University of Engineering & Technology Peshawar, Pakistan

CE301: Structure Analysis II

Module 03:

Analysis of S.I Frames Using Flexibility method

By:
Prof. Dr. Bashir Alam
Civil Engineering Department
UET , Peshawar

Topics to be Covered

- Introduction
- Prerequisites for using flexibility method for Frames
- Revision of Unit Load Method
- Flexibility method procedure for frame analysis
- Analysis of frame Example 1
- Example 2
- Example 3
- Assignment

Flexibility Method for Frames Analysis

❑ Introduction:

Frames are analyzed with flexibility method due to

- To solve the problem in matrix notation, which is more systematic
- To compute reactions at all the supports.
- To compute internal resisting axial, shear & bending moment at any section of the indeterminate Frame.

Flexibility Method for Frames Analysis

The force method of analysis can readily be employed to analyze the indeterminate frames. The basic steps in the analysis of indeterminate frame by force method are the same as that discussed in the analysis of indeterminate beams in the previous lessons. Under the action of external loads, the frames undergo axial and bending deformations. Since the axial rigidity of the members is much higher than the bending rigidity, the axial deformations are much smaller than the bending deformations and are normally not considered in the analysis. The compatibility equations for the frame are written with respect to bending deformations only.

Flexibility Method for Frames Analysis

- **Prerequisites for Analysis with Flexibility method:**

  It is necessary that students must have strong background of the following concepts before starting analysis with flexibility or any other matrix method.

- Enough concept of Matrix Algebra
- Must be able to find the Statical Indeterminacy
- Concept of Deflection methods (Unit load method for displacement)

Flexibility Method for Frames Analysis

❑ Unit Load Method for Displacement

- This Method is based on the principle of virtual work.
- Best suitable to find the slope & deflection of primary structure when subjected to applied loads .
- Displacement due to external loads at any point *i* is given by

$$\Delta_n = \sum_{i=1}^{m} \frac{M_i\, mni}{E_i I_i}$$

Flexibility Method for Frames Analysis

- Displacement at any point *i* due to a unit load at j is given by (also called flexibility coefficient)

- Where M are the equations of bending moment in primary structure subjected to external loads & m are the equations of BM subjected to unit loads.

$$\delta_{ij} = \sum_{i=1}^{n} \frac{m_i \, mj}{E_i I_i} dx$$

Flexibility Method for Frames Analysis

❑ Analysis Procedure

The following steps should be followed to analyze a statical indeterminate frame using flexibility method.

Start with finding the degree of statical indeterminacy S.I .

- **Step # 01:** Identify the redundants and obtain BDS also compute [DRS] values.

  No. of redundants = D.S.I

Assume # of redundants = 2
Just to understand the procedure

Note: for choosing the redundant actions there may be more than one possible options but choose that one as redundant which makes the calculation handy.
DRS is initial support rotation or settlement corresponding to redundant locations

Flexibility Method for Frames Analysis

- **Step # 02:** Analyze the BDS (primary structure) under the following loading conditions

i. Analyze the BDS when acted upon by the external loads and write the equations of bending moment M for different segments.

ii. Analyze the BDS when acted upon by the unit load at redundant location 1 & write the equations of BM m_1 for different segments.

iii. Analyze the BDS when acted upon by the unit load at redundant location 2 & write the equations of BM m_2 for different segments.

iv. And so on if redundants are more than 2.

Note: To write an equation take a section in each segment of loading & then write BM equation with proper sign convention.

Flexibility Method for Frames Analysis

- **Step # 03:** Develop Bending moment equation table . BM equations table will be consist of following things which will make the calculation work easy.

Segments	Segments means different members of frame like	
	AB	BC
Origin	A or B any one can be the origin	B or C any one can be the origin
Limits (length of segment)	0 to x feet	0 to y feet
I (Moment of inertia)	Value of I for segment AB	Value of I for segment BC
M	Equation of M for segment AB	Equation of M for segment BC
m_1	Equation of m_1 for segment AB	Equation of m_1 for segment BC
m_2	Equation of m_2 for segment AB	Equation of m_2 for segment BC

Flexibility Method for Frames Analysis

- **Step # 04:** Find BDS Displacements due to external loads

  or Compute the values of DRL

$$DRL_1 = \sum_{i=1}^{n} \int \frac{M\, m_1}{E_i I_i} dx$$

$$DRL_2 = \sum_{i=1}^{n} \int \frac{M\, m_2}{E_i I_i} dx$$

Flexibility Method for Frames Analysis

- **Step # 05:** Find BDS Displacements due to unit loads or Compute the values of flexibility coefficients & flexibility matrix.

$$f_{11} = \sum_{i=1}^{n} \int \frac{m_1 \, m_1}{E_i I_i} dx \qquad f_{22} = \sum_{i=1}^{n} \int \frac{m_2 \, m_2}{E_i I_i} dx$$

$$f_{12} = f_{21} = \sum_{i=1}^{n} \int \frac{m_1 \, m_2}{E_i I_i} dx$$

$$[\, f \,] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$$

Note: $f_{12} = f_{21}$

Flexibility Method for Frames Analysis

Step # 06: Apply compatibility and principle of superposition at the location of the removed redundants and solve the matrices to compute the values of redundant actions AR. As we know

$$[DRS] = [DRL] + [f] \bullet [AR]$$

From this

$$[AR] = [f]^{-1} \bullet [DRS - DRL]$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

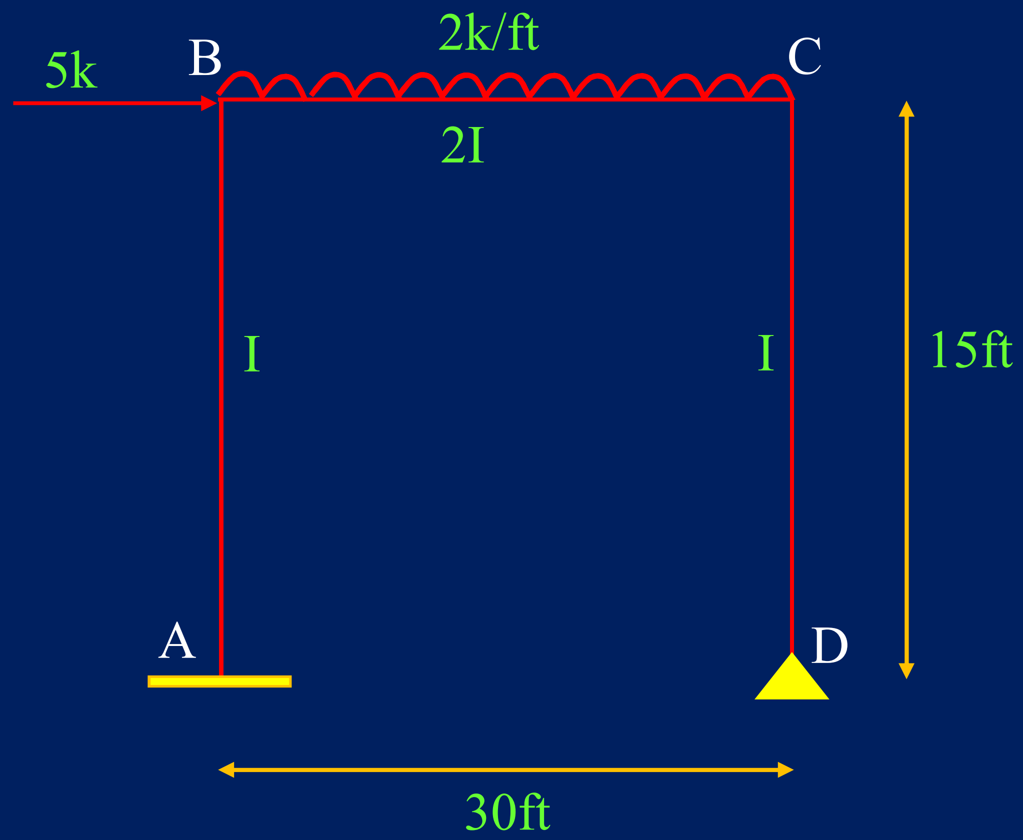
Flexibility Method for Frames Analysis

Step # 07: Compute the member end actions & draw shear force and bending moment diagrams.

$$[AM] = [AML] + [AMR][AR]$$

Flexibility Method for Frames Analysis

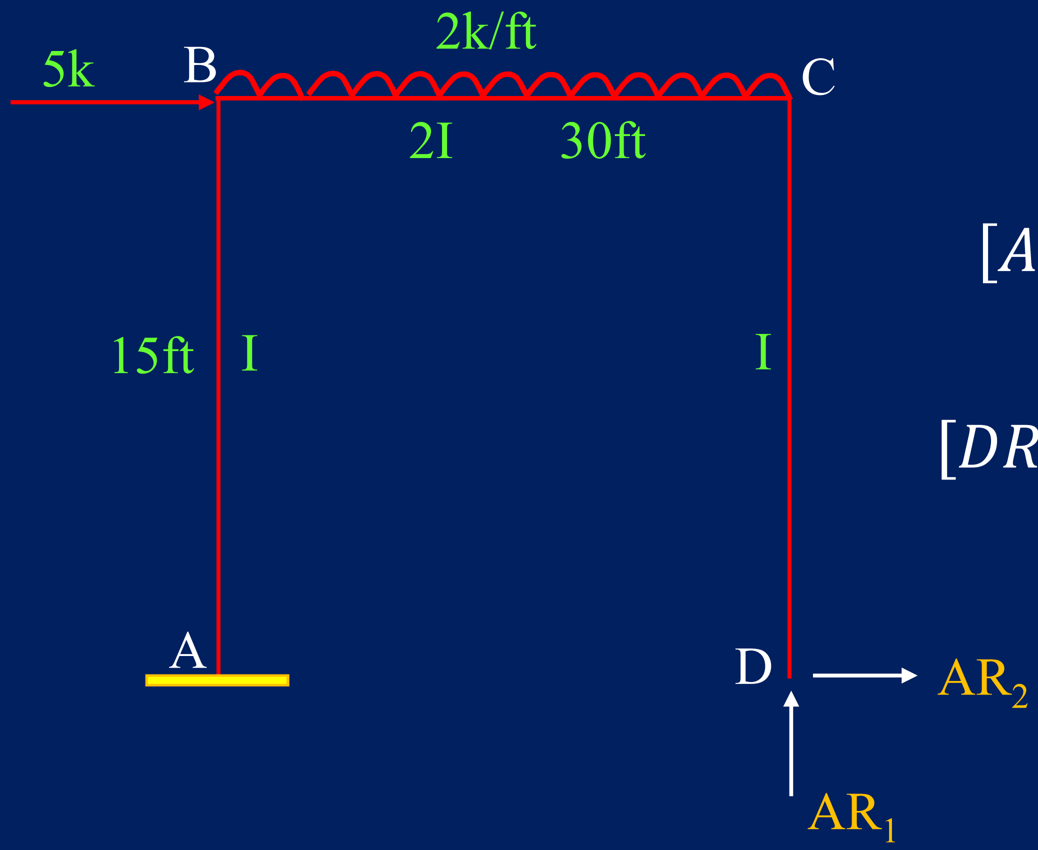
Problem 01: Analyze the given frame using flexibility method.



S.I = 2 degree
So two redundant actions should be chosen.
Choose those as redundant which makes the calculation handy.

Flexibility Method for Frames Analysis

- **Step # 01:** Identify the redundants and obtain BDS also compute [DRS] values.

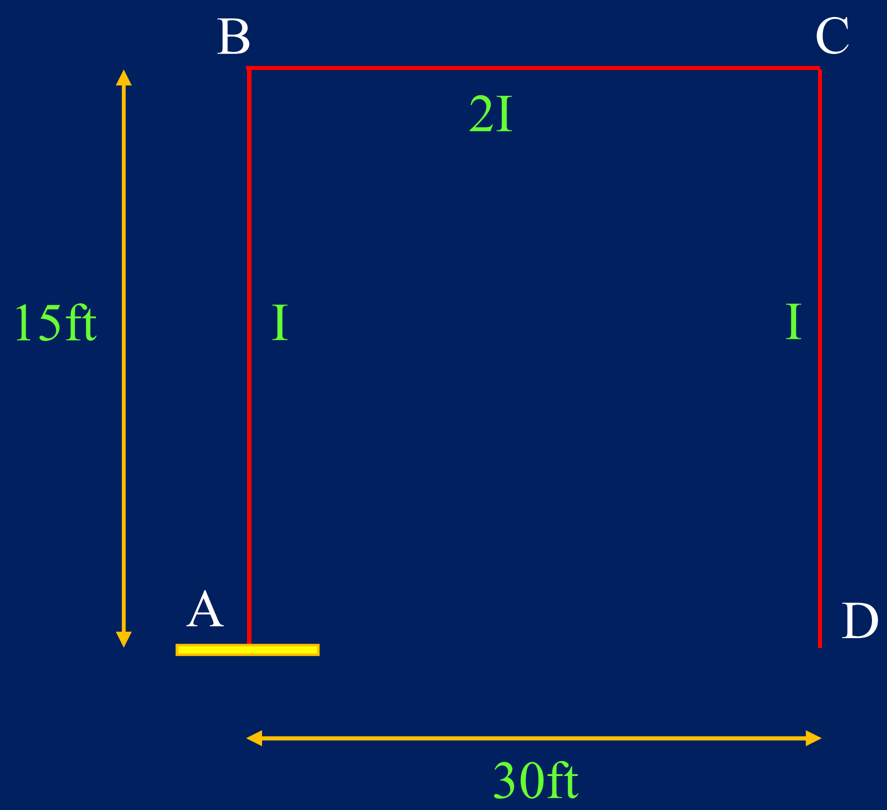


$$[AR] = \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[DRS] = \begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Vertical & horizontal reaction at D are chosen as redundant actions

Flexibility Method for Frames Analysis



Basic determinate structure (BDS) or Primary structure or Released structure

Flexibility Method for Frames Analysis

- **Step # 02:** Analyze the BDS (primary structure) under the following loading conditions

i. Analyze the BDS when acted upon by the external loads and write the equations of bending moment M for different segments.

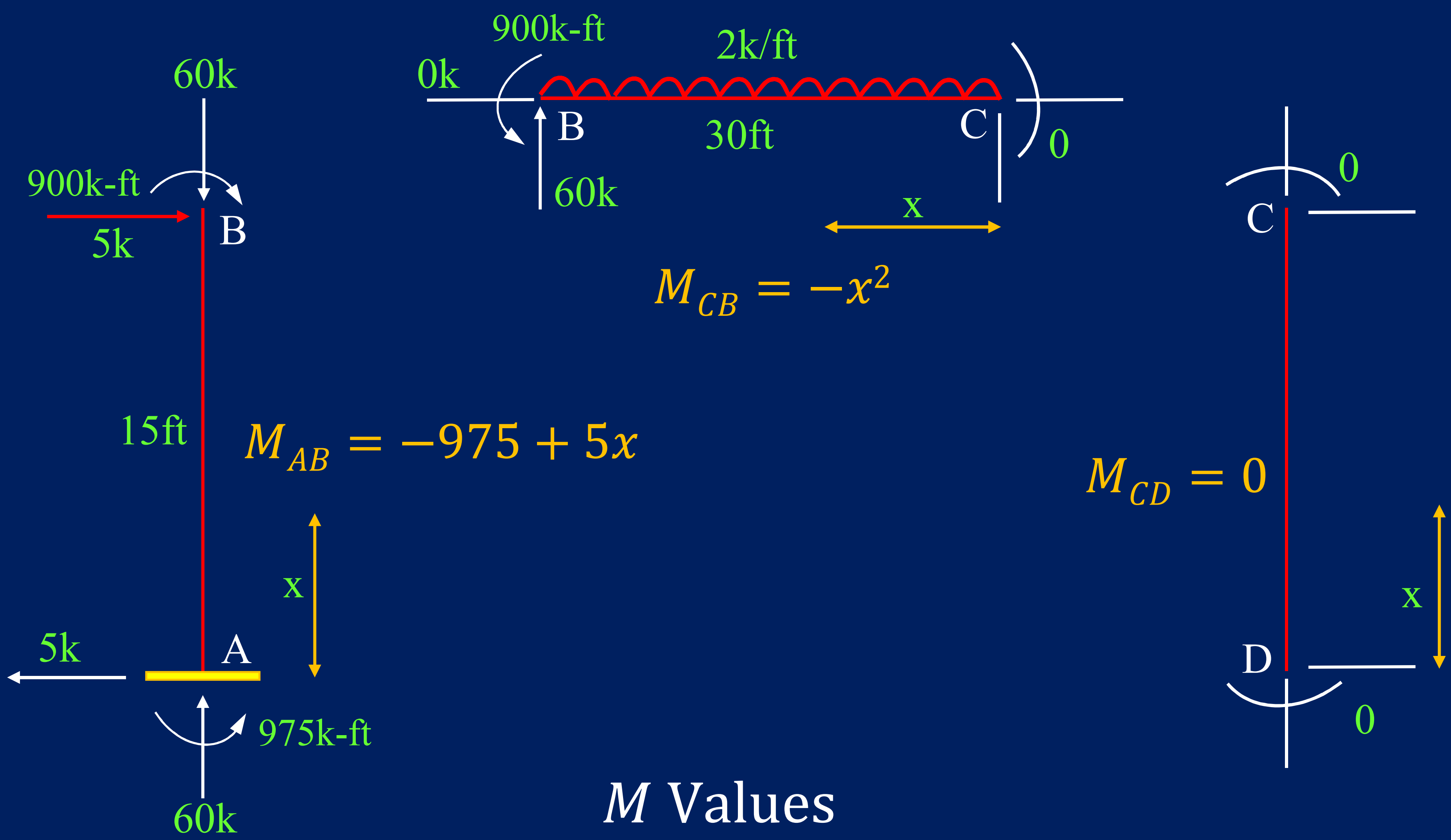
ii. Analyze the BDS when acted upon by the unit load at redundant location 1 & write the equations of BM m_1 for different segments.

iii. Analyze the BDS when acted upon by the unit load at redundant location 2 & write the equations of BM m_2 for different segments.

Note: To write an equation take a section in each segment of loading & then write BM equation with proper sign convention.

Flexibility Method for Frames Analysis

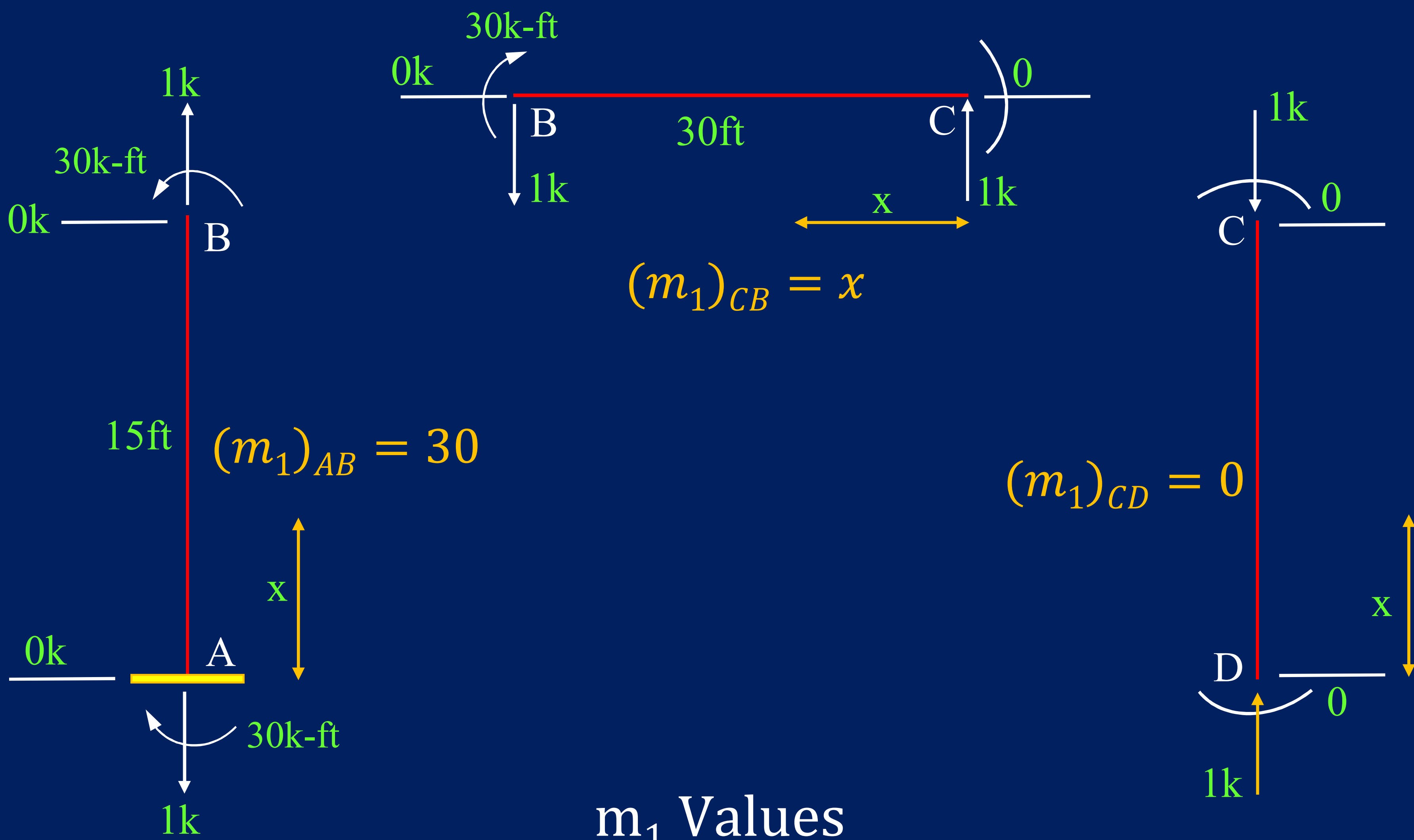
i. BDS acted upon by the external loads



M Values

Flexibility Method for Beams Analysis

i. BDS acted upon by unit load at redundant location 1.



$$(m_1)_{AB} = 30$$

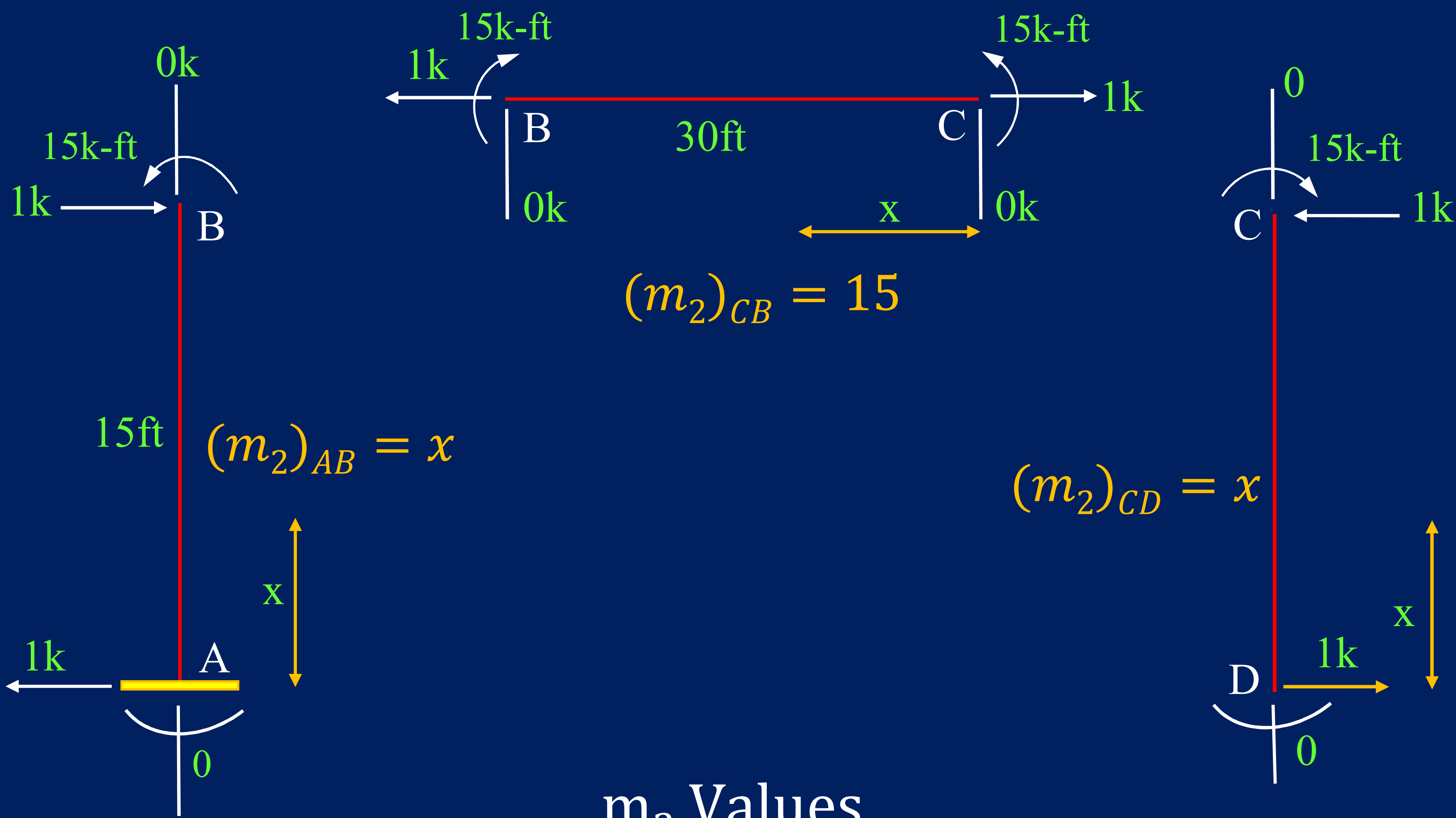
$$(m_1)_{CB} = x$$

$$(m_1)_{CD} = 0$$

m_1 Values

Flexibility Method for Beams Analysis

i. BDS acted upon by unit load at redundant location 2.



$(m_2)_{AB} = x$

$(m_2)_{CB} = 15$

$(m_2)_{CD} = x$

m_2 Values

Flexibility Method for Beams Analysis

- **Step # 03:** Develop Bending moment equation table.

