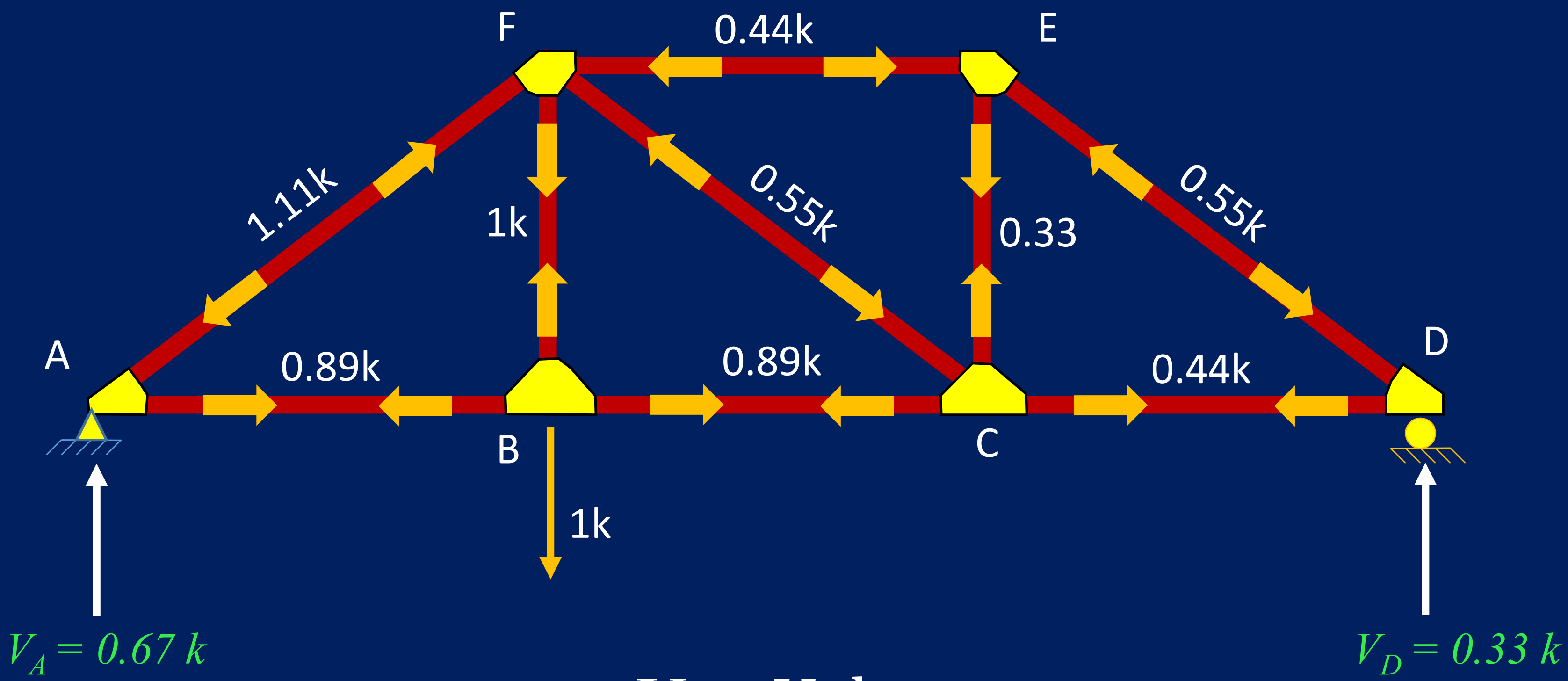
$$V_A = 0.67\ k$$

$$\sum Fx = 0$$

$$H_A = 0k$$

Flexibility Method for Trusses Analysis

Now find all the member forces using method of joints & we will get

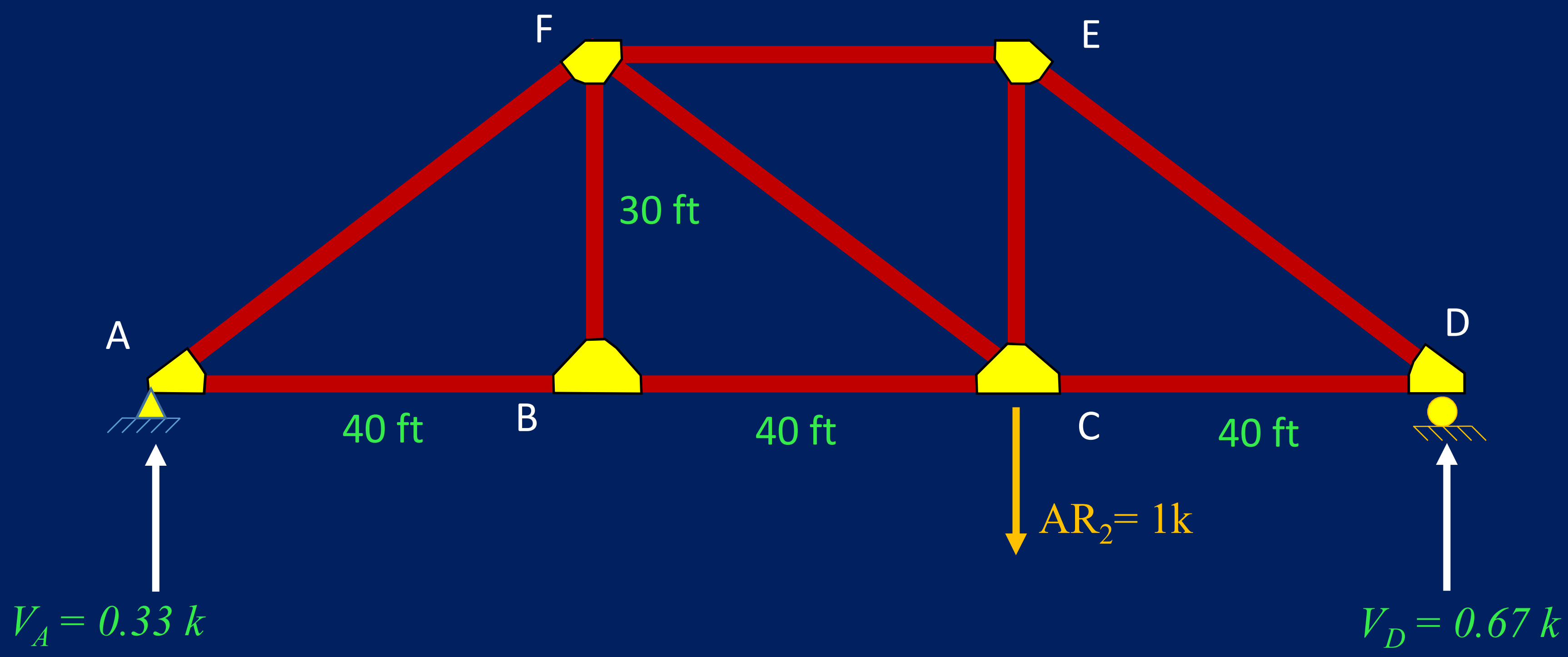


U_{1i} - Values

Class Activity: Verify the member forces using method of joints

Flexibility Method for Trusses Analysis

iii. BDS acted upon by the Unit load at redundant location 2.



$$\sum M_A = 0$$

$$1 * 40 = VD * 120$$

$$V_D = 0.67\ k$$

$$\sum Fy = 0$$

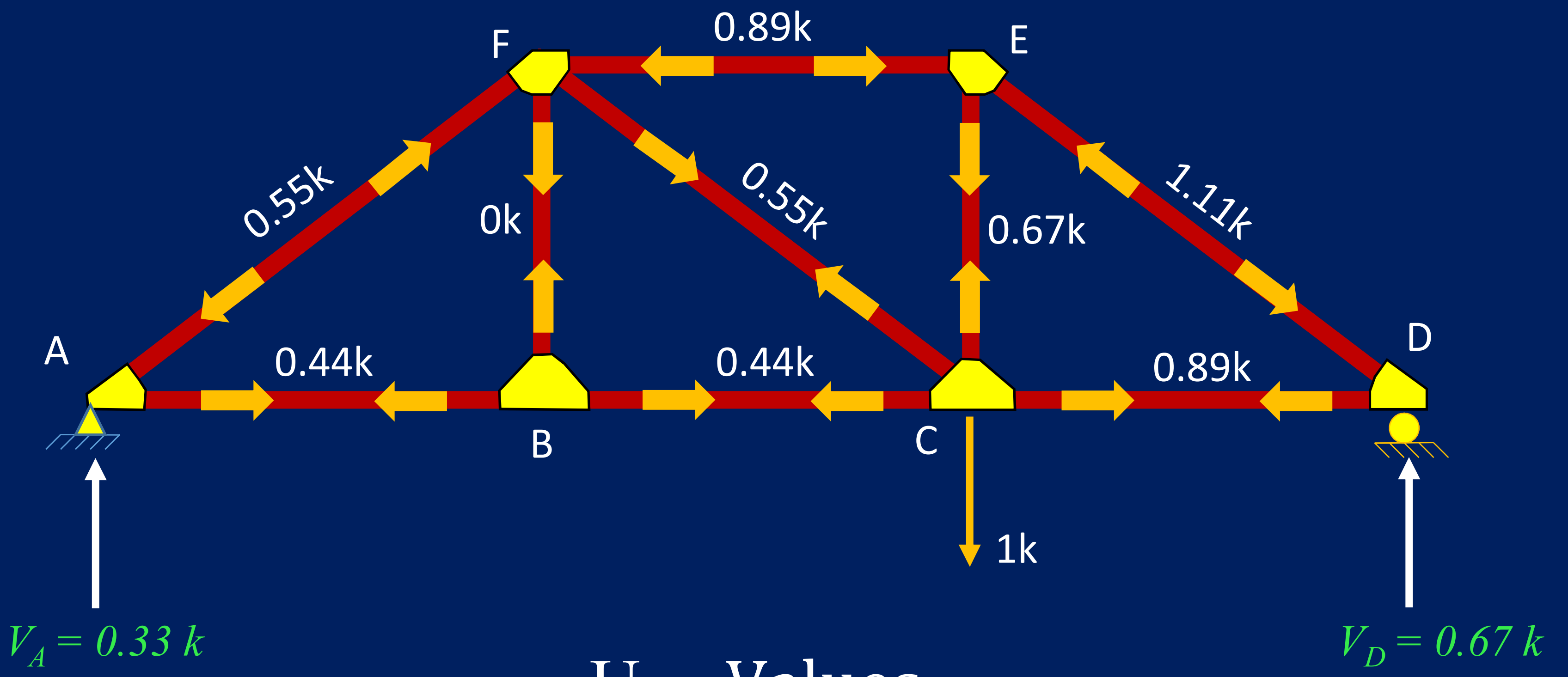
$$V_A = 0.33\ k$$

$$\sum Fx = 0$$

$$H_A = 0k$$

Flexibility Method for Trusses Analysis

Now find all the member forces using method of joints & we will get



U_{2i} - Values

Class Activity: Verify the member forces using method of joints

Flexibility Method for Trusses Analysis

- **Step # 03:** Develop Member forces table.

