

Design of Rectangular Beam:-

(1)

① Design a rectangular beam for a 22 ft simple span if a dead load of 1 k/ft (not including the beam weight) and a live load of 2 k/ft are to be supported. Use $f_c' = 4000$ psi and $f_y = 60,000$ psi.

Estimating Beam Dimensions and Weight:-

① Assume Height $h = 10\%$ of span length

$$h = 0.10 \times 22$$

$$h = 2.2 \text{ ft}$$

$$h = 2.2 \times 12 = 26.4''$$

$$\text{Say } h = 27''$$

$$d = 27 - 2.5$$

$$\boxed{d = 24.5''}$$

② Assume width $b = \frac{h}{2} = \frac{27}{2} = 13.5''$

$$\boxed{b = 14''}$$

③ Beam weight $= b \times h \times \gamma_c = \left(\frac{14 \times 27}{12 \times 12} \right) \times (150) = 393.75 \frac{\text{lb}}{\text{ft}}$

$$= 394 \frac{\text{lb}}{\text{ft}} = 0.394 \text{ k/ft}$$

$$\boxed{\text{Beam wt} = 0.394 \text{ k/ft}}$$

Factored load W_u and Moment M_u :-

$$W_u = 1.2 \text{ DL} + 1.6 \text{ LL}$$

$$W_u = 1.2(1 + 0.394) + 1.6(2)$$

$$\boxed{W_u = 4.873 \text{ k/ft}}$$

Factored load =

$$\text{Factored Moment} = M_u = \frac{WuL^2}{8} = \frac{4.873 \times (22)^2}{8}$$

$$M_u = 294.8 \text{ ft-k}$$

Reinforcement:-

Assuming strength Reduction Factor $\phi = 0.90$

ρ is calculated from expression which was derived in Module #2

$$\rho = \frac{0.85 f_c'}{f_y} \left(1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}} \right) \rightarrow \textcircled{1}$$

$$R_n = \frac{M_u}{\phi b d^2} = \frac{294.8 \times 1000 \times 12}{0.90 \times 14 \times (24.5)^2}$$

$$R_n = 467.7 \text{ psi}$$

Put R_n in eq. ①

$$\rho = \frac{0.85 \times 4000}{60,000} \left[1 - \sqrt{1 - \frac{2 \times 467.7}{0.85 \times 4000}} \right]$$

$$\rho = 0.00842$$

Selecting Reinforcing:-

$$A_s = \rho b d = 0.00842 \times 14 \times 24.5$$

$$A_s = 2.89 \text{ in}^2$$

Referring to Table (A-4) — Area of Groups of standard Bars (in²) — US Customary units

$$\text{Use } 3\#9 \text{ bar } A_s = (3.00 \text{ in}^2)$$

Checking Solution:-

[ρ values from Appendix A, Table A.7]

$$\textcircled{1} \rho = \frac{A_s}{bd} = \frac{3}{14 \times 24.5} = 0.00875 > \rho_{min} = 0.0033$$

$$< \rho_{max} = 0.0181$$

∴ Section is ductile and $\phi = 0.90$ **OK**

$$\textcircled{2} a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3 \times 60}{0.85 \times 4 \times 14}$$

a = 3.78 in

$\phi M_n > M_u$

$\phi M_n = \phi A_s f_y (d - \frac{a}{2})$

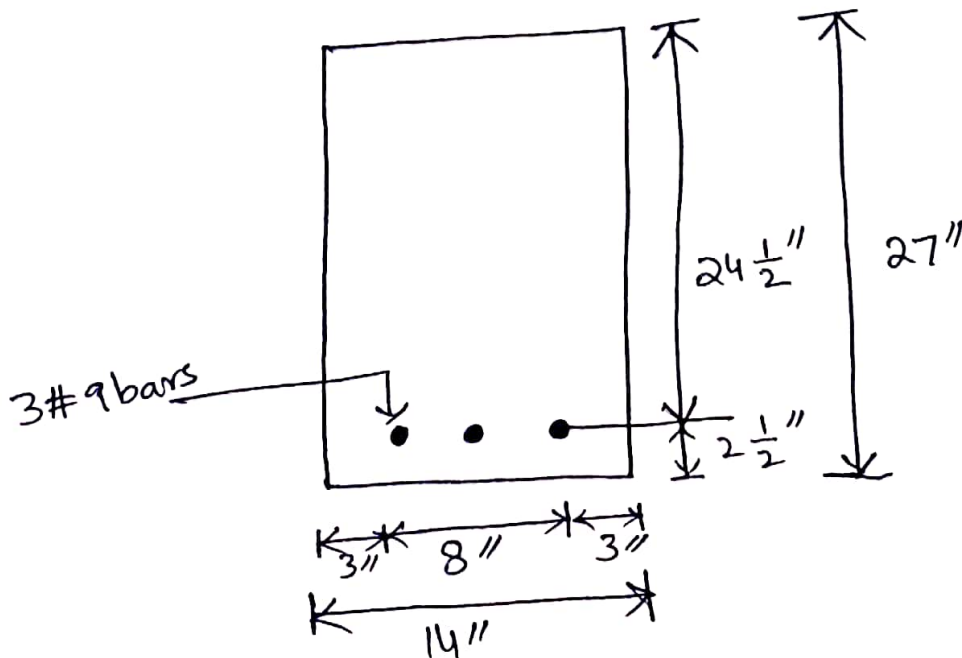
$\phi M_n = 0.90 \times 3 \times 60 (24.5 - \frac{3.78}{2})$