Segment	AB	BC	CD
Origin	A	C	D
Limits	0 to 15 ft	0 to 30 ft	0 to 15ft
Moment of inertia , I	I	2I	I
M values	$5x - 975$	$-x^2$	0
m_1 values	30	x	0
m_2 values	x	15	x

Flexibility Method for Beams Analysis

- **Step # 04:** Find BDS Displacements due to external loads

  or Compute the values of DRL matrix.

$$\boldsymbol{DRL_1 = \sum_{i=1}^{n} \int \frac{M\, m_1}{E_i I_i} dx}$$

$$DRL_1 = \frac{1}{EI}\left[\int_0^{15} (5x - 975)(30)dx + \int_0^{30} \frac{(-x^2)(x)}{2}\, dx + 0\right]$$

$$DRL_1 = \frac{-523125}{EI}$$

Flexibility Method for Beams Analysis

$$\boldsymbol{DRL_2 = \sum_{i=1}^{n} \int \frac{M\, m_2}{E_i I_i} dx}$$

$$DRL_2 = \frac{1}{EI}\left[\int_0^{15} (5x - 975)(x)dx + \int_0^{30} \frac{(-x^2)(15)}{2}\, dx + 0\right]$$

$$DRL_2 = \frac{-171562.5}{EI}$$

$$DRL_1 = \frac{-523125}{EI} \qquad DRL_2 = \frac{-171562.5}{EI}$$

$$[DRL] = \begin{bmatrix} DRL_1 \\ DRL_2 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -523125 \\ -171562.5 \end{bmatrix}$$

Flexibility Method for Frames Analysis

- **Step # 05:** Find BDS Displacements due to unit loads

  or Compute the values of flexibility coefficients

  & flexibility matrix.

$$f_{11} = \sum_{i=1}^{n} \int \frac{m_1 \, m_1}{E_i I_i} dx$$

$$f_{11} = \frac{1}{EI}\left[\int_0^{15} (30)(30)dx + \int_0^{30} \frac{(x)(x)}{2} \, dx + 0\right]$$

$$f_{11} = \frac{18000}{EI}$$

Flexibility Method for Frames Analysis

$$f_{22} = \sum_{i=1}^{n} \int \frac{m_2\, m_2}{E_i I_i} dx$$

$$f_{12} = f_{21} = \frac{1}{EI}\left[\int_0^{15} (30)(x)dx + \int_0^{30} \frac{(15)(x)}{2}\, dx + 0\right]$$

$$f_{12} = f_{21} = \frac{6750}{EI}$$

$$f_{12} = f_{21} = \sum_{i=1}^{n} \int \frac{m_1\, m_2}{E_i I_i} dx$$

$$f_{22} = \frac{1}{EI}\left[\int_0^{15} (x)(x)dx + \int_0^{30} \frac{(15)(15)}{2}\, dx + \int_0^{15} (x)(x)dx\right]$$

$$f_{22} = \frac{5625}{EI}$$

Flexibility Method for Frames Analysis

$$f_{11} = \frac{18000}{EI} \qquad f_{22} = \frac{5625}{EI}$$

$$f_{12} = f_{21} = \frac{6750}{EI}$$

$$[f] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$$

$$[f] = \frac{1}{EI}\begin{bmatrix} 18000 & 6750 \\ 6750 & 5625 \end{bmatrix}$$

Flexibility Method for Frames Analysis

Step # 06: Apply compatibility and principle of superposition at the locations of the removed redundants and solve the matrices to compute the values of redundant actions AR. As we know

$$[DRS] = [DRL] + [f] \bullet [AR]$$

From this

$$[AR] = [f]^{-1} \bullet [DRS - DRL]$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

Flexibility Method for Frames Analysis

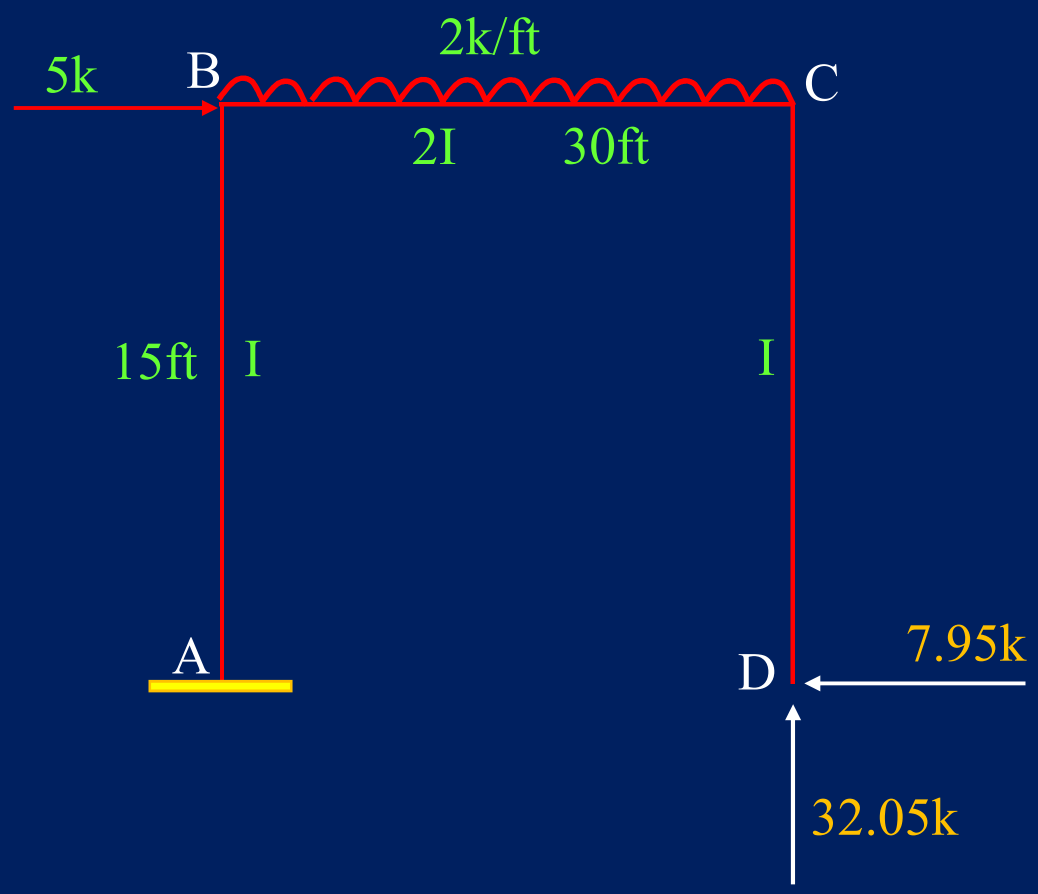
$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = EI \begin{bmatrix} 18000 & 6750 \\ 6750 & 5625 \end{bmatrix}^{-1} \begin{bmatrix} 0-(-523125) \\ 0-(-171562.5) \end{bmatrix} \times \frac{1}{EI}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 32.05 \\ -7.95 \end{bmatrix}$$

-ive sign shows that our assumed redundant action direction is wrong

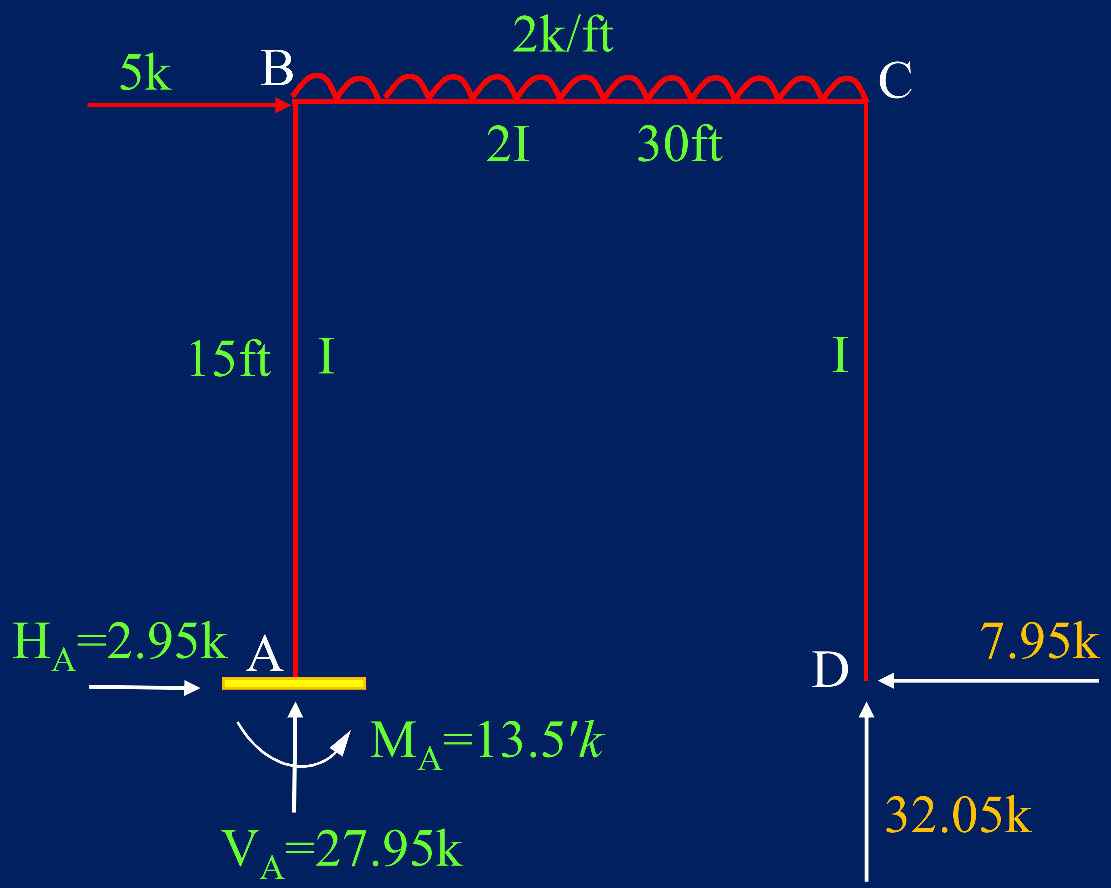
Flexibility Method for Frames Analysis



Final determinate structure

Flexibility Method for Frames Analysis

Using Equilibrium equations one can compute the remaining support reactions.



$$\sum Fx = 0$$

$$HA + 5 = 7.95$$

$$HA = 2.95\,k$$

$$\sum Fy = 0$$

$$VA + 32.05 = 60$$

$$VA = 27.95$$

Now

$$MA = 32.05 * 30 - 60 * 15 - 5 * 15$$

$$MA = 13.5'k$$

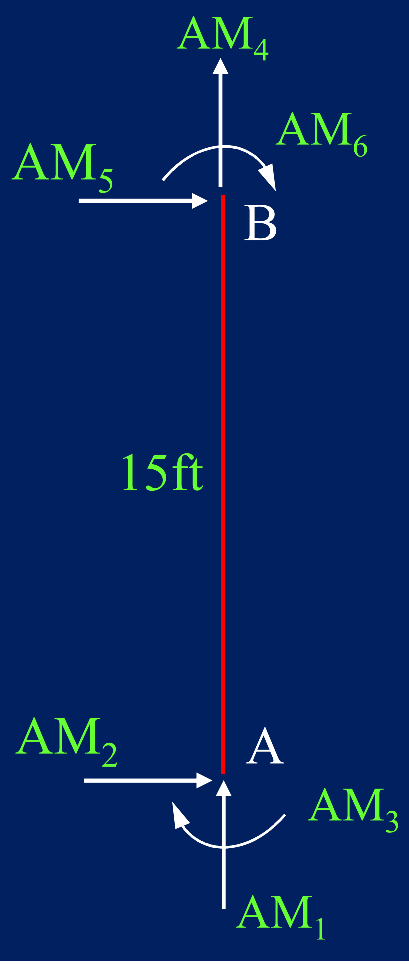
Flexibility Method for Frames Analysis

Step # 07: Compute the member end actions & draw SFD & BMD As we know that

$$[AM] = [AML] + [AMR][AR]$$

Member AB:

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix}$$

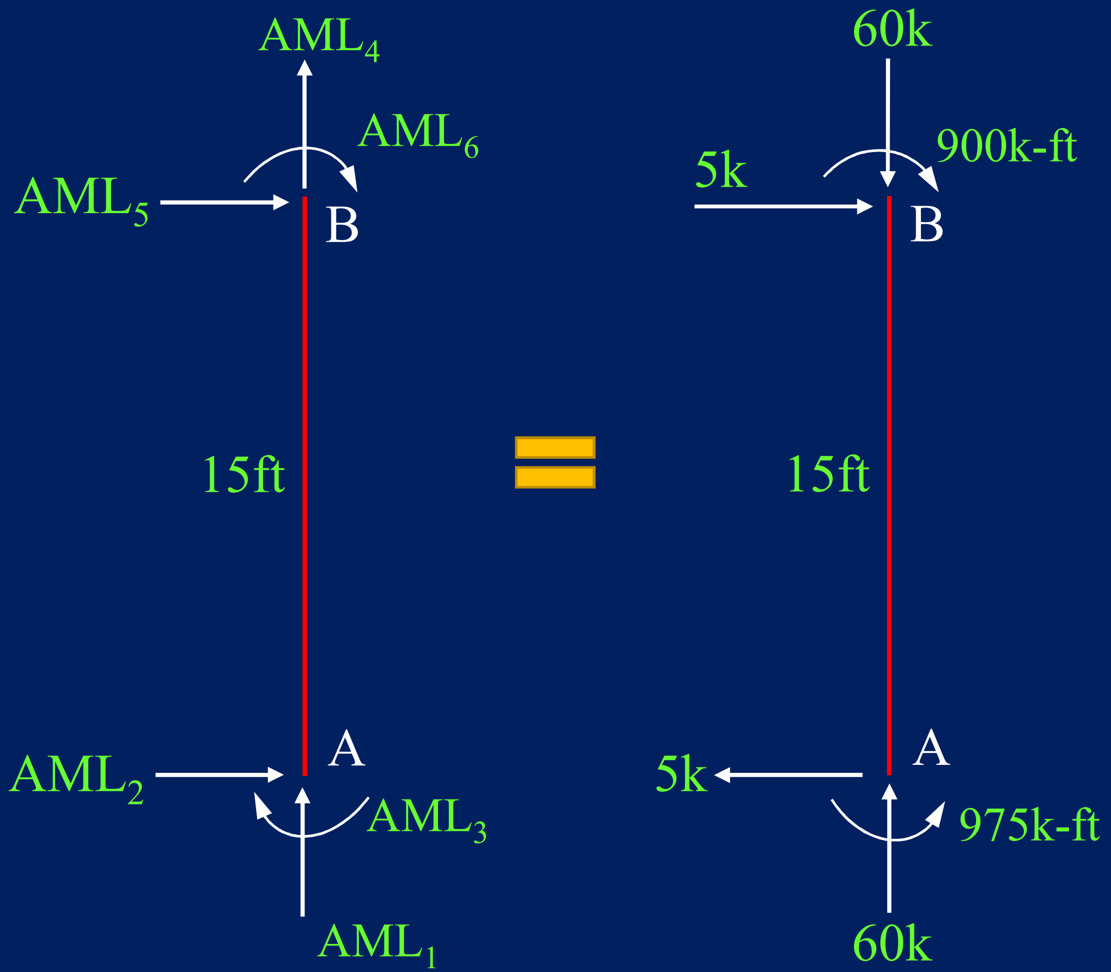


Sign convention:

- Horizontal force in +ive X direction will be taken as positive.
- Upward vertical force will be taken as positive.
- Clockwise moment will be taken as positive.

Flexibility Method for Frames Analysis

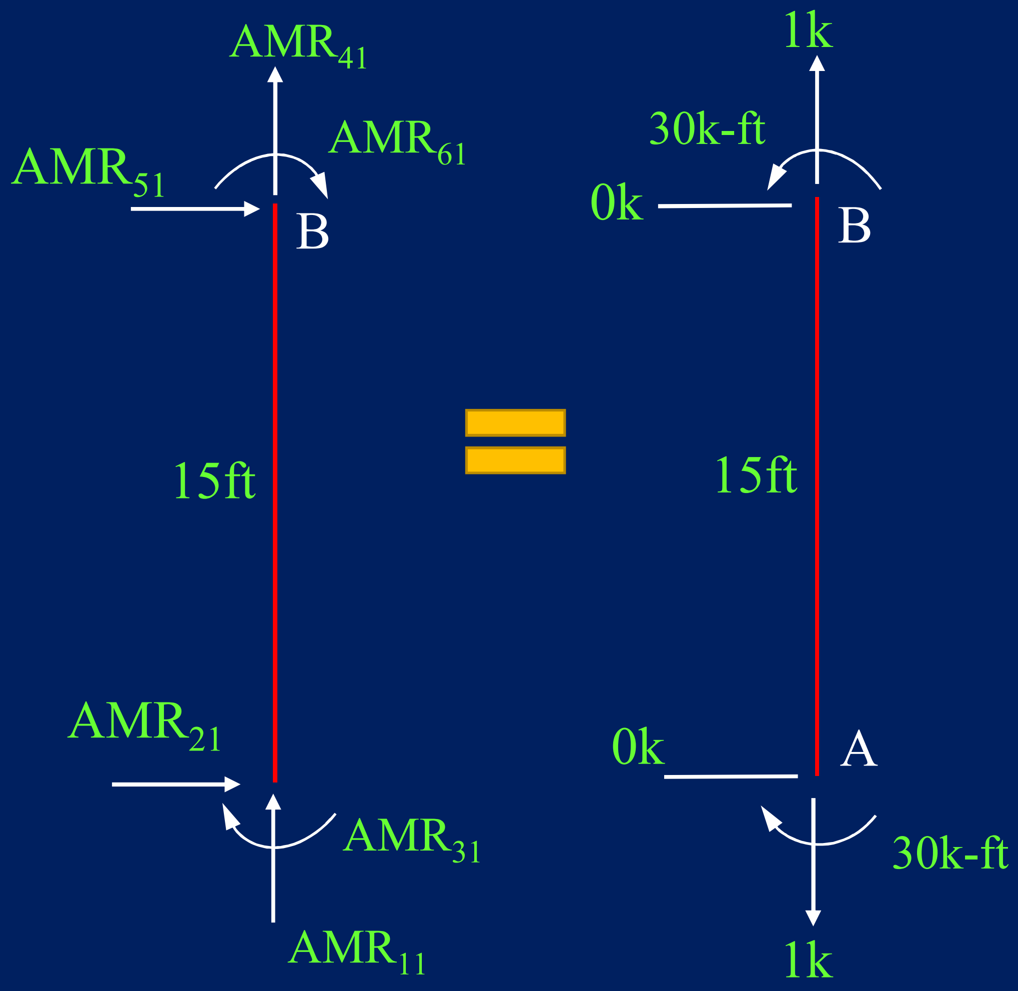
a) Compute AML values.



$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} = \begin{bmatrix} 60 \\ -5 \\ -975 \\ -60 \\ 5 \\ 900 \end{bmatrix}$$

Flexibility Method for Frames Analysis

b) Compute the AMR values: when unit is applied at redundant location 1.

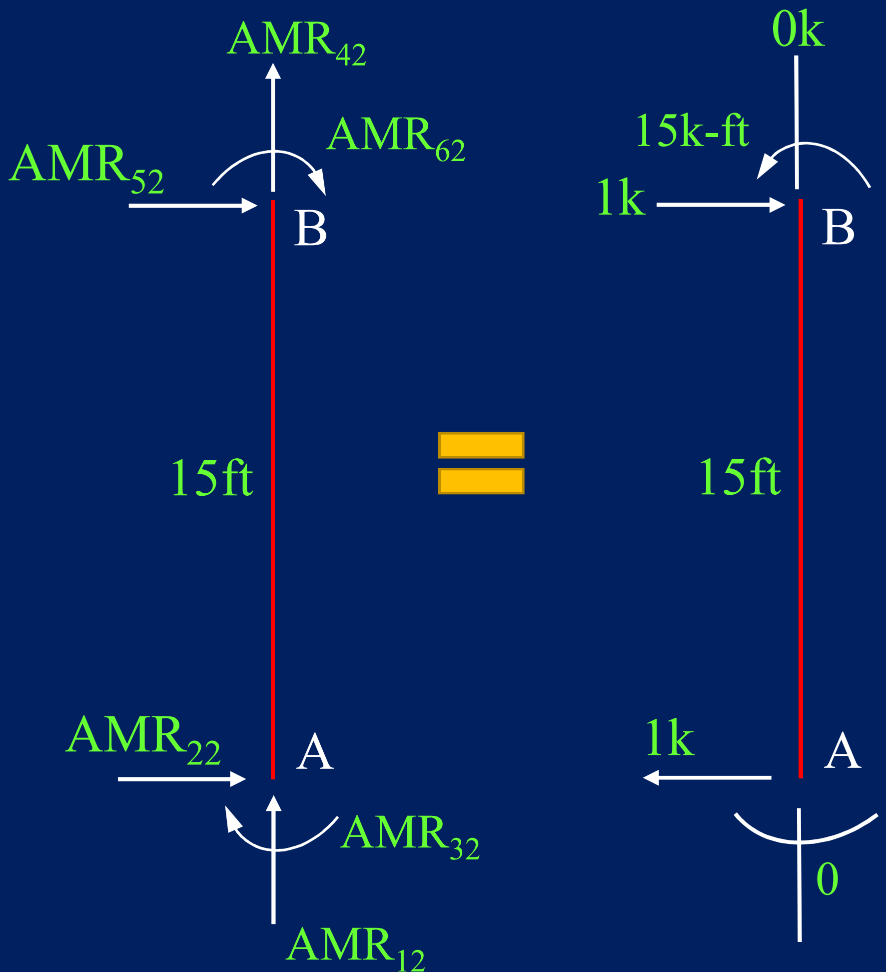


$$[AMR] = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & ? \\ 0 & ? \\ 30 & ? \\ 1 & ? \\ 0 & ? \\ -30 & ? \end{bmatrix}$$

Flexibility Method for Frames Analysis

b) Compute the AMR values: when unit is applied at redundant location 2.



$$[AMR] = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 30 & 0 \\ 1 & 0 \\ 0 & 1 \\ -30 & -15 \end{bmatrix}$$

Flexibility Method for Frames Analysis

$$[AMR]_{6*2} = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 30 & 0 \\ 1 & 0 \\ 0 & 1 \\ -30 & -15 \end{bmatrix}$$

Now member end actions will be computed as given below

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} + \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix} \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix}$$

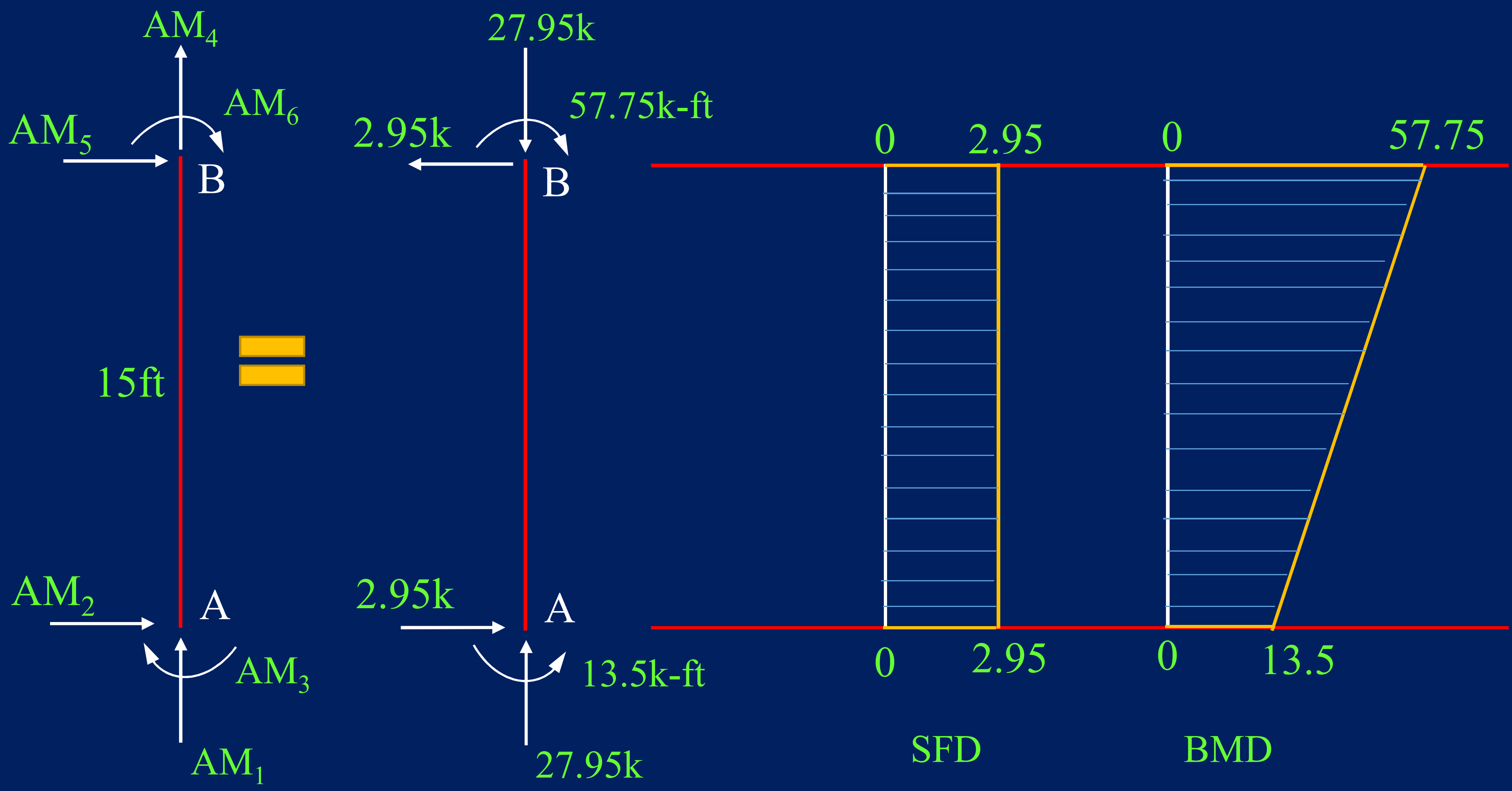
Flexibility Method for Frames Analysis

So member end actions will be

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} 60 \\ -5 \\ -975 \\ -60 \\ 5 \\ 900 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 30 & 0 \\ 1 & 0 \\ 0 & 1 \\ -30 & -15 \end{bmatrix} \begin{bmatrix} 32.05 \\ -7.95 \end{bmatrix} = \begin{bmatrix} 27.95 \\ 2.95 \\ -13.5 \\ -27.95 \\ -2.95 \\ 57.75 \end{bmatrix}$$

Flexibility Method for Frames Analysis

So member end actions, Shear force & Bending moment diagrams of member AB



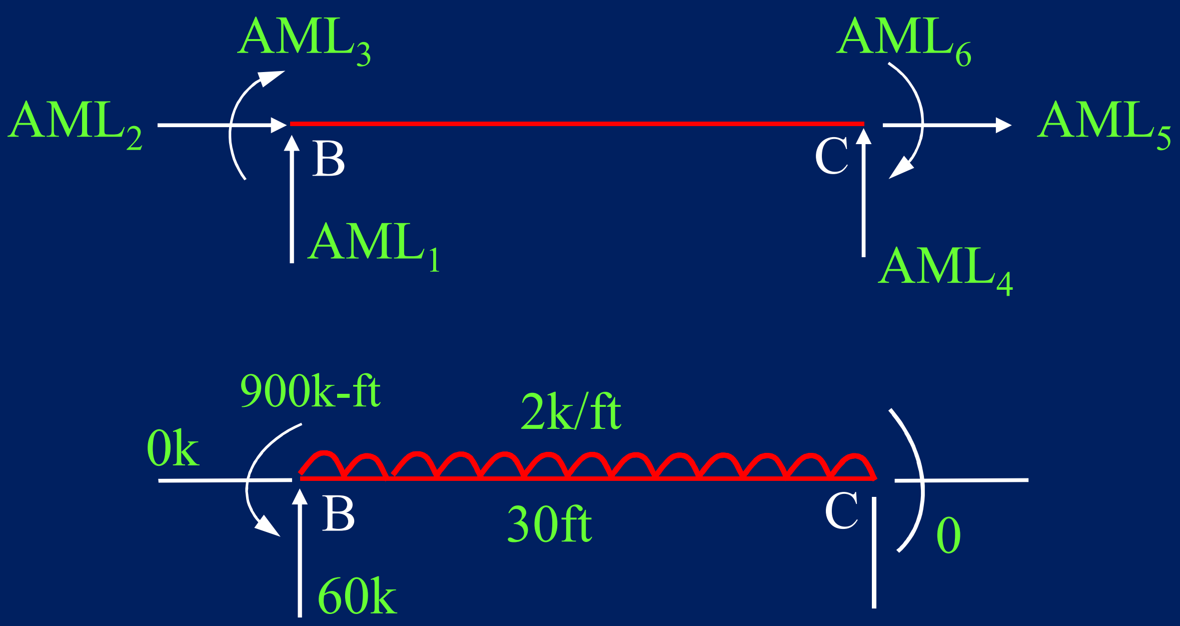
Flexibility Method for Frames Analysis

Member BC: member end actions for member BC are specified below

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix}$$

Flexibility Method for Frames Analysis

a) Compute AML values.



