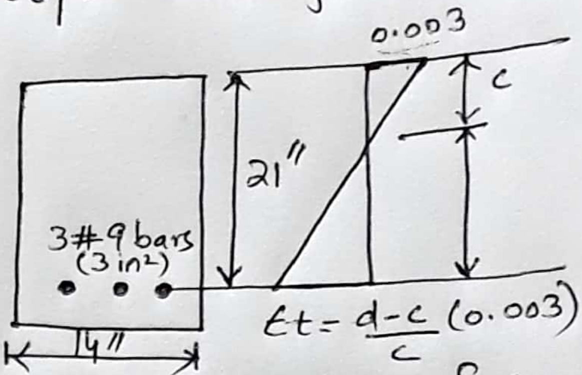
Example 3.1

Determine the values of a, c and ϵ_t for the beam shown in figure $f_y = 60{,}000\,psi$ and $f_c' = 3000\,psi$



Beam Cross Section for Example 3.1

Solution:-

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

$$a = \frac{3 \times 60}{0.85 \times 3 \times 14}$$

$$\boxed{a = 5.04\ in}$$

$\beta_1 = 0.85$ for $f_c' = 3000\,psi$ Concrete.

$$c = \frac{a}{\beta_1} = \frac{5.04}{0.85} = 5.93\ in$$

$$\boxed{c = 5.93\ in}$$

$$\epsilon_t = \frac{d-c}{c}(0.003) = \left(\frac{21 - 5.93}{5.93}\right)(0.003)$$

$$\boxed{\epsilon_t = 0.00762}$$

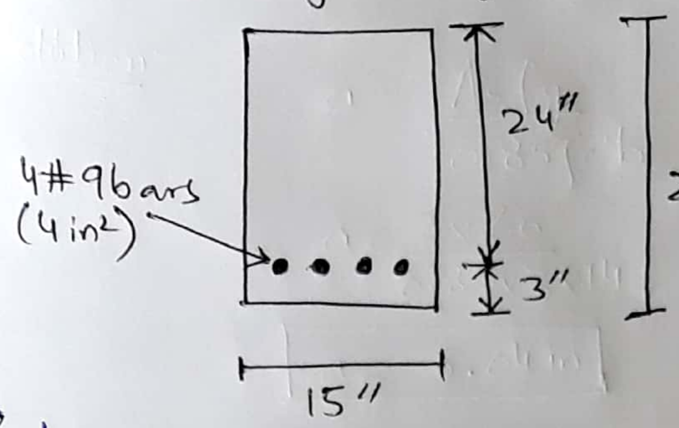
The value of strain is much Greater than the yield strain of 0.002.

This is an Indication of ductile behaviour of the beam

The steel is well into its yield plateau before Concrete crushes.

Example 3.2

Determine the ACI design moment capacity ϕM_n of the shown in Figure if $f_c' = 4000$ psi and $f_y = 60{,}000$ psi



Solution:-

① Checking Steel Percentage

$$\rho = \frac{A_s}{bd} = \frac{4}{15 \times 24}$$

$$\rho = 0.0111 \quad > \rho_{min} = 0.0033 \quad < \rho_{max} = 0.0181$$

both from Appendix A Table A.7

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{4 \times 60000}{0.85 \times 4000 \times 15}$$

$$\boxed{a = 4.71 \text{ in}}$$

$\beta_1 = 0.85$ for 4000 psi Concrete

$$c = \frac{a}{\beta_1} = \frac{4.71}{0.85}$$

$$\boxed{c = 5.54 \text{ in}}$$

② Drawing strain Diagram

$$\epsilon_t = \frac{d-c}{c}(0.003) = \frac{24 - 5.54}{5.54}(0.003)$$

$$\epsilon_t = 0.0100 > 0.005$$ [Tension Controlled]