Member	AE	Length, L_i (ft)	P_i values	U_{1i} Values	U_{2i} values
AB	Constant	40	26.67	0.89	0.44
BC		40	26.67	0.89	0.44
CD		40	26.67	0.44	0.89
DE		50	-33.33	-0.55	-1.11
EF		40	-26.67	-0.44	-0.89
AF		50	-33.33	-1.11	-0.55
BF		30	0	1	0
CF		50	0	-0.55	0.55
CE		30	0	0.33	0.67

Flexibility Method for Trusses Analysis

- **Step # 04:** Find BDS Displacements due to external loads or Compute the values of DRL matrix.

$$\boldsymbol{DRL_1 = \sum_{i=1}^{m} \frac{P_i\, U_{1i}\, Li}{EA_i}}$$

$$DRL_1 = \frac{1}{EA}\begin{bmatrix} (40 * 26.67 * 0.89) + (40 * 26.67 * 0.89) + (40 * 26.67 * 0.44) \\ + (50 * -33.33 * -0.55) + (40 * -26.67 * -0.44) \\ + (50 * -33.33 * -1.11) + 0 + 0 + 0 \end{bmatrix}$$

$$DRL_1 = \frac{5622.3}{EA}$$

Flexibility Method for Trusses Analysis

$$DRL_2 = \sum_{i=1}^{m} \frac{P_i\, U_{2i}\, Li}{EA_i}$$

$$DRL_2 \;=\; \frac{1}{EA}\begin{bmatrix} (40*26.67*0.44)+(40*26.67*0.44)+(40*26.67*0.89) \\ +(50*-33.33*-1.11)+(40*-26.67*-0.89) \\ +(50*-33.33*-0.55)+0+0+0 \end{bmatrix}$$

$$DRL_2 = \frac{5622.3}{EA}$$

$$DRL_1 = \frac{5622.3}{EA} \qquad DRL_2 = \frac{5622.3}{EA}$$

$$[DRL] = \begin{bmatrix} DRL_1 \\ DRL_2 \end{bmatrix} = \frac{1}{EA}\begin{bmatrix} 5622.3 \\ 5622.3 \end{bmatrix}$$

Flexibility Method for Trusses Analysis

- **Step # 05:** Find BDS Displacements due to unit loads or Compute the values of flexibility coefficients & flexibility matrix.

$$f_{11} = \sum_{i=1}^{m} \frac{U_{1i} U_{1i} L_i}{EA_i}$$

$$f_{11} = \frac{1}{EA}\begin{bmatrix} (40 * 0.89 * 0.89) + (40 * 0.89 * 0.89) + (40 * 0.44 * 0.44) \\ + (50 * -0.55 * -0.55) + (40 * -0.44 * -0.44) \\ + (50 * -1.11 * -1.11) + (50 * -1.11 * -1.11) \\ +(50 * -0.55 * -0.55) + (30 * 0.33 * 0.33) \end{bmatrix}$$

$$f_{11} = \frac{205.2}{EA}$$

Flexibility Method for Trusses Analysis

$$f_{12} = f_{21} = \sum_{i=1}^{m} \frac{U_{1i} U_{2i} L_i}{EA_i}$$

$$f_{12} = f_{21} = \frac{1}{EA}\begin{bmatrix} (40 * 0.89 * 0.44) + (40 * 0.89 * 0.44) + (40 * 0.44 * 0.89) \\ + (50 * -0.55 * -1.11) + (40 * -0.44 * -0.89) \\ + (50 * -1.11 * -0.55) + 0 + (50 * -0.55 * 0.55) \\ +(30 * 0.33 * 0.67) \end{bmatrix}$$

$$f_{12} = f_{21} = \frac{115.8}{EA}$$

Flexibility Method for Trusses Analysis

$$f_{22} = \sum_{i=1}^{m} \frac{U_{2i} U_{2i} L_i}{EA_i}$$

$$f_{22} = \frac{1}{EA}\begin{bmatrix} (40*0.44*0.44)+(40*0.44*0.44)+(40*0.89*0.89) \\ +(50*-1.11*-1.11)+(40*-0.89*-0.89) \\ +(50*-0.55*-0.55)+0+(50*0.55*0.55) \\ +(30*0.67*0.67) \end{bmatrix}$$

$$f_{22} = \frac{185.39}{EA}$$

$$[f] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \Rightarrow [f] = \frac{1}{EA}\begin{bmatrix} 205.2 & 115.8 \\ 115.8 & 185.39 \end{bmatrix}$$

Flexibility Method for Trusses Analysis

Step # 06: Apply compatibility and principle of superposition at the location of the removed redundants and solve the matrices to compute the values of redundant actions AR. As we know

From this

$$[DRS] = [DRL] + [f] \bullet [AR]$$

$$[AR] = [f]^{-1} \bullet [DRS - DRL]$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

Flexibility Method for Trusses Analysis

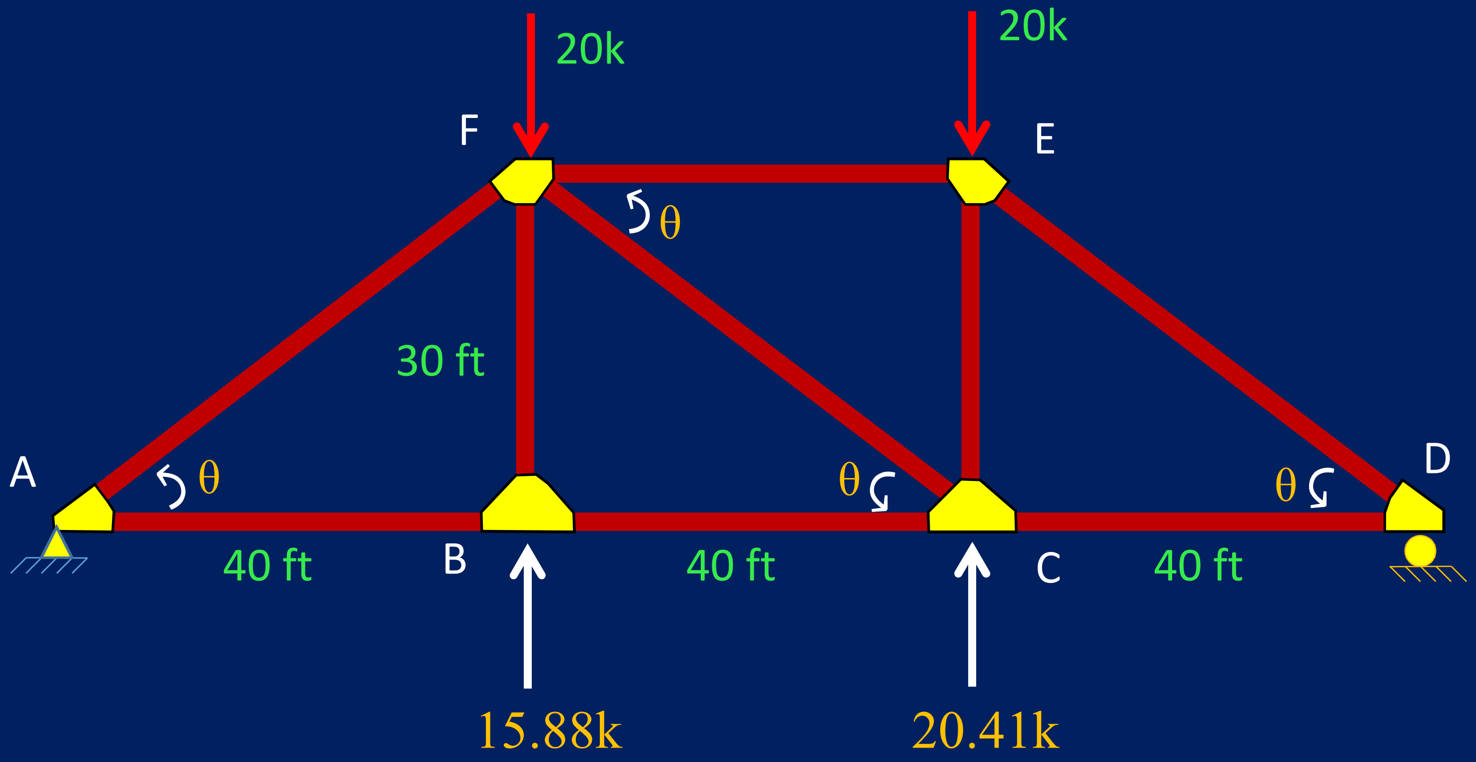
$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = EA \begin{bmatrix} 205.2 & 115.8 \\ 115.8 & 185.39 \end{bmatrix}^{-1} \begin{bmatrix} 0 - (5622.3) \\ 0 - (5622.3) \end{bmatrix} \times \frac{1}{EA}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} -15.88 \\ -20.41 \end{bmatrix}$$

