University of Engineering & Technology Peshawar, Pakistan

CE301: Structure Analysis II

Module 02:

Analysis of S.I Beams Using Flexibility method

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Topics to be Covered

- Introduction
- Prerequisites for using flexibility method
- Revision of conjugate beam concept
- Member end actions
- Analysis of beam Example 1
- Example 2
- Example 3

Flexibility Method for Beams Analysis

❑ Introduction:

Beams are analyzed with flexibility method due to

- To solve the problem in matrix notation, which is more systematic
- To compute reactions at all the supports.
- To compute internal resisting shear & bending moment at any section of the continuous beam.

Flexibility Method for Beams Analysis

- **Prerequisites for Analysis with Flexibility method:**

  It is necessary that students must have strong background of the following concepts before starting analysis with flexibility or any other matrix method.

- Enough concept of Matrix Algebra
- Must be able to find the Statical Indeterminacy
- Concept of Deflection methods (Conjugate Beam & Unit load methods)

Flexibility Method for Beams Analysis

❑ Conjugate Beam Method:

Conjugate beam is defined as the imaginary beam with the same dimensions (length) as that of the original beam but load at any point on the conjugate beam is equal to the bending moment at that point divided by EI. The conjugate-beam method is an engineering method to derive the slope and displacement of a beam.

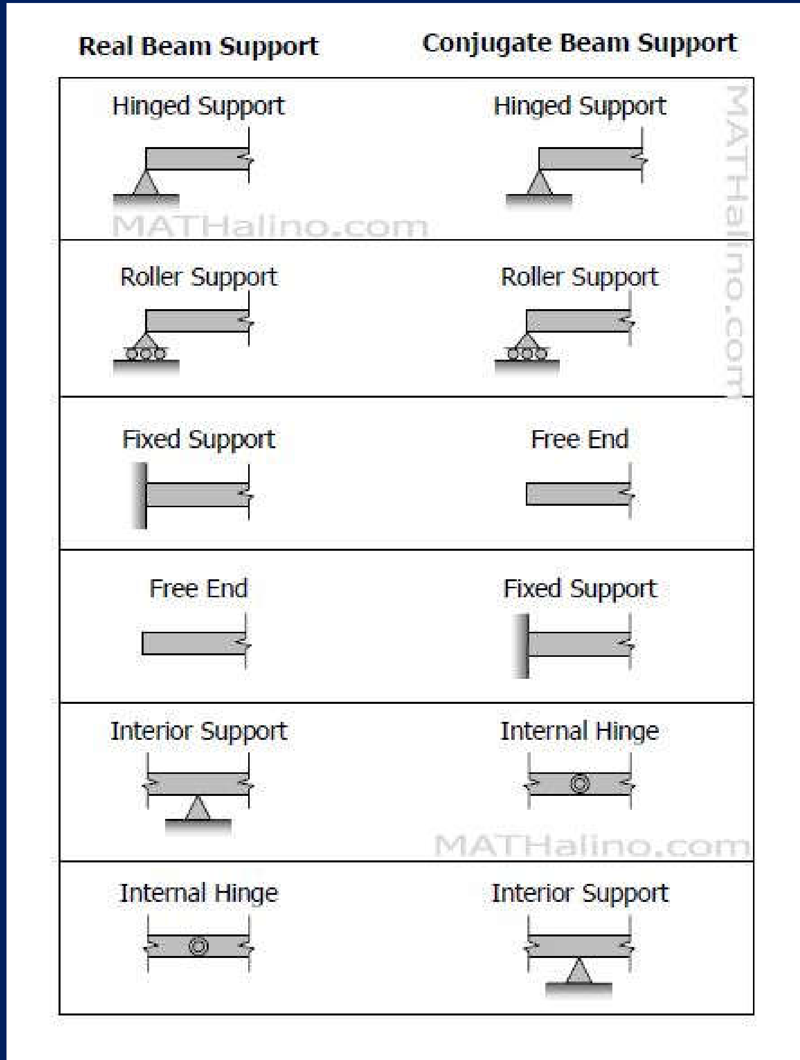
Slope on real beam = Shear on conjugate beam