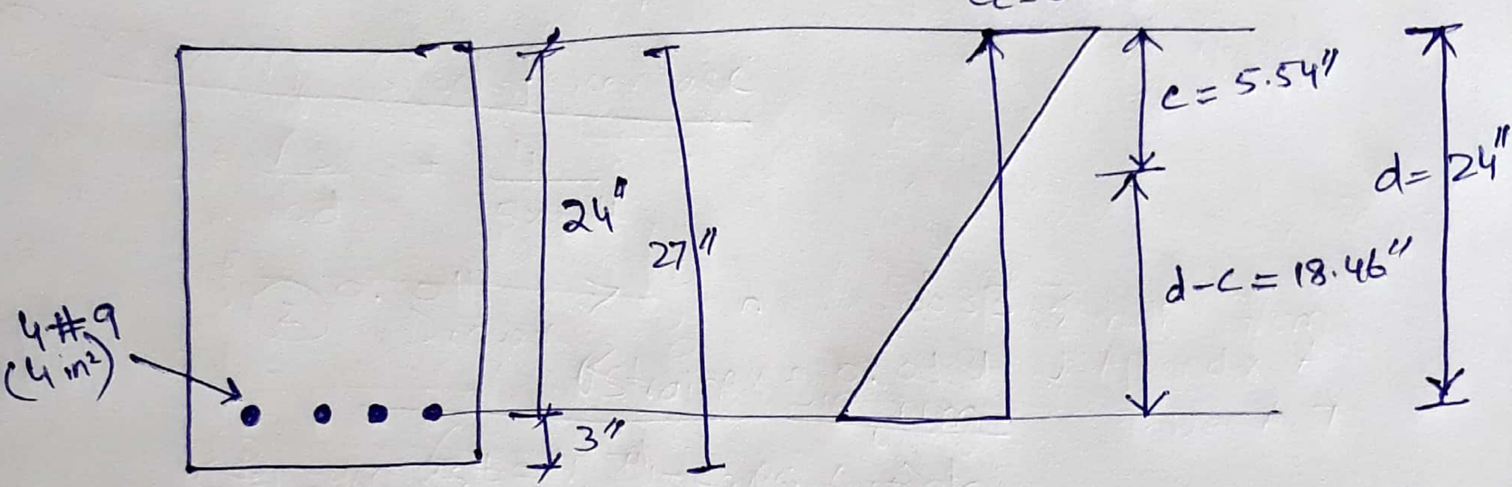
$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 4 \times 60 \left(24 - \frac{4.71}{2}\right)$$

$$M_n = 5194.8 \text{ in-K}$$

$$\boxed{M_n = 432.9 \text{ ft-K}}$$

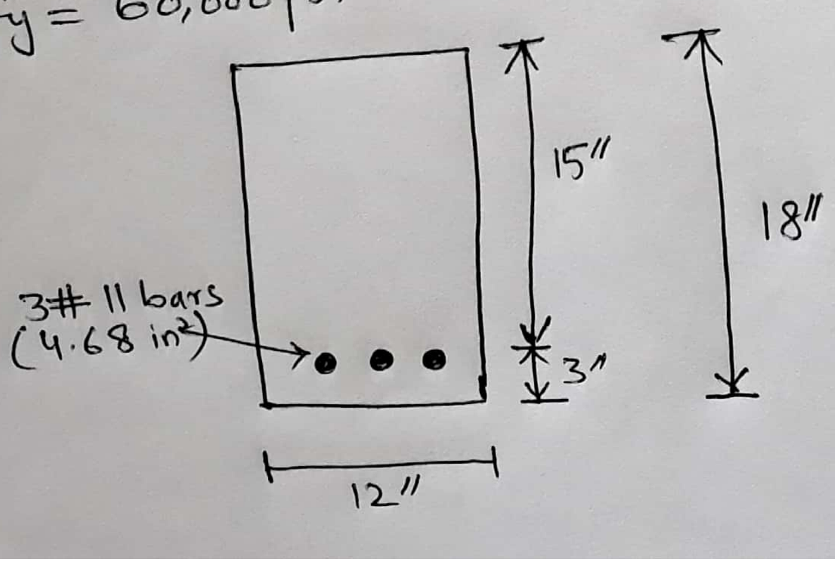
$$\phi M_n = \phi M_n = 0.9 \times 432.9$$

$$\boxed{\phi M_n = 389.6 \text{ ft-K}}$$ By applying strength reduction factor.