-ive sign shows that our assumed redundant action direction is wrong

Flexibility Method for Trusses Analysis



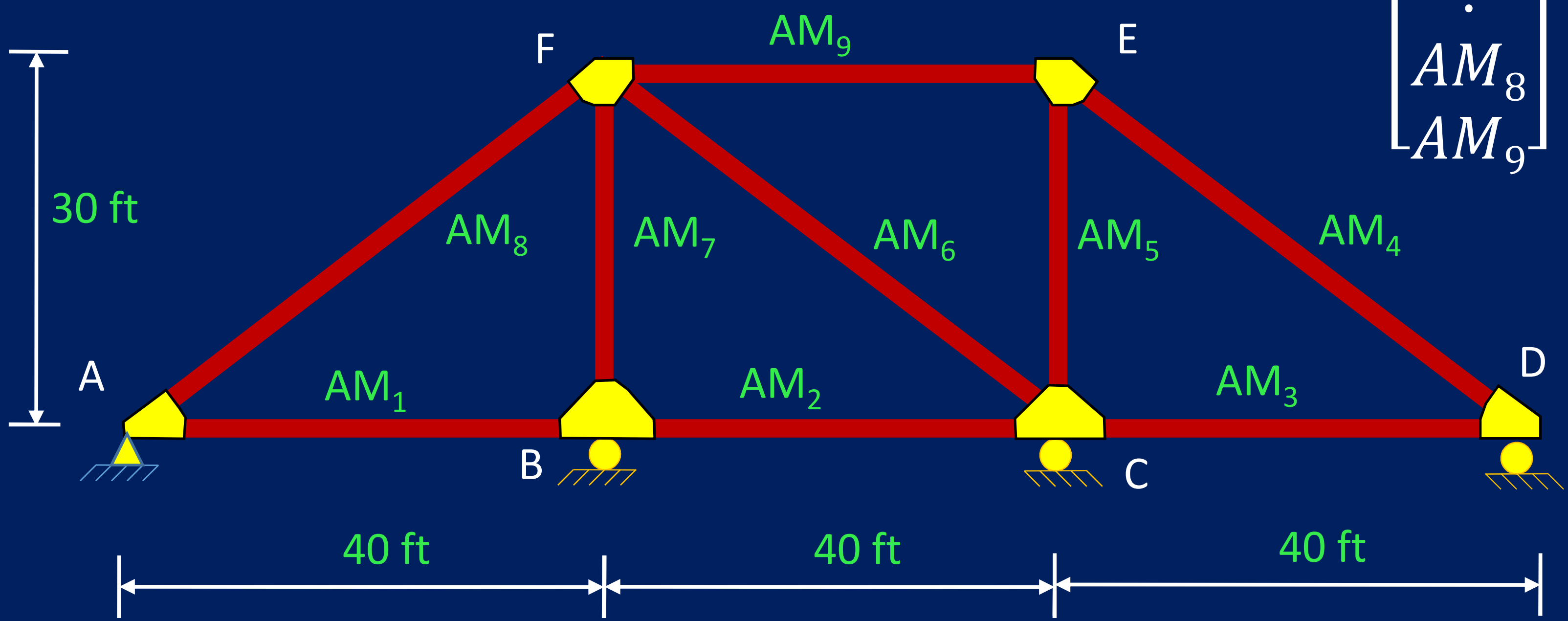
Final determinate structure

Flexibility Method for Trusses Analysis

Step # 07: Compute the member end actions. As we know that

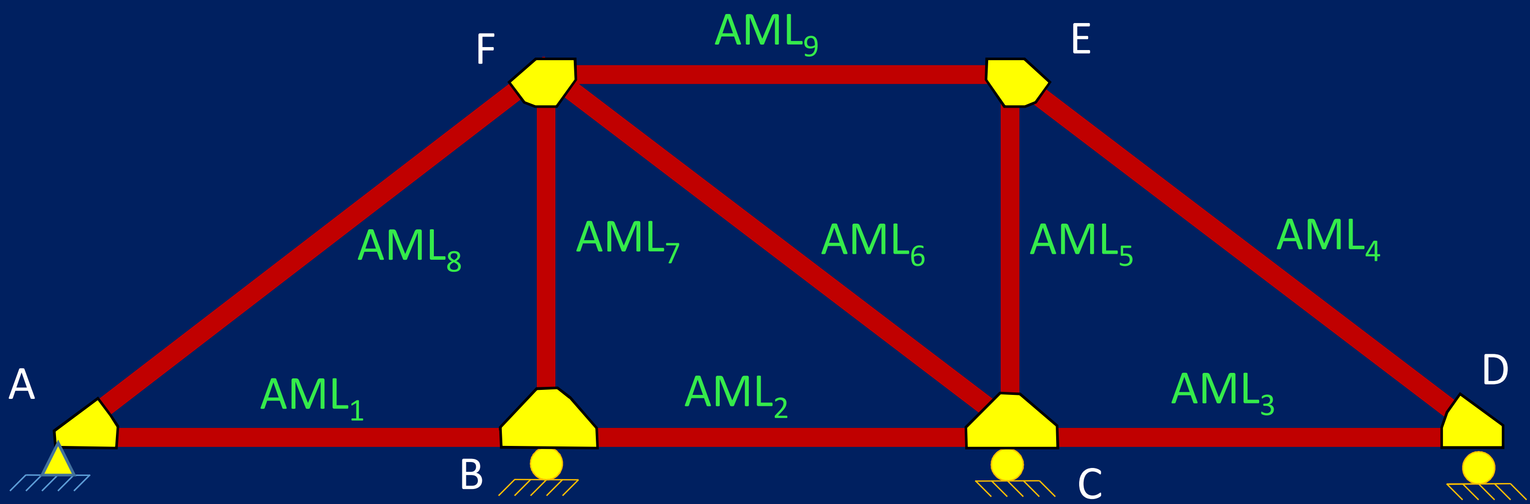
$$[AM] = [AML] + [AMR][AR]$$

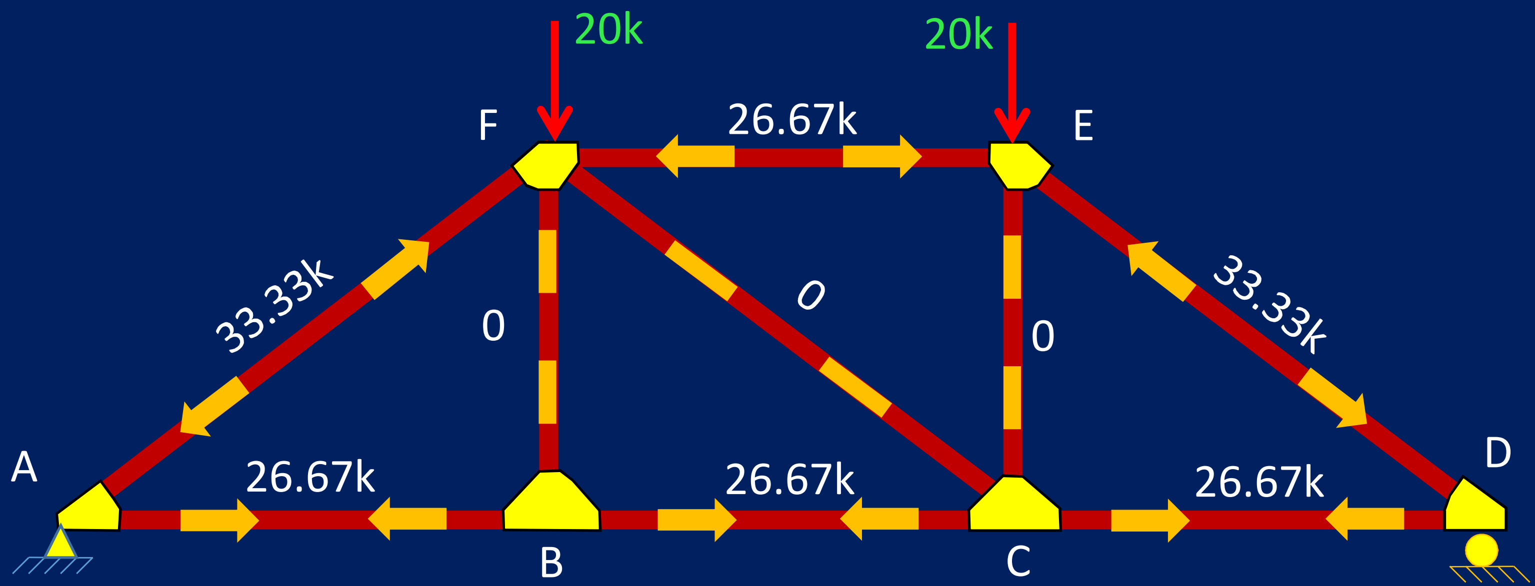
$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ \vdots \\ AM_8 \\ AM_9 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ \vdots \\ ? \\ ? \end{bmatrix}$$



Flexibility Method for Trusses Analysis

a) Compute AML values.





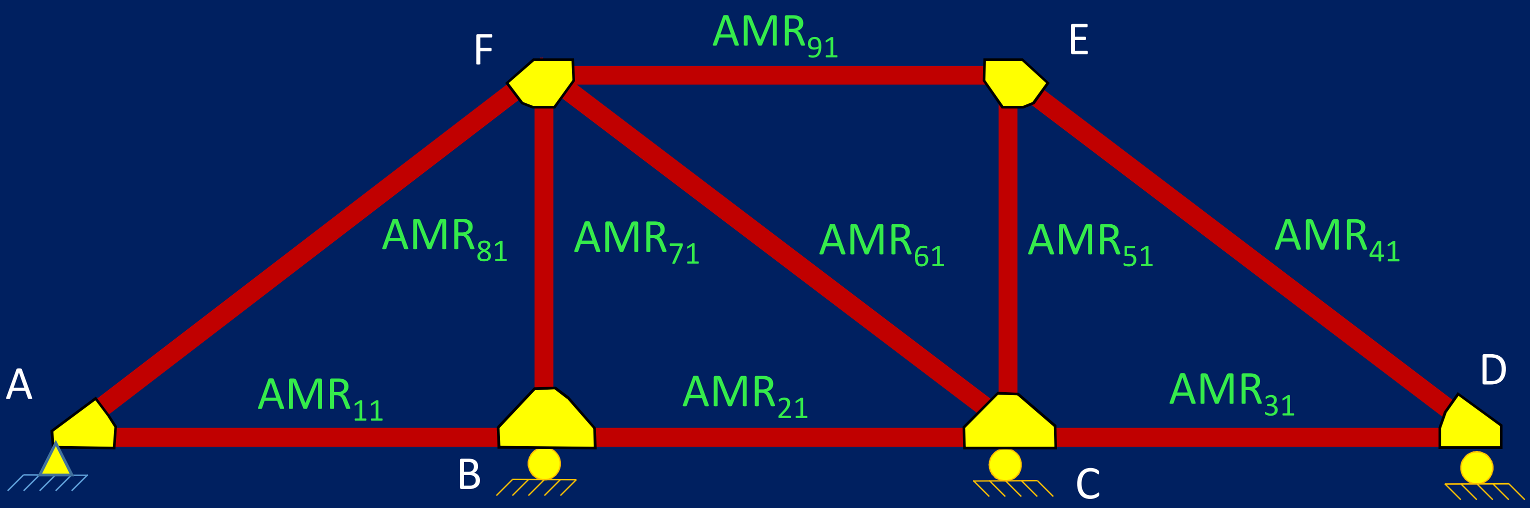
Flexibility Method for Trusses Analysis

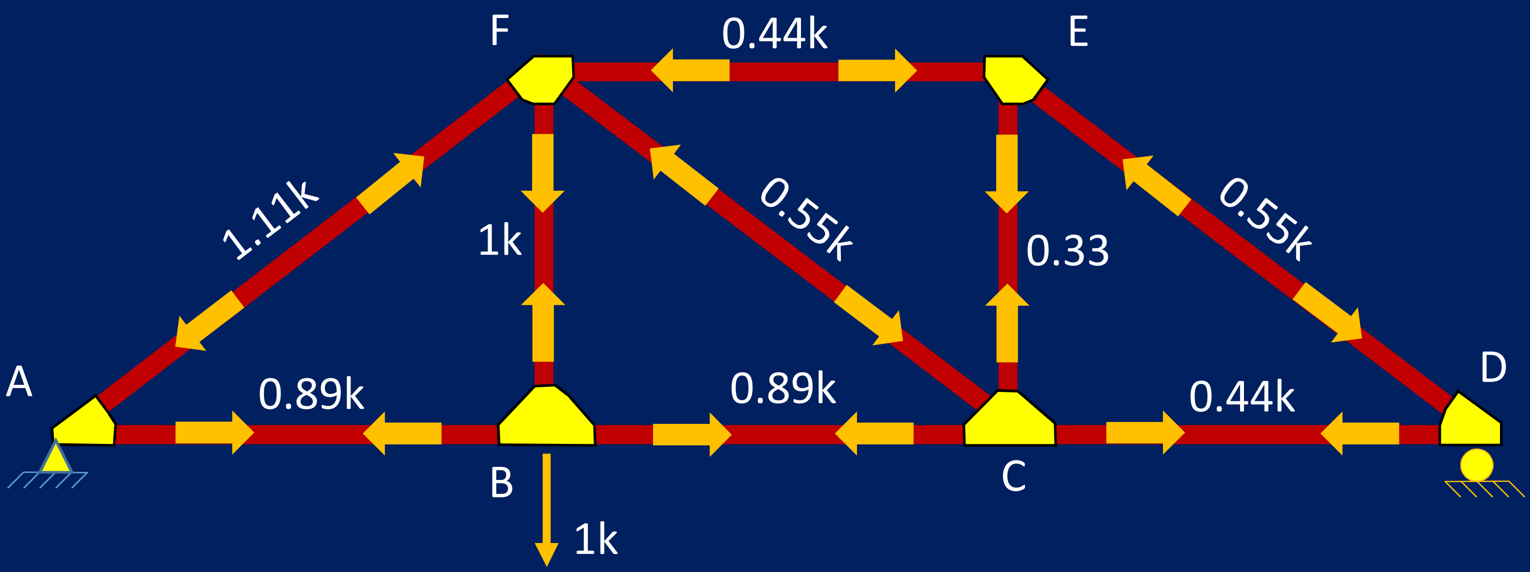
So AML values from the previous slide is

$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \\ AML_7 \\ AML_8 \\ AML_9 \end{bmatrix} = \begin{bmatrix} 26.67 \\ 26.67 \\ 26.67 \\ -33.33 \\ 0 \\ 0 \\ 0 \\ -33.33 \\ -26.67 \end{bmatrix}$$

Flexibility Method for Trusses Analysis

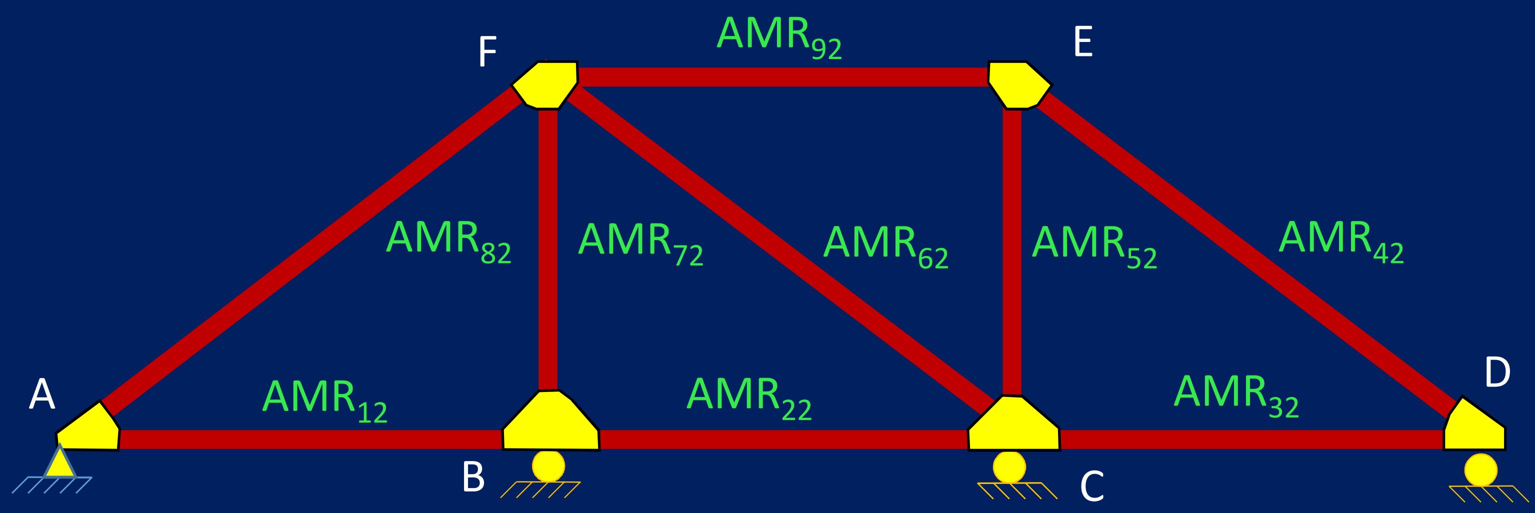
b) Compute the AMR values: when unit is applied at redundant location 1.