$\phi M_n = \frac{3662 \text{ in-k}}{12}$

$\phi M_n = 305.2 \text{ ft-k} > M_u = 294.8 \text{ ft-k}$

OK

Final Section:-



(2) A beam is to be Selected with $\rho \leq 0.0120$, $M_u = 600 \text{ ft-k}$
 $f_y = 60,000 \text{ psi}$ and $f_c' = 4000 \text{ psi}$

Solution:- Assume $\phi = 0.90$

Substituting values in equation

$$\frac{M_u}{\phi b d^2} = \rho f_y \left(1 - \frac{1}{1.7} \frac{\rho f_y}{f_c'}\right)$$

$$\frac{600 \times 1000 \times 12}{0.9 b d^2} = 0.0120 \times 60,000 \left(1 - \frac{1}{1.7} \left(\frac{0.0120 \times 60,000}{4000}\right)\right)$$

$$b d^2 = 12427 \text{ in}^3 \rightarrow \textcircled{x}$$

Now Selection of "b" and "d"

b"	d"	d/b
12"	32.18	2.68
14"	29.79	2.12
16"	27.87	1.74
18"	26.28	1.46
⋮		

Reasonable $b = 12$

$$d^2 = \frac{12427 \text{ in}^3}{b}$$

$$d = \sqrt{\frac{12427}{b}}$$

$$b = 12, d = \sqrt{\frac{12427}{12}} = 32.18''$$

$$b = 14, d = \sqrt{\frac{12427}{14}} = 29.79''$$

$$b = 16, d = \sqrt{\frac{12427}{16}} = 27.87''$$

$$b = 18, d = \sqrt{\frac{12427}{18}} = 26.28''$$

Select

$$b = 14 \quad d = 30''$$

Alternatively:- Referring to table A.13 $\left[\begin{matrix} f_y = 60,000 \text{ psi} \\ f_c' = 4000 \text{ psi} \end{matrix} \right]$

When $\rho = 0.0120 \Rightarrow \frac{M_u}{\phi b d^2} = 643.5 \text{ psi}$

$$b d^2 = \frac{M_u}{\phi 643.5}$$

$$b d^2 = \frac{600 \times 1000 \times 12}{0.90 \times 643.5}$$

$$b d^2 = 12432 \text{ in}^3 \rightarrow \textcircled{Y}$$

X and Y are approximately equal OK

So beam dimension Selected

width = $b = 14''$

depth = $d = 30''$

Reinforcement:-

$$A_s = \rho b d = 0.0120 \times 14 \times 30$$

$$A_s = 5.04 \text{ in}^2$$

Use 4 # 10 ($A_s = 5.06 \text{ in}^2$)

Checking Solution:-

$$\textcircled{1} \rho = \frac{A_s}{b d} = \frac{5.06}{14 \times 30}$$

$$\rho = 0.01205 > \rho_{\min} = 0.0033$$

$$< \rho_{\max} = 0.0181$$

$$\textcircled{2} a = \frac{A_s f_y}{0.85 f_c' b} = \frac{5.06 \times 60}{0.85 \times 4 \times 14}$$

$$a = 6.38''$$

$$\phi M_n > M_u$$

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2} \right)$$

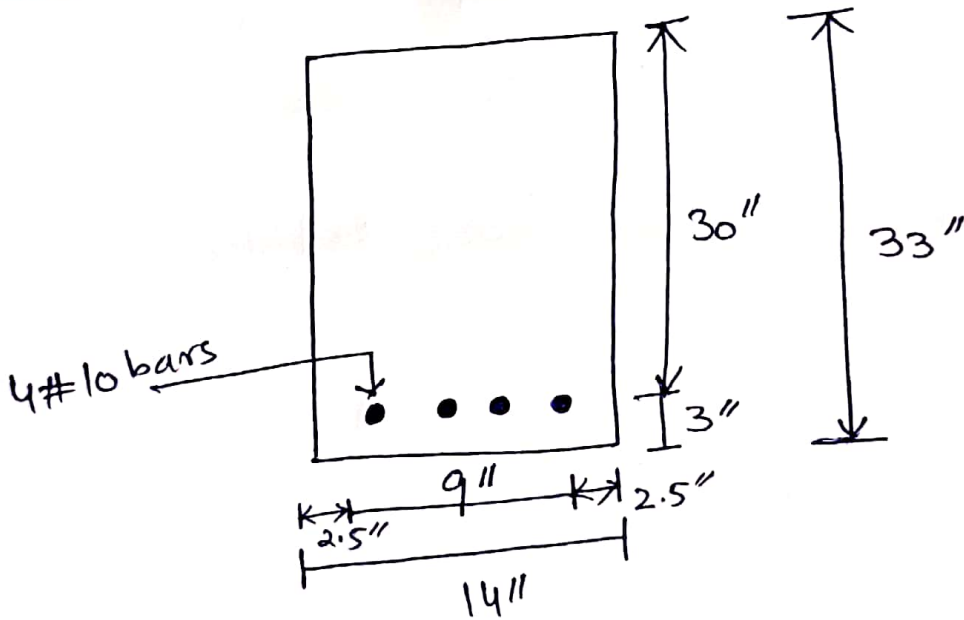
$$\phi M_n = 0.90 \times 5.06 \times 60 \left(30 - \frac{6.38}{2} \right)$$

$$\phi M_n = \frac{7325.56 \text{ in-k}}{12}$$

$$\phi M_n = 610.47 \text{ ft-k} > M_u = 600 \text{ ft-k}$$

OK

Final Section:-



(7)