Deflection on real beam = Moment on conjugate beam

Flexibility Method for Beams Analysis

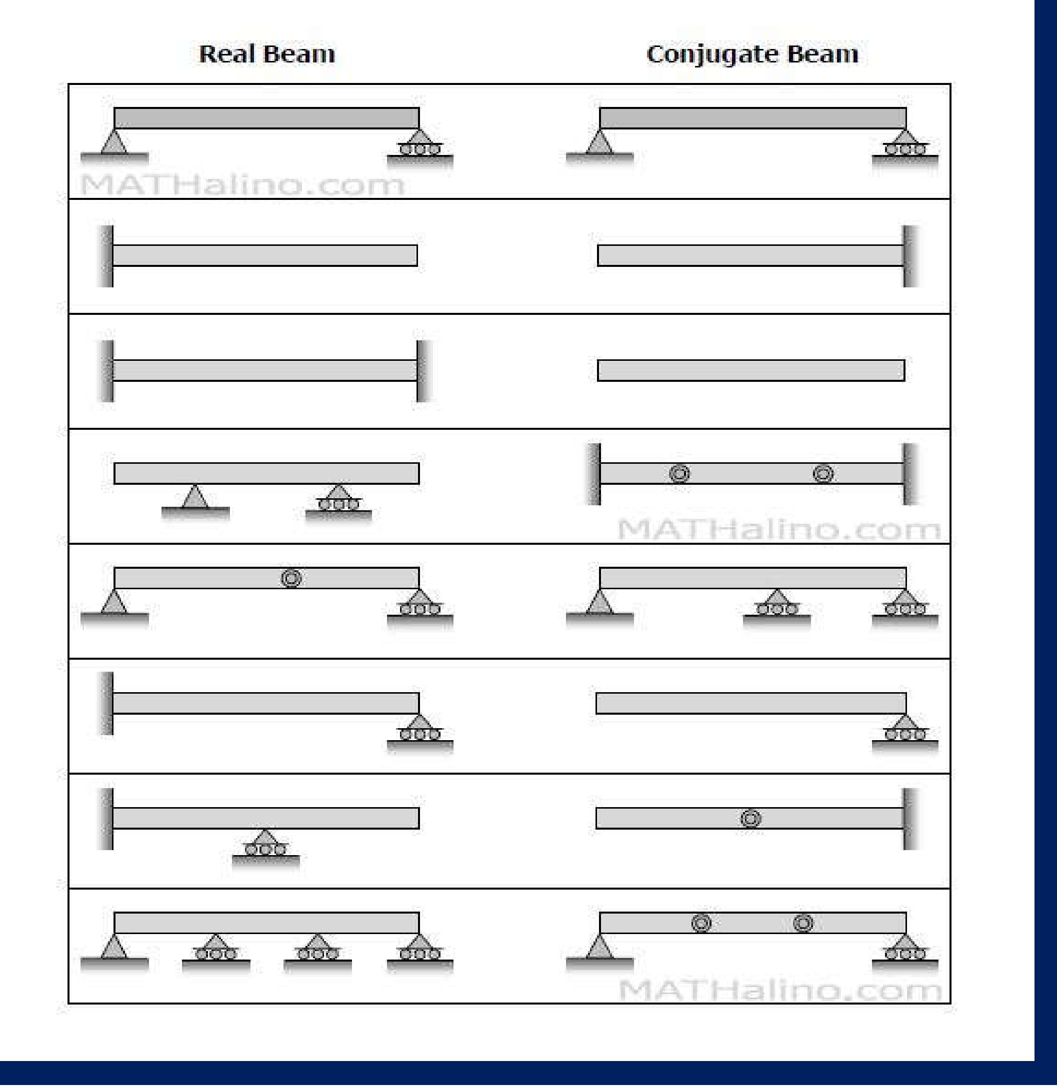
Properties of Conjugate Beam:

1. The load on the conjugate beam is the M/EI diagram of the loads on the actual beam.
2. A simple support for the real beam remains simple support for the conjugate beam.
3. A fixed end for the real beam becomes free end for the conjugate beam.
4. The point of zero shear for the conjugate beam corresponds to a point of zero slope for the real beam.
5. The point of maximum moment for the conjugate beam corresponds to a point of maximum deflection for the real beam.