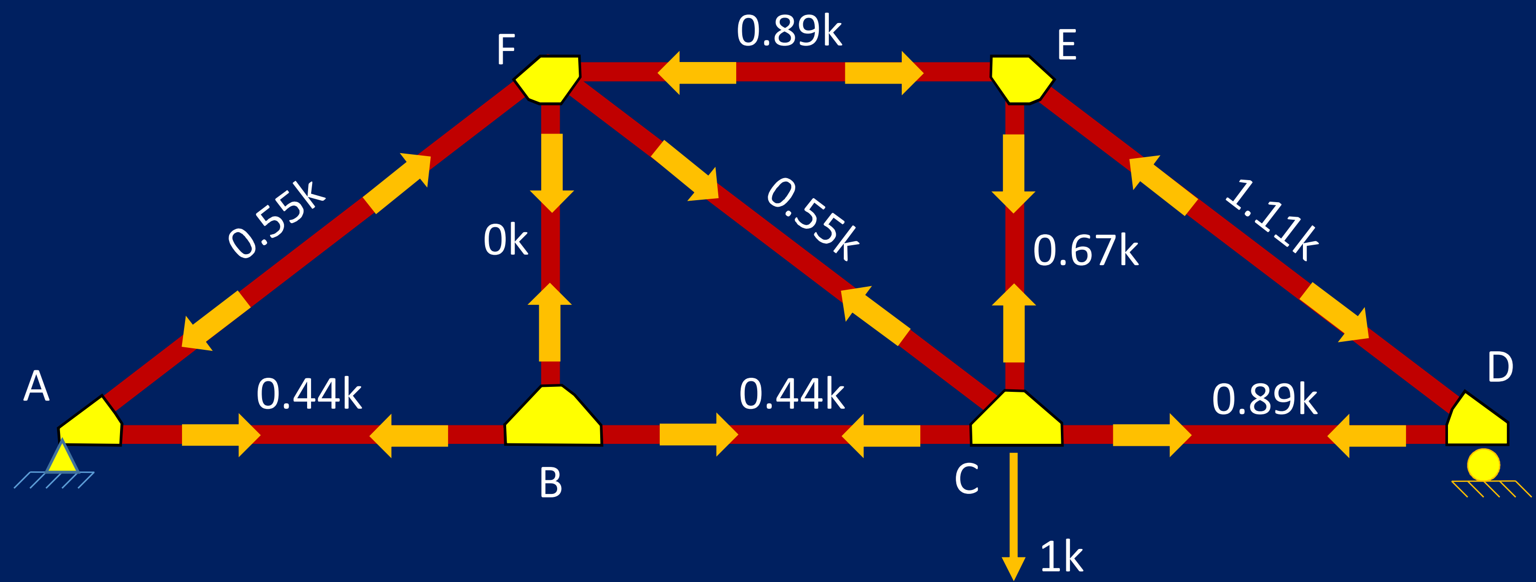




Flexibility Method for Trusses Analysis

b) Compute the AMR values: when unit is applied at redundant location 2.





Flexibility Method for Trusses Analysis

So AMR matrix

$$[AMR]_{9\,*\,2} = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \\ AMR_{71} & AMR_{72} \\ AMR_{81} & AMR_{82} \\ AMR_{91} & AMR_{92} \end{bmatrix} = \begin{bmatrix} 0.89 & 0.44 \\ 0.89 & 0.44 \\ 0.44 & 0.89 \\ -0.55 & -1.11 \\ 0.33 & 0.67 \\ -0.55 & 0.55 \\ 1 & 0 \\ -1.11 & -0.55 \\ -0.44 & -0.89 \end{bmatrix}$$

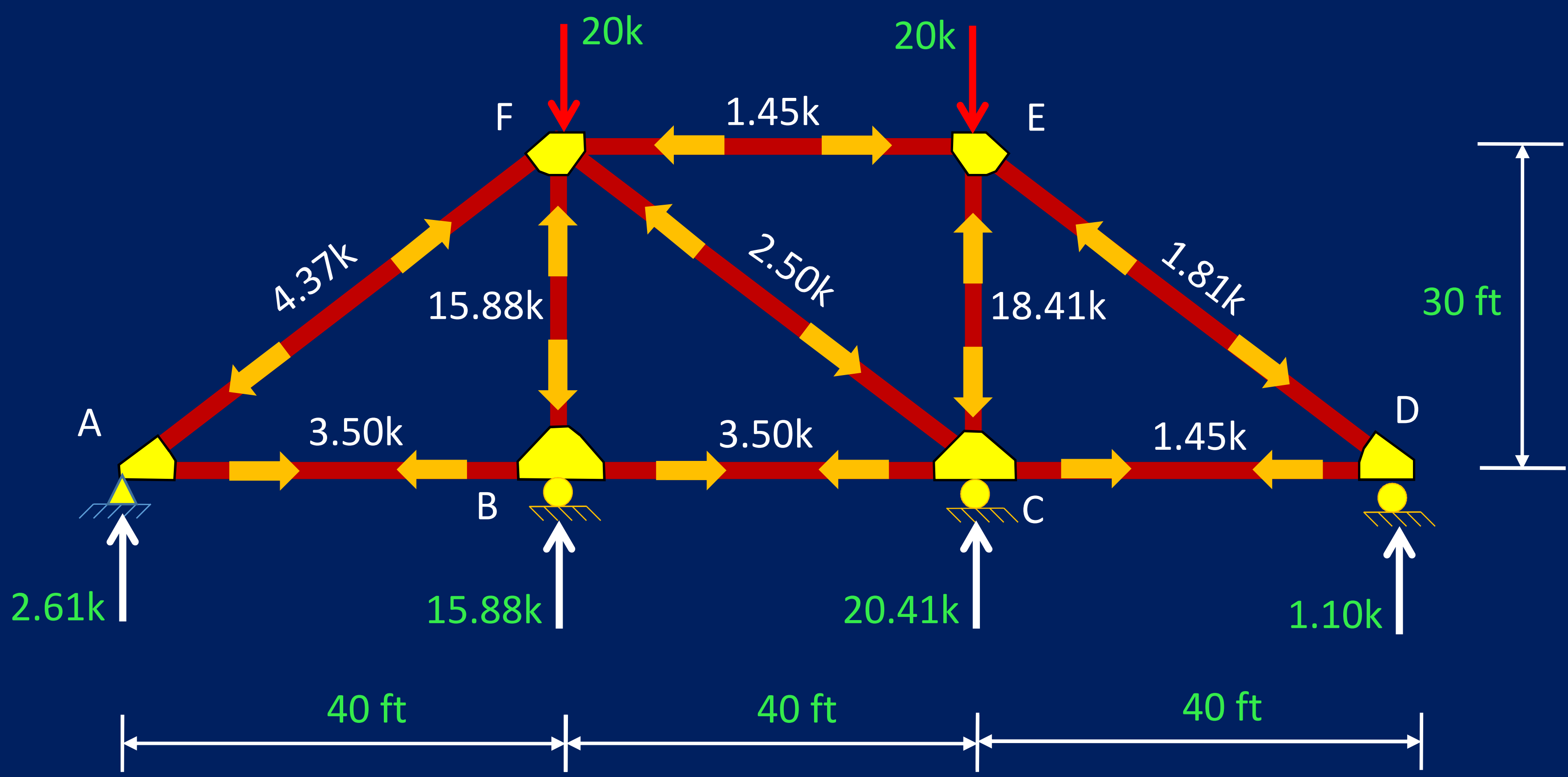
Flexibility Method for Trusses Analysis

Now member end actions

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \\ AM_7 \\ AM_8 \\ AM_9 \end{bmatrix} = \begin{bmatrix} 26.67 \\ 26.67 \\ 26.67 \\ -33.33 \\ 0 \\ 0 \\ 0 \\ -33.33 \\ -26.67 \end{bmatrix} + \begin{bmatrix} 0.89 & 0.44 \\ 0.89 & 0.44 \\ 0.44 & 0.89 \\ -0.55 & -1.11 \\ 0.33 & 0.67 \\ -0.55 & 0.55 \\ 1 & 0 \\ -1.11 & -0.55 \\ -0.44 & -0.89 \end{bmatrix} \begin{bmatrix} -15.14 \\ -20.41 \end{bmatrix} = \begin{bmatrix} 3.50 \\ 3.50 \\ 1.45 \\ -1.81 \\ -18.41 \\ -2.50 \\ -15.88 \\ -4.37 \\ -1.45 \end{bmatrix}$$

Flexibility Method for Trusses Analysis

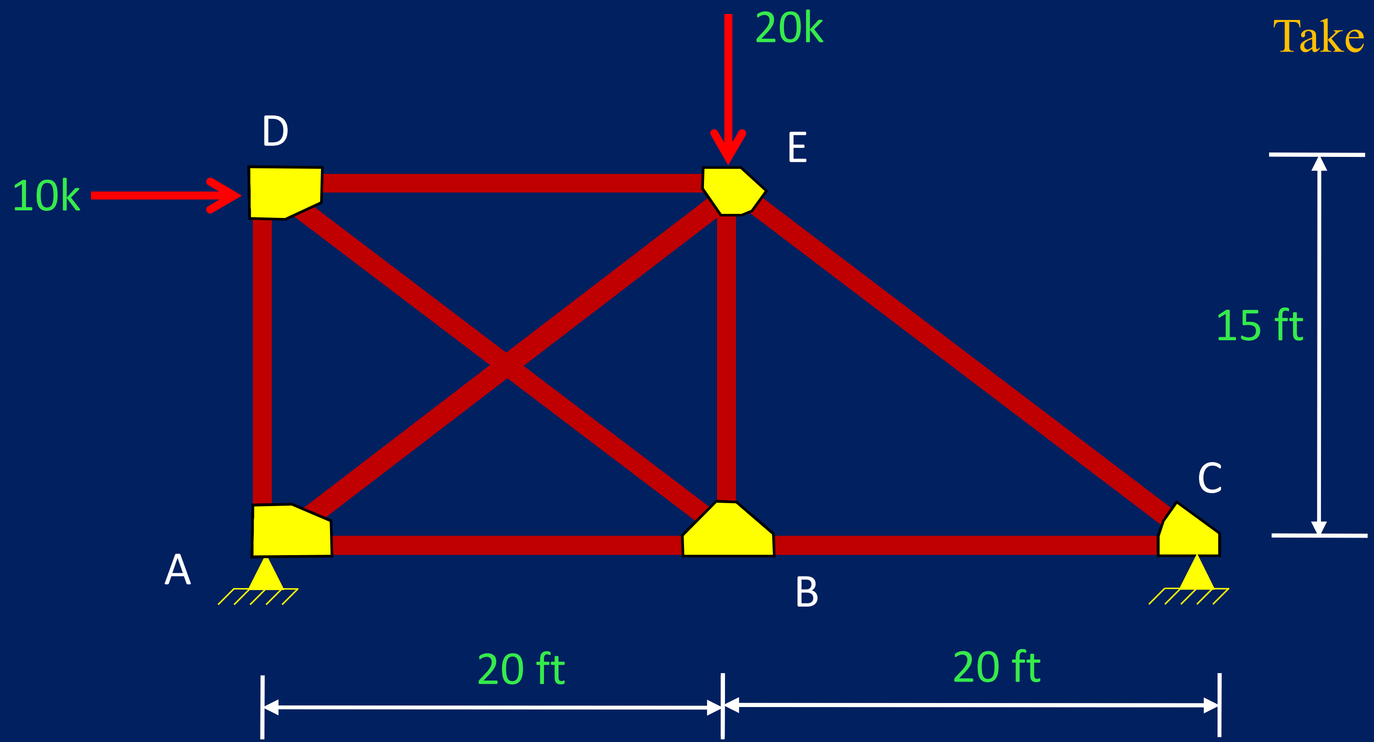
To find the support reaction simply apply equilibrium equations and we will get



Completely analyzed structure

Flexibility Method for Trusses Analysis

Problem 03: Analyze the given truss using flexibility method.



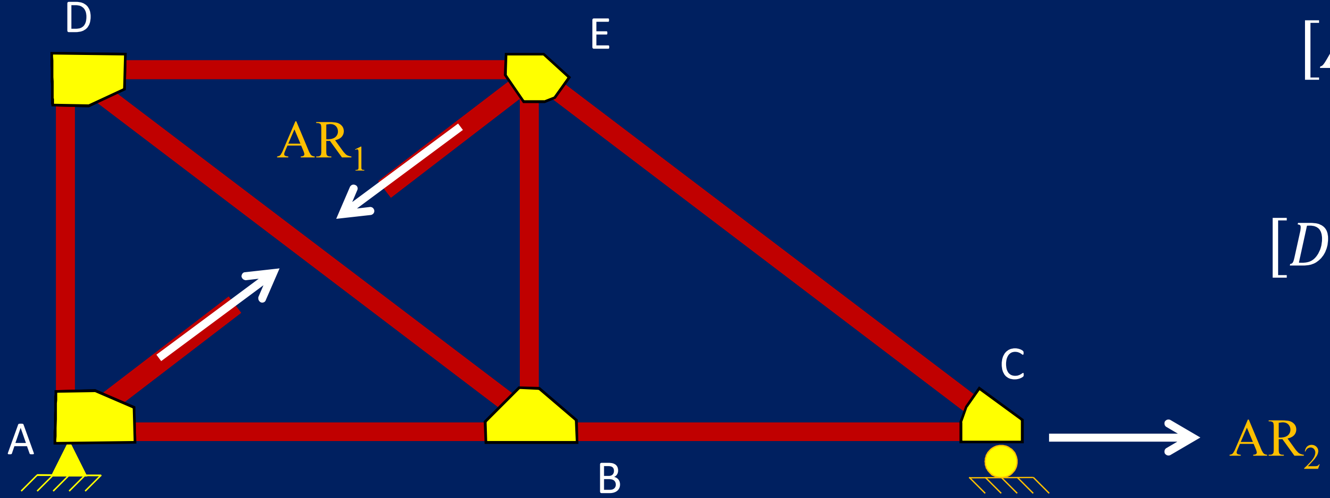
Take EI = constant

E.I = 4 – 3 = 1°
I.I = 8 + 3 – 2(5) = 1°
T.I = 2 degrees

S.I = 2 degrees
So two redundant actions should be chosen.