③ A rectangular beam is to be sized with $f_y = 60,000 \text{ psi}$ and $f_c' = 3000 \text{ psi}$ and ρ approximately equal to $0.18 \frac{f_c'}{f_y}$. It is to have a 25' simple span and to support a dead load, in addition to its own weight equal to 2 k/ft and a live load equal to 3 k/ft

Solution:-

① Assume Beam weight = 400 lb/ft = 0.400 k/ft

$$W_u = 1.2 \text{ DL} + 1.6 \text{ L.L}$$

$$W_u = 1.2 (2 + 0.40) + 1.6 (3)$$

$$\boxed{W_u = 7.68 \text{ k/ft}}$$

$$M_u = \frac{W_u L^2}{8} = \frac{7.68 \times (25)^2}{8}$$

$$\boxed{M_u = 600 \text{ ft-k}}$$

$$\rho = 0.18 \frac{f_c'}{f_y} = \frac{0.18 \times 3}{60}$$

$$\boxed{\rho = 0.009}$$

$$\frac{M_u}{\phi b d^2} = 482.6 \text{ psi}$$

[from Table A.12)

$$b d^2 = \frac{M_u}{\phi 482.6} = \frac{600 \times 1000 \times 12}{0.9 \times 482.6}$$

$$\boxed{b d^2 = 16577 \text{ in}^3}$$

$$d = \sqrt{\frac{16577}{b}}$$

b"	d"
16	32.19
18	30.35 ← selected
20	28.79

$$d = \sqrt{\frac{16577}{16}} = 32.19$$

$$d = \sqrt{\frac{16577}{18}} = 30.35$$

$$d = \sqrt{\frac{16577}{20}} = 28.79$$

b = 18" d = 30.35"

Try 18" x 33" (d = 30.50")

Beam weight = $\frac{18 \times 33}{12 \times 12} \times 150 = 619 \text{ lb/ft}$
 > 400 lb/ft

Not Good

② Now Assuming Beam weight = 650 lb/ft

$$W_u = 1.2DL + 1.6LL$$

$$W_u = 1.2(2 + 0.650) + 1.6(3)$$

$$W_u = 7.98 \text{ k/ft}$$

$$M_u = \frac{W_u l^2}{8} = \frac{7.98 \times 25^2}{8}$$

$$M_u = 623.4 \text{ ft-k}$$

For $\rho = 0.009$ $\frac{M_u}{\phi b d^2} = 482.6 \text{ psi}$
 (from Table A.12)

$$b d^2 = \frac{M_u}{\phi 482.6} = \frac{623 \times 1000 \times 12}{0.9 \times 482.6}$$

$$b d^2 = 17223 \text{ in}^3$$

b"	d"
16	32.81
18	30.93
20	29.35

$$d = \sqrt{\frac{17223}{16}} = 32.81$$

$$d = \sqrt{\frac{17223}{18}} = 30.93$$

$$d = \sqrt{\frac{17223}{20}} = 29.35$$

Try 18" x 34" beam (d = 31 in)

$$\text{Beam wt} = \frac{18 \times 34}{12 \times 12} \times (150) = 637.5 \text{ lb/ft} < 650 \text{ lb/ft} \quad \boxed{\text{OK}}$$

$$A_s = \rho b d = 0.009 \times 18 \times 31$$

$$\boxed{A_s = 5.02 \text{ in}^2}$$

use 5 # 9 bar (Table A.5)

$$\boxed{A_s = 5.00 \text{ in}^2}$$

Note:- Normally a bar selection should exceed the theoretical value of A_s . In this case, the area chosen was less than but very close to the theoretical area and it will be checked to be sure it has enough capacity.

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{5 \times 60}{0.85 \times 3 \times 18}$$

$$\boxed{a = 6.54 \text{ in}}$$

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2} \right) = 0.90 \times 5 \times 60 \left(31 - \frac{6.54}{2} \right)$$

$$\phi M_n = \frac{7487.6 \text{ in lb}}{12}$$

$$\phi M_n = 623.97 \text{ ft-k} > M_u = 623.47 \text{ ft-k} \quad \boxed{\text{OK}}$$

The reason a beam with less reinforcing steel than calculated is acceptable is that a value of d exceeding the theoretical value was selected ($d = 31 \text{ in} > 30.93 \text{ in}$) - whenever the value of b and d selected results in a bd^2 that exceeds the calculated value based on the assumed f , the actual value of f will be lower than the assumed value.