$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ -900 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Flexibility Method for Frames Analysis

b) Compute the AMR values: when unit is applied at redundant location 1.

$$[AMR] = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & ? \\ 0 & ? \\ 30 & ? \\ 1 & ? \\ 0 & ? \\ 0 & ? \end{bmatrix}$$

Flexibility Method for Frames Analysis

b) Compute the AMR values: when unit is applied at redundant location 2.

$$[AMR] = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 30 & 15 \\ 1 & 0 \\ 0 & 1 \\ 0 & -15 \end{bmatrix}$$

Flexibility Method for Frames Analysis

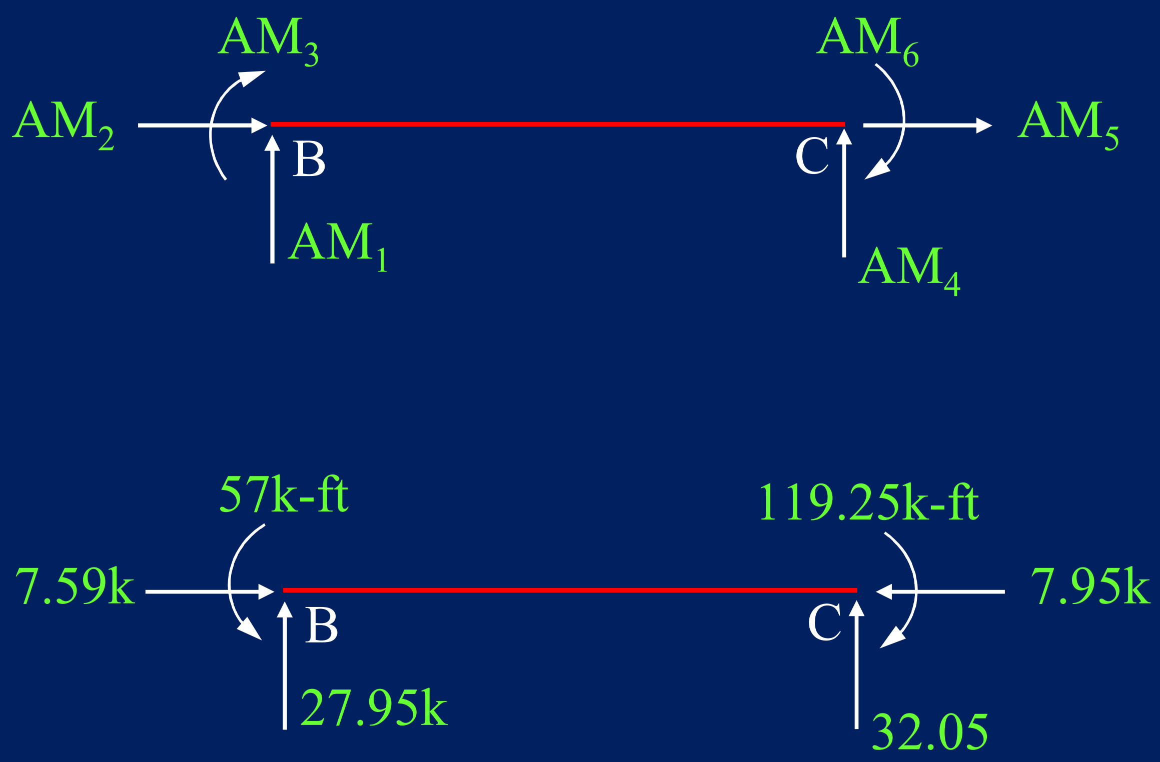
$$[AMR]_{6*2} = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 30 & 15 \\ 1 & 0 \\ 0 & 1 \\ 0 & -15 \end{bmatrix}$$

So member end actions will be

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ -900 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 30 & 15 \\ 1 & 0 \\ 0 & 1 \\ 0 & -15 \end{bmatrix} \begin{bmatrix} 32.05 \\ -7.95 \end{bmatrix} = \begin{bmatrix} 27.95 \\ 7.59 \\ -57.75 \\ 32.05 \\ -7.95 \\ 119.25 \end{bmatrix}$$

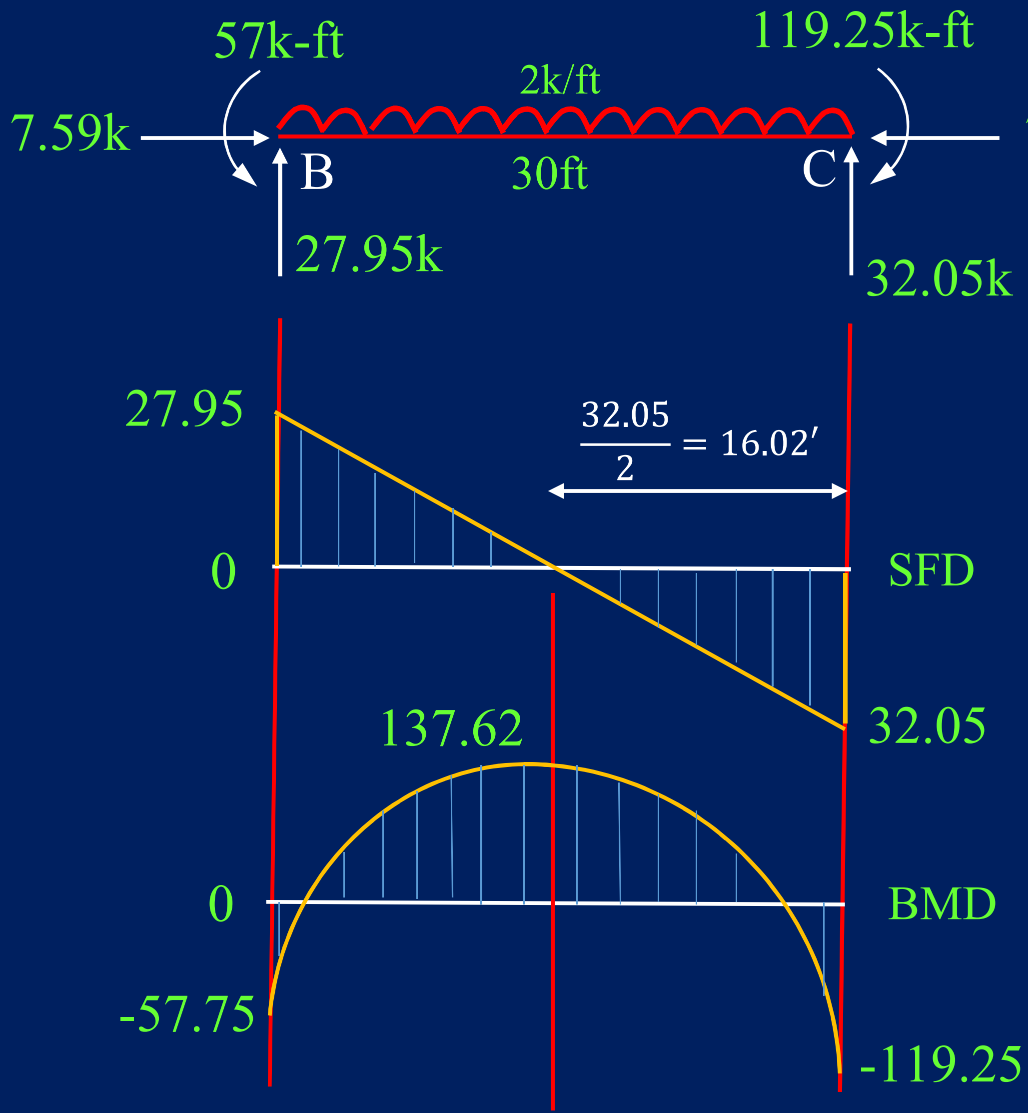
Flexibility Method for Frames Analysis

So member end actions of member BC



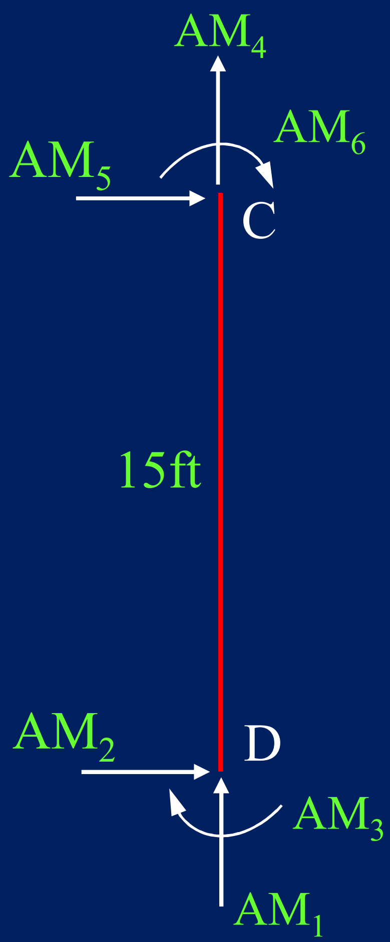
Flexibility Method for Frames Analysis

Member BC:



Flexibility Method for Frames Analysis

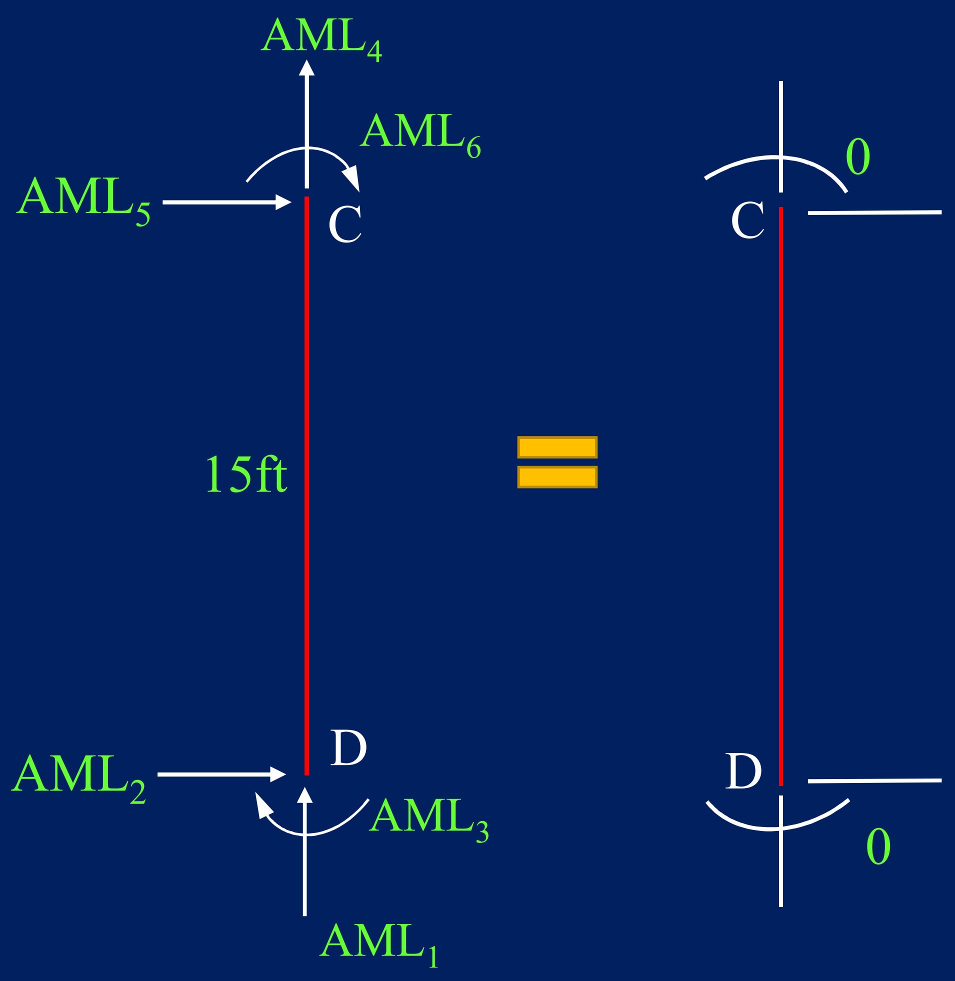
Member CD: member end actions for member CD are specified below



$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix}$$

Flexibility Method for Frames Analysis

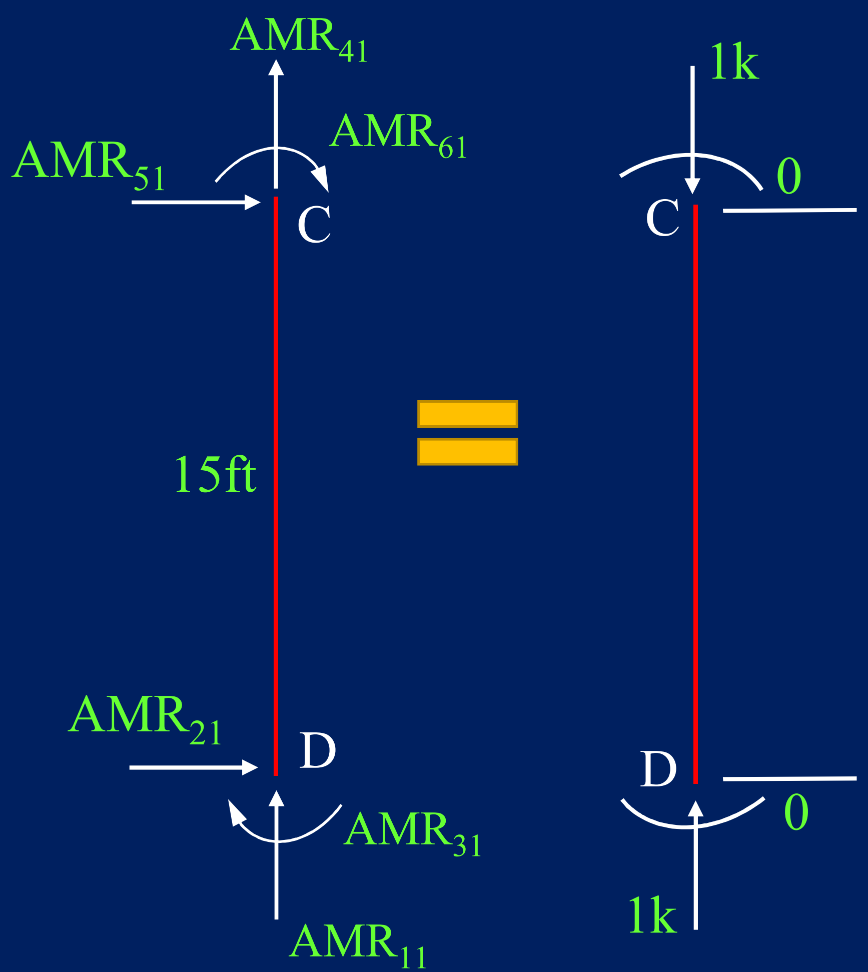
a) Compute AML values.



$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Flexibility Method for Frames Analysis

b) Compute the AMR values: when unit is applied at redundant location 1.

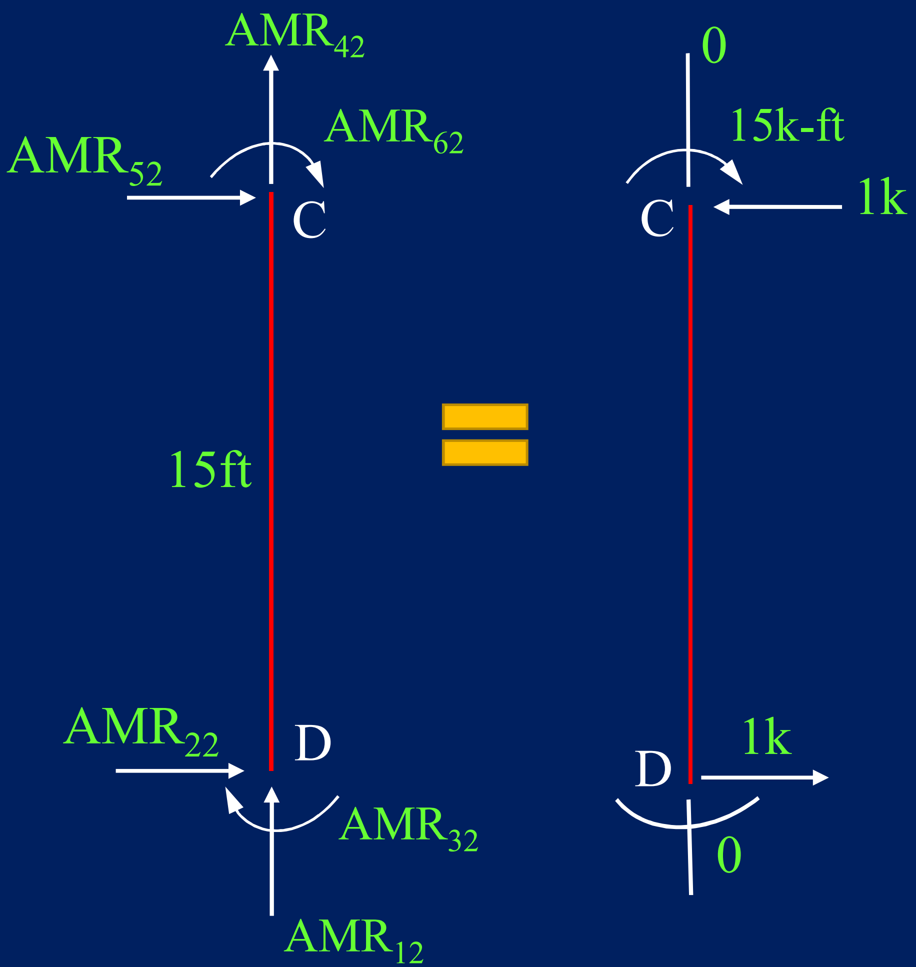


$$[AMR] = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & ? \\ 0 & ? \\ 0 & ? \\ -1 & ? \\ 0 & ? \\ 0 & ? \end{bmatrix}$$

Flexibility Method for Frames Analysis

b) Compute the AMR values: when unit is applied at redundant location 2.



$$[AMR] = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 15 \end{bmatrix}$$

Flexibility Method for Frames Analysis

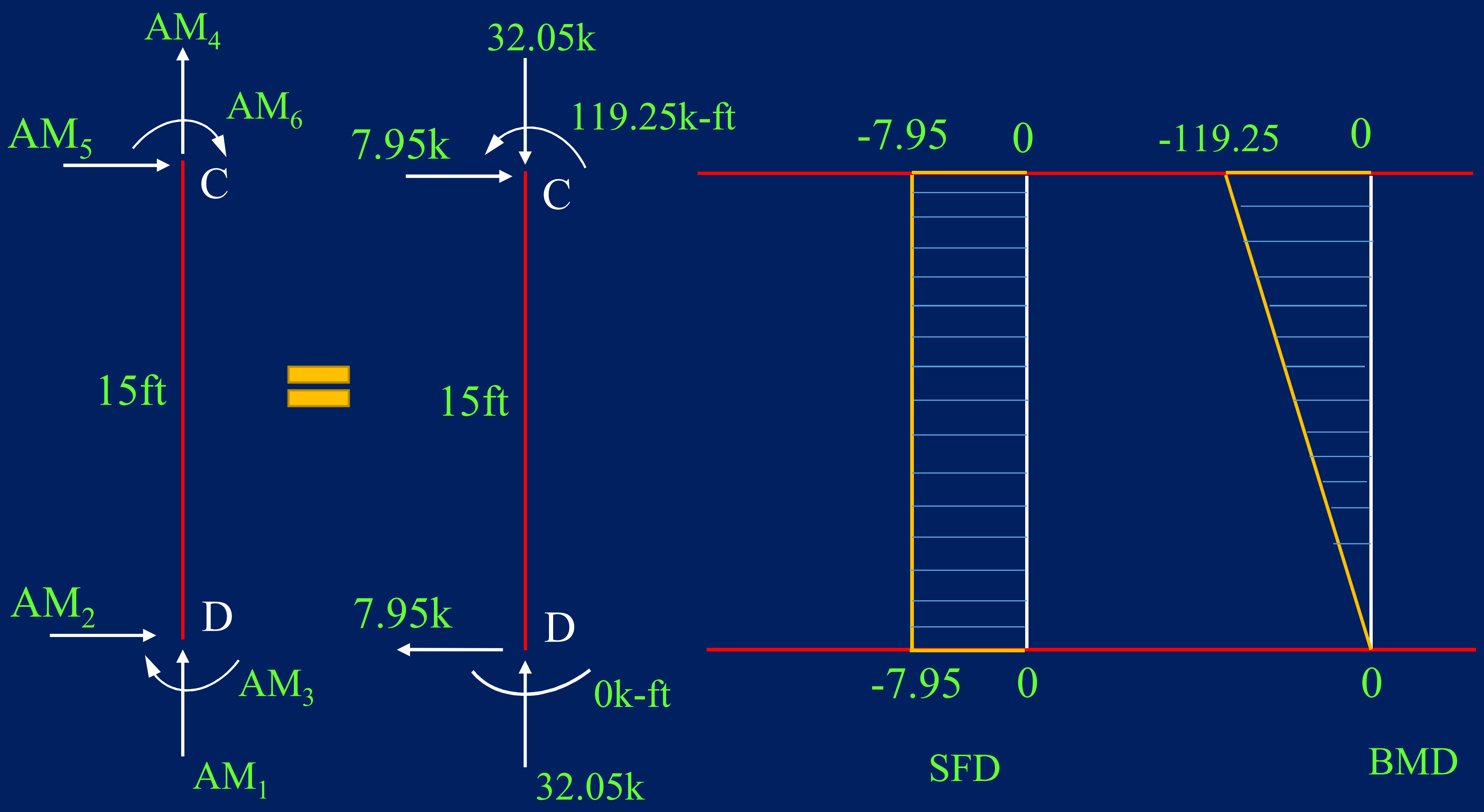
$$[AMR]_{6\,*\,2} = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 15 \end{bmatrix}$$

So member end actions will be

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 15 \end{bmatrix} \begin{bmatrix} 32.05 \\ -7.95 \end{bmatrix} = \begin{bmatrix} 32.05 \\ -7.95 \\ 0 \\ -32.05 \\ 7.95 \\ -119.25 \end{bmatrix}$$

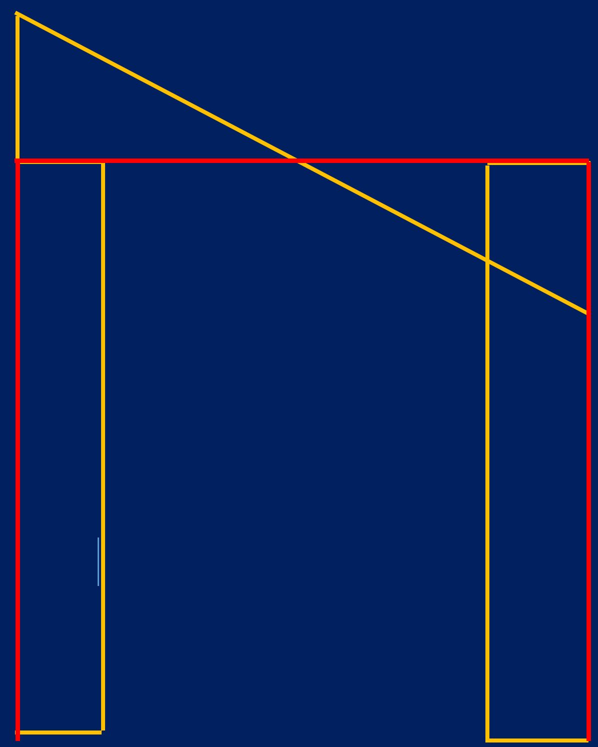
Flexibility Method for Frames Analysis

So member end actions, Shear force & bending moment diagrams of member CD

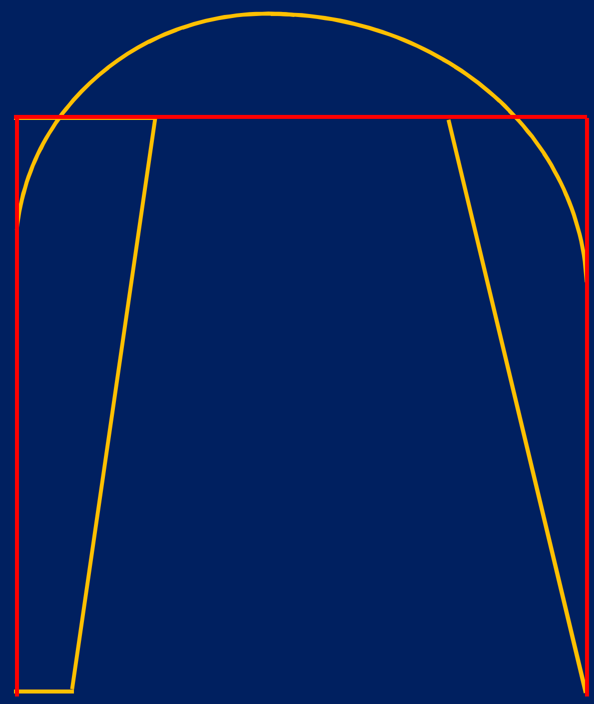


Flexibility Method for Frames Analysis

Combined shear force & bending moment diagrams:



SFD



BMD

Flexibility Method for Frames Analysis

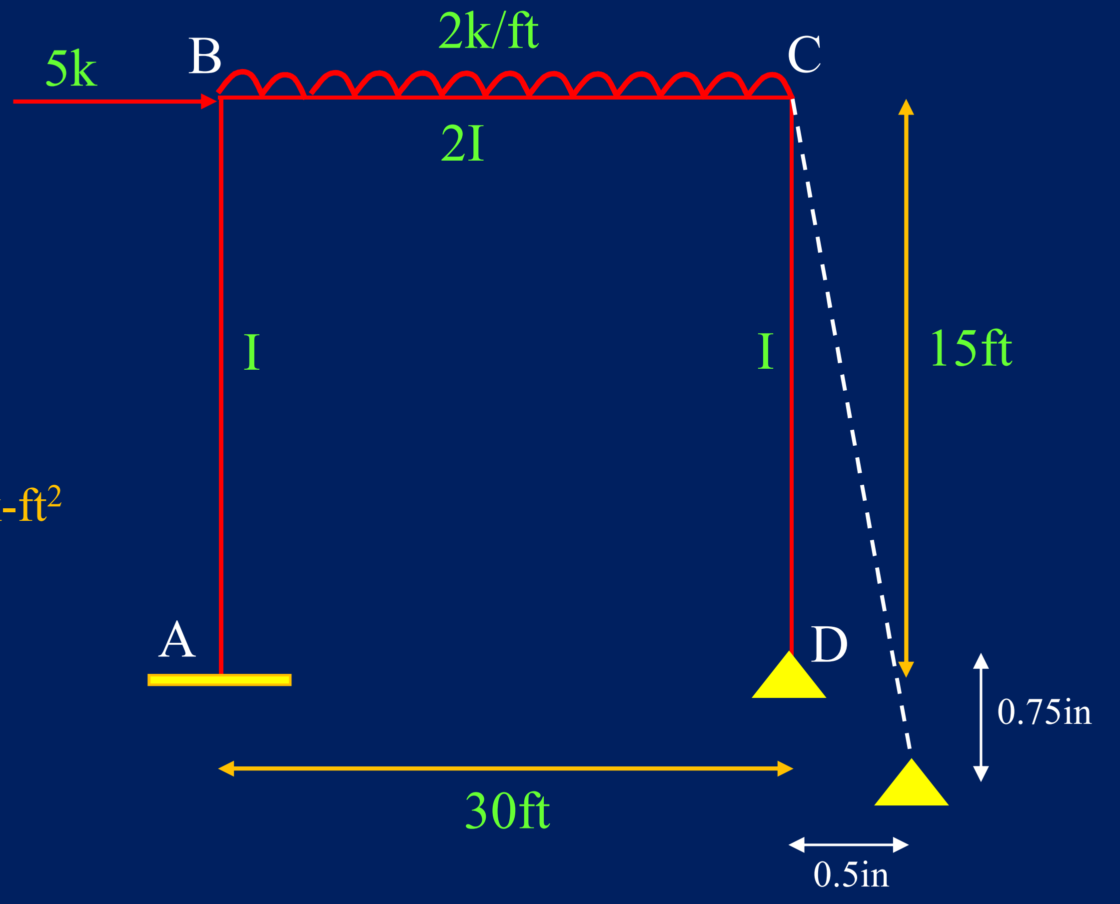
Problem 02: Analyze the given frame using flexibility method such that support D translate to right by 0.5in and also settle down by 0.75in.