Example 3.3:-

Determine the ACI design Moment capacity ϕM_n of the beam shown in Figure if $f_c' = 4000$ psi and $f_y = 60{,}000$ psi



Solution:-

Checking steel Percentage:-

$$\rho = \frac{A_s}{bd} = \frac{4.68}{12 \times 15} = 0.026 > \rho_{min} = 0.0033$$

$> \rho_{max}\ 0.0181$
($\epsilon_t < 0.005$ will be)

Computing value of ϵ_t

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{4.68 \times 60000}{0.85 \times 4000 \times 12}$$

$$\boxed{a = 6.88 in}$$

$\beta_1 = 0.85$ for 4000 psi Concrete.

$$c = \frac{a}{\beta_1} = \frac{6.88}{0.85}$$

$$\boxed{c = 8.09 in}$$

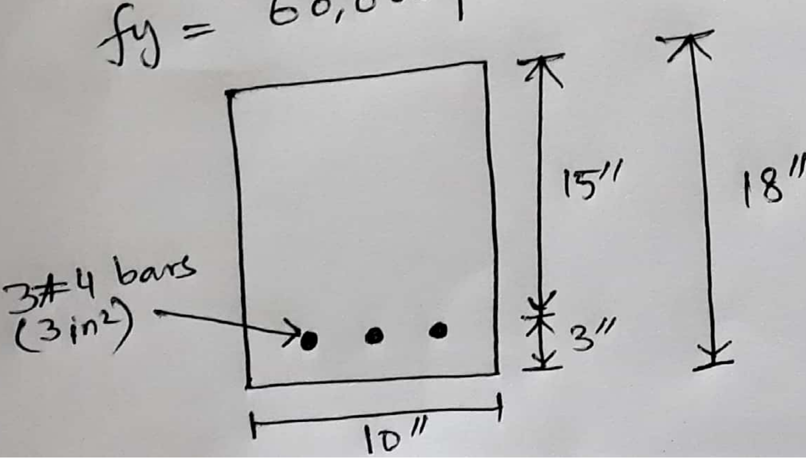
$$\epsilon_t = \frac{d-c}{c}(0.003) = \frac{15-8.09}{8.09}(0.003)$$

$$\epsilon_t = 0.00256 < 0.004$$

Section is not ductile and may not be used as per ACI section 10.3.5.

Example 3.4 :-

Determine the ACI design moment capacity ϕM_n for the beam of the Figure if $f_c' = 4000 psi$ and $f_y = 60,000 psi$



Solution:-

① Checking steel Percentage:-

$$\rho = \frac{A_s}{bd} = \frac{3}{10 \times 15} = 0.020 > \rho_{min} = 0.003$$
$$\cong \rho_{max} = 0.0181$$
$$(\epsilon_t \cong 0.005)$$

② Computing value of ϵ_t :-

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3 \times 60{,}000}{0.85 \times 4000 \times 10}$$

$$\boxed{a = 5.29''}$$

$\beta_1 = 0.85$ for 4000 psi Concrete.

$$c = \frac{a}{\beta_1} = \frac{5.29}{0.85}$$

$$\boxed{c = 6.22 \text{ in}}$$

$$\epsilon_t = \frac{d-c}{c}(0.003) = \frac{15-6.22}{6.22}(0.003) = 0.00423$$

$\epsilon_t > 0.004$
$\epsilon_t < 0.005$

Hence Beam is in transition Zone.

$$\phi = 0.65 + (\epsilon_t - 0.002)\frac{250}{3} \quad \text{(from Figure 3.5)}$$
$$\phi = 0.65 + (0.00423 - 0.002)\left(\frac{250}{3}\right)$$
$$\phi =$$
$$\boxed{\phi = 0.836}$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = 3 \times 60\left(15 - \frac{5.29}{2}\right)$$
$$M_n = 2223.9 \text{ in K} \Rightarrow M_n = 185.3 \text{ ft-K}$$
$$\phi M_n = 0.836 \times 185.3$$
$$\boxed{\phi M_n = 154.9 \text{ ft-K}}$$