If a value of $b = 18''$ and $d = 30''$ had been selected, the result would have been that the actual value of f would be greater than the assumed value of 0.009 . Using the actual values of b and d to recalculate f .

$$\frac{M_u}{\phi b d^2} = \frac{623.4 \times 1000 \times 12}{0.9 \times 18 \times (30)^2} = 513.1 \text{ psi}$$

Referring to Appendix A Table A.12 $f = 0.00965$ which exceed $f = 0.009$ -

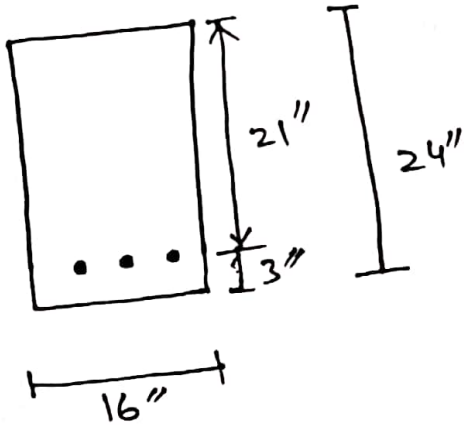
$$A_s = f b d = 0.00965 \times 18 \times 30$$

$$\boxed{A_s = 5.21 \text{ in}^2}$$

use 7#8 bars, ($A_s = 5.50 \text{ in}^2$)

Either design is acceptable - This kind of flexibility is sometimes perplexing to the student who simply wants to know the right answer. One of the best features of reinforced concrete is that there is so much flexibility in the choices that can be made.

- ④ The dimensions of the beam shown in figure have been selected for architectural reasons. Determine the reinforcing steel Area by each trial and Error Method.



$$M_u = 160 \text{ ft-k}$$

$$f_c' = 3000 \text{ psi}$$

$$f_y = 60,000 \text{ psi}$$

Solution:-

$$\textcircled{1} \frac{M_u}{\phi b d^2} = \frac{160 \times 1000 \times 12}{0.9 \times 16 \times (21)^2} = 302.3 \text{ psi}$$

$$f. \text{ From Table A.12} \quad \rho = 0.0054$$

$$A_s = \rho b d = 0.0054 \times 16 \times 21$$

$$\boxed{A_s = 1.81 \text{ in}^2}$$

$$\textcircled{2} R_n = \frac{M_u}{\phi b d^2} = 302.3$$

$$\rho = \frac{0.85 f_c'}{f_y} \left(1 - \sqrt{1 - \frac{2 R_n}{0.85 f_c'}} \right)$$

$$\rho = \frac{0.85 \times 3000}{60000} \left(1 - \sqrt{1 - \frac{2 \times 302.3}{0.85 \times 3000}} \right)$$

$$\rho = 0.00538$$

Trial and Error (Iterative Method)

Assume $a = 2''$

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2} \right)} = \frac{160 \times 1000 \times 12}{0.9 \times 60000 \left(21 - \frac{2}{2} \right)} = 1.78 \text{ in}^2$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1.78 \times 60000}{0.85 \times 3000 \times 16} = 2.62''$$

Assume $a = 2.62''$

$$A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{160 \times 1000 \times 12}{0.9 \times 60000 (21 - \frac{2.62}{2})}$$

$$A_s = 1.81 \text{ in}^2$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1.81 \times 60000}{0.85 \times 3000 \times 16}$$

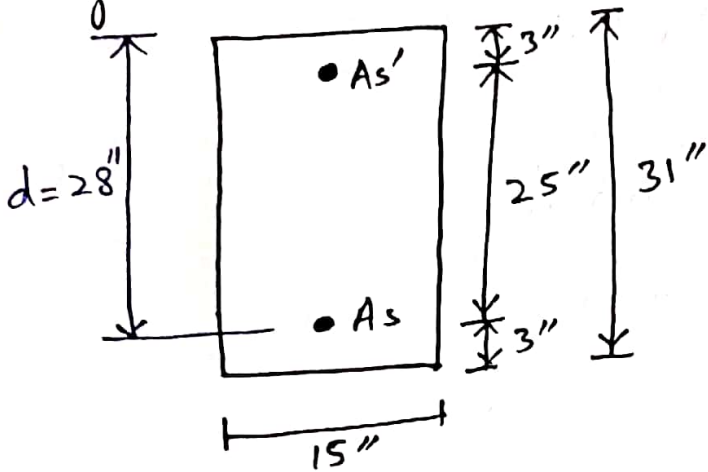
$$a = 2.66''$$

Close enough to previous (a) if not repeat the same process.

Based on this Method use a theoretical value of $A_s = 1.81 \text{ in}^2$

Design of Doubly Reinforced Beams:-

① Design a rectangular beam for $M_D = 325 \text{ ft-k}$ and $M_L = 400 \text{ ft-k}$ - if $f_c' = 4000 \text{ psi}$ and $f_y = 60,000 \text{ psi}$ - the maximum permissible beam dimensions are shown in Figure.