Flexibility Method for Trusses Analysis

- **Step # 01:** Identify the redundants and obtain BDS also compute [DRS] values.



$$[AR] = \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[DRS] = \begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

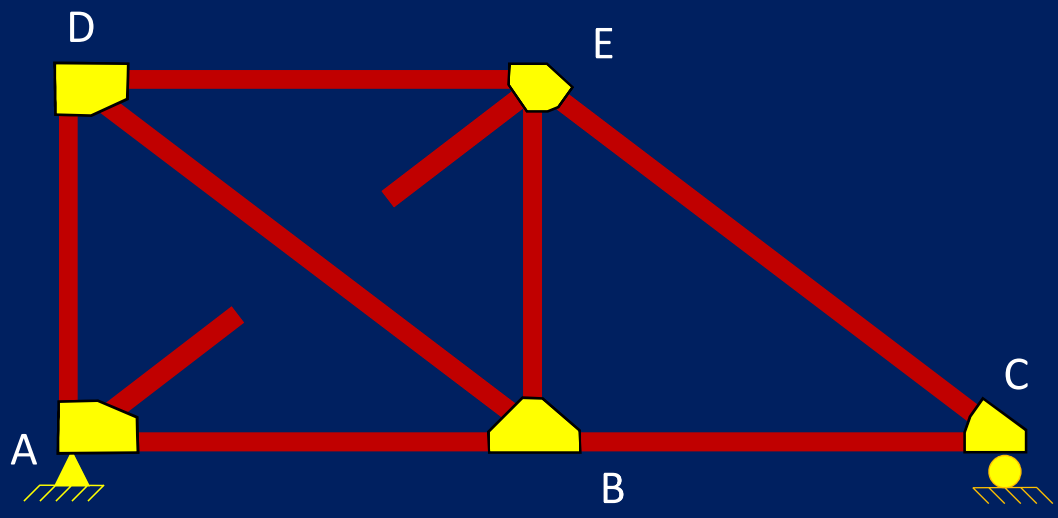
$$\tan^{-1}\frac{15}{20} = 36.86°$$

$$\sin\theta = 0.6$$

$$\cos\theta = 0.8$$

Horizontal reaction at C & member force in member AE are chosen as redundant actions

Flexibility Method for Trusses Analysis



Basic determinate structure (BDS) or Primary structure or Released structure

Flexibility Method for Trusses Analysis

- **Step # 02:** Analyze the BDS (primary structure) under the following loading conditions

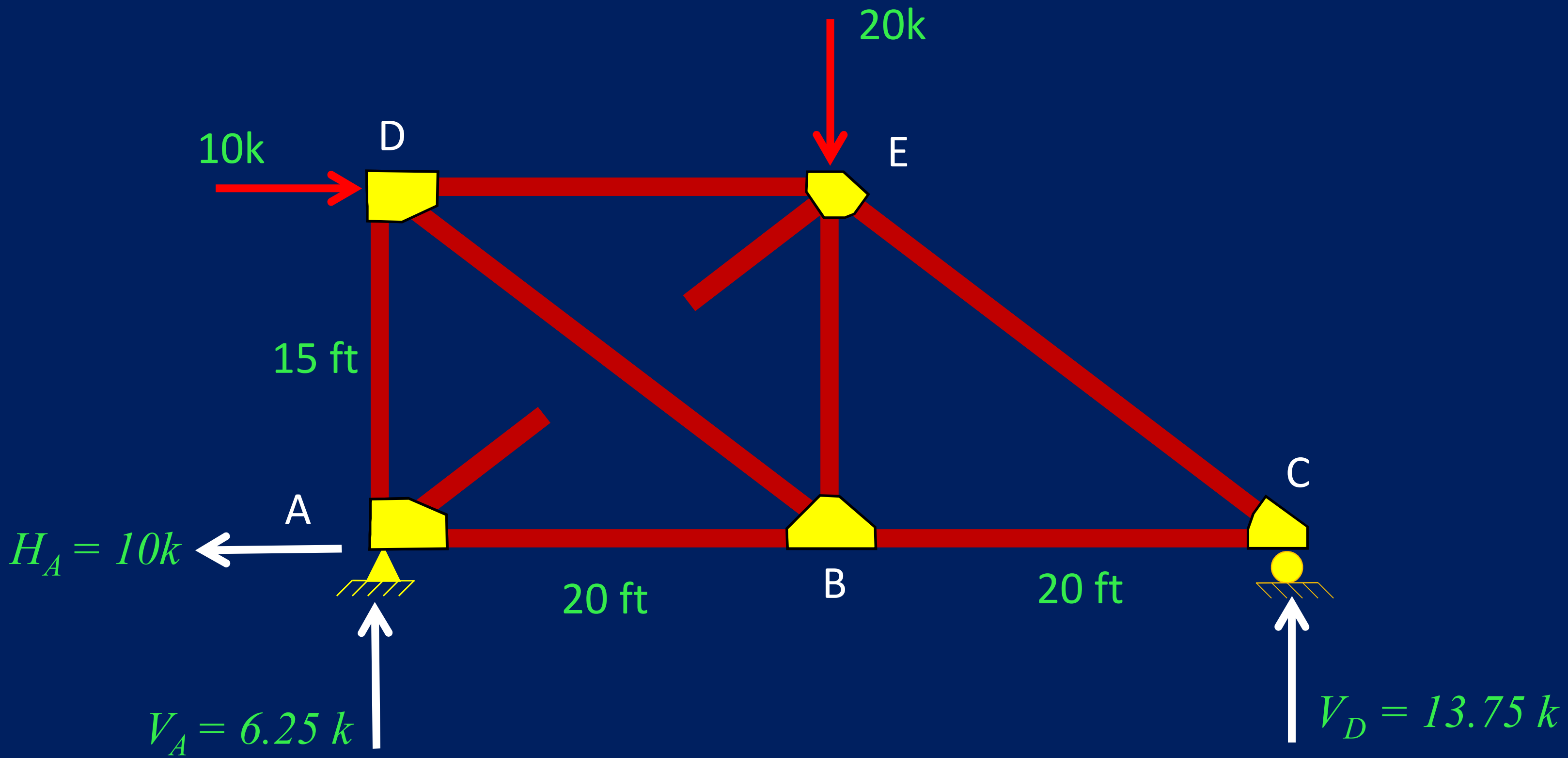
i. Analyze the BDS when acted upon by the actual loads and compute the values of member forces P_i.

ii. Analyze the BDS when acted upon by the unit load at redundant location 1 & compute the values of member forces U_{1i}.

iii. Analyze the BDS when acted upon by the unit load at redundant location 2 & compute the values of member forces U_{2i}.

Flexibility Method for Trusses Analysis

i. BDS acted upon by the external loads



$$\sum M_A = 0$$

$$20 * 20 + 10 * 15 - V_D * 40 = 0$$
$$V_D = 13.75\ k$$

$$\sum Fy = 0$$

$$VA = 20 - 13.75$$

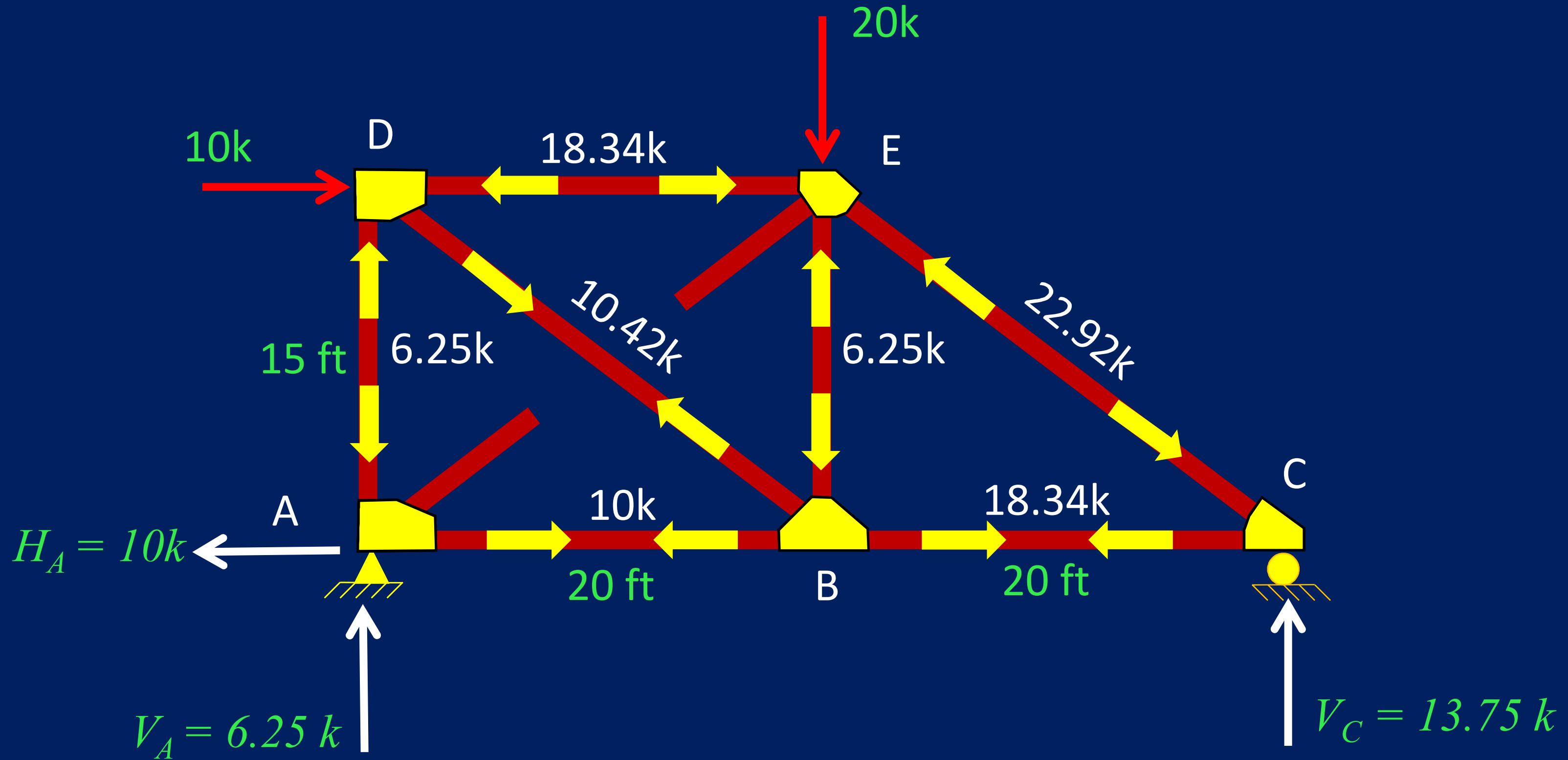
$$V_A = 6.25\ k$$

$$\sum Fx = 0$$

$$H_A = 10\ k$$

Flexibility Method for Trusses Analysis

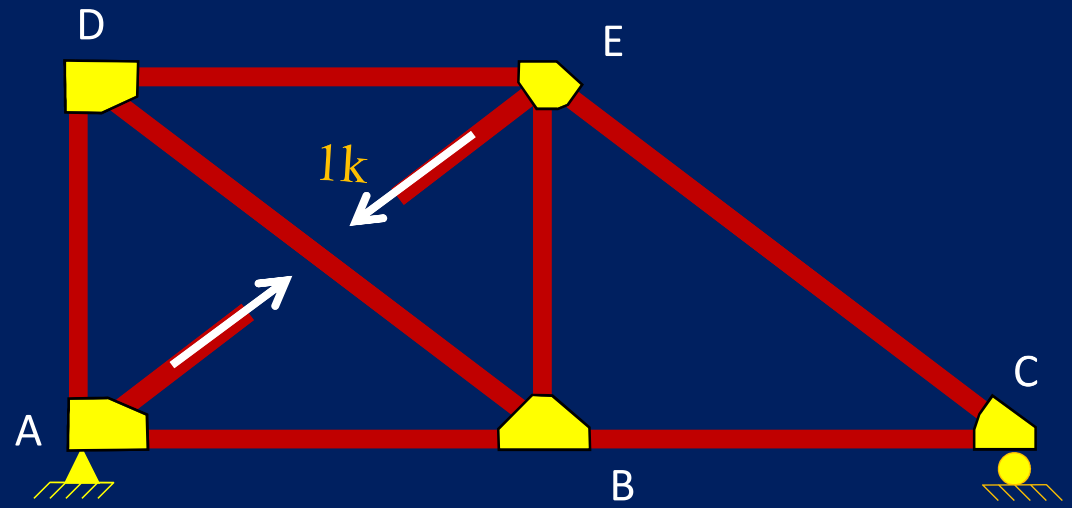
Now find all the member forces using method of joints & we will get