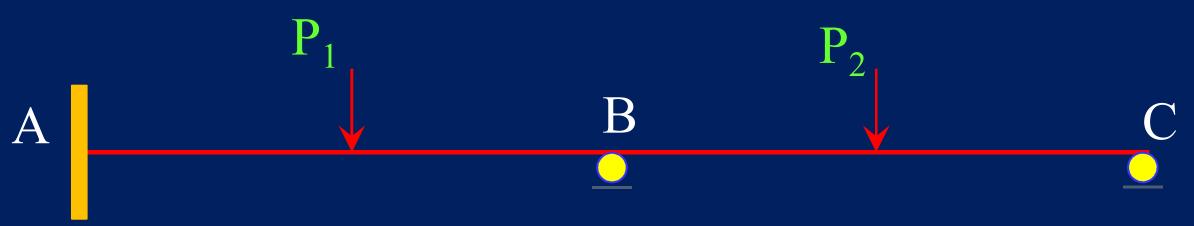
Flexibility Method for Beams Analysis

❑ Real beams and its Conjugate Beams



Flexibility Method for Beams Analysis

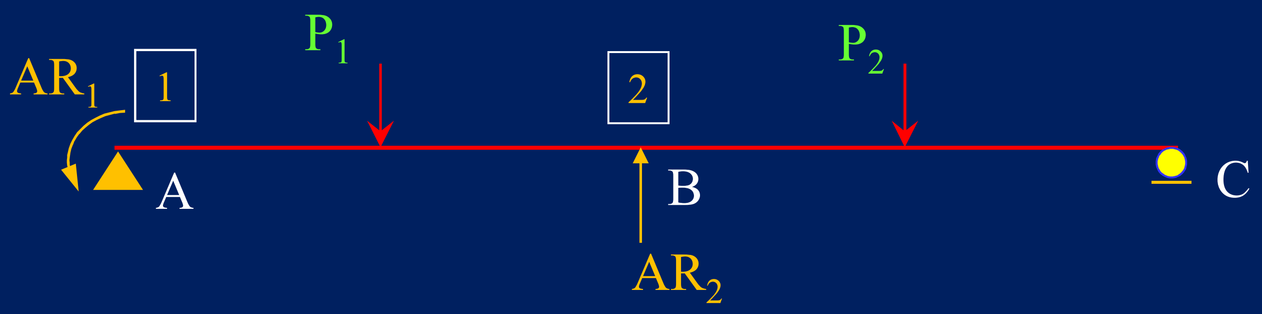
- **Member End Actions:** Member end actions are the moments and shear at each point where its going to be changed. We can find the member end actions using flexibility method when the redundant actions are known.
- Let us illustrate the procedure of finding member actions by flexibility method with the help of Example 2 in Module 1 as shown in fig.



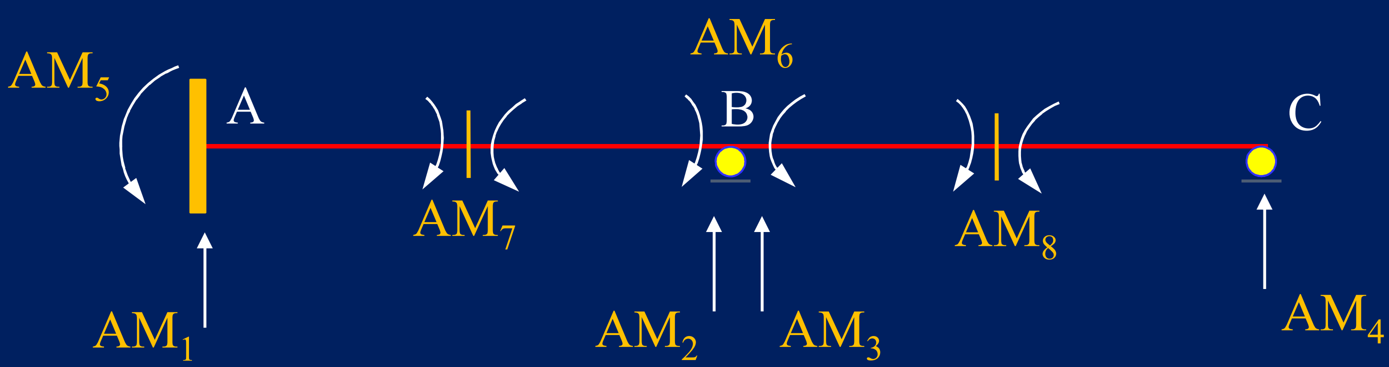
S.I = 2 degrees

Flexibility Method for Beams Analysis

Vertical reactions at B & moment at A is taken as redundant.