Solution:-

① Factored Moment:-

$$M_u = 1.2 M_D + 1.6 M_L$$

$$M_u = 1.2 \times 325 + 1.6 \times 400$$

$$M_u = 1030 \text{ ft-k}$$

② Nominal Moment "Mn" :-

$$M_n = \frac{M_u}{\phi} = \frac{1030}{0.90}$$

$$M_n = 1144.4 \text{ ft-k}$$

Assuming maximum possible tensile steel with no compression steel and computing beam's nominal strength moment

$$\rho_{\max} (\text{from Appendix A, Table A.7}) = 0.0181$$

$$A_{s1} = \rho_{\max} b d = 0.0181 \times 15 \times 28$$

$$A_{s1} = 7.60 \text{ in}^2$$

For $\rho_{\max} = 0.0181$ $\frac{M_u}{\phi b d^2} = 912 \text{ psi}$ (from Table A.13)

$$M_{u1} = 912 \times \phi b d^2 = 912 \times 0.9 \times 15 \times (28)^2$$

$$M_{u1} = \frac{9652608}{12} \text{ in-lb} \Rightarrow M_{u1} = 804.4 \text{ ft-k}$$

$$M_{n1} = \frac{M_{u1}}{\phi} = \frac{804.4}{0.90}$$

$$M_{n1} = 893.8 \text{ ft-k}$$

$$M_{n2} = M_n - M_{n1} = 1144.4 - 893.8$$

$$M_{n2} = 250.6 \text{ ft-k}$$

③ Theoretical A_s' required:-

$$A_s' = \frac{M_{n2}}{f_y(d-d')} = \frac{250.6 \times 12}{60(28-3)}$$

Try 2#9 (2.00 in²)

$$A_s' = 2.00 \text{ in}^2$$

$$A_s' f_s' = A_{s2} f_y$$

$$A_{s2} = \frac{A_s' f_s'}{f_y} = \frac{2 \times 60}{60} = 2.00 \text{ in}^2$$

$$A_{s2} = 2 \text{ in}^2$$

$$A_s = A_{s1} + A_{s2}$$

$$A_s = 7.60 + 2$$

$$A_s = 9.60 \text{ in}^2$$

Try 8#10 (10.12 in²)

Note that the actual value of A_s' is exactly the same as the theoretical value.
 The Actual value of A_s however is higher than the theoretical value by $10.12 - 9.6 = 0.52 \text{ in}^2$.

If new bar selection for A_s' is made whereby the actual value of A_s' exceeds the theoretical value by about this much (0.52 in^2), the design will be adequate.

Select 3 # 8 bars ($A_s' = 2.36 \text{ in}^2$) and repeat the previous steps.

Assuming $f_s' = f_y$

$$\textcircled{1} \frac{(A_s - A_s') f_y}{0.85 f_c' b \beta_1} = \frac{(10.12 - 2.36) \times 60}{0.85 \times 4 \times 15 \times 0.85} = 10.74 \text{ in}$$

$$\textcircled{2} \epsilon_s' = \left(\frac{c - d'}{c} \right) (0.003) = \left(\frac{10.74 - 3}{10.74} \right) (0.003) = 0.00216 > \epsilon_y$$

$$\textcircled{3} \epsilon_t = \left(\frac{d - c}{c} \right) (0.003) = \left(\frac{28 - 10.74}{10.74} \right) (0.003) = 0.00482 < 0.005$$

$$\beta \neq 0.90$$

$$\phi = 0.65 + (\epsilon_t - 0.002) \frac{250}{3}$$

$$\phi = 0.65 + (0.00482 - 0.002) \frac{250}{3}$$

$$\boxed{\phi = 0.88}$$

$$A_{s2} = \frac{A_s' f_s'}{f_y} = \frac{2.36 \times 60}{60} = 2.36 \text{ in}^2$$

$$A_{s1} = A_s - A_{s2} = 10.12 - 2.36 = 7.76 \text{ in}^2$$

$$M_{n1} = A_s1 f_y \left(d - \frac{a}{2} \right) = 7.76 \times 60 \left[28 - \frac{0.85 \times 10.74}{2} \right]$$