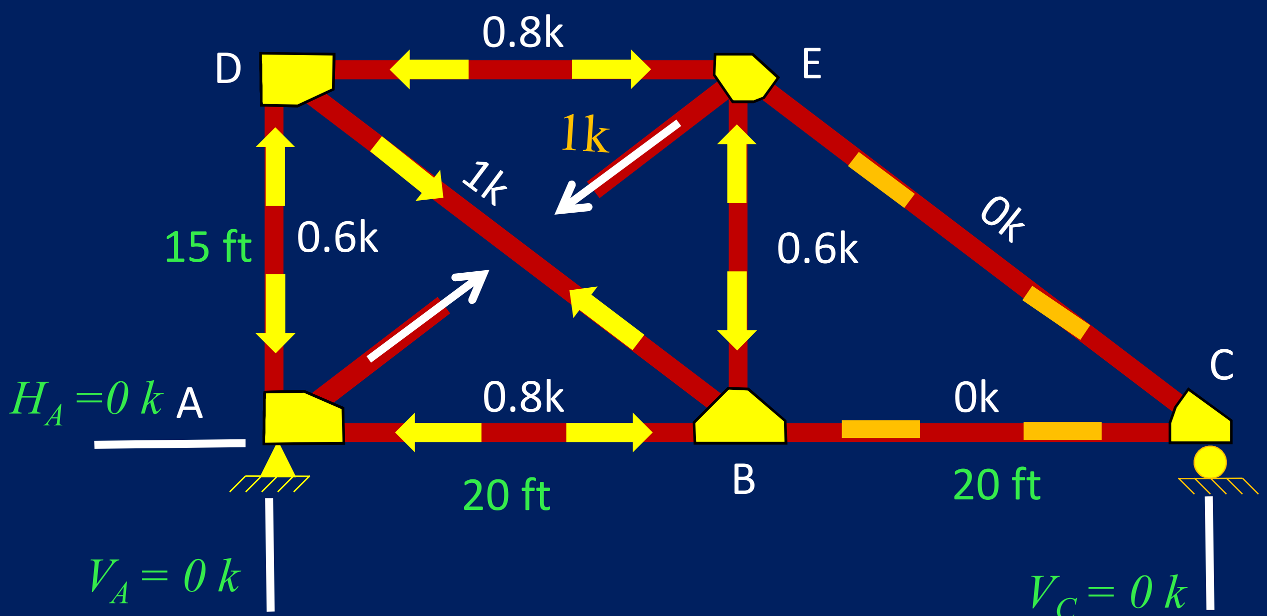


P_i -Values

Flexibility Method for Trusses Analysis

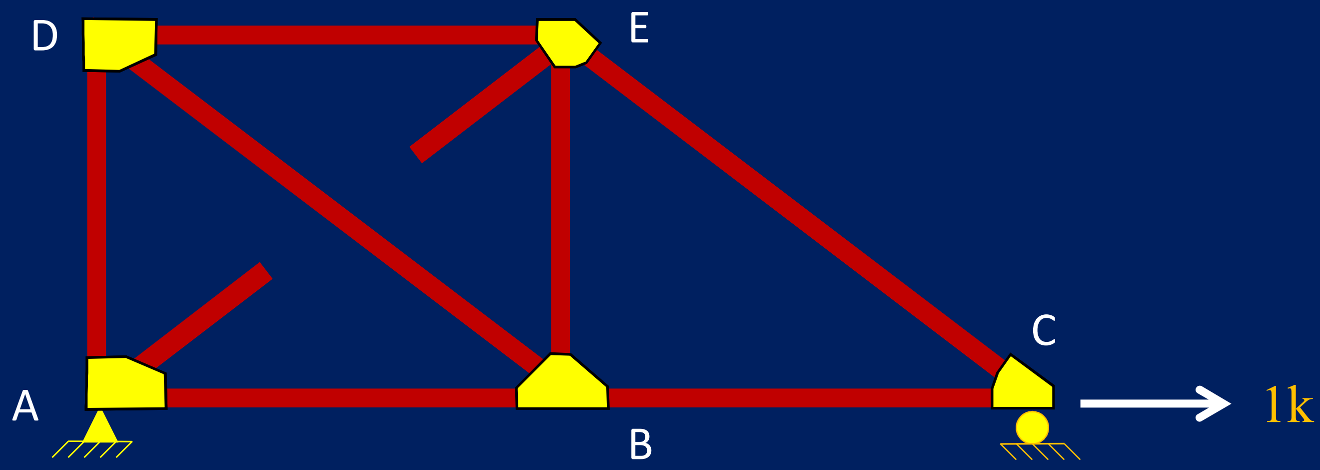
ii. BDS acted upon by the Unit load at redundant location 1.(U_{1i} values)

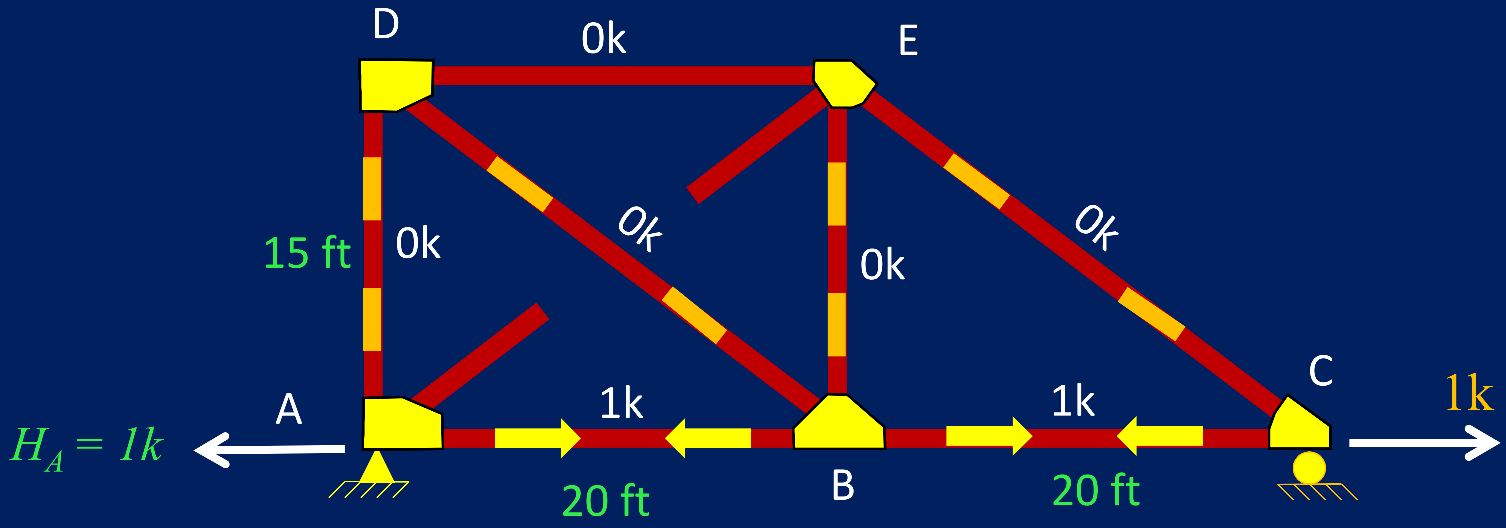




Flexibility Method for Trusses Analysis

iii. BDS acted upon by the Unit load at redundant location 2.(U_{2i} values)





Flexibility Method for Trusses Analysis

- **Step # 03:** Develop Member forces table.

Member	AE	Length, L_i (ft)	P_i values	U_{1i} Values	U_{2i} values
AB	Constant	20	10	-0.8	1
BC		20	18.34	0	1
CD		25	-22.92	0	0
DE		20	-18.34	-0.8	0
AD		15	-6.25	-0.6	0
BD		25	10.42	1	0
AE		25	0	1	0
BE		15	-6.25	-0.6	0

Flexibility Method for Trusses Analysis

- **Step # 04:** Find BDS Displacements due to external loads

  or Compute the values of DRL matrix.

$$\boldsymbol{DRL_1 = \sum_{i=1}^{m} \frac{P_i\, U_{1i}\, Li}{EA_i}}$$

$$DRL_1 \;=\; \frac{1}{EA}\begin{bmatrix} (20 * 10 * -0.8) + 0 + 0 + (20 * -18.34 * -0.8) \\ + (15 * -6.25 * -0.6) + (25 * 10.42 * 1) \\ + 0 + (15 * -6.25 * -0.6) \end{bmatrix}$$

$$DRL_1 = \frac{506.44}{EA}$$

Flexibility Method for Trusses Analysis

$$\boldsymbol{DRL_2 = \sum_{i=1}^{m} \frac{P_i\, U_{2i}\, Li}{EA_i}}$$

$$DRL_2 = \frac{1}{EA}\begin{bmatrix}(20 * 10 * 1) + (20 * -18.34 * 1) \\ +0 + 0 + 0 + 0 + 0 + 0 + 0\end{bmatrix}$$

$$DRL_2 = \frac{566.8}{EA}$$

$$DRL_1 = \frac{506.44}{EA} \qquad DRL_2 = \frac{566.8}{EA}$$

$$[DRL] = \begin{bmatrix} DRL_1 \\ DRL_2 \end{bmatrix} = \frac{1}{EA}\begin{bmatrix} 506.44 \\ 566.8 \end{bmatrix}$$

Flexibility Method for Trusses Analysis

- **Step # 05:** Find BDS Displacements due to unit loads

  or Compute the values of flexibility coefficients

  & flexibility matrix.

$$f_{11} = \sum_{i=1}^{m} \frac{U_{1i} U_{1i} L_i}{EA_i}$$

$$f_{11} = \frac{1}{EA} \begin{bmatrix} (20 * -0.8 * -0.8) + 0 + 0 + (20 * -0.8 * -0.8) \\ + (15 * -0.6 * -0.6) + (25 * 1 * 1) \\ + (25 * 1 * 1) + (15 * -0.6 * -0.6) \end{bmatrix}$$

$$f_{11} = \frac{86.4}{EA}$$

Flexibility Method for Trusses Analysis

$$f_{12} = f_{21} = \sum_{i=1}^{m} \frac{U_{1i} U_{2i} L_i}{EA_i}$$

$$f_{12} \;=\; \frac{1}{EA}[(20 * -0.8 * 1) + 0 + 0 + 0 + 0 + 0 + 0 + 0]$$

$$f_{12} = f_{21} = \frac{-16}{EA}$$

$$f_{22} = \sum_{i=1}^{m} \frac{U_{2i} U_{2i} L_i}{EA_i}$$

$$f_{22} \;=\; \frac{1}{EA}[(20 * 1 * 1) + (20 * 1 * 1) + 0 + 0 + 0 + 0 + 0 + 0]$$

$$f_{22} = \frac{40}{EA}$$

$$[\,f\,] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \;\Rightarrow\; [\,f\,] = \frac{1}{EA}\begin{bmatrix} 86.4 & -16 \\ -16 & 40 \end{bmatrix}$$

Flexibility Method for Trusses Analysis

Step # 06: Apply compatibility and principle of superposition at the location of the removed redundants and solve the matrices to compute the values of redundant actions AR. As we know

$$[DRS] = [DRL] + [f] \bullet [AR]$$

From this

$$[AR] = [f]^{-1} \bullet [DRS - DRL]$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