S.I = 2 degree

Take E = 30000ksi

I = 600 in^4

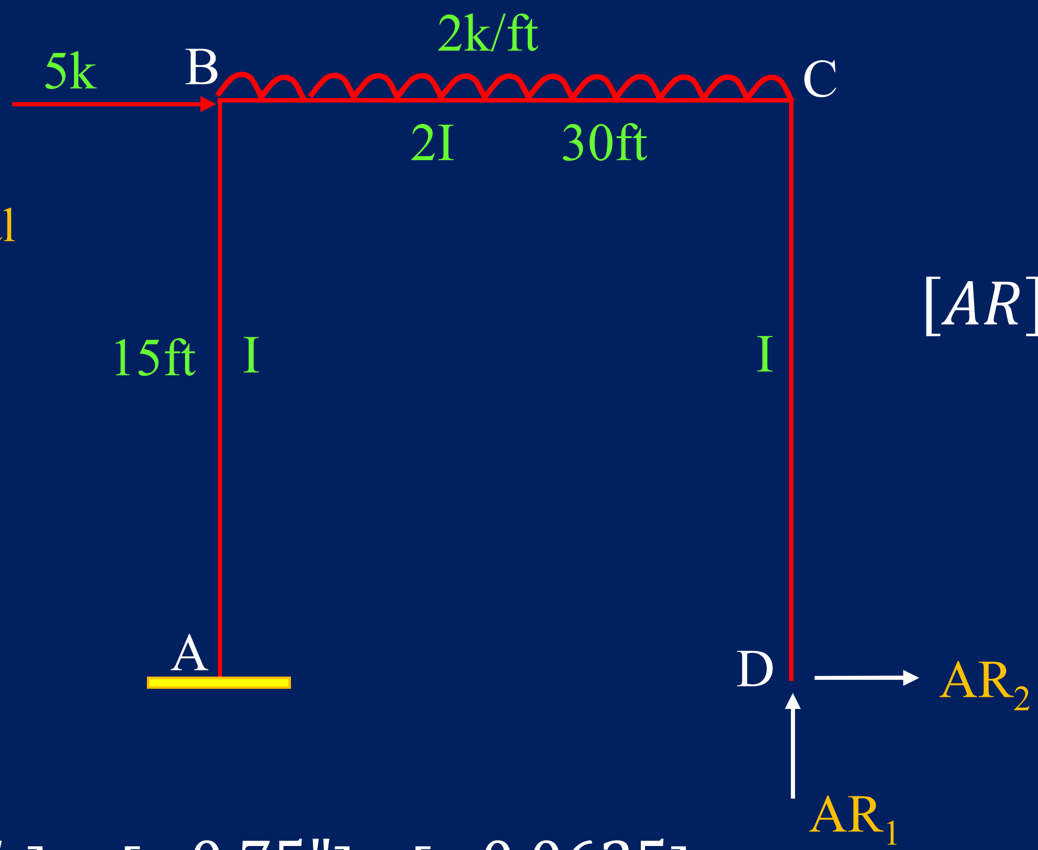
EI = 125000 k-ft^2



Flexibility Method for Frames Analysis

- **Step # 01:** Identify the redundants and obtain BDS also compute [DRS] values.

Vertical & horizontal reaction at D are chosen as redundant actions



$$[AR] = \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[DRS] = \begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} -0.75'' \\ 0.5'' \end{bmatrix} = \begin{bmatrix} -0.0625 \\ 0.0416 \end{bmatrix}$$

Flexibility Method for Frames Analysis

- **Step # 02:** Analyze the BDS (primary structure) under the following loading conditions

i. Analyze the BDS when acted upon by the external loads and write the equations of bending moment M for different segments.

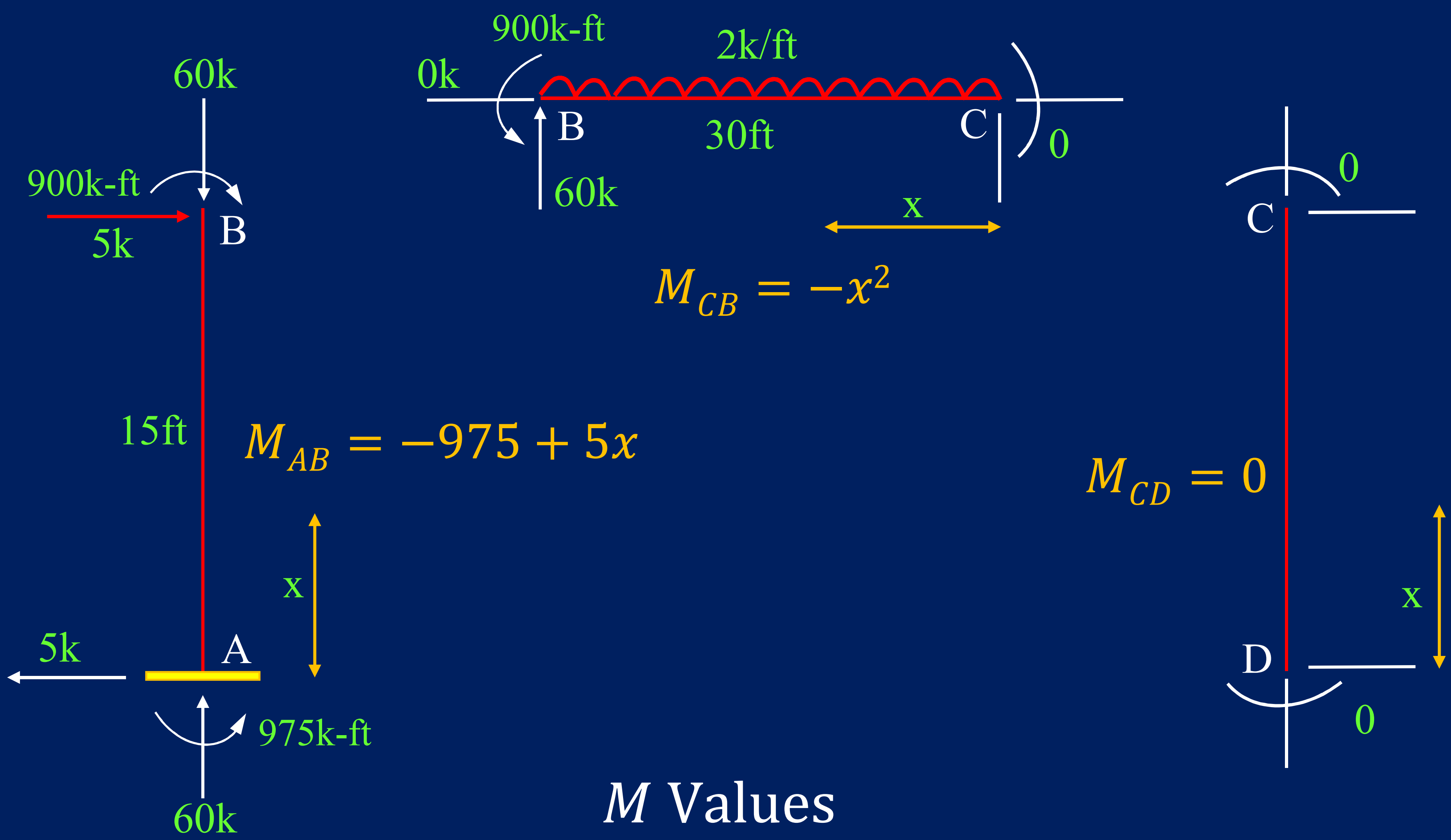
ii. Analyze the BDS when acted upon by the unit load at redundant location 1 & write the equations of BM m_1 for different segments.

iii. Analyze the BDS when acted upon by the unit load at redundant location 2 & write the equations of BM m_2 for different segments.

Note: To write an equation take a section in each segment of loading & then write BM equation with proper sign convention.

Flexibility Method for Frames Analysis

i. BDS acted upon by the external loads



$$M_{CB} = -x^2$$

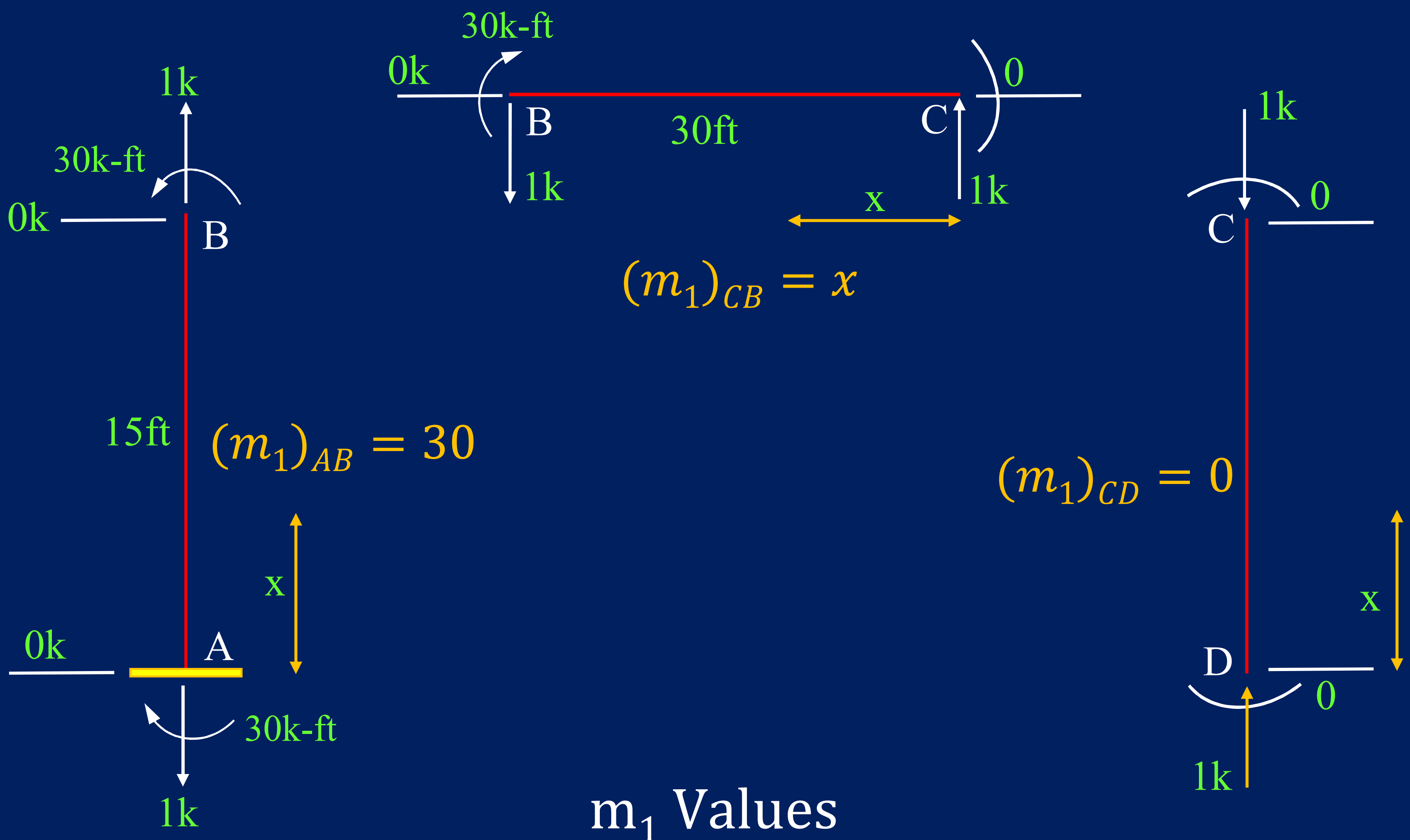
$$M_{AB} = -975 + 5x$$

$$M_{CD} = 0$$

M Values

Flexibility Method for Beams Analysis

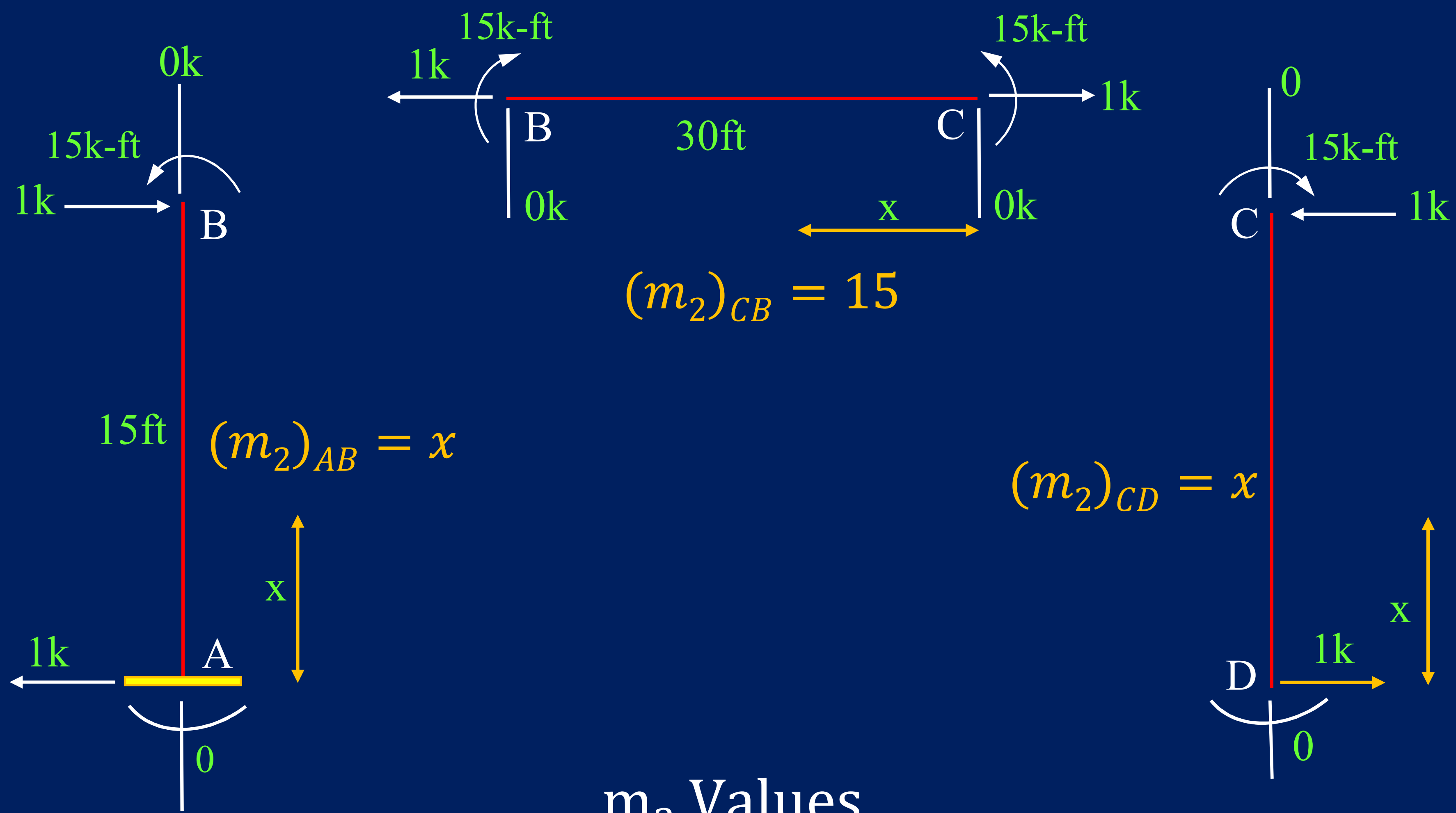
i. BDS acted upon by unit load at redundant location 1.



m_1 Values

Flexibility Method for Beams Analysis

i. BDS acted upon by unit load at redundant location 2.



m_2 Values

Flexibility Method for Beams Analysis

- **Step # 03:** Develop Bending moment equation table.

Segment	AB	BC	CD
Origin	A	B	C
Limits	0 to 15 ft	0 to 30 ft	0 to 15ft
Moment of inertia , I	I	2I	I
M values	$-975 + 5x$	$-x^2$	0
m_1 values	30	x	0
m_2 values	x	15	x

Flexibility Method for Beams Analysis

- **Step # 04:** Find BDS Displacements due to external loads

  or Compute the values of DRL matrix.

$$\boldsymbol{DRL_1 = \sum_{i=1}^{n} \int \frac{M\, m_1}{E_i I_i} dx}$$

$$DRL_1 = \frac{1}{EI}\left[\int_0^{15} (5x - 975)(30)dx + \int_0^{30} \frac{(-x^2)(x)}{2}\, dx + 0\right]$$

$$DRL_1 = \frac{-523125}{EI}$$

Flexibility Method for Beams Analysis

$$\boldsymbol{DRL_2 = \sum_{i=1}^{n} \int \frac{M\, m_2}{E_i I_i}\, dx}$$

$$DRL_2 = \frac{1}{EI}\left[\int_0^{15} (5x - 975)(x)\,dx + \int_0^{30} \frac{(-x^2)(15)}{2}\,dx + 0\right]$$

$$DRL_2 = \frac{-171562.5}{EI}$$

$$[DRL] = \begin{bmatrix} DRL_1 \\ DRL_2 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -523125 \\ -171562.5 \end{bmatrix} = \frac{1}{125000}\begin{bmatrix} -523125 \\ -171562.5 \end{bmatrix}$$

$$[DRL] = \begin{bmatrix} DRL_1 \\ DRL_2 \end{bmatrix} = \begin{bmatrix} -4.185 \\ -1.372 \end{bmatrix}$$

Flexibility Method for Frames Analysis

- **Step # 05:** Find BDS Displacements due to unit loads

  or Compute the values of flexibility coefficients

  & flexibility matrix.

$$f_{11} = \sum_{i=1}^{n} \int \frac{m_1 \, m_1}{E_i I_i} dx$$

$$f_{11} = \frac{1}{EI} \left[\int_0^{15} (30)(30) dx + \int_0^{30} \frac{(x)(x)}{2} \, dx + 0 \right]$$

$$f_{11} = \frac{18000}{EI}$$

Flexibility Method for Frames Analysis

$$f_{22} = \sum_{i=1}^{n} \int \frac{m_2\, m_2}{E_i I_i} dx$$

$$f_{12} = f_{21} = \frac{1}{EI}\left[\int_0^{15} (30)(x)dx + \int_0^{30} \frac{(15)(x)}{2} dx + 0\right]$$

$$f_{12} = f_{21} = \frac{6750}{EI}$$

$$f_{12} = f_{21} = \sum_{i=1}^{n} \int \frac{m_1\, m_2}{E_i I_i} dx$$

$$f_{22} = \frac{1}{EI}\left[\int_0^{15} (x)(x)dx + \int_0^{30} \frac{(15)(15)}{2} dx + \int_0^{15} (x)(x)dx\right]$$

$$f_{22} = \frac{5625}{EI}$$

Flexibility Method for Frames Analysis

$$[f] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$$

$$[f] = \frac{1}{EI}\begin{bmatrix} 18000 & 6750 \\ 6750 & 5625 \end{bmatrix}$$

$$[f] = \frac{1}{125000}\begin{bmatrix} 18000 & 6750 \\ 6750 & 5625 \end{bmatrix}$$

$$[f] = \begin{bmatrix} 0.144 & 0.054 \\ 0.054 & 0.045 \end{bmatrix}$$

Flexibility Method for Frames Analysis

Step # 06: Apply compatibility and principle of superposition at the location of the removed redundants and solve the matrices to compute the values of redundant actions AR. As we know

$$[DRS] = [DRL] + [f] \bullet [AR]$$

From this

$$[AR] = [f]^{-1} \bullet [DRS - DRL]$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

Flexibility Method for Frames Analysis

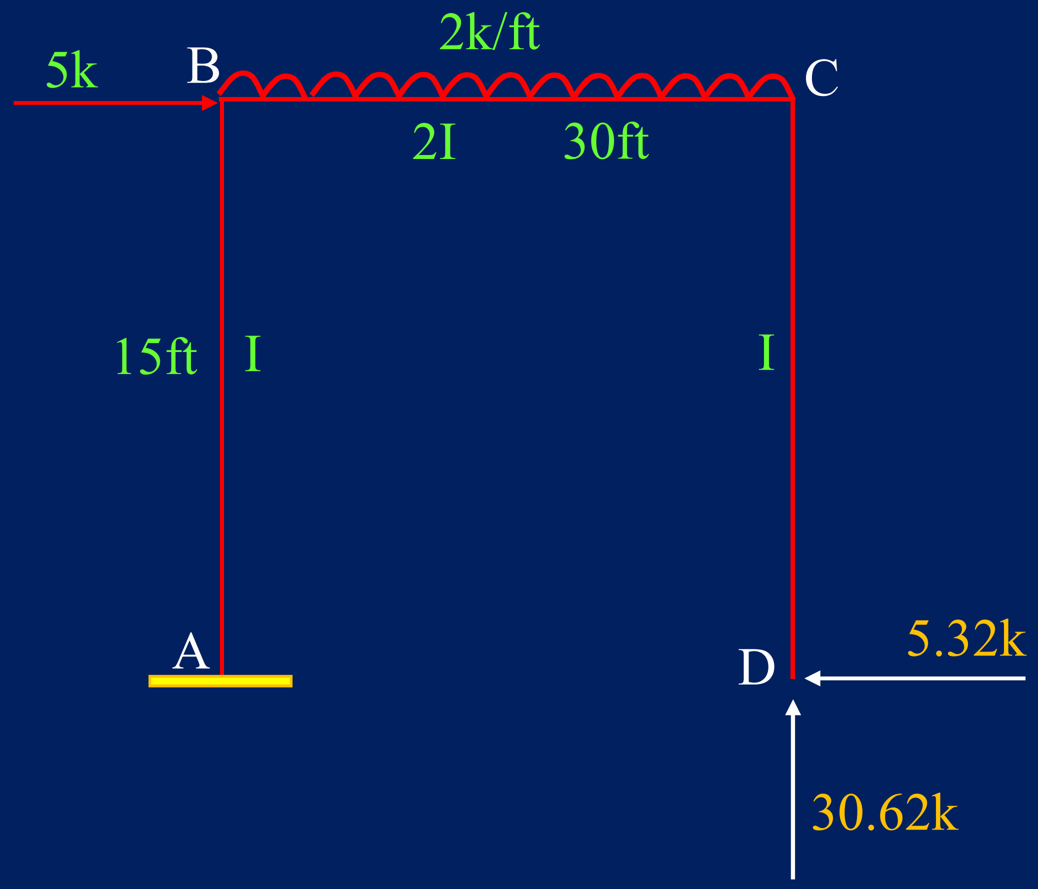
$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = EI \begin{bmatrix} 0.144 & 0.054 \\ 0.054 & 0.045 \end{bmatrix}^{-1} \begin{bmatrix} -0.0625 - (-4.185) \\ 0.04167 - (-1.3725) \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 30.62 \\ -5.32 \end{bmatrix}$$

-ive sign shows that our assumed redundant action direction is wrong

Flexibility Method for Frames Analysis



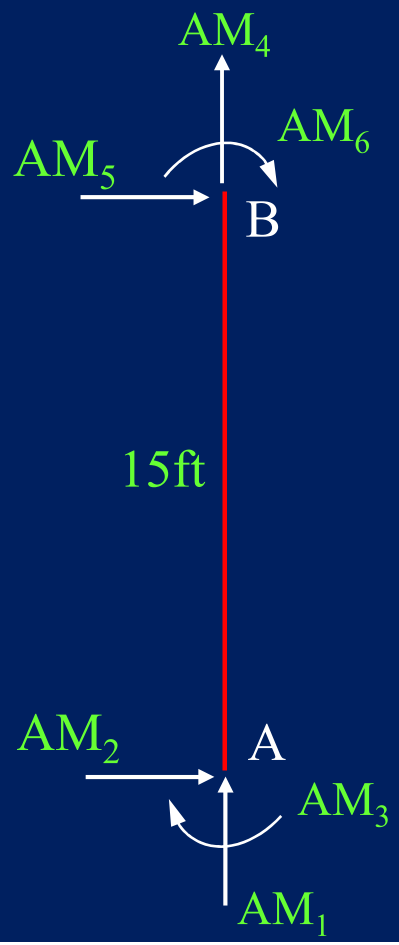
Final determinate structure

Flexibility Method for Frames Analysis

Step # 07: Compute the member end actions also draw the SFD & BMD As we know that

$$[AM] = [AML] + [AMR][AR]$$

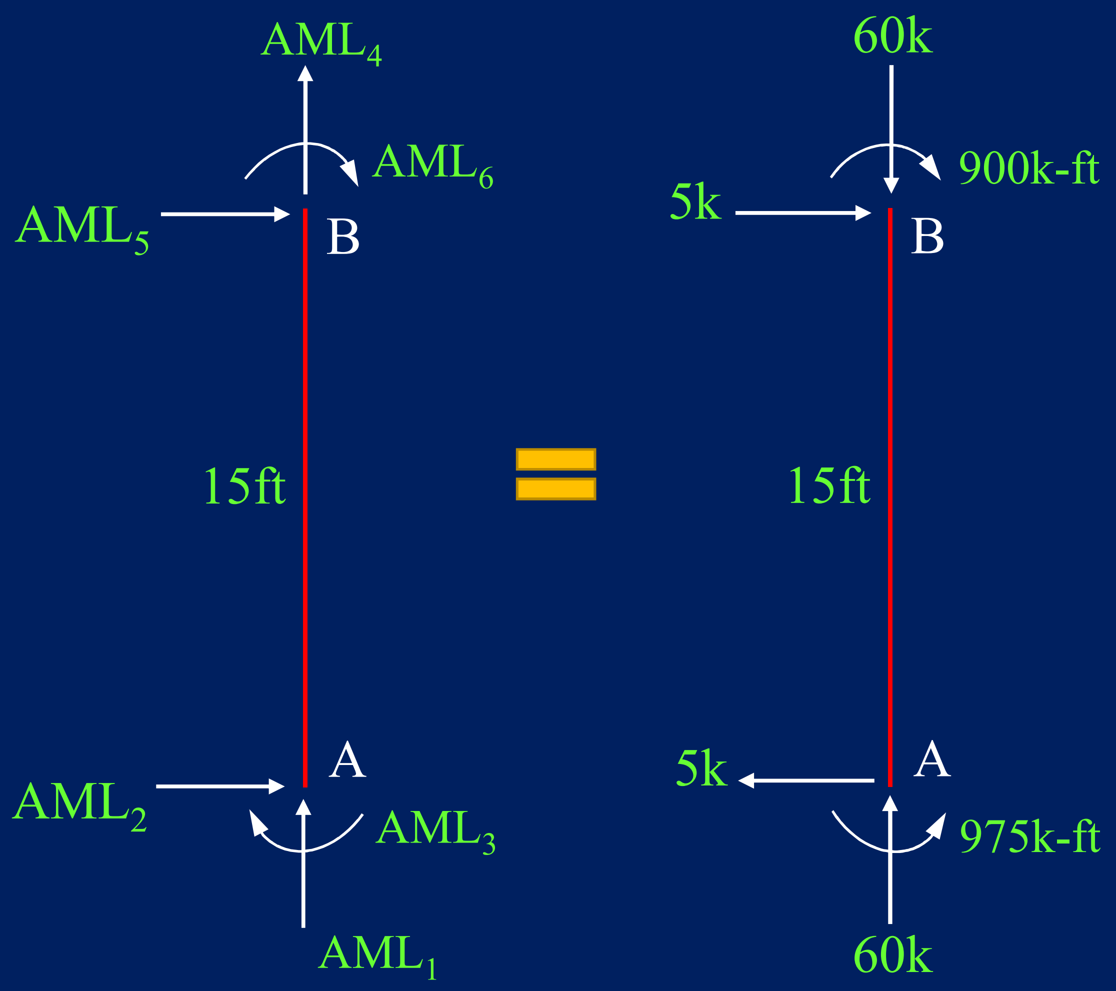
Member AB:



$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix}$$

Flexibility Method for Frames Analysis

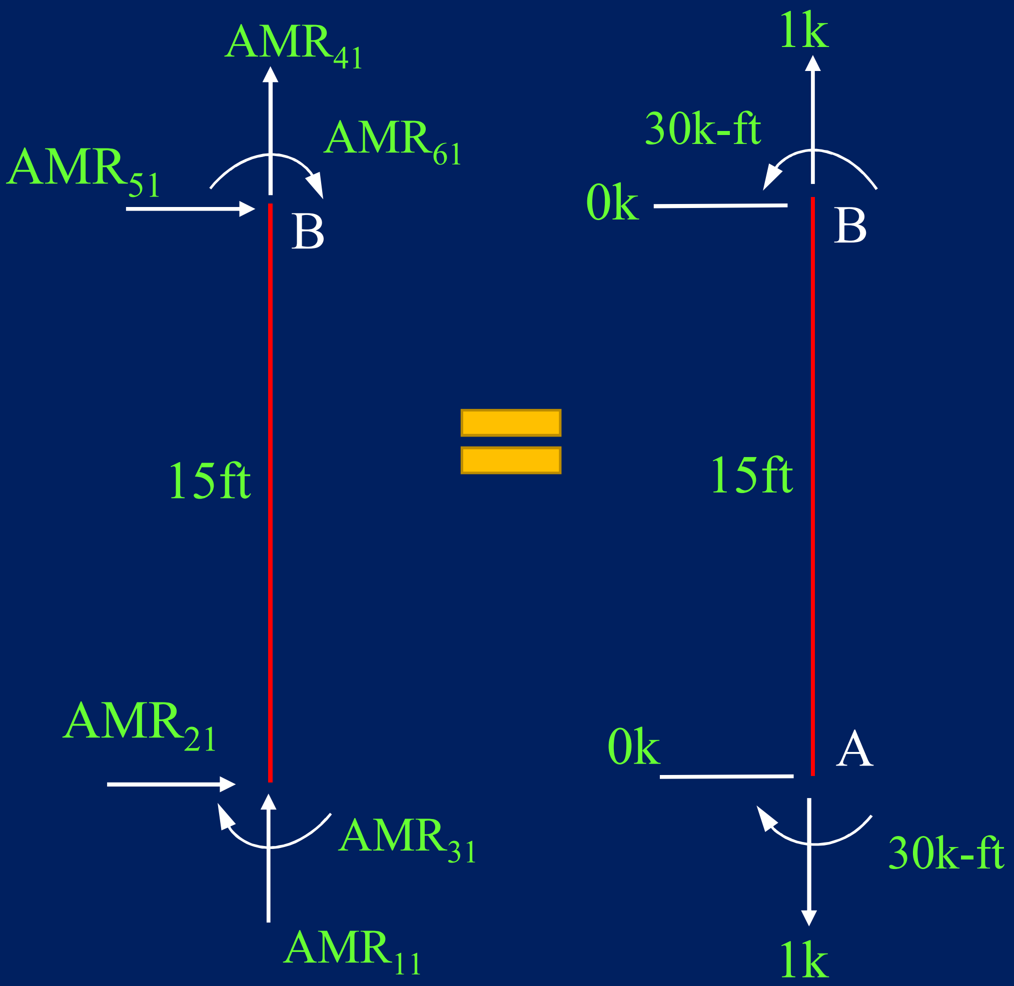
a) Compute AML values.



$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} = \begin{bmatrix} 60 \\ -5 \\ -975 \\ -60 \\ 5 \\ 900 \end{bmatrix}$$

Flexibility Method for Frames Analysis

b) Compute the AMR values: when unit is applied at redundant location 1.

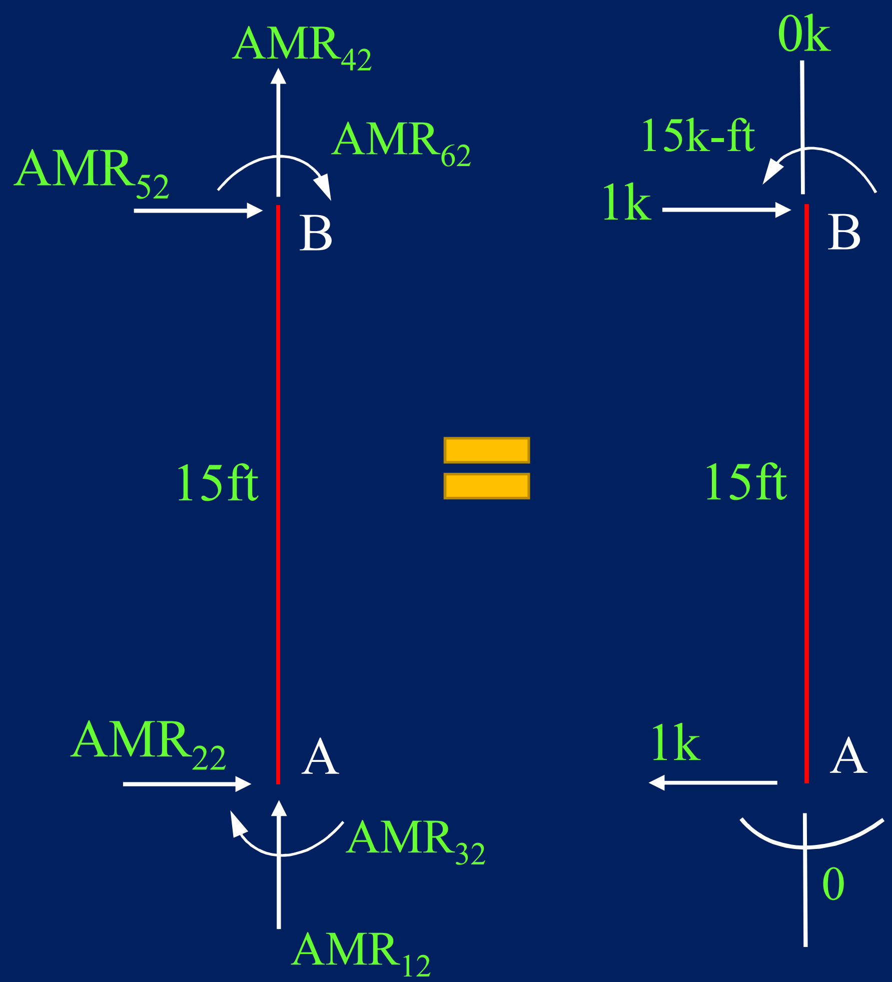


$$[AMR] = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & ? \\ 0 & ? \\ 30 & ? \\ 1 & ? \\ 0 & ? \\ -30 & ? \end{bmatrix}$$

Flexibility Method for Frames Analysis

b) Compute the AMR values: when unit is applied at redundant location 2.



$$[AMR] = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 30 & 0 \\ 1 & 0 \\ 0 & 1 \\ -30 & -15 \end{bmatrix}$$

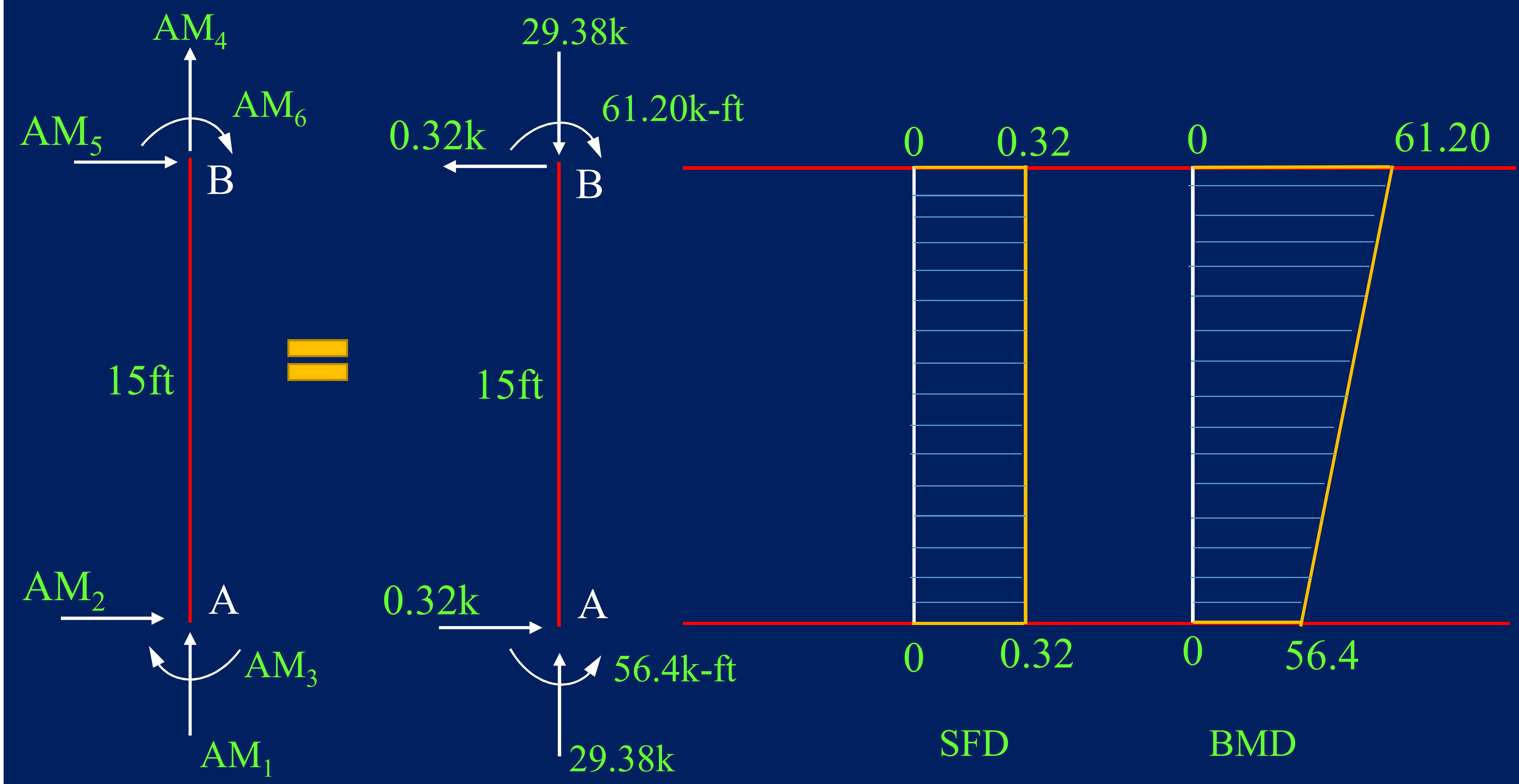
Flexibility Method for Frames Analysis

$$[AMR]_{6*2} = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 30 & 0 \\ 1 & 0 \\ 0 & 1 \\ -30 & -15 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} 60 \\ -5 \\ -975 \\ -60 \\ 5 \\ 900 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 30 & 0 \\ 1 & 0 \\ 0 & 1 \\ -30 & -15 \end{bmatrix} \begin{bmatrix} 30.62 \\ -5.32 \end{bmatrix} = \begin{bmatrix} 29.38 \\ 0.32 \\ -56.4 \\ -29.38 \\ -0.32 \\ 61.20 \end{bmatrix}$$

Flexibility Method for Frames Analysis

So member end actions, Shear force & Bending moment diagrams of member AB



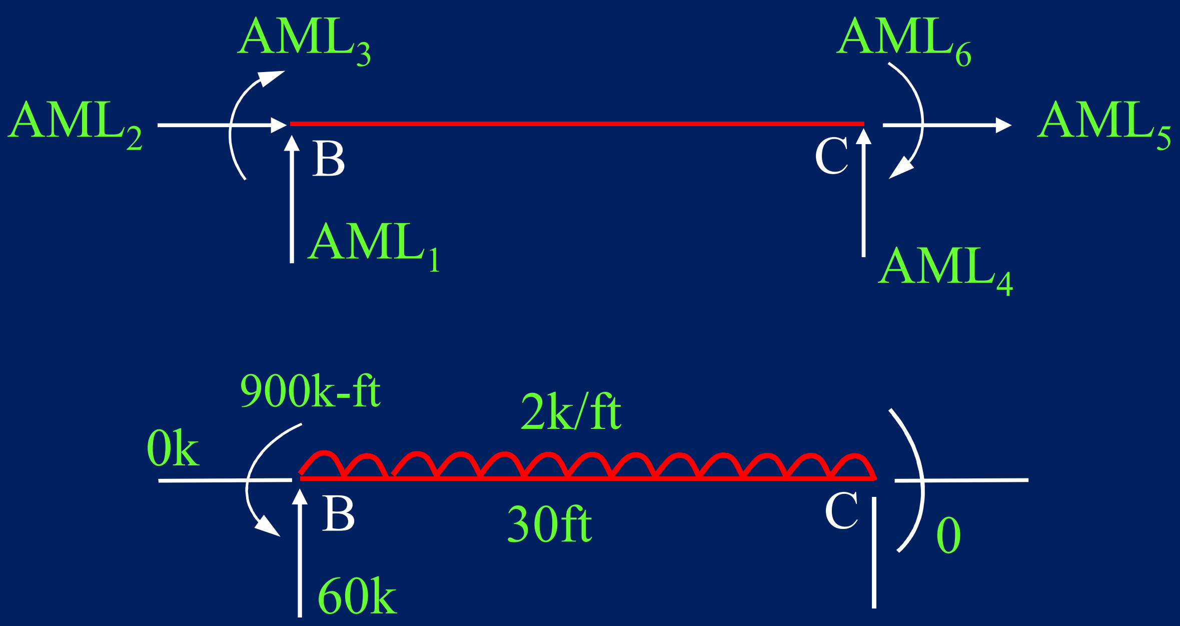
Flexibility Method for Frames Analysis

Member BC: member end actions for member BC are specified below

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix}$$

Flexibility Method for Frames Analysis

a) Compute AML values.



$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ -900 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Flexibility Method for Frames Analysis

b) Compute the AMR values: when unit is applied at redundant location 1.

$$[AMR] = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & ? \\ 0 & ? \\ 30 & ? \\ 1 & ? \\ 0 & ? \\ 0 & ? \end{bmatrix}$$

Flexibility Method for Frames Analysis

b) Compute the AMR values: when unit is applied at redundant location 2.

$$[AMR] = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 30 & 15 \\ 1 & 0 \\ 0 & 1 \\ 0 & -15 \end{bmatrix}$$

Flexibility Method for Frames Analysis

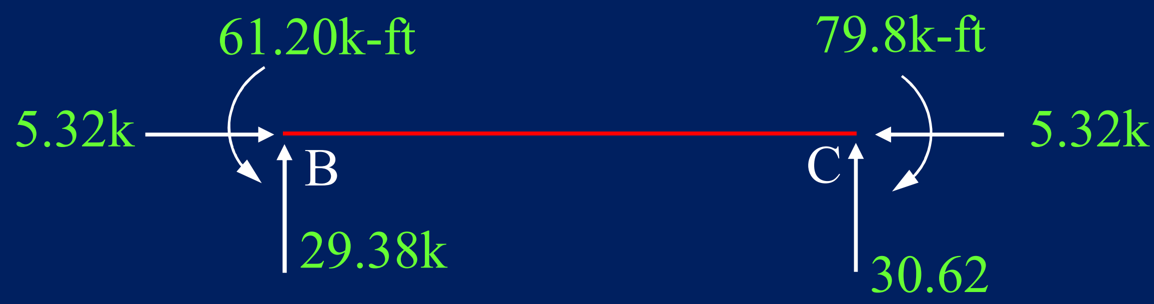
$$[AMR]_{6\,*\,2} = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 30 & 15 \\ 1 & 0 \\ 0 & 1 \\ 0 & -15 \end{bmatrix}$$

So member end actions will be

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ -900 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 30 & 15 \\ 1 & 0 \\ 0 & 1 \\ 0 & -15 \end{bmatrix} \begin{bmatrix} 30.62 \\ -5.32 \end{bmatrix} = \begin{bmatrix} 29.38 \\ 5.32 \\ -61.20 \\ 30.62 \\ -5.32 \\ 79.8 \end{bmatrix}$$

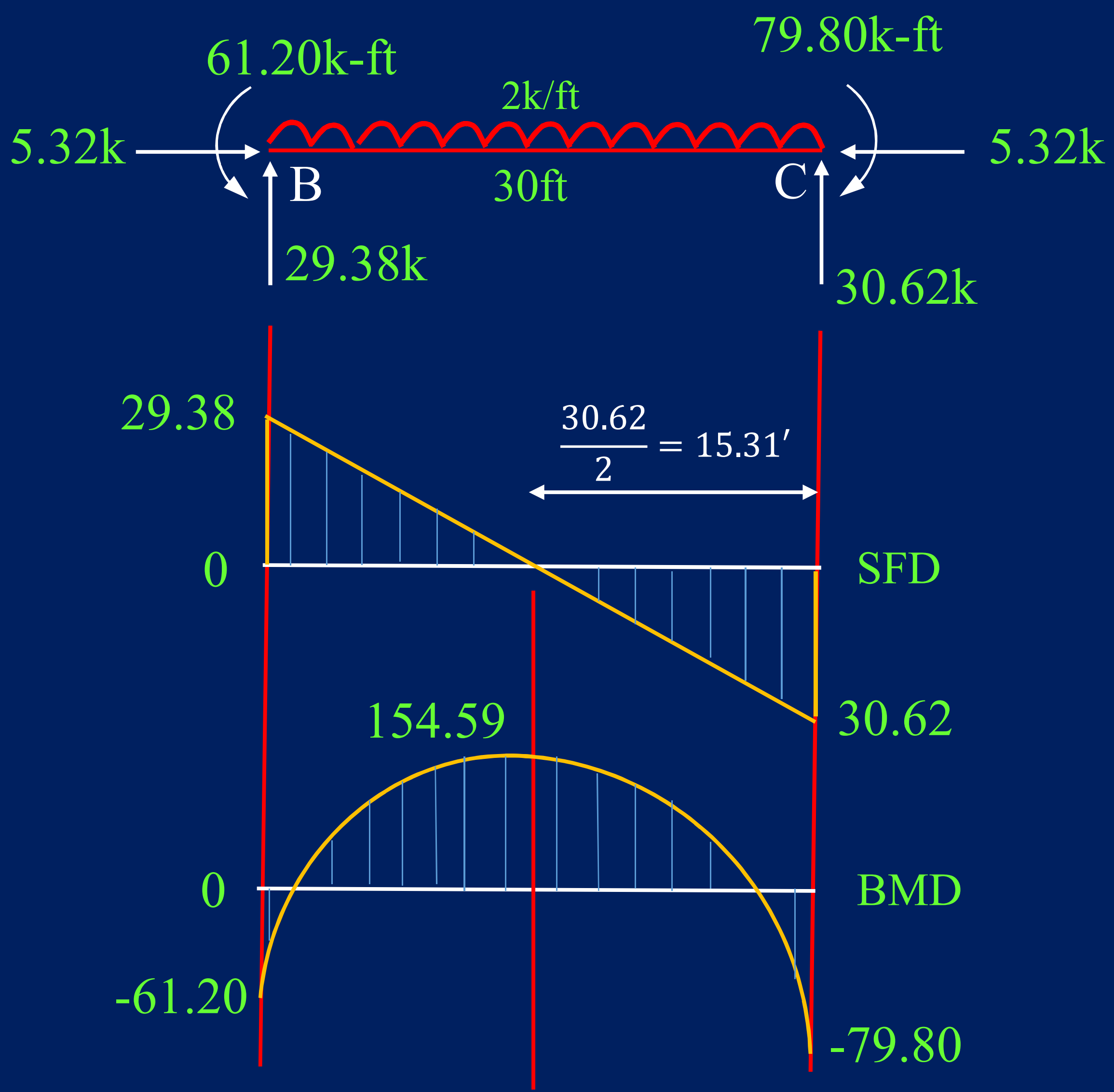
Flexibility Method for Frames Analysis

So member end actions of member BC



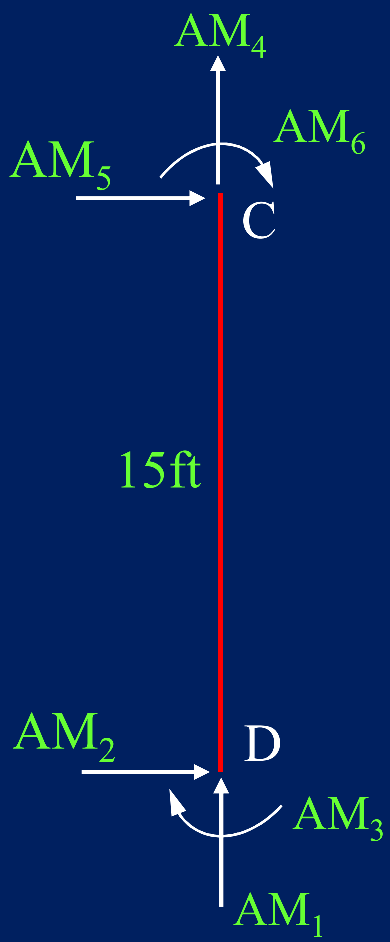
Flexibility Method for Frames Analysis

Member BC:



Flexibility Method for Frames Analysis

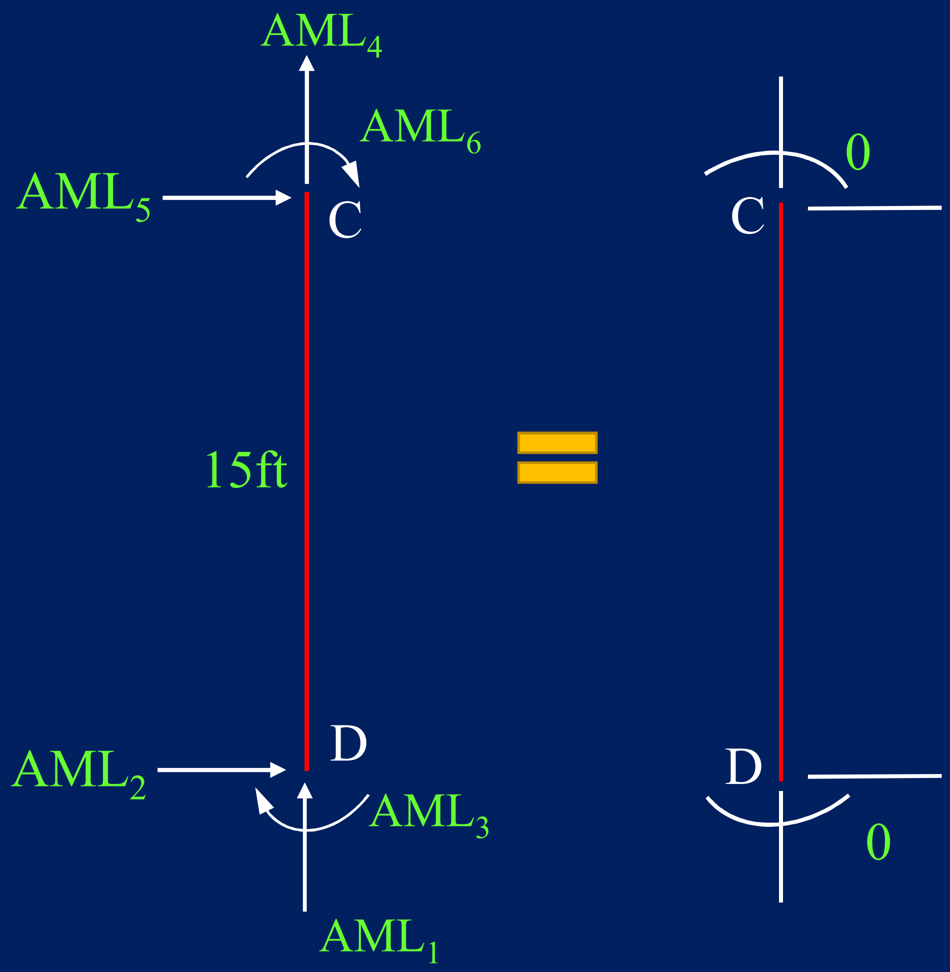
Member CD: member end actions for member CD are specified below



$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix}$$

Flexibility Method for Frames Analysis

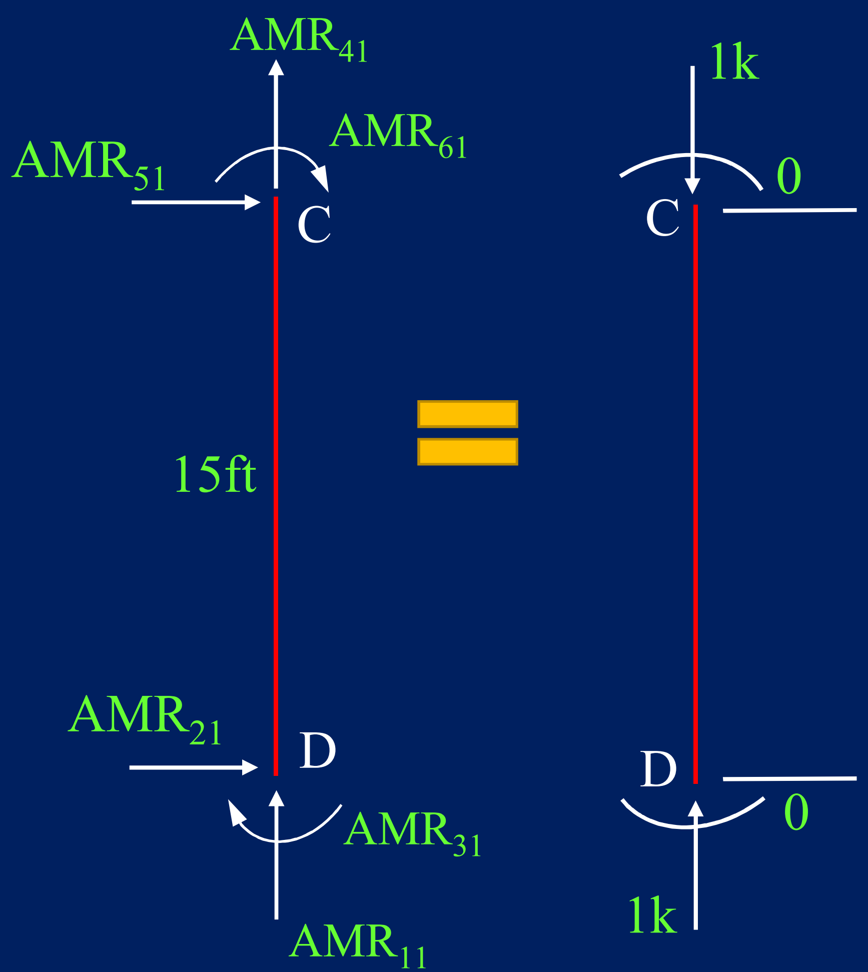
a) Compute AML values.