$$M_{n1} = \frac{10912}{12} \text{ in-k}$$

$$M_{n1} = 909.3 \text{ ft-k}$$

$$M_{n2} = A_s2 f_y (d - d') = (2.36)(60)(28 - 3)$$

$$M_{n2} = \frac{3540}{12} \text{ in-k}$$

$$M_{n2} = 295 \text{ ft-k}$$

$$M_n = M_{n1} + M_{n2} = 909.3 + 295$$

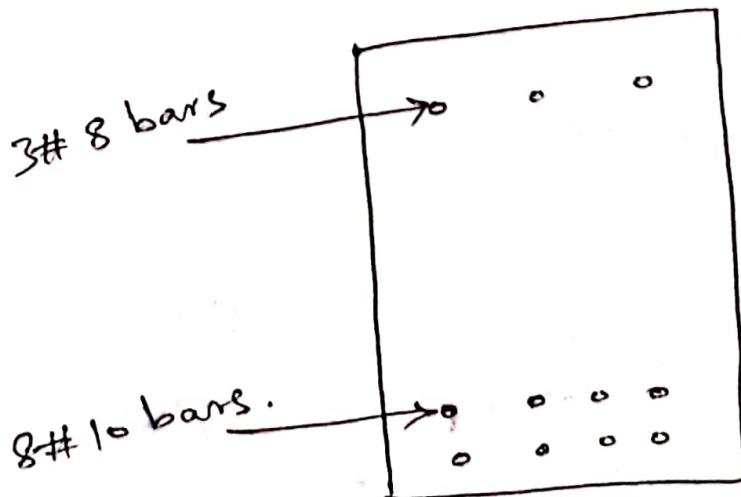
$$M_n = 1204.3 \text{ ft-k}$$

$$\phi M_n = 0.88 \times 1204.3$$

$$\phi M_n = 1059.9 \text{ ft-k} > M_u \quad \boxed{\text{OK}}$$

$$A_s' = 2.36 \text{ in}^2 \quad (3 \# 8 \text{ bars})$$

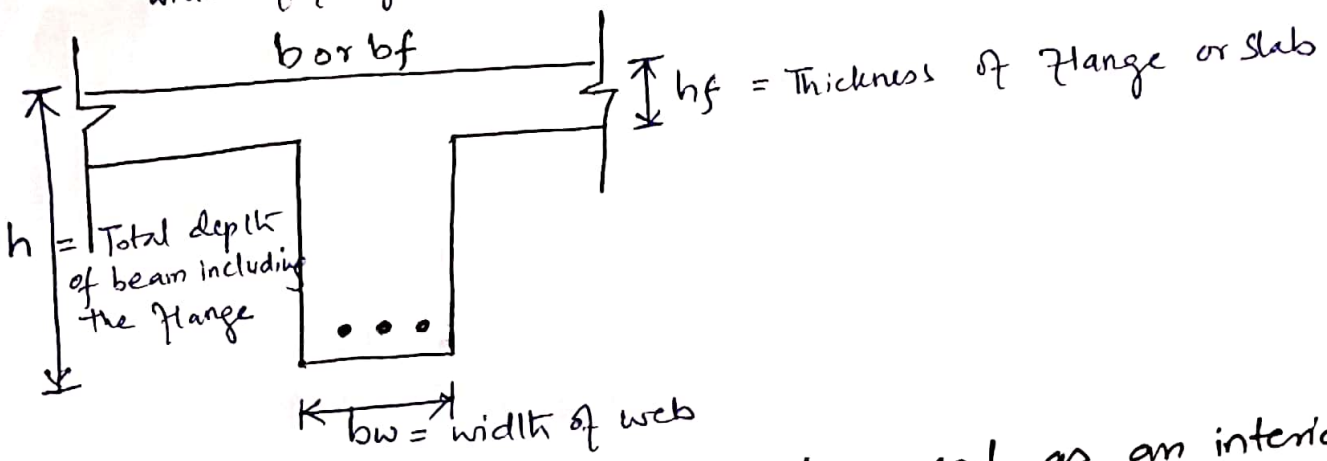
$$A_s = 10.12 \text{ in}^2 \quad (8 \# 10 \text{ bars})$$



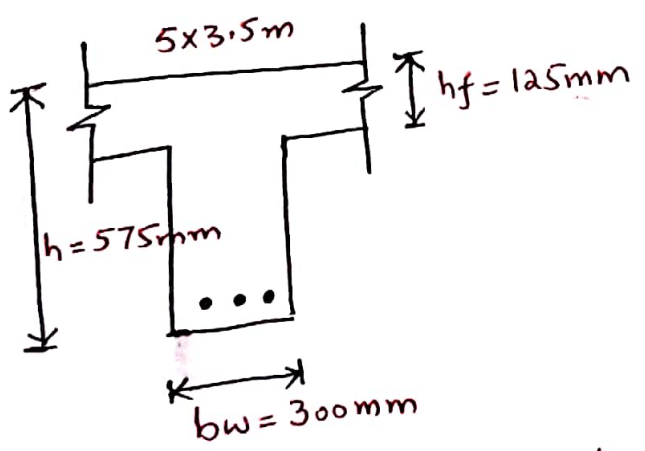
Design of T beam:-

①

width of flange or portion of slab acting on T beam .



① Design a T beam Section can be used as an interior simply supported beam of span 5m - The slab panels on both sides of the beam are 5m x 3.5m. Factored load is to 15 kN/m². $f_c' = 17.25 \text{ MPa}$, $f_y = 420 \text{ MPa}$, $h_f = 125 \text{ mm}$, $b_w = 300 \text{ mm}$ and $h = 575 \text{ mm}$.
 ① Design the beam for the calculated loading.



$l_y = 5 \text{ m}$
 $l_x = 3.5 \text{ m}$
 Factored slab load = 15 kN/m²
 $h = 575 \text{ mm}$
 $b_w = 300 \text{ mm}$
 $h_f = 125 \text{ mm}$
 $f_c' = 17.25 \text{ MPa}$
 $F_y = 420 \text{ MPa}$

① Factored self weight of beam:- $F_s = 1.2 b_w (h - h_f) \times \text{unit of concrete}$

$$F_s = 1.2 b_w (h - h_f) \times 2400 \frac{\text{kg}}{\text{m}^3} \times 0.00981$$

$$F_s = 1.2 \times \frac{300}{1000} \frac{(575 - 125)}{1000} \times 2400 \times 0.00981$$

$$\left[\frac{1 \text{ kg}}{\text{m}^3} = 0.00981 \frac{\text{kN}}{\text{m}^3} \right]$$

$$F_s = 3.82 \text{ kN/m}$$

② width of slab supported by the beam:-

$$b_s = \left[1 - \frac{R^2}{3} \right] l_x = \left[1 - \frac{0.7^2}{3} \right] 3.5$$

$$b_s = 2.93 \text{ m}$$

$$R = \frac{l_x}{l_y}$$

$$R = \frac{3.5}{5}$$

$$R = 0.7$$

③ Factored Slab load:-

Factored slab load = width of Slab supported by beam x Factored Slab load

$$F_1 = b_s \times 15$$

$$F_1 = 2.93 \times 15$$

$$F_1 = 43.95 \text{ kN/m}$$

④ Total Factored load:-

$$F_T = F_1 + F_s$$

$$F_T = 43.95 + 3.83$$

$$F_T = 47.77 \text{ kN/m} = W_u$$

⑤ Moment:-

$$M_u = \frac{W_u l^2}{8} = \frac{47.77 \times 5^2}{8}$$

$$M_u = 149.3 \text{ kN-m}$$

⑥ Effective Slab width "b"