Flexibility Method for Trusses Analysis

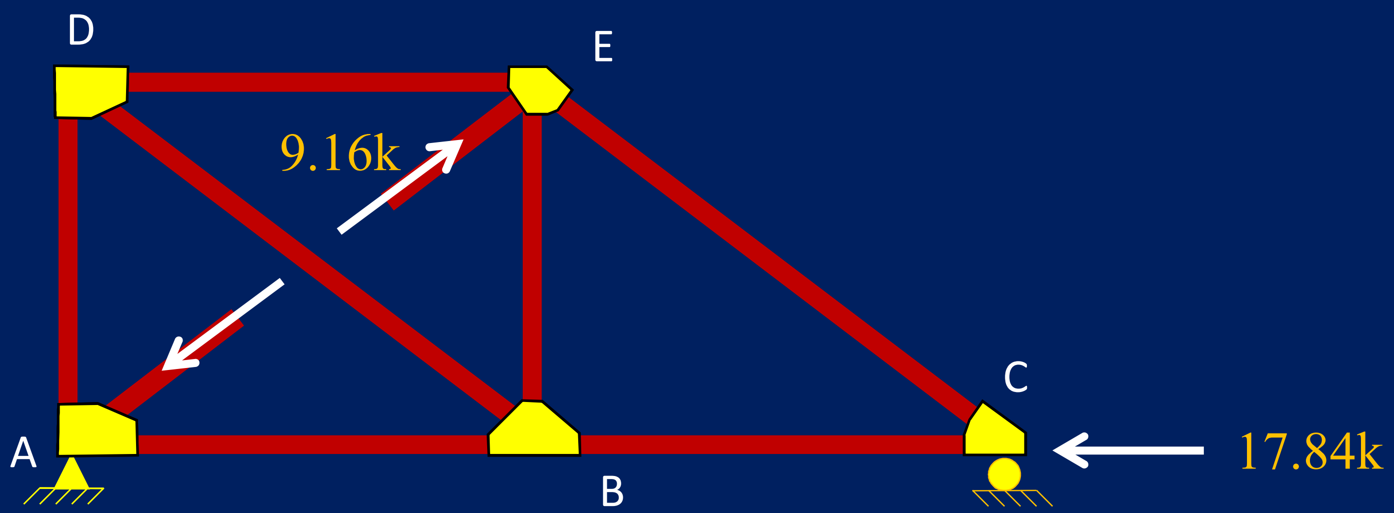
$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

$$\begin{bmatrix} AR1 \\ AR2 \end{bmatrix} = EA \begin{bmatrix} 86.4 & -16 \\ -16 & 40 \end{bmatrix}^{-1} \begin{bmatrix} 0 - (506.44) \\ 0 - (566.8) \end{bmatrix} \times \frac{1}{EA}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} -9.16 \\ -17.84 \end{bmatrix}$$

-ive sign shows that our assumed redundant action direction is wrong

Flexibility Method for Trusses Analysis

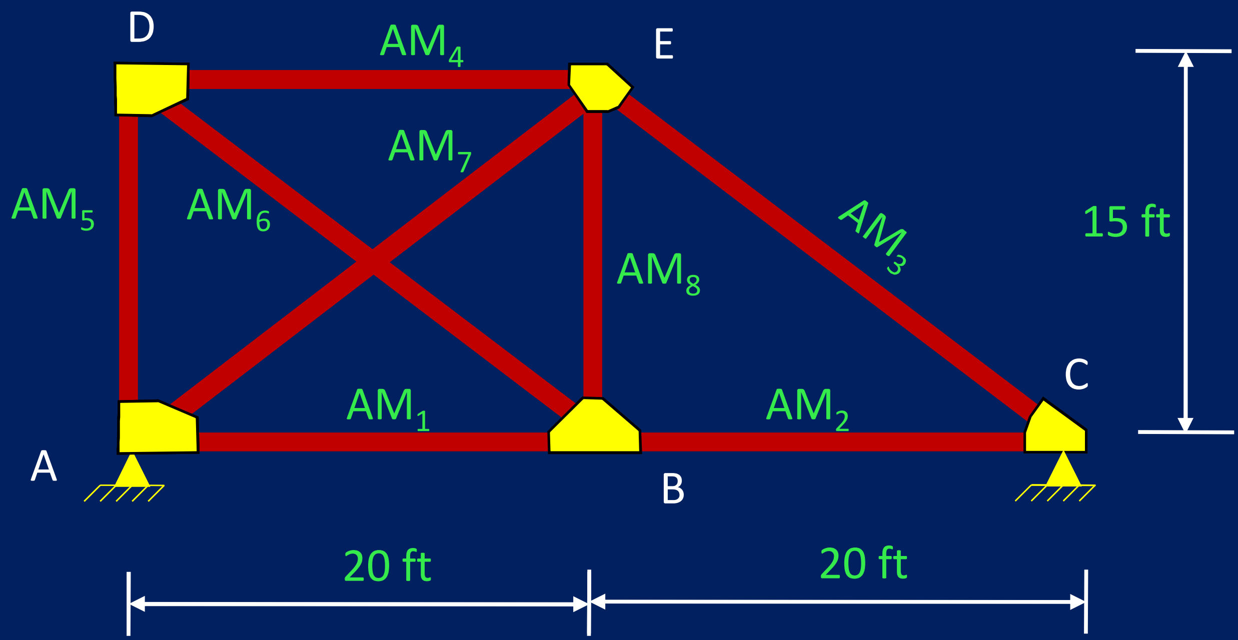


Final determinate structure

Flexibility Method for Trusses Analysis

Step # 07: Compute the member end actions. As we know that

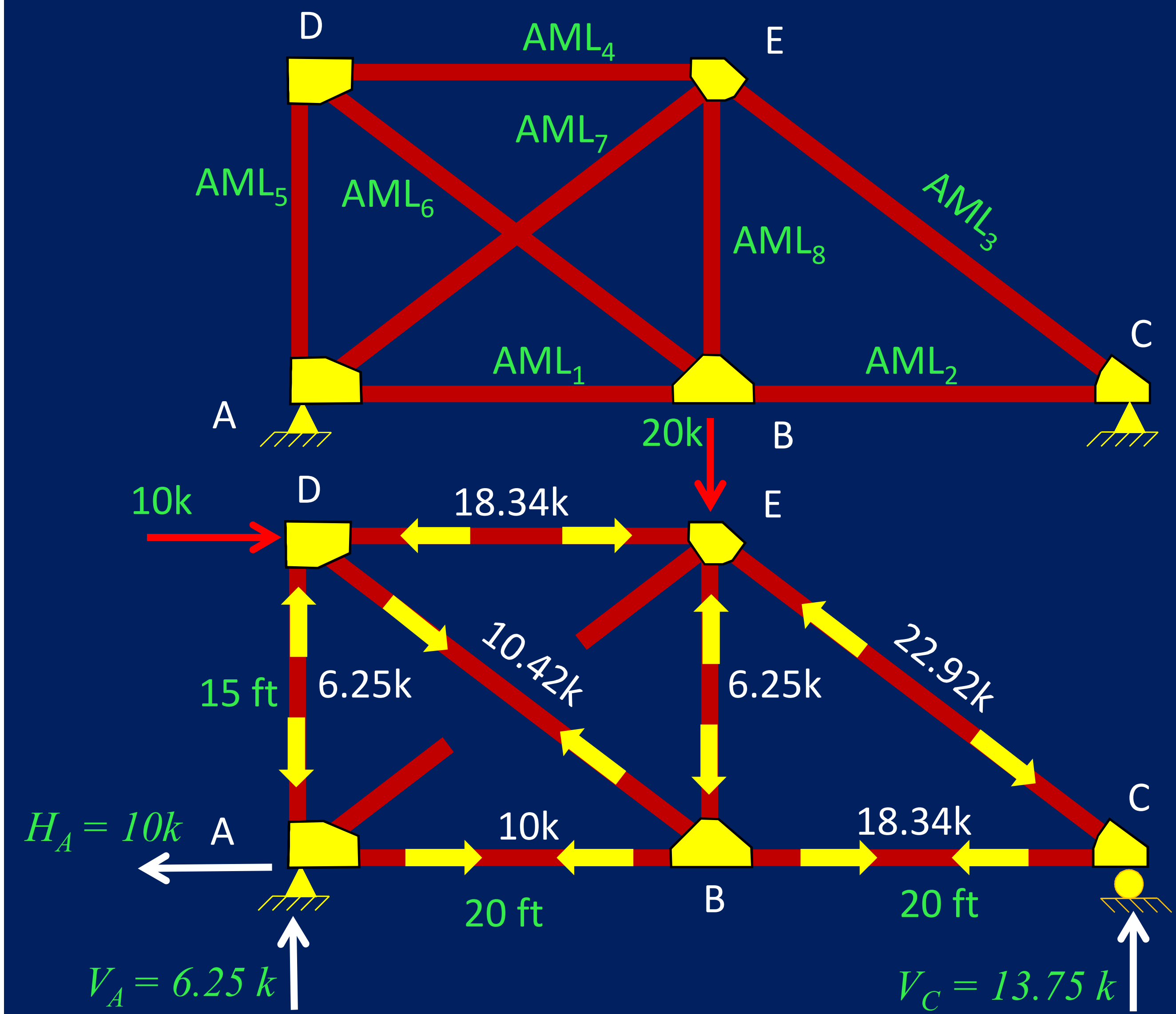
$$[AM] = [AML] + [AMR][AR]$$



$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ \vdots \\ AM_8 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ \vdots \\ ? \end{bmatrix}$$

Flexibility Method for Trusses Analysis

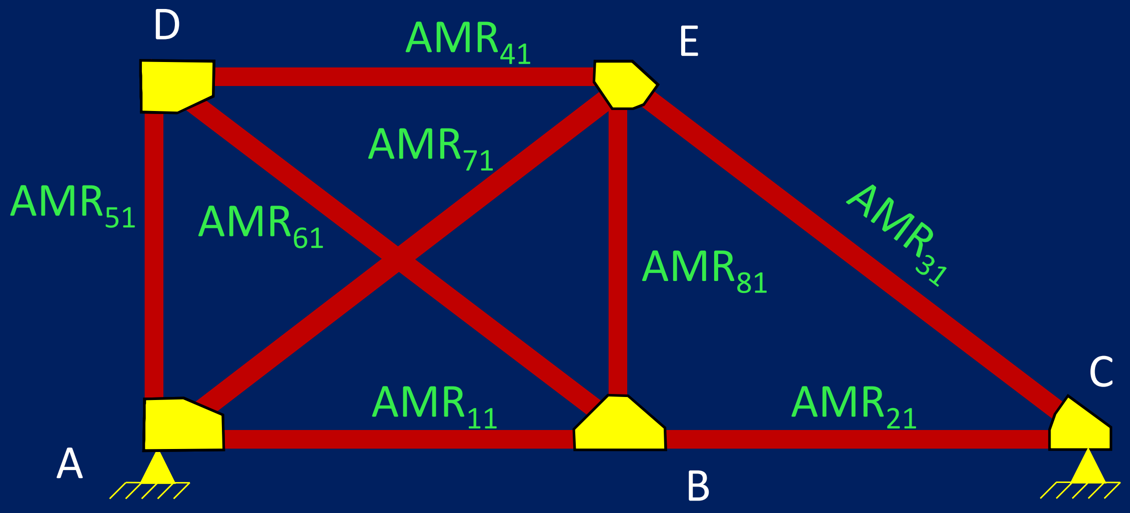
a) Compute AML values.

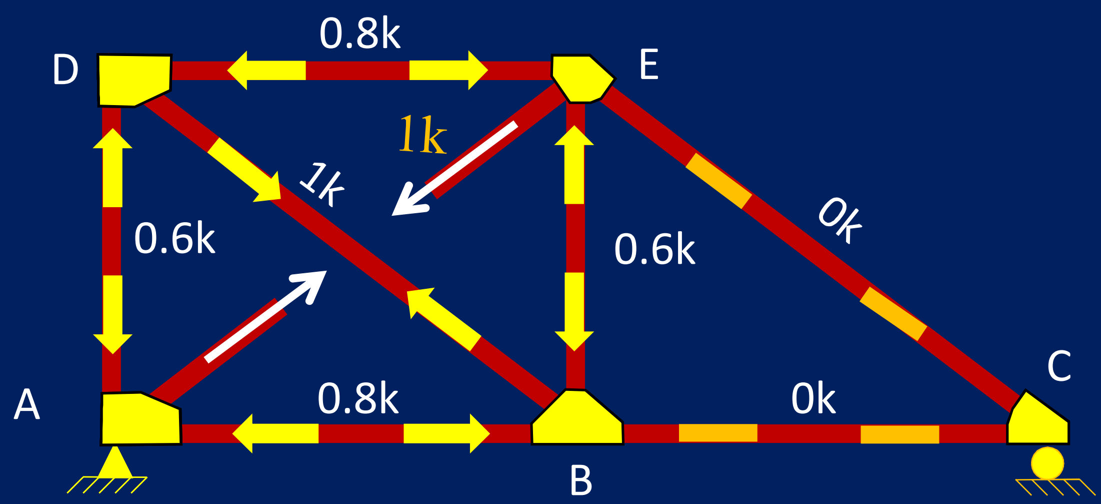


$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \\ AML_7 \\ AML_8 \end{bmatrix} = \begin{bmatrix} 10 \\ 18.34 \\ -22.92 \\ -18.34 \\ -6.25 \\ 10.42 \\ 0 \\ -6.25 \end{bmatrix}$$