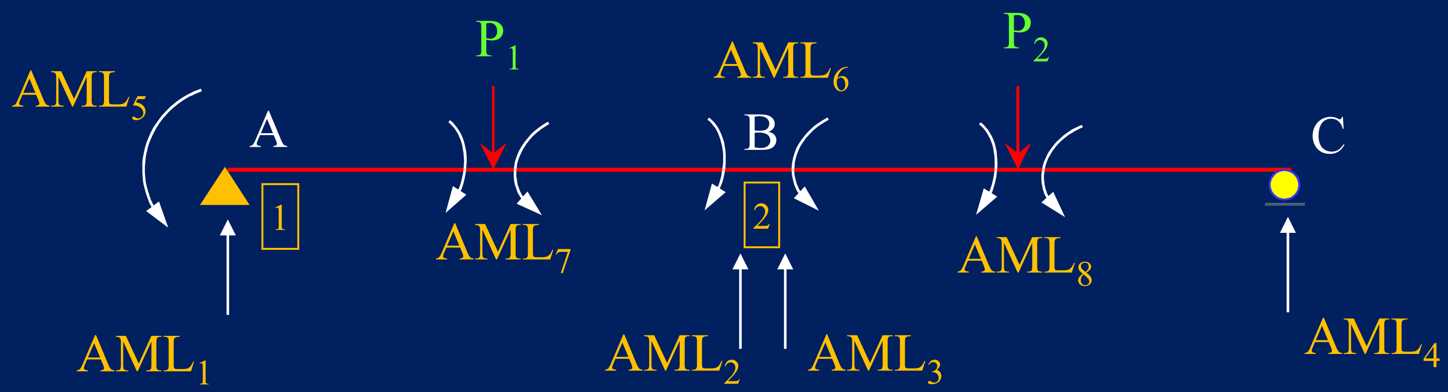
Step # 01: Specify the member end actions AM_n by assigning a No. to them. e.g. in the above problem



AM_n = is the member end action in the indeterminate structure at different location specified with a number "n". n shows the number of member end actions.

Flexibility Method for Beams Analysis

Step # 02: Determine the member end actions (shear & moments) in the released structure under the external applied loads (AML_n) at the specified locations.

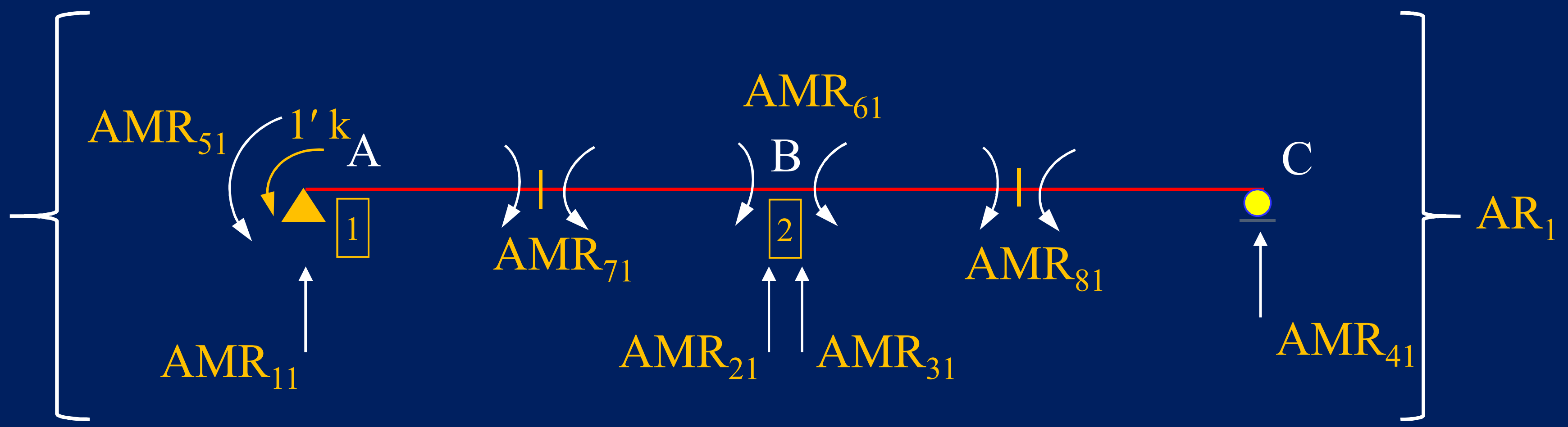


AML_n = is the member end actions in the released structure at location "n" when acted upon by the external applied loads.