$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Flexibility Method for Frames Analysis

b) Compute the AMR values: when unit is applied at redundant location 1.

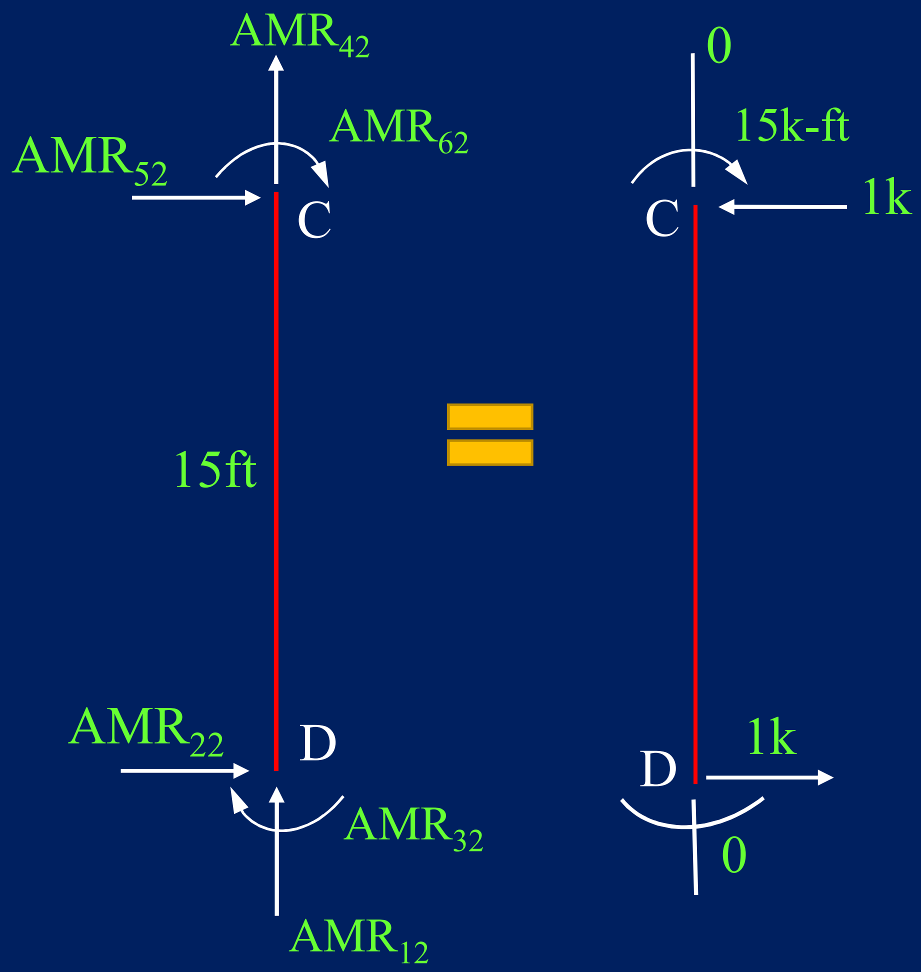


$$[AMR] = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & ? \\ 0 & ? \\ 0 & ? \\ -1 & ? \\ 0 & ? \\ 0 & ? \end{bmatrix}$$

Flexibility Method for Frames Analysis

b) Compute the AMR values: when unit is applied at redundant location 2.



$$[AMR] = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 15 \end{bmatrix}$$

Flexibility Method for Frames Analysis

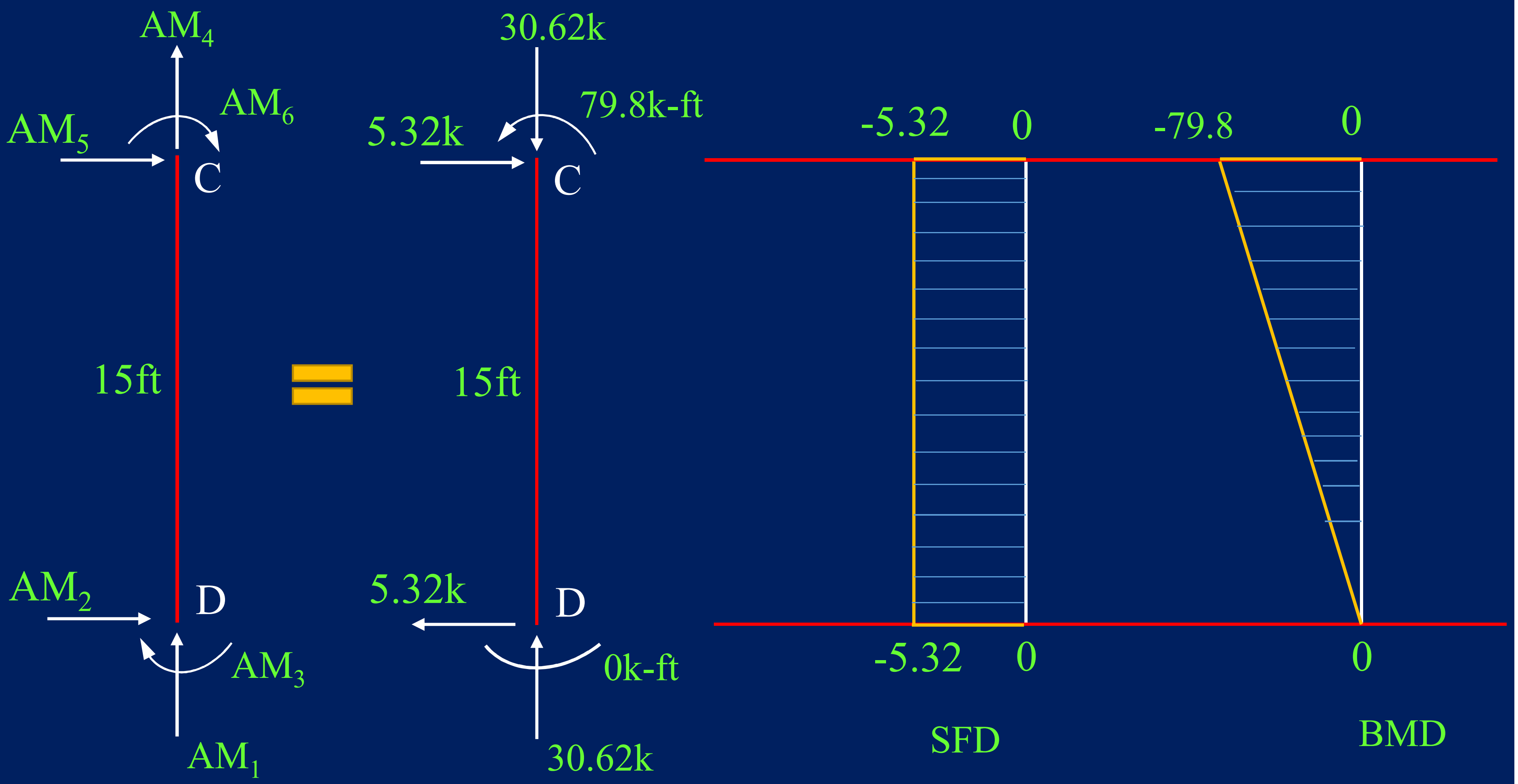
$$[AMR]_{6\,*\,2} = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 15 \end{bmatrix}$$

So member end actions will be

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 15 \end{bmatrix} \begin{bmatrix} 30.62 \\ -5.32 \end{bmatrix} = \begin{bmatrix} 30.62 \\ -5.32 \\ 0 \\ -30.62 \\ 5.32 \\ -79.8 \end{bmatrix}$$

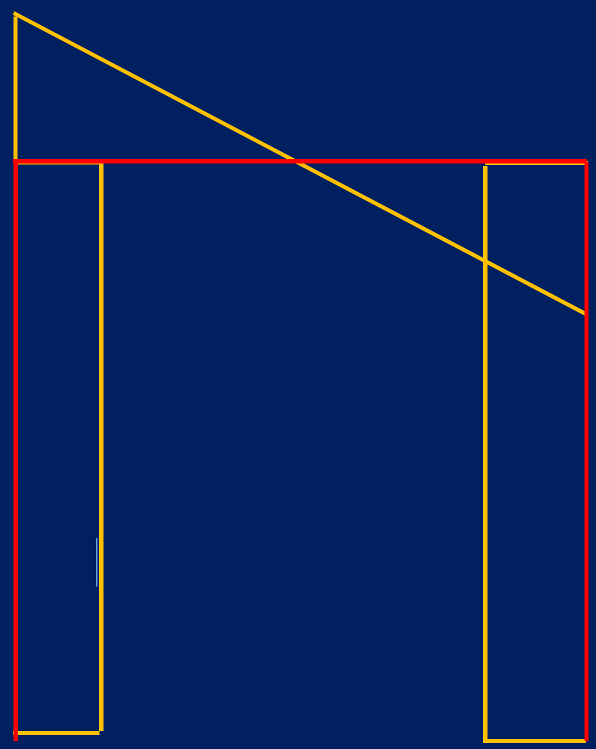
Flexibility Method for Frames Analysis

So member end actions, Shear force & bending moment diagrams of member CD

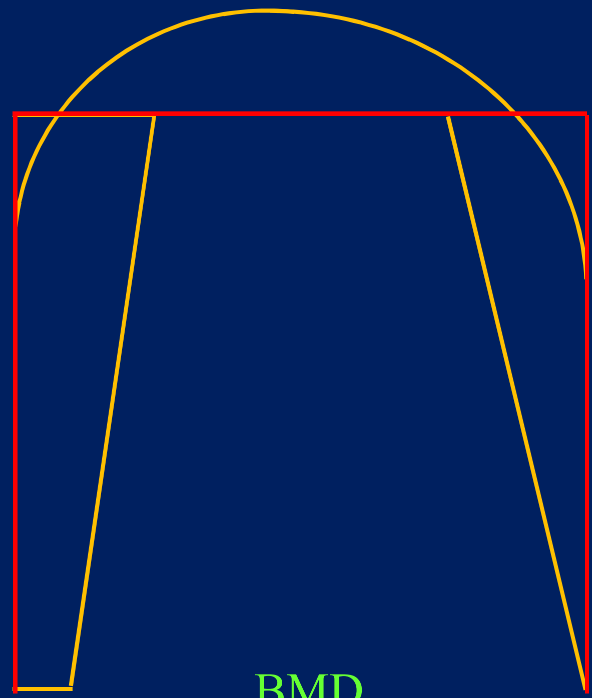


Flexibility Method for Frames Analysis

Combined shear force & bending moment diagrams:



SFD

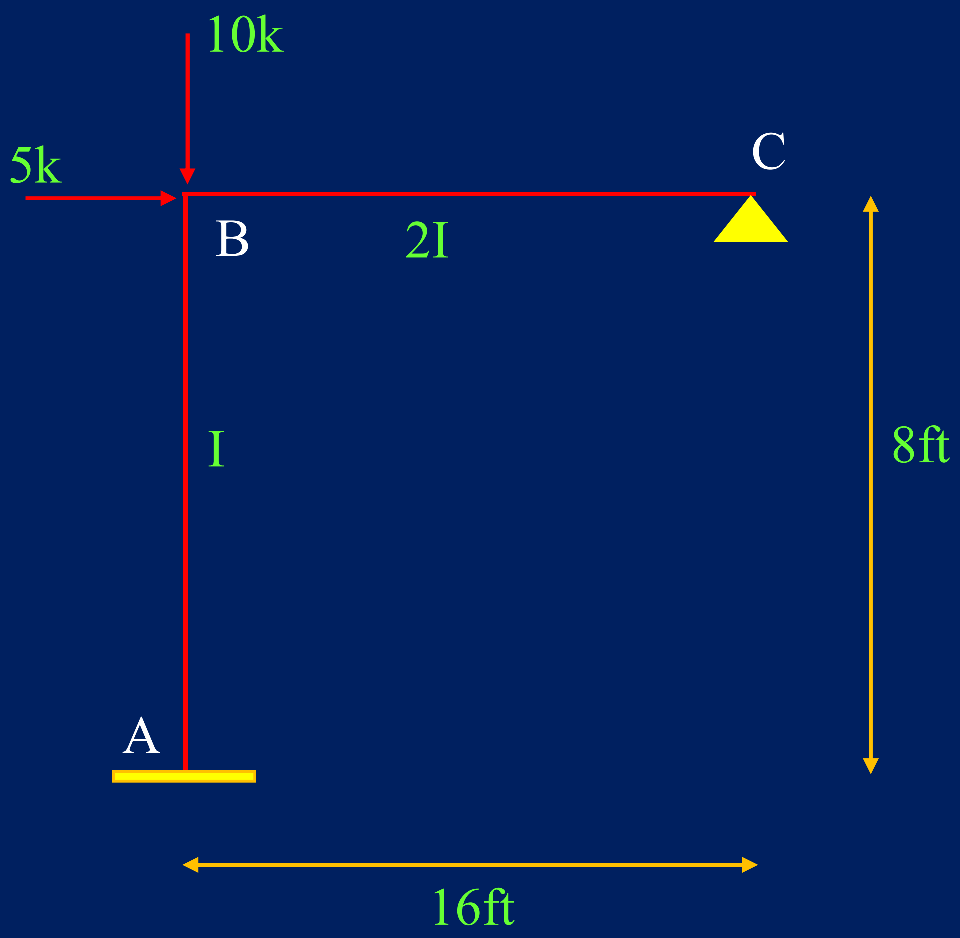


BMD

Flexibility Method for Frames Analysis

Problem 03: Analyze the given frame using flexibility method.

Take EI = constant



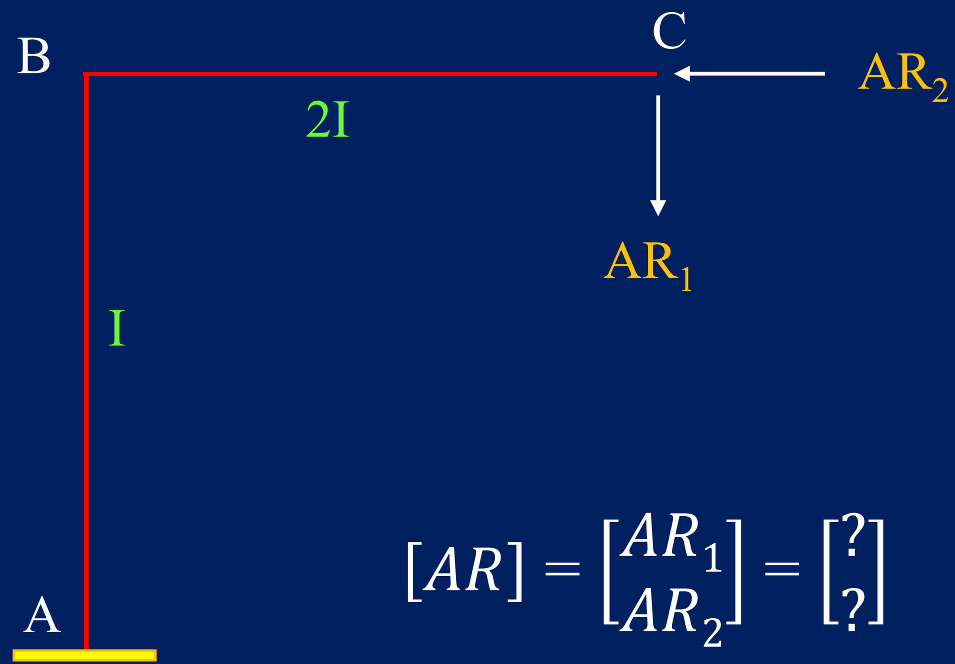
S.I = 2 degree

So two redundant actions should be chosen.

Flexibility Method for Frames Analysis

- **Step # 01:** Identify the redundants and obtain BDS also compute [DRS] values.

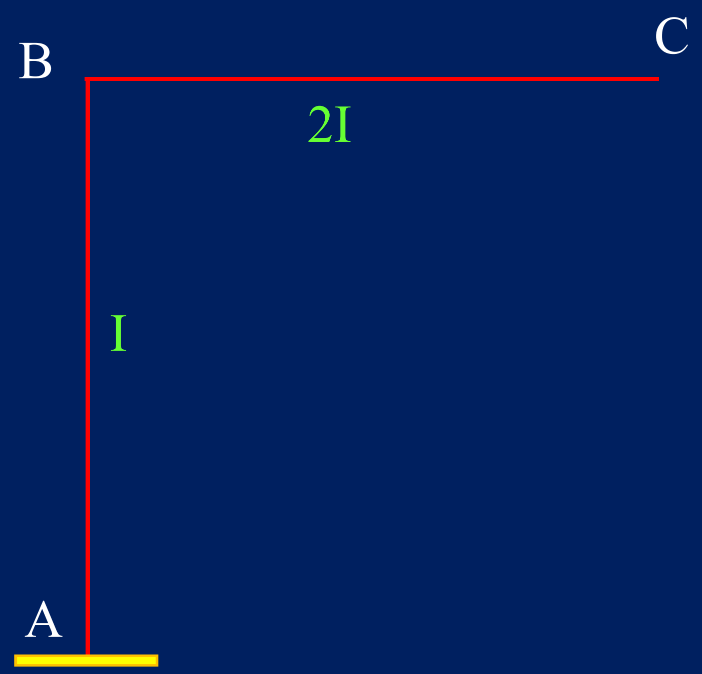
Vertical & horizontal reaction at C are chosen as redundant actions



$$[AR] = \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[DRS] = \begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Flexibility Method for Frames Analysis



Basic determinate structure (BDS) or Primary structure or Released structure

Flexibility Method for Frames Analysis

- **Step # 02:** Analyze the BDS (primary structure) under the following loading conditions

i. Analyze the BDS when acted upon by the external loads and write the equations of bending moment M for different segments.

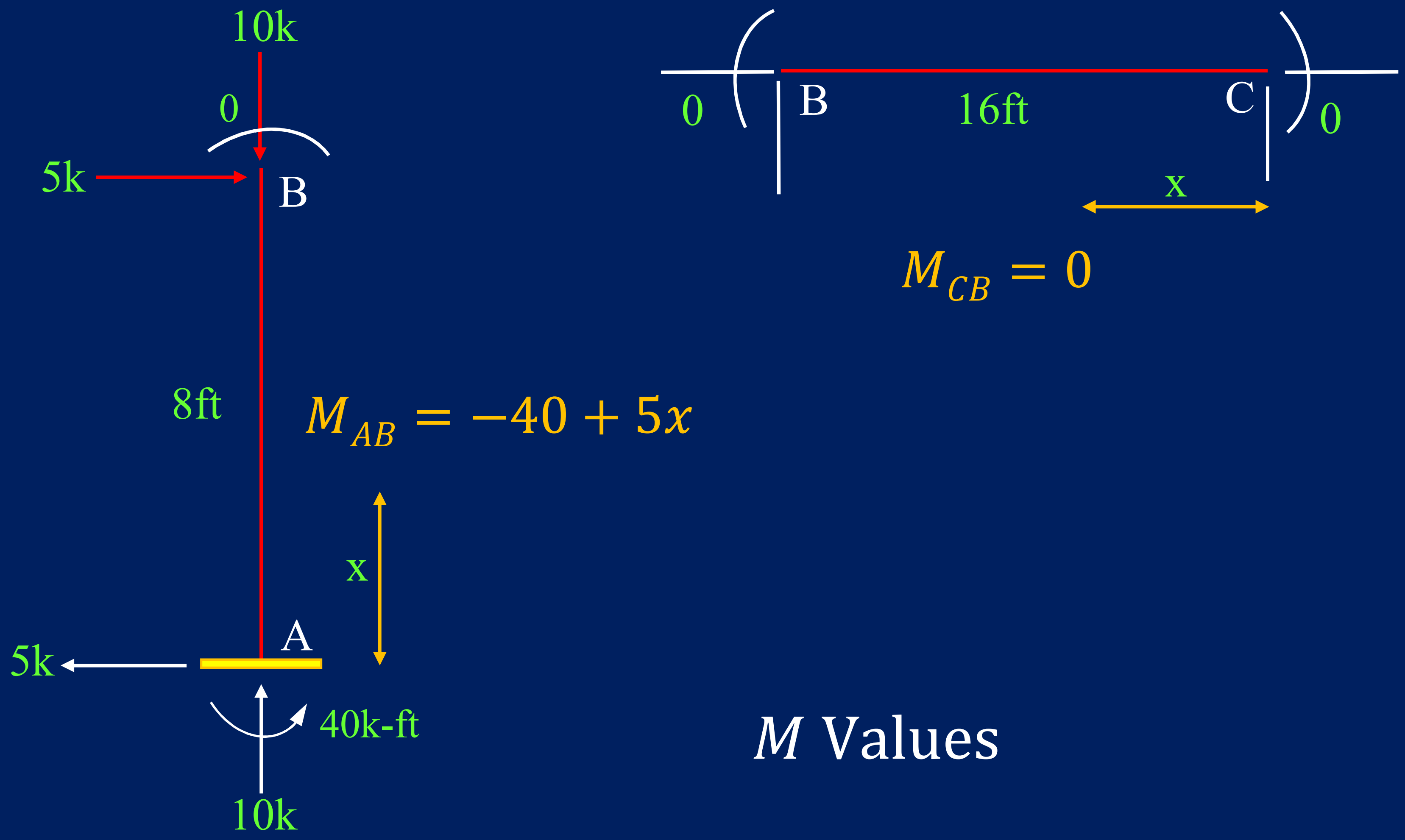
ii. Analyze the BDS when acted upon by the unit load at redundant location 1 & write the equations of BM m_1 for different segments.

iii. Analyze the BDS when acted upon by the unit load at redundant location 2 & write the equations of BM m_2 for different segments.

Note: To write an equation take a section in each segment of loading & then write BM equation with proper sign convention.

Flexibility Method for Frames Analysis

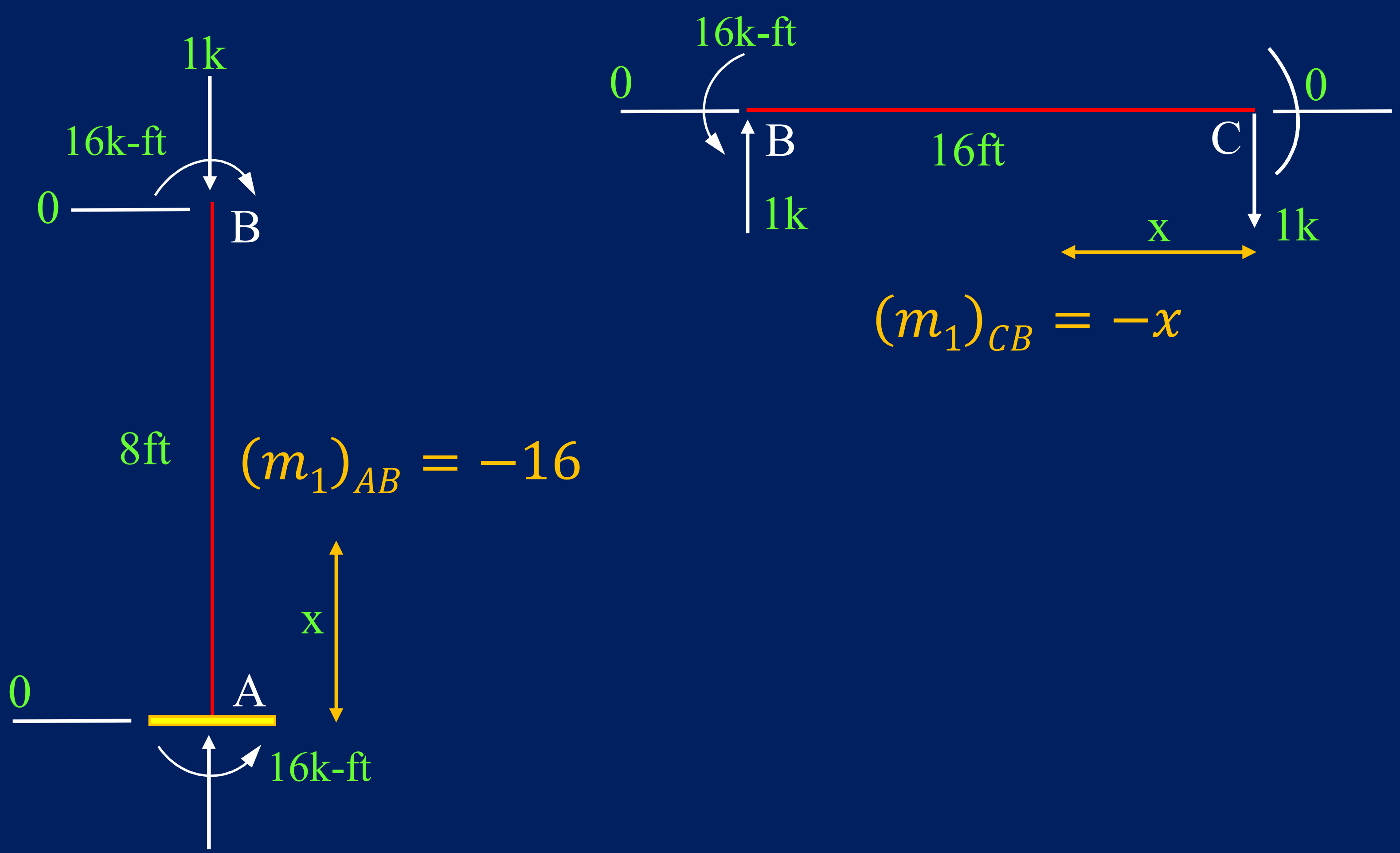
i. BDS acted upon by the external loads



M Values

Flexibility Method for Beams Analysis

i. BDS acted upon by unit load at redundant location 1.