The Smallest of the following:-

① $\frac{l_y}{4} = \frac{5000}{4} = 1250 \text{ mm} \rightarrow \textcircled{1}$

② $16h_f + b_w = 16 \times (125) + 300 = 2300 \text{ mm} \rightarrow \textcircled{2}$

③ $b_w + [0.5(l_x - b_w)] \times 2 = 300 + [0.5 \times (3500 - 300)] \times 2 = 3500 \text{ mm} \rightarrow \textcircled{3}$

Smallest of ① ② and ③ .

$$b = 1250 \text{ mm}$$

⑦ Steel Reinforcement A_s by Trial and Error Method:-

Assuming

$$a = \beta h_f$$

$$a = 0.85 \times 125$$

$$a = 106.3 \text{ mm}$$

$$A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{149.3 \times 1000 \times 1000}{0.9 \times 420 (500 - \frac{10.6 \cdot 3}{2})}$$

$$\begin{matrix} d = 575 - 75 \\ \boxed{d = 500 \text{ mm}} \end{matrix}$$

$$\boxed{A_s = 884 \text{ mm}^2}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{884 \times 420}{0.85 \times 17.25 \times 1250}$$

$$\boxed{a = 20.33 \text{ mm}}$$

$$c = \frac{a}{\beta_1} = \frac{20.33}{0.85}$$

$$\boxed{c = 23.9 \text{ mm}} < h_f \quad [\text{N.A. lies within the flange}] \quad \boxed{\text{OK}}$$

$$\rightarrow a = 20.33 \text{ mm}$$

$$A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{149.3 \times 10^6}{0.9 \times 420 (500 - \frac{20.3}{2})} = \boxed{A_s = 806 \text{ mm}^2}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{806 \times 420}{0.85 \times 17.25 \times 1250} \Rightarrow \boxed{a = 18.5 \text{ mm}}$$

$$a_l = \beta \times \frac{3}{8} d = 0.85 \times \frac{3}{8} \times 500$$

$$\boxed{a_l = 159.4 \text{ mm}}$$

Check $a < a_l$ The limiting tensile strain is produced in the steel $\phi_b = 0.9$ OK

Check for ρ :-

$$\rho = \frac{A_s}{b w d} = \frac{806}{300 \times 500}$$

$$\boxed{\rho = 0.0053}$$

$$\rho_{\min} = \frac{1.4}{f_y} = \frac{1.4}{420}$$

$$\boxed{\rho_{\min} = 0.00333}$$

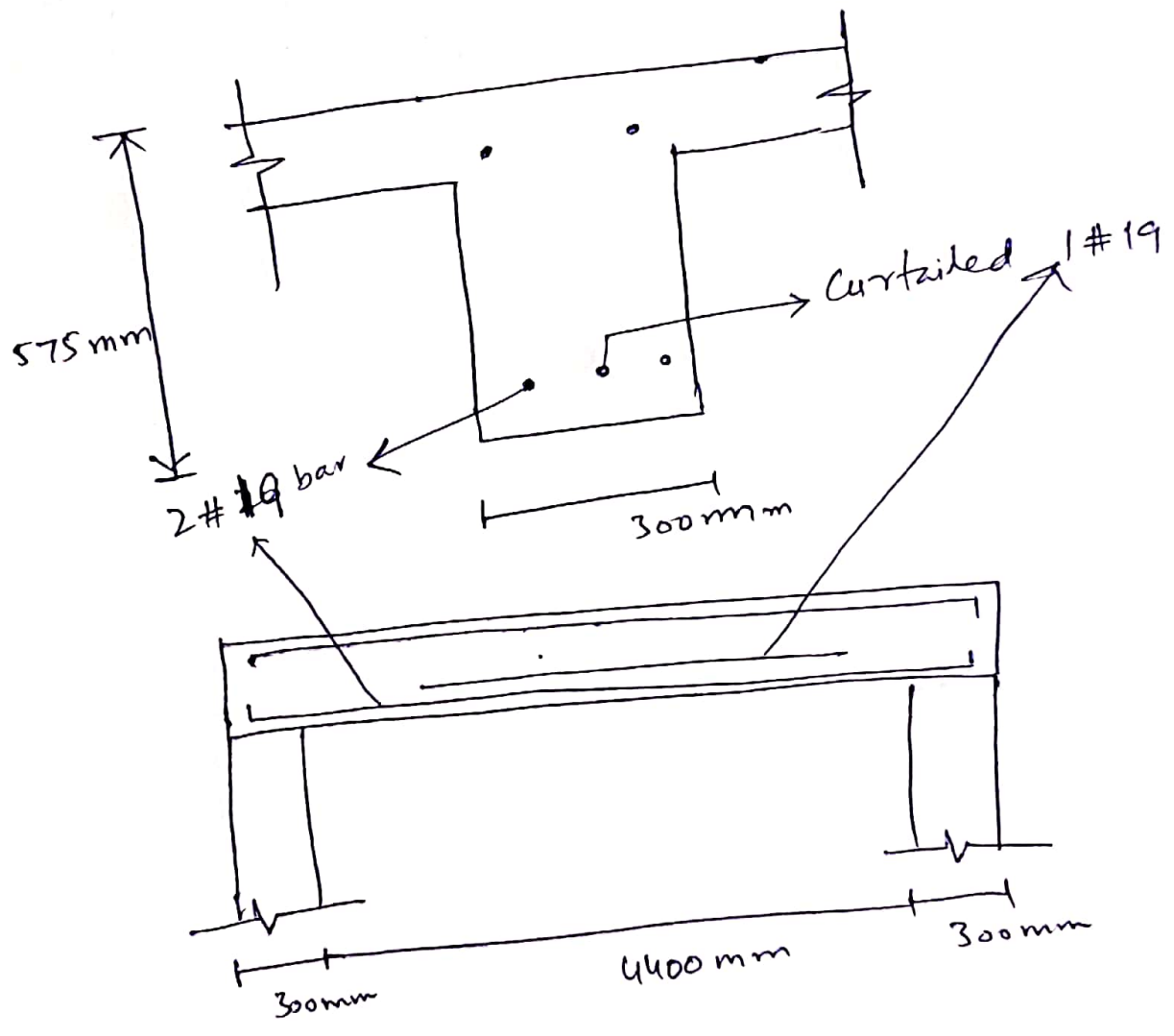
$$\rho > \rho_{\min} \quad \boxed{\text{OK}}$$