Flexibility Method for Beams Analysis

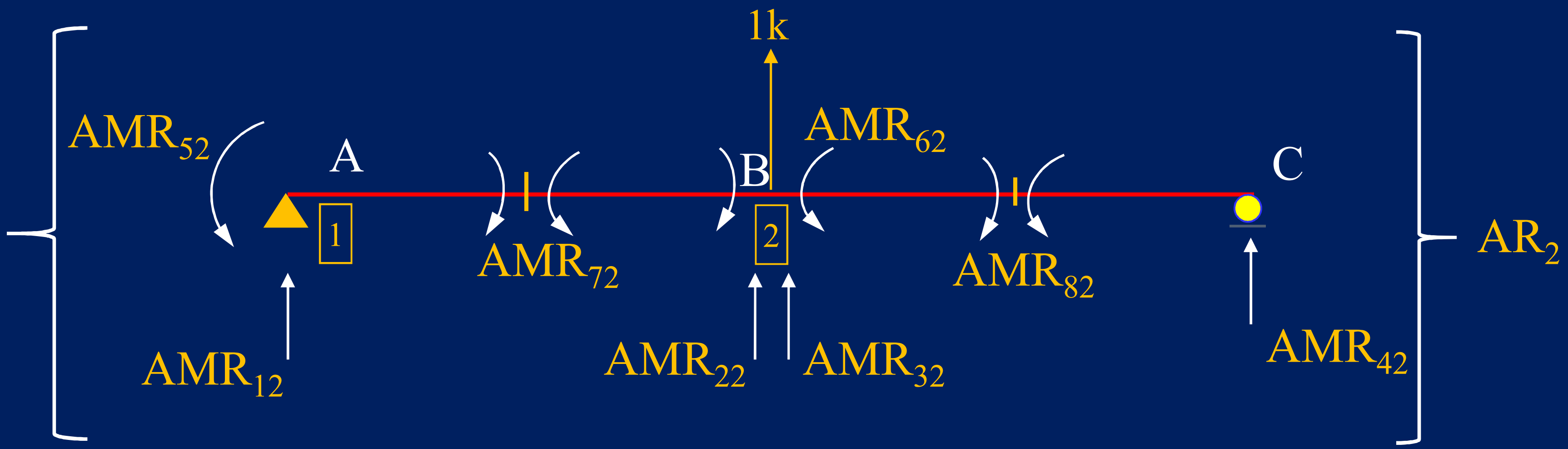
Step # 03: Determine the member end actions (shear & moments) in the released structure when acted upon by a unit at action 1st at redundant location 1 and then at 2 (AMR_{n1} & AMR_{n2}).

When a unit action is applied at the redundant location 1 as shown.



Flexibility Method for Beams Analysis

When a unit action is applied at the redundant location 2 as shown.



AMR_{n1} & AMR_{n2} = is the member end actions in the released structure when acted upon by a unit action.
n shows the number member action and 1 & 2 shows the unit action/ load position.

Flexibility Method for Beams Analysis

$$AM_1 = AML_1 + AMR_{11}AR_1 + AMR_{12}AR_2$$

$$AM_2 = AML_2 + AMR_{21}AR_1 + AMR_{22}AR_2$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$AM_8 = AML_8 + AMR_{81}AR_1 + AMR_{82}AR_2$$

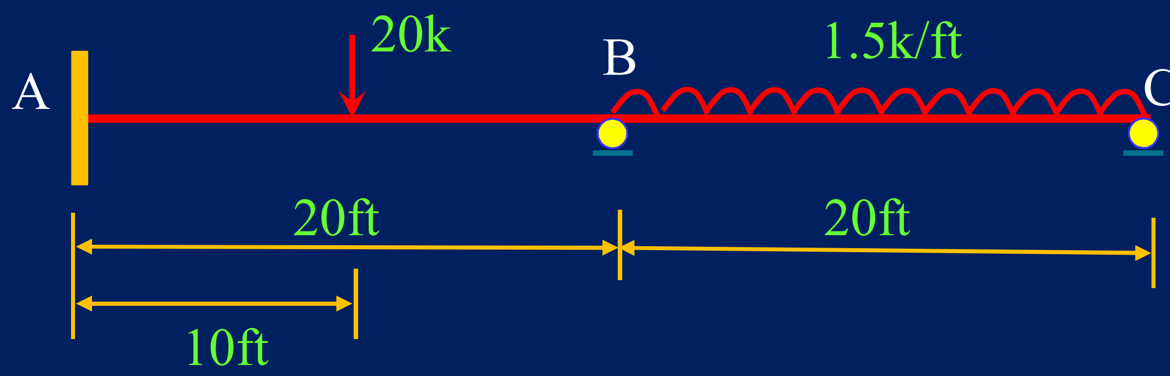
$$\begin{bmatrix} AM_1 \\ AM_2 \\ \vdots \\ AM_8 \end{bmatrix} = \begin{bmatrix} AML_1 \\ AML_2 \\ \vdots \\ AML_8 \end{bmatrix} + \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ \vdots & \vdots \\ AMR_{81} & AMR_{82} \end{bmatrix} \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix}$$