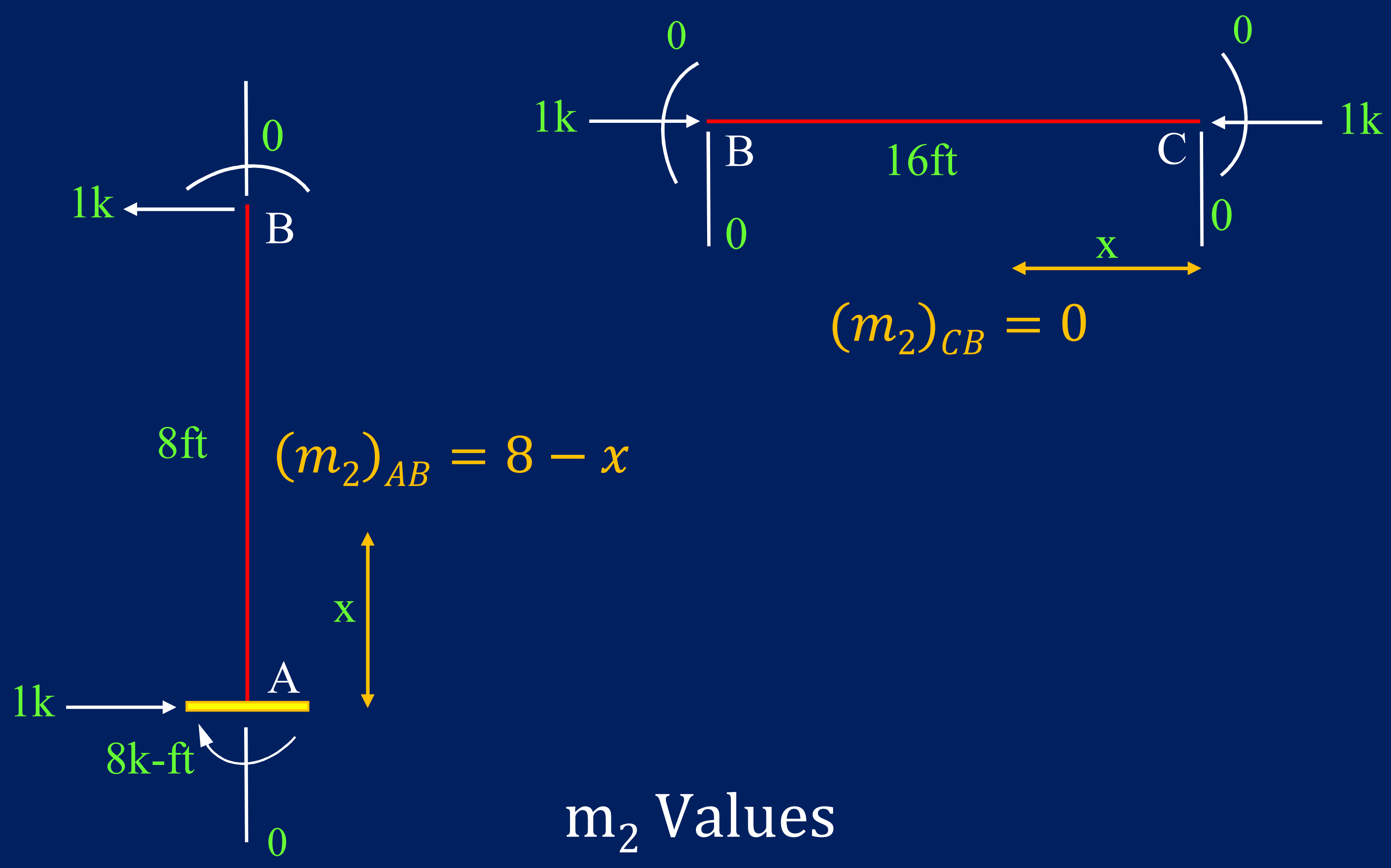
$$(m_1)_{AB} = -16$$

$$(m_1)_{CB} = -x$$

m_1 Values

Flexibility Method for Beams Analysis

i. BDS acted upon by unit load at redundant location 2.



m_2 Values

Flexibility Method for Beams Analysis

- **Step # 03:** Develop Bending moment equation table.

Segment	AB	BC
Origin	A	B
Limits	0 to 8 ft	0 to 16 ft
Moment of inertia , I	I	2I
M values	-40 + 5x	0
m_1 values	-16	-x
m_2 values	8 - x	0

Flexibility Method for Beams Analysis

- **Step # 04:** Find BDS Displacements due to external loads

  or Compute the values of DRL matrix.

$$\boldsymbol{DRL_1 = \sum_{i=1}^{n} \int \frac{M\, m_1}{E_i I_i} dx}$$

$$DRL_1 = \frac{1}{EI}\left[\int_0^8 (5x - 40)(-16)dx + 0\right]$$

$$DRL_1 = \frac{2560}{EI}$$

Flexibility Method for Beams Analysis

$$\boldsymbol{DRL_2 = \sum_{i=1}^{n} \int \frac{M\, m_2}{E_i I_i} dx}$$

$$DRL_2 = \frac{1}{EI}\left[\int_0^8 (5x - 40)(8 - x)dx + 0\right]$$

$$DRL_2 = \frac{-853.33}{EI}$$

$$DRL_1 = \frac{2560}{EI} \qquad DRL_2 = \frac{-853.33}{EI}$$

$$[DRL] = \begin{bmatrix} DRL_1 \\ DRL_2 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 2560 \\ -853.33 \end{bmatrix}$$

Flexibility Method for Frames Analysis

- **Step # 05:** Find BDS Displacements due to unit loads

  or Compute the values of flexibility coefficients

  & flexibility matrix.

$$f_{11} = \sum_{i=1}^{n} \int \frac{m_1 \, m_1}{E_i I_i} dx$$

$$f_{11} = \frac{1}{EI}\left[\int_0^8 (-16)(-16)dx + \int_0^{16} \frac{(-x)(-x)}{2} dx\right]$$

$$f_{11} = \frac{2730.67}{EI}$$

Flexibility Method for Frames Analysis

$$f_{22} = \sum_{i=1}^{n} \int \frac{m_2\, m_2}{E_i I_i} dx$$

$$f_{12} = f_{21} = \frac{1}{EI}\left[\int_0^8 (-16)(8-x)dx + 0\right]$$

$$f_{12} = f_{21} = \frac{-512}{EI}$$

$$f_{12} = f_{21} = \sum_{i=1}^{n} \int \frac{m_1\, m_2}{E_i I_i} dx$$

$$f_{22} = \frac{1}{EI}\left[\int_0^8 (8-x)(8-x)dx + 0\right]$$

$$f_{22} = \frac{170.67}{EI}$$

Flexibility Method for Frames Analysis

$$f_{11} = \frac{2730.67}{EI} \qquad f_{22} = \frac{170.67}{EI}$$

$$f_{12} = f_{21} = \frac{-512}{EI}$$

$$[\,f\,] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$$

$$[\,f\,] = \frac{1}{EI}\begin{bmatrix} 2730.67 & -512 \\ -512 & 170.67 \end{bmatrix}$$

Flexibility Method for Frames Analysis

Step # 06: Apply compatibility and principle of superposition at the location of the removed redundants and solve the matrices to compute the values of redundant actions AR. As we know

$$[DRS] = [DRL] + [f] \bullet [AR]$$

From this

$$[AR] = [f]^{-1} \bullet [DRS - DRL]$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

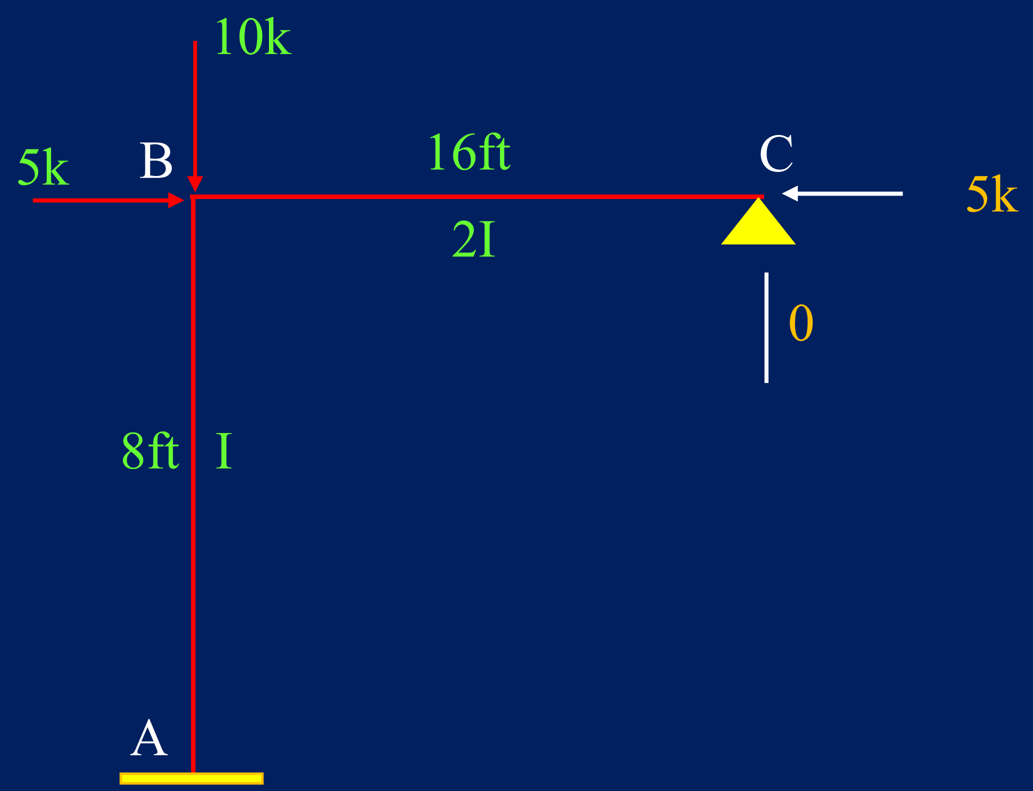
Flexibility Method for Frames Analysis

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

$$\begin{bmatrix} AR1 \\ AR2 \end{bmatrix} = EI \begin{bmatrix} 2730.67 & -512 \\ -512 & 170.67 \end{bmatrix}^{-1} \begin{bmatrix} 0 - (2560) \\ 0 - (-853.33) \end{bmatrix} \times \frac{1}{EI}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

Flexibility Method for Frames Analysis



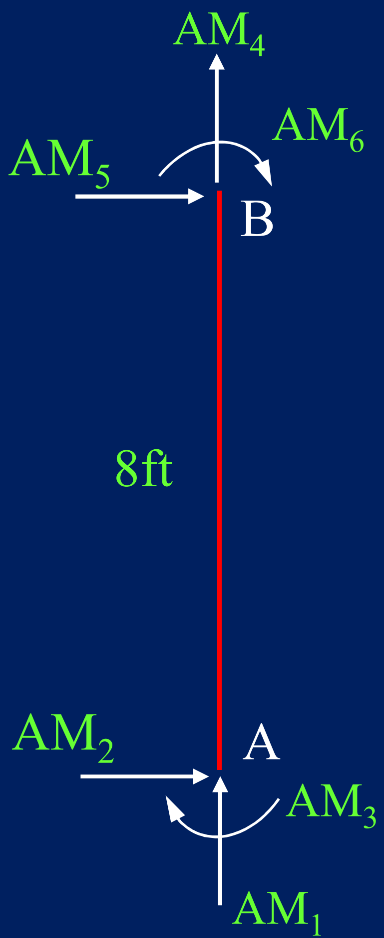
Final determinate structure

Flexibility Method for Frames Analysis

Step # 07: Compute the member end actions. Also Draw shear force and bending moment diagrams As we know that

$$[AM] = [AML] + [AMR][AR]$$

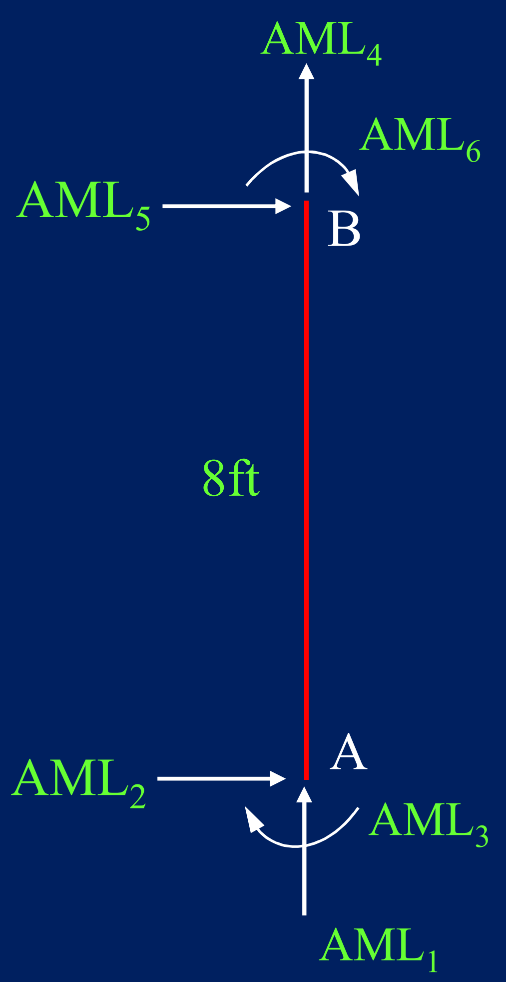
Member AB:



$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix}$$

Flexibility Method for Frames Analysis

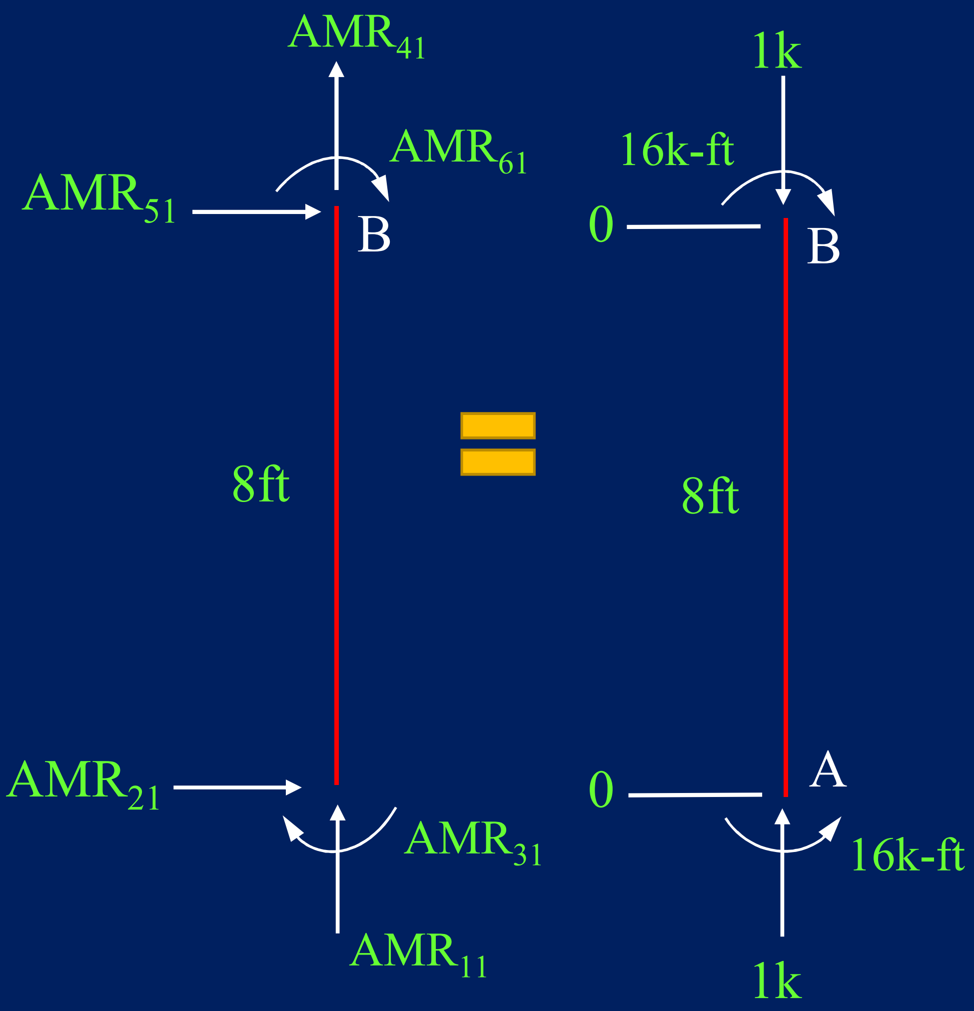
a) Compute AML values.



$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \\ -40 \\ -10 \\ 5 \\ 0 \end{bmatrix}$$

Flexibility Method for Frames Analysis

b) Compute the AMR values: when unit is applied at redundant location 1.

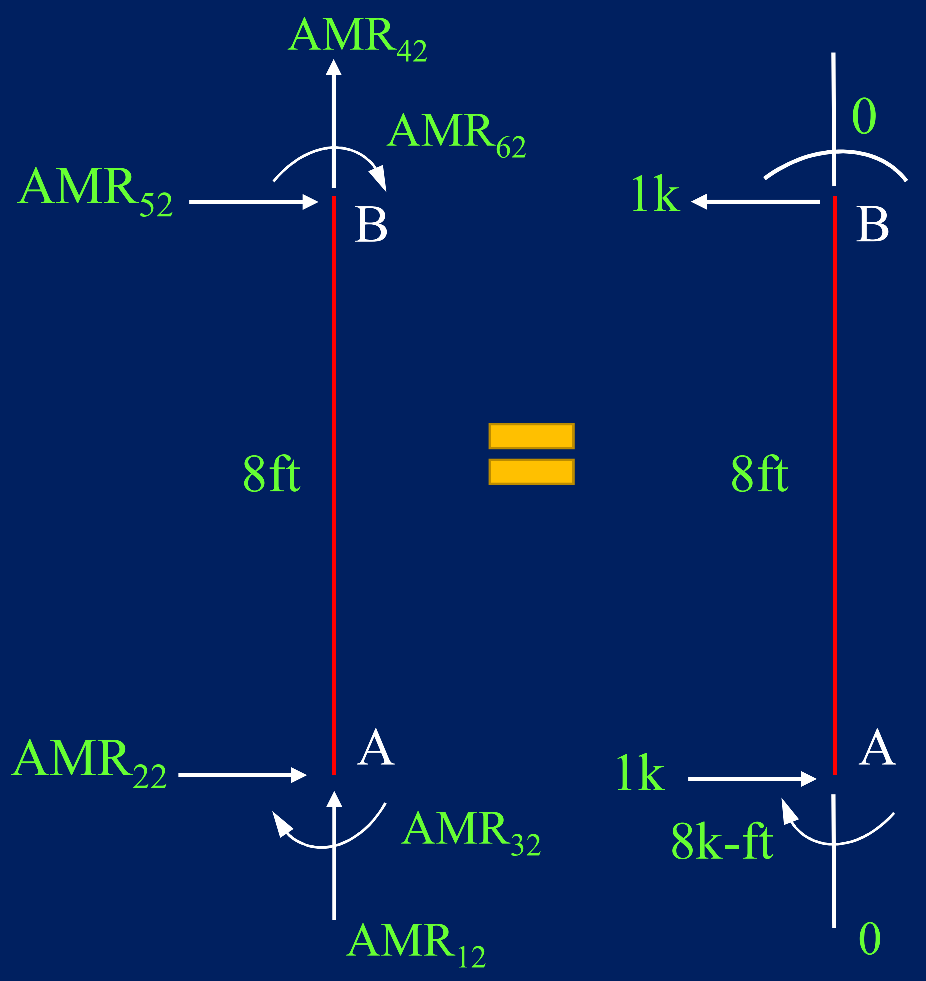


$$[AMR] = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & ? \\ 0 & ? \\ -16 & ? \\ -1 & ? \\ 0 & ? \\ 16 & ? \end{bmatrix}$$

Flexibility Method for Frames Analysis

b) Compute the AMR values: when unit is applied at redundant location 2.



$$[AMR] = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -16 & 8 \\ -1 & 0 \\ 0 & -1 \\ 16 & 0 \end{bmatrix}$$

Flexibility Method for Frames Analysis

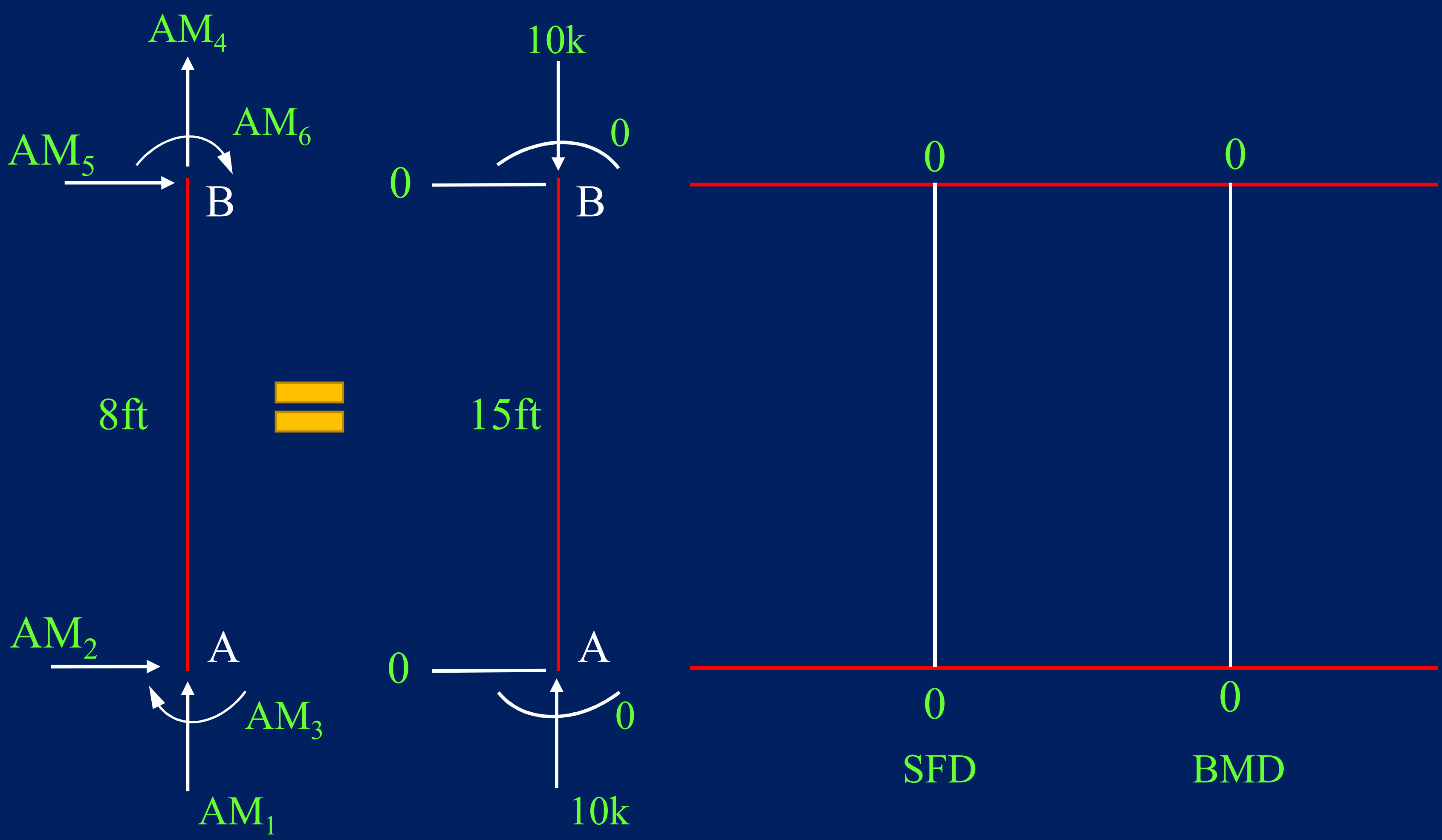
$$[AMR]_{6*2} = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -16 & 8 \\ -1 & 0 \\ 0 & -1 \\ 16 & 0 \end{bmatrix}$$

So member end actions will be

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \\ -40 \\ -10 \\ 5 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -16 & 8 \\ -1 & 0 \\ 0 & -1 \\ 16 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ -10 \\ 0 \\ 0 \end{bmatrix}$$

Flexibility Method for Frames Analysis

So member end actions, Shear force & bending moment diagrams of member AB



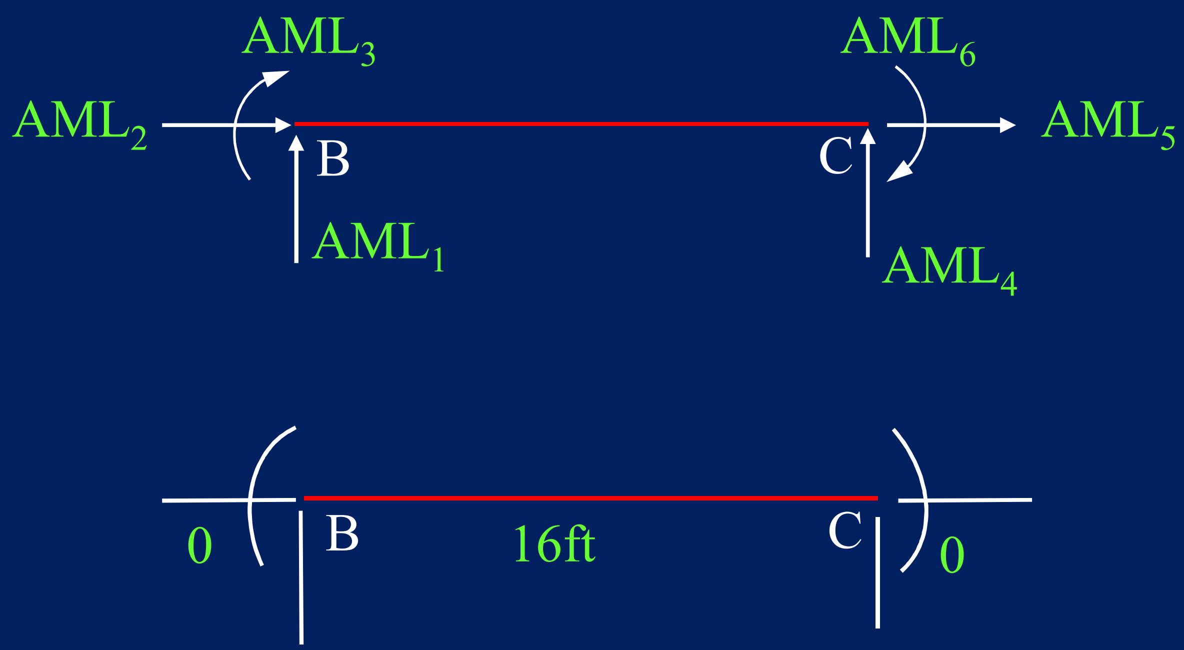
Flexibility Method for Frames Analysis

Member BC: member end actions for member BC are specified below

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix}$$

Flexibility Method for Frames Analysis

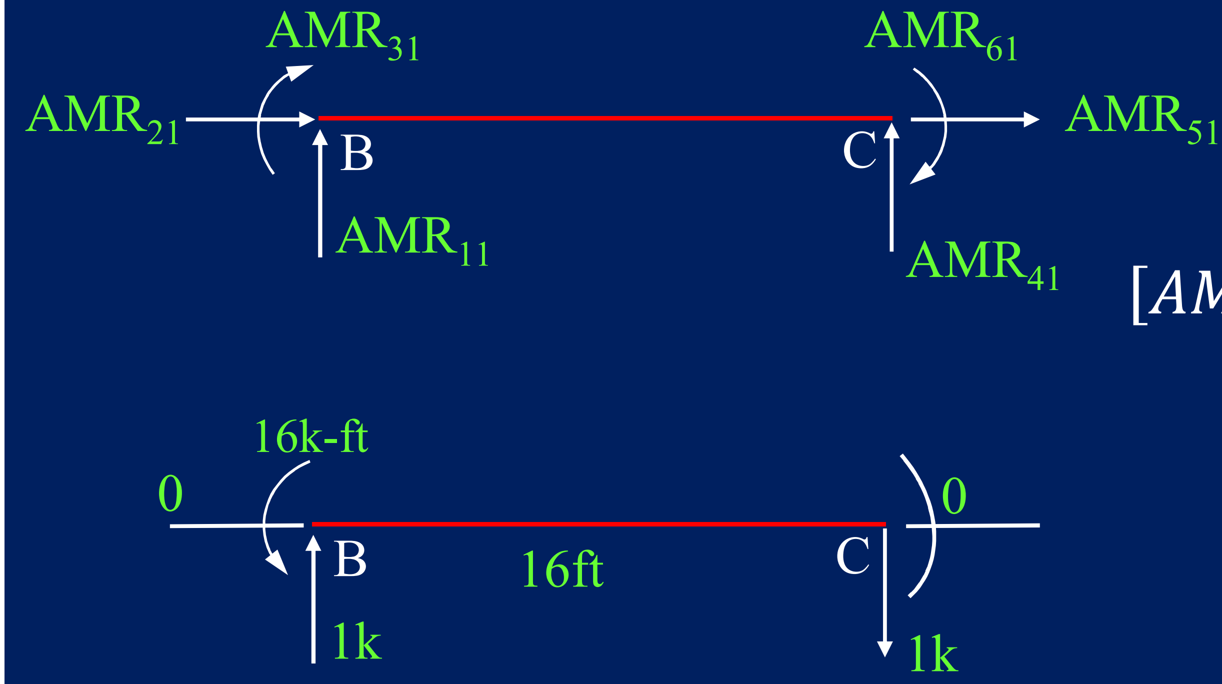
a) Compute AML values.