Reinforcement:-

$$A_s = 806 \text{ mm}^2$$

Referring to Table B.4 (Area of Groups of Standard Metric Bars (mm²).

3 # 19 bar ($A_s = 852 \text{ mm}^2$).



Case (ii):- Design the beam if the factored positive moment is 800 kN-m

$$M_u = 800 \text{ kN-m}$$

Assuming $a = \beta_1 h_f$

$$a = 0.85 \times 125$$

$$a = 106.3 \text{ mm}$$

Trial 1:-

$$A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{800 \times 10^6}{0.9 \times 420 (500 - \frac{106.3}{2})}$$

$$A_s = 4736 \text{ mm}^2$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{4736}{0.85 \times 17.25 \times 1250}$$

$$a = 108.5 \text{ mm}$$

$$c = a / \beta_1 = \frac{108.5}{0.85} \Rightarrow c = 128 \text{ mm}$$

$$c = 128 \text{ mm} > h_f = 125 \text{ mm}$$

N.A lies outside the flange

Extra steel will be added

Area of steel of flange:-

$$A_{sf} = 0.85 \beta_1 h_f \frac{f_c'}{f_y} (b - b_w) = 0.85 \times 0.85 \times 125 \times \frac{17.25}{420} (1250 - 300)$$

$$A_{sf} = 3524 \text{ mm}^2$$

Moment at flange:-

$$M_f = \phi M_{sf}$$

$$M_f = 0.9 \times A_{sf} \cdot f_y (d - \frac{a}{2})$$

$$a = \beta_1 h_f$$

$$M_f = 0.9 \times 3524 \times 420 (500 - \frac{0.85 \times 125}{2}) / 10^6$$

$$M_f = 595.3 \text{ kN-m}$$

Moment at web:-

$$M_u = M_w + M_f \Rightarrow$$

$$M_w = M_u - M_f$$

$$M_w = 800 - 595.3$$

$$M_u = 204.7 \text{ KN}\cdot\text{m}$$

Area of steel of web:-

$$a = \beta_1 h_f = 0.85 \times 125 = 106.3 \text{ mm}$$

$$\text{Trial } A_{sw} = \frac{M_w}{\phi f_y (d - \frac{a}{2})} = \frac{204.7 \times 10^6}{0.9 \times 420 (500 - \frac{106.3}{2})} = 1212 \text{ mm}^2$$

$$a = \frac{A_{sw} f_y}{0.85 f_c' b w} = \frac{1212 \times 420}{0.85 \times 17.25 \times 300} = 115.7 \text{ mm}$$

$$\text{Again } A_{sw} = \frac{M_u}{\phi b f_y (d - \frac{a}{2})} = \frac{204.7 \times 10^6}{0.9 \times 420 (500 - \frac{115.7}{2})} = 1225 \text{ mm}^2$$

$$a = \frac{A_{sw} f_y}{0.85 f_c' b w} = \frac{1225 \times 420}{0.85 \times 17.25 \times 300} = 117 \text{ mm}$$

$$A_{sw} = \frac{M_w}{\phi f_y (d - \frac{a}{2})} = \frac{204.7 \times 10^6}{0.9 \times 420 (500 - \frac{117}{2})} = 1227 \text{ mm}^2$$

Total Area of Steel = Area of steel of Flange + web

$$A_s = A_{sf} + A_{sw}$$

$$A_s = 3524 + 1227$$

$$A_s = 4751 \text{ mm}^2$$

Referring to Table B.4

use 10# 25 bar

$$A_s = 5100 \text{ mm}^2$$