$$[AM] = [AML] + [AMR][AR]$$

This method will be explained again in the coming problem

Flexibility Method for Beams Analysis

Problem 01: Analyze the given beam using flexibility method.



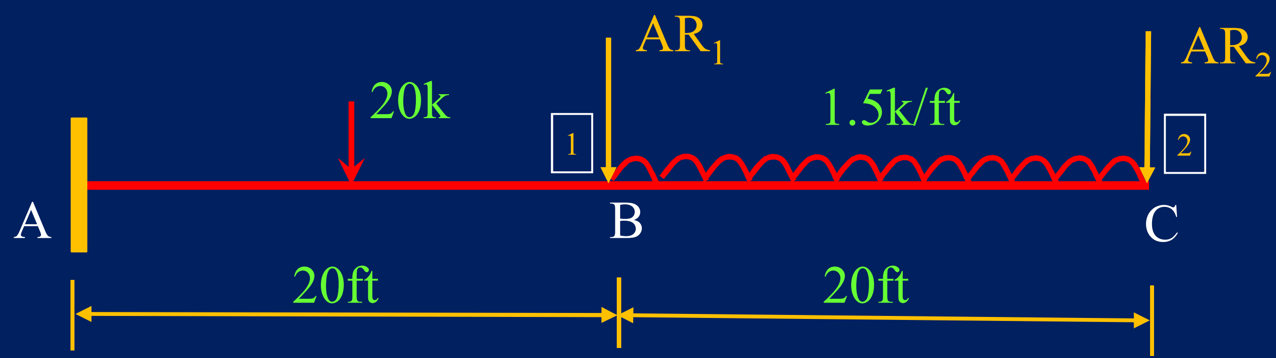
Take EI = constant

S.I = 2 degree
So two redundant actions should be chosen.
Choose those as redundant which makes the calculation handy.

Flexibility Method for Beams Analysis

- **Step # 01:** Select the redundant actions and assign coordinates at those locations.

  Vertical reactions at B & C are taken as redundant.



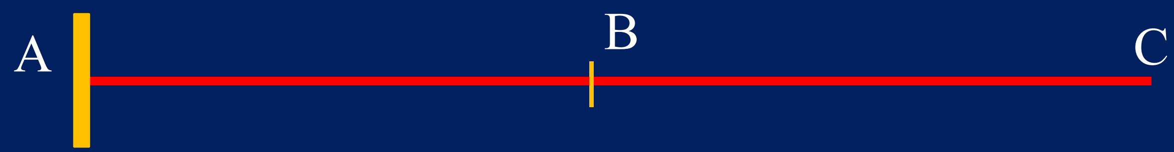
$$[AR]_{2*1} = \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[DRS]_{2*1} = \begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- DRS_1 & DRS_2 are the initial support rotation & settlement corresponding to the redundant actions 1 & 2 which is zero in this case.

Flexibility Method for Beams Analysis

- Remove all the loads and redundant constrains to get the primary structure.