$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Flexibility Method for Frames Analysis

b) Compute the AMR values: when unit is applied at redundant location 1.

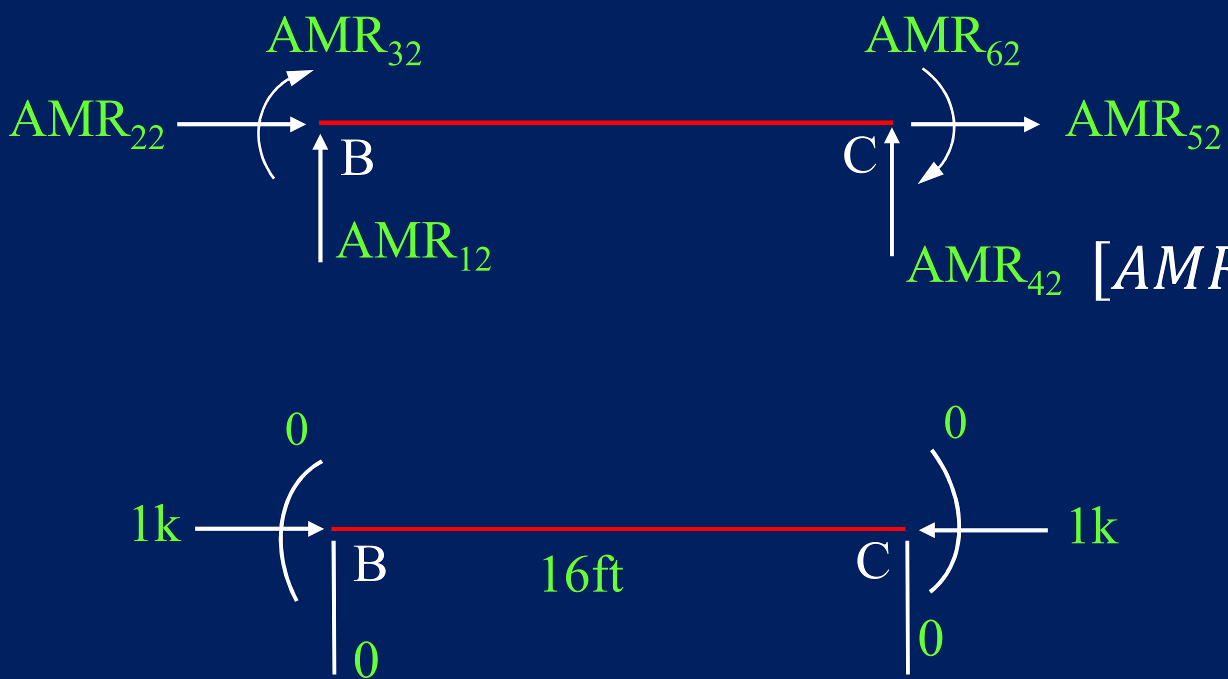


$$[AMR] = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & ? \\ 0 & ? \\ -16 & ? \\ -1 & ? \\ 0 & ? \\ 0 & ? \end{bmatrix}$$

Flexibility Method for Frames Analysis

b) Compute the AMR values: when unit is applied at redundant location 2.



$$[AMR] = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -16 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}$$

Flexibility Method for Frames Analysis

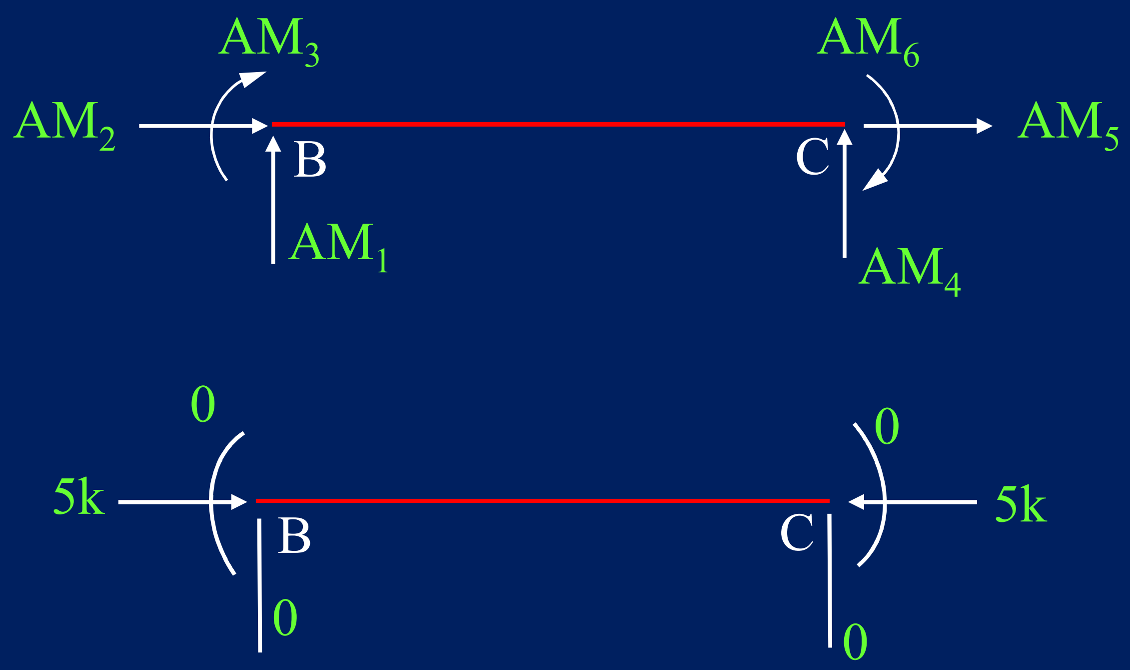
$$[AMR]_{6*2} = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -16 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}$$

So member end actions will be

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -16 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 0 \\ 0 \\ -5 \\ 0 \end{bmatrix}$$

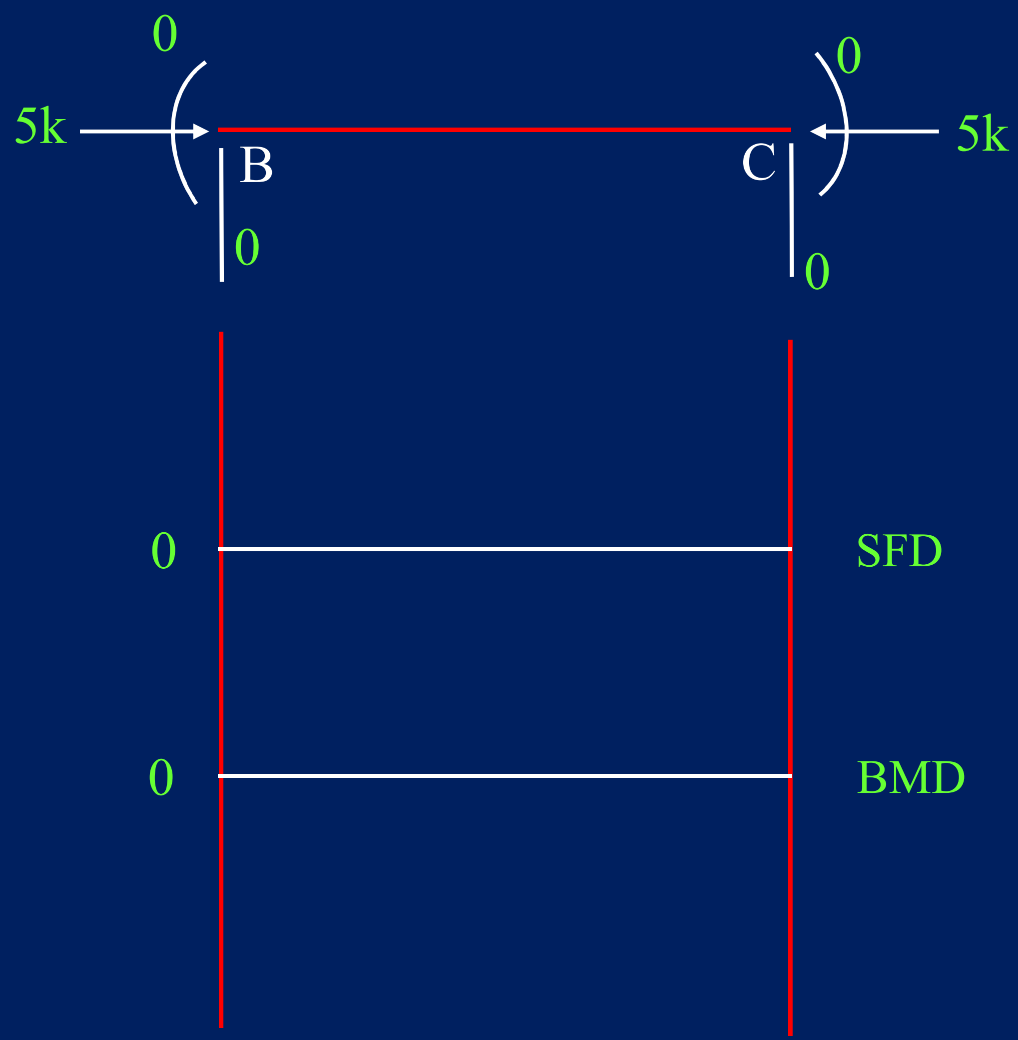
Flexibility Method for Frames Analysis

So member end actions of member BC

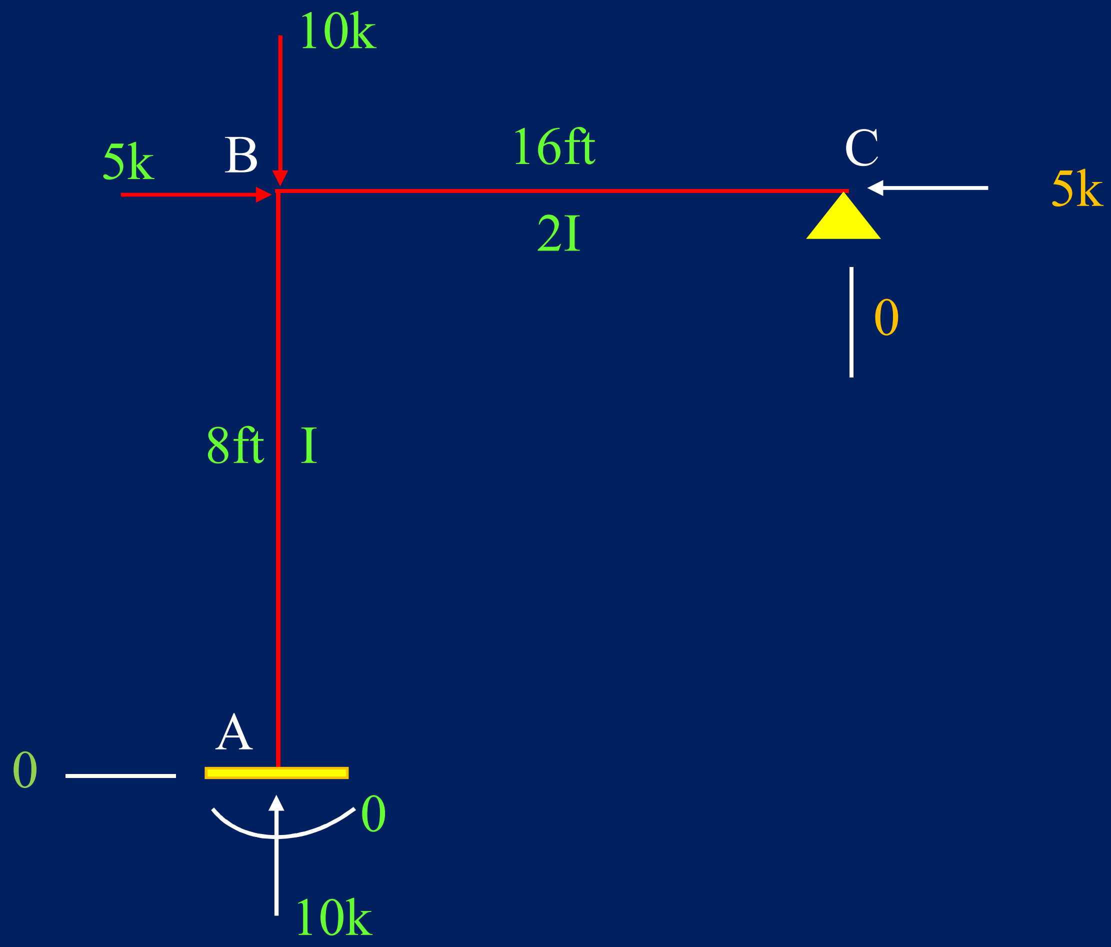


Flexibility Method for Frames Analysis

Shear force & bending moment diagrams of member BC



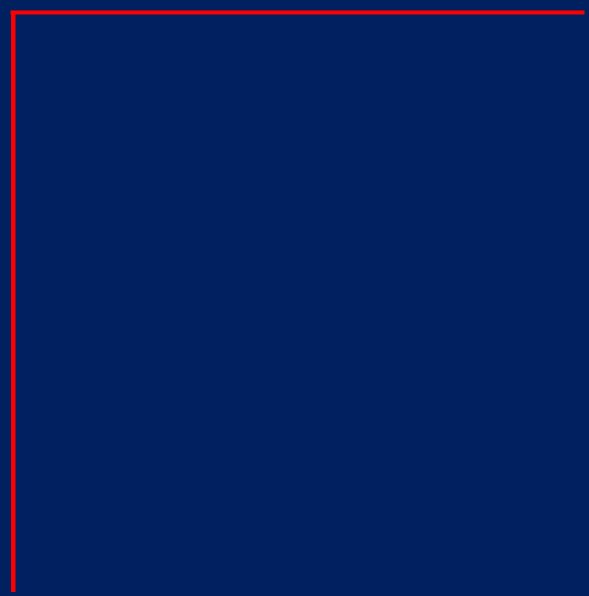
Flexibility Method for Frames Analysis



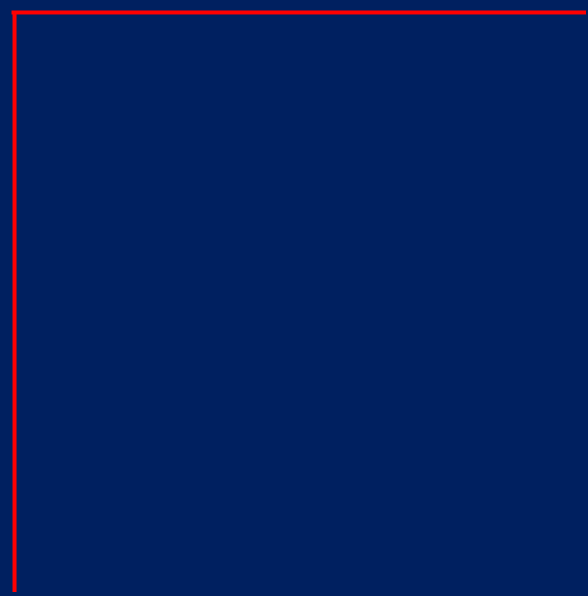
Complete analyzed structure

Flexibility Method for Frames Analysis

Combined shear force & bending moment diagram:



SFD



BMD

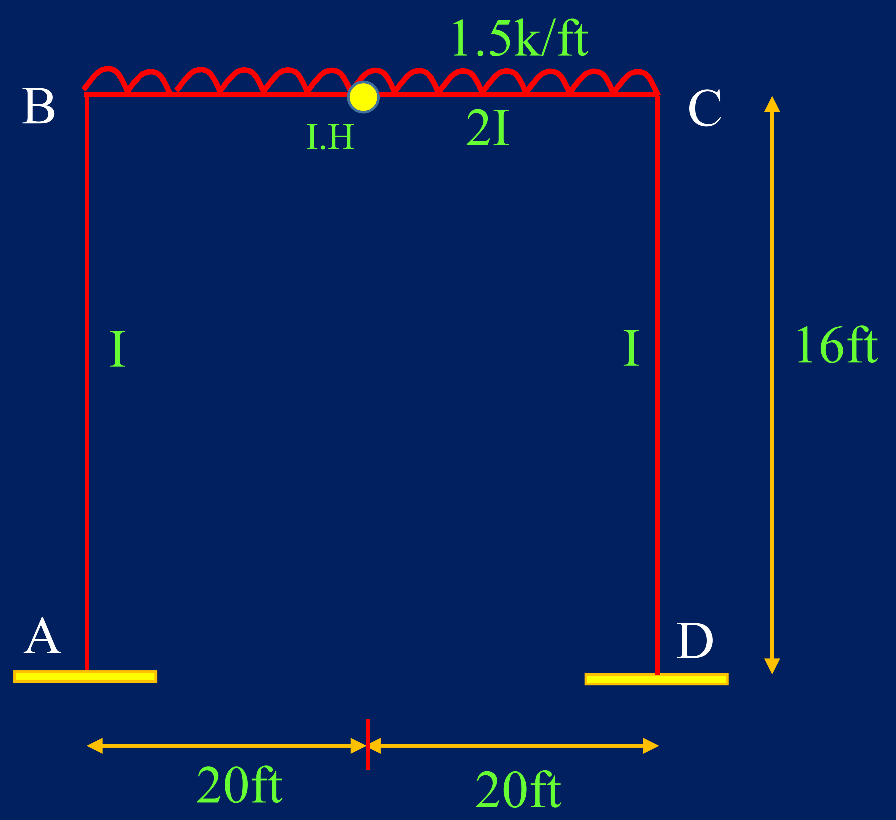
Flexibility Method for Frames Analysis

Assignment 02: Analyze the given frames using flexibility method

Note:

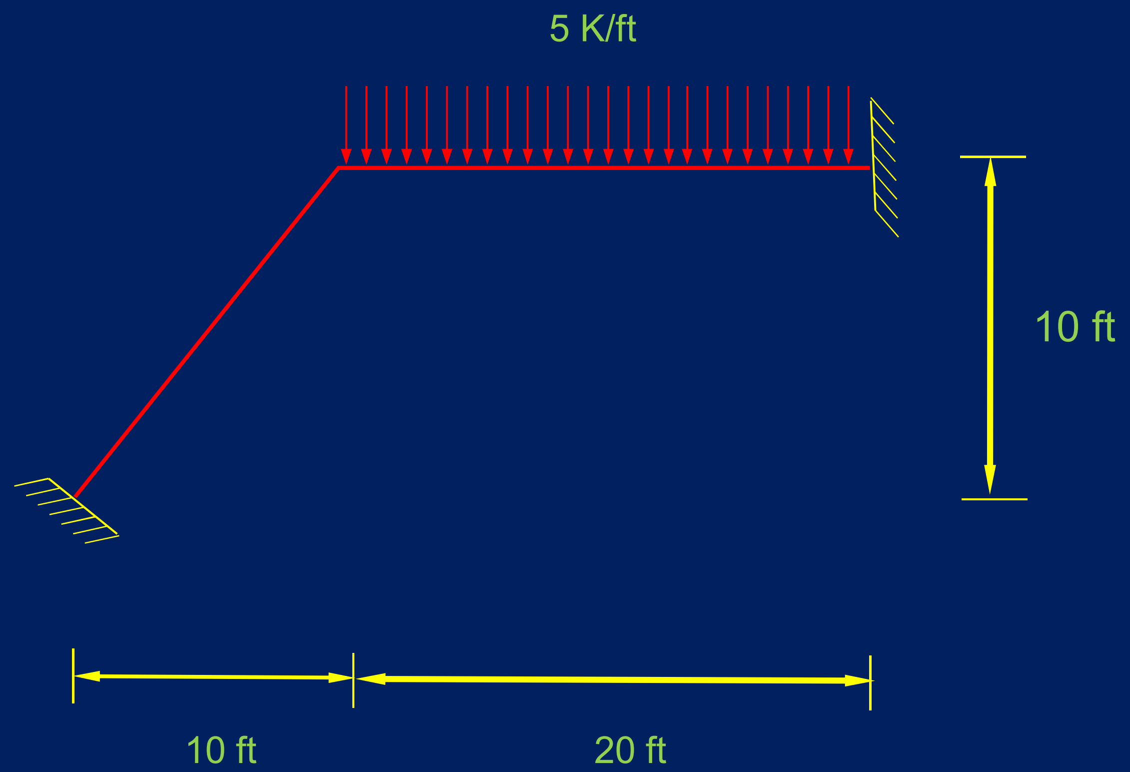
- Section B have to solve problem 4 & 5.
- Section C have to solve problem 6 & 7.

Problem 04:



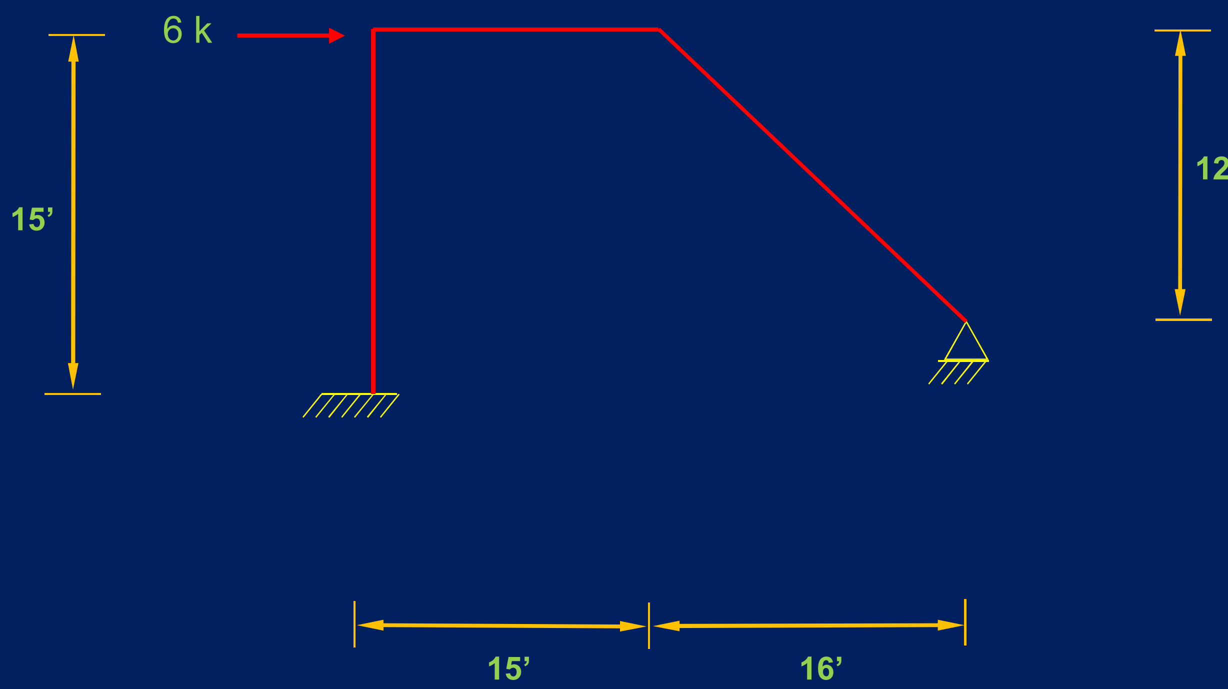
Flexibility Method for Frames Analysis

Problem 05:



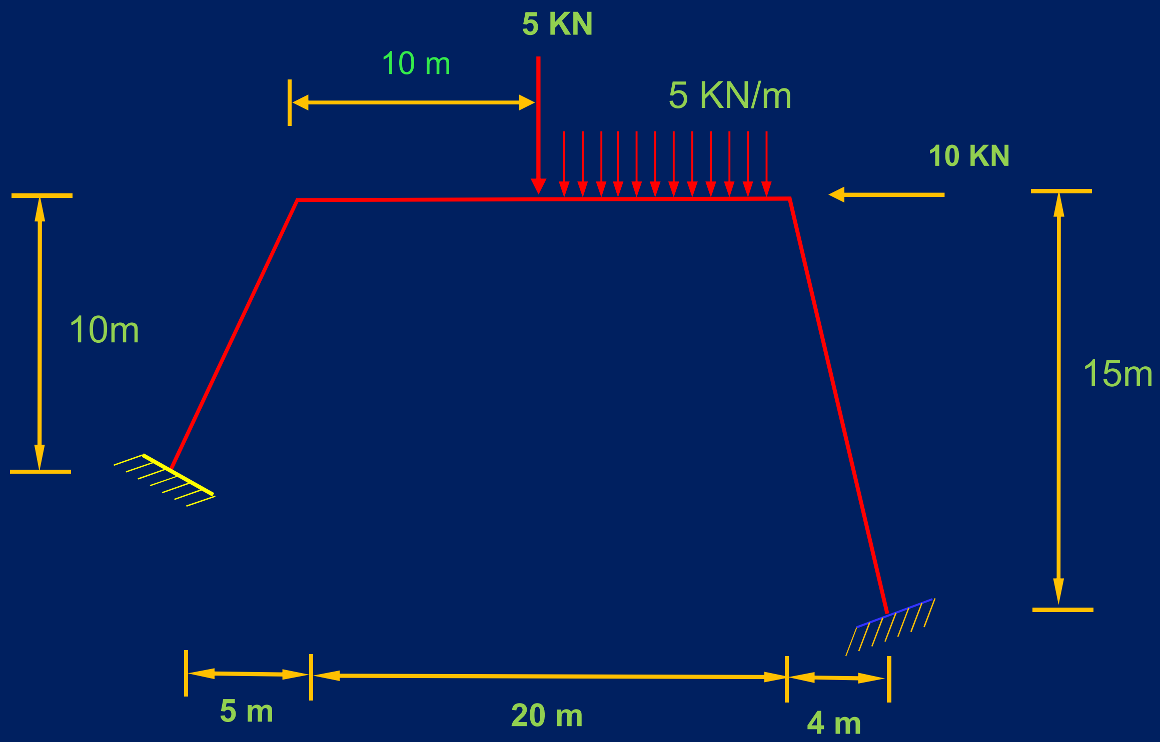
Flexibility Method for Frames Analysis

Problem 06:



Flexibility Method for Frames Analysis

Problem 07:



References

- Structural Analysis by R. C. Hibbeler
- Matrix structural analysis by William Mc Guire
- Matrix analysis of frame structures by William Weaver
- Online Civil Engineering blogs