Flexibility Method for Trusses Analysis

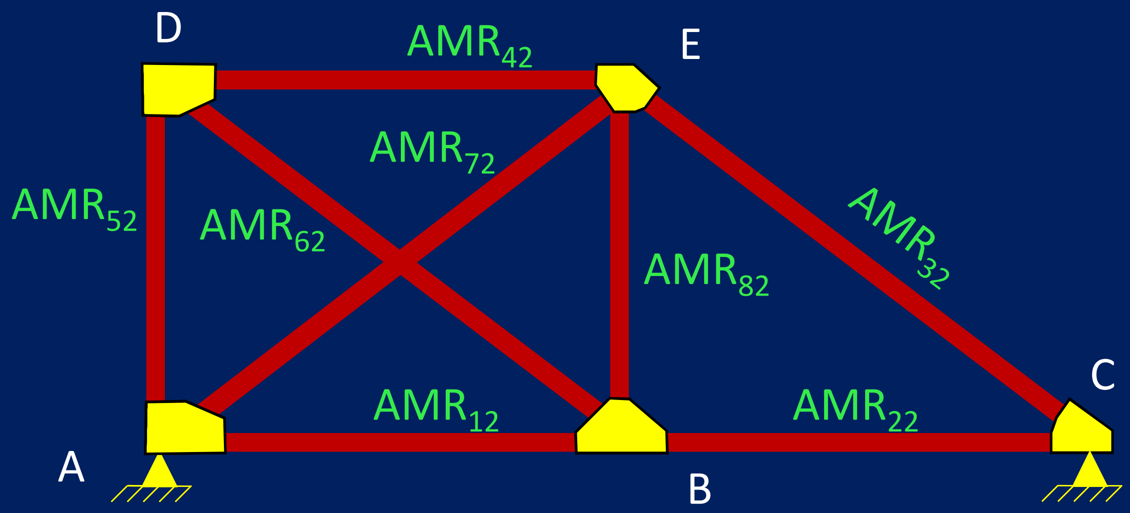
b) Compute the AMR values: when unit is applied at redundant location 1.

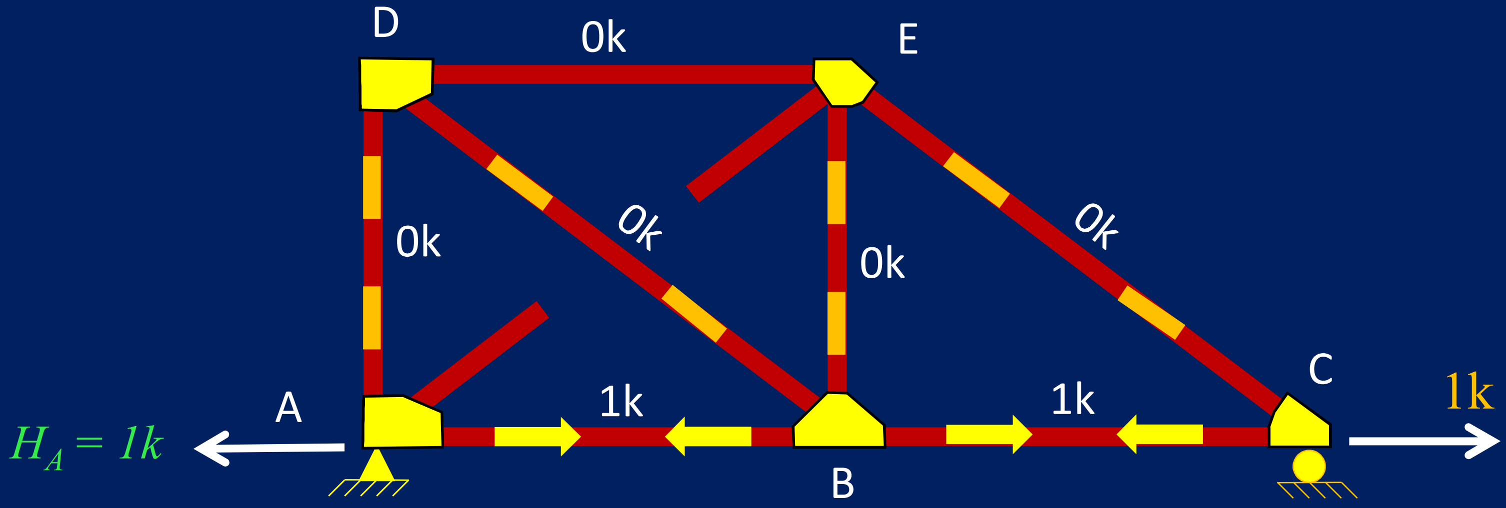




Flexibility Method for Trusses Analysis

b) Compute the AMR values: when unit is applied at redundant location 2.





Flexibility Method for Trusses Analysis

So AMR matrix

$$[AMR]_{9\,*\,2} = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \\ AMR_{71} & AMR_{72} \\ AMR_{81} & AMR_{82} \end{bmatrix} = \begin{bmatrix} -0.8 & 1 \\ 0 & 1 \\ 0 & 0 \\ -0.8 & 0 \\ -0.6 & 0 \\ 1 & 0 \\ 1 & 0 \\ -0.6 & 0 \end{bmatrix}$$

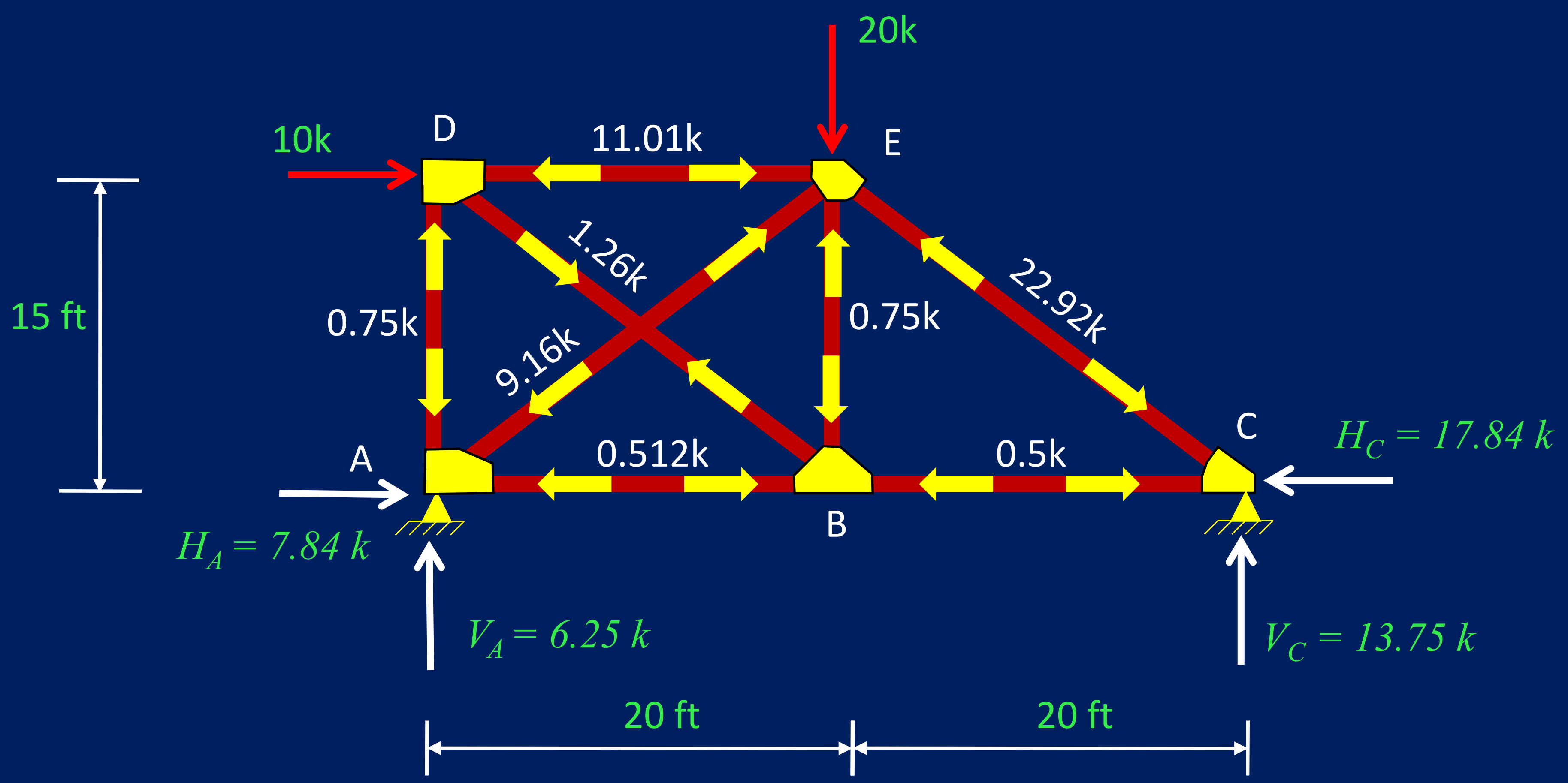
Flexibility Method for Trusses Analysis

Now member end actions

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \\ AM_7 \\ AM_8 \end{bmatrix} = \begin{bmatrix} 10 \\ 18.34 \\ -22.92 \\ -18.34 \\ -6.25 \\ 10.42 \\ 0 \\ -6.25 \end{bmatrix} + \begin{bmatrix} -0.8 & 1 \\ 0 & 1 \\ 0 & 0 \\ -0.8 & 0 \\ -0.6 & 0 \\ 1 & 0 \\ 1 & 0 \\ -0.6 & 0 \end{bmatrix} \begin{bmatrix} -9.16 \\ -17.84 \end{bmatrix} = \begin{bmatrix} -0.512 \\ 0.5 \\ -22.92 \\ -11.01 \\ -0.75 \\ 1.26 \\ -9.16 \\ -0.75 \end{bmatrix}$$

Flexibility Method for Trusses Analysis

To find the support reaction simply apply equilibrium equations and we will get

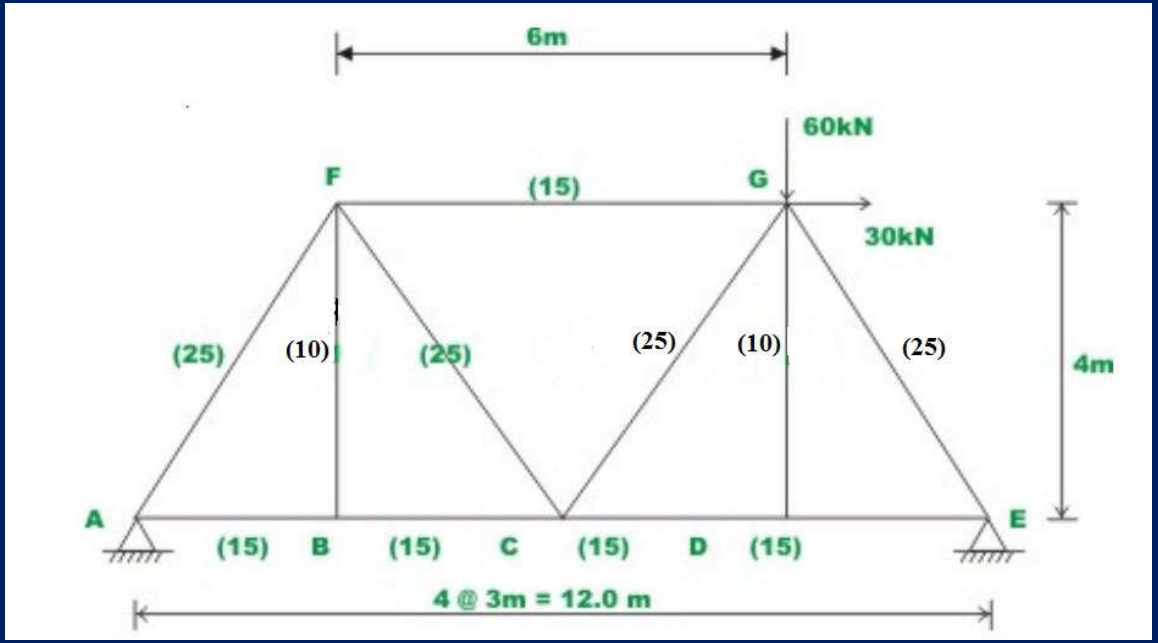


Completely analyzed structure

Flexibility Method for Trusses Analysis

Assignment # 03(a)

Problem 01: Analyze the given truss using flexibility method ,The cross sectional areas of the members in square centimeters are shown in parenthesis. Assume $E = 2 * 10^5$ N/mm^2



References

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- Matrix structural analysis by William Mc Guire
- Matrix analysis of frame structures by William Weaver
- Online Civil Engineering blogs