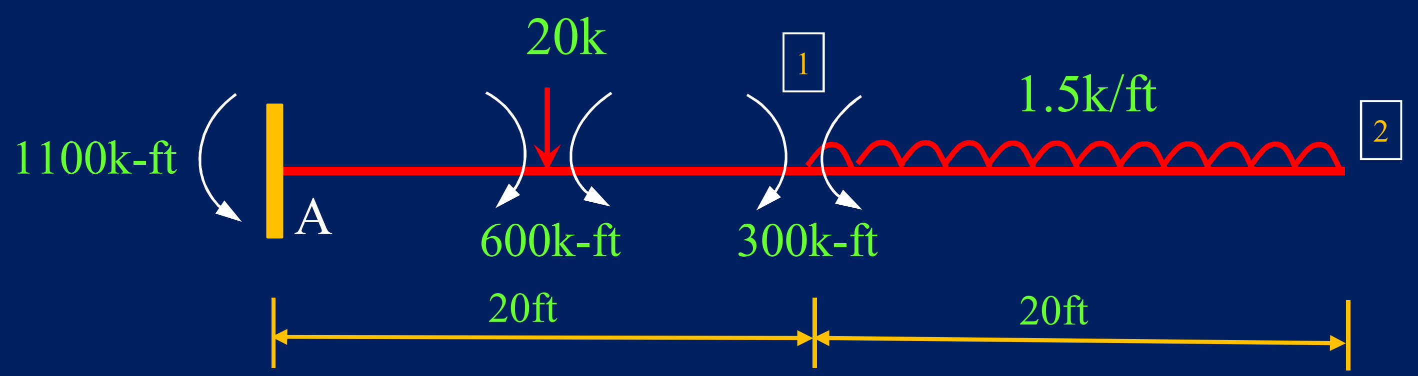


Basic determinate structure (BDS) or
released structure or
Primary structure

Note: when the support settlement and redundant actions are in opposite direction then DRS will have –ve sign.

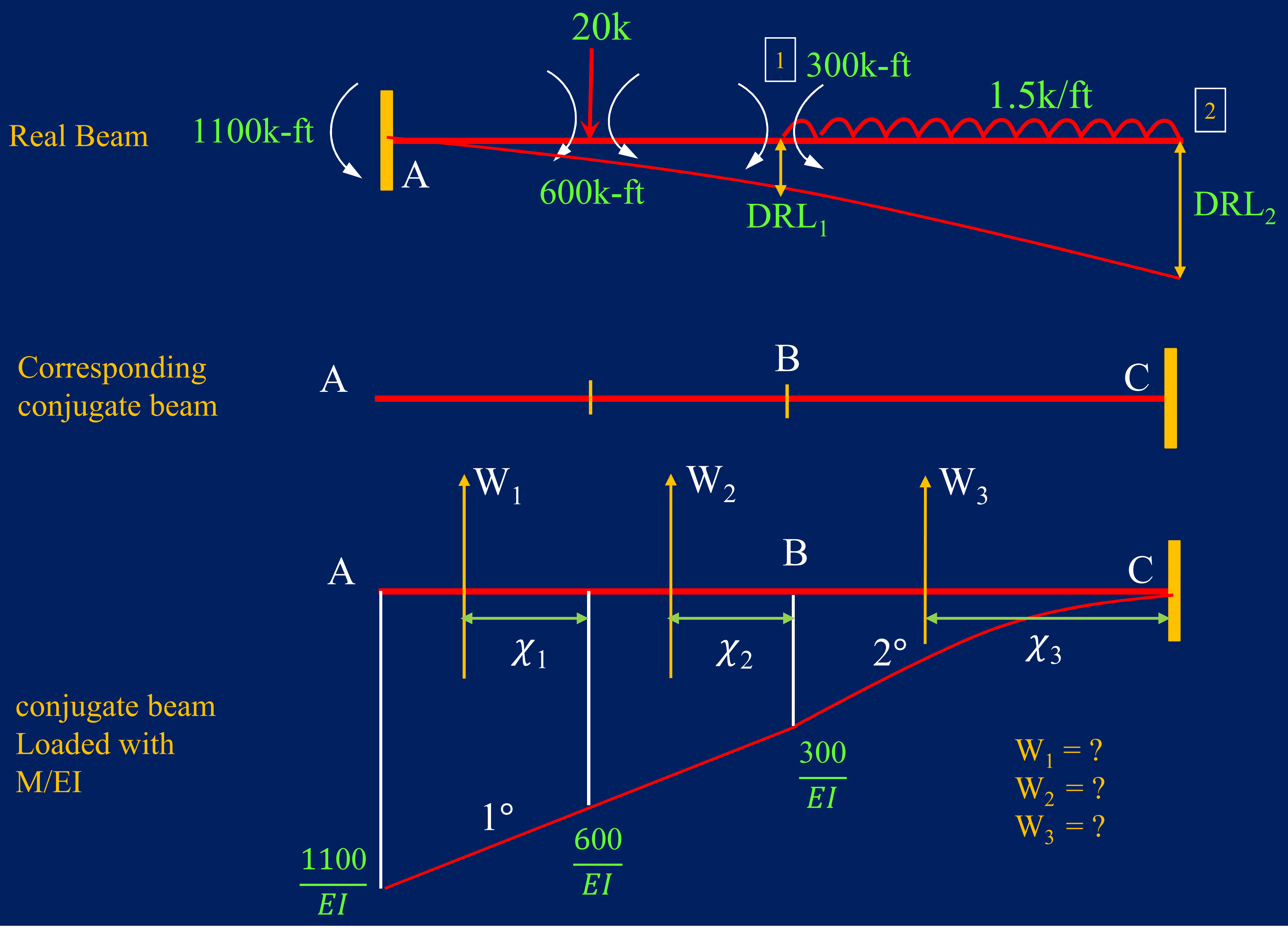
Flexibility Method for Beams Analysis

- **Step # 02 :** Compute the values of displacements (DRL) in the primary structure corresponding to the redundant locations when primary structure acted upon by the actual loads or Compute the values of [DRL].



1st Compute the moments at different locations in the released structure

Flexibility Method for Beams Analysis



Flexibility Method for Beams Analysis

Finding the values of W_1, W_2 & W_3 from the fig shown in previous slide

$$W_1 = \frac{1}{2}\left(\frac{1100+600}{EI}\right)10 = \frac{8500}{EI}\text{k}$$

$$W_2 = \frac{1}{2}\left(\frac{600+300}{EI}\right)10 = \frac{4500}{EI}\text{k}$$

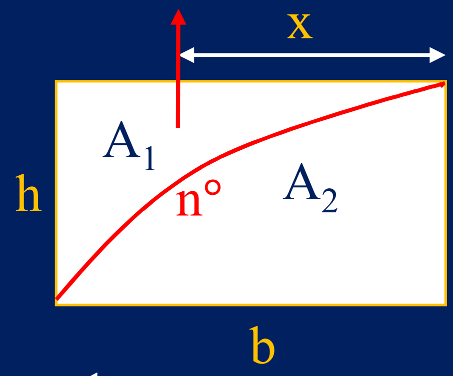
$$W_2 = \frac{1}{3}\left(\frac{300}{EI}\right)20 = \frac{2000}{EI}\text{k}$$

$$X_1 = \frac{10}{3}\left(\frac{600+2(1100)}{600+1100}\right) = 5.49'$$

$$X_2 = \frac{10}{3}\left(\frac{300+2(600)}{300+600}\right) = 5.56'$$

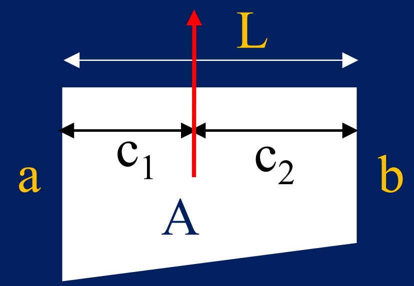
$$X_2 = \frac{3}{4}(20) = 15'$$

Note



$$A_1 = \frac{1}{n+1}(bh),\ A_2 = \frac{n}{n+1}(bh)$$

$$\text{x} = \frac{n+1}{n+2}(b)$$



$$A = \left(\frac{a+b}{2}\right)L$$

$$\text{C}_1 = \frac{L}{3}\left(\frac{a+2b}{a+b}\right) \quad C_2 = \frac{L}{3}\left(\frac{2a+b}{a+b}\right)$$

Flexibility Method for Beams Analysis

$$DRL_1 = W_1(X_1 + 10) + W_2(X_2)$$

$$= \frac{8500}{\mathrm{EI}}(5.49 + 10) + \frac{4500}{\mathrm{EI}}(5.56)$$

$$= \frac{156685}{\mathrm{EI}}$$

$$DRL_2 = W_1(X_1 + 30) + W_2(X_2 + 20) + W_3X_3$$

$$= \frac{8500}{\mathrm{EI}}(5.49 + 30) + \frac{4500}{\mathrm{EI}}(5.56 + 20) + \frac{2000}{\mathrm{EI}}(15)$$

$$= \frac{446685}{\mathrm{EI}}$$

$$[DRL] = \begin{bmatrix} DRL_1 \\ DRL_2 \end{bmatrix} = \begin{bmatrix} 156685 \\ 446685 \end{bmatrix} \times \frac{1}{EI}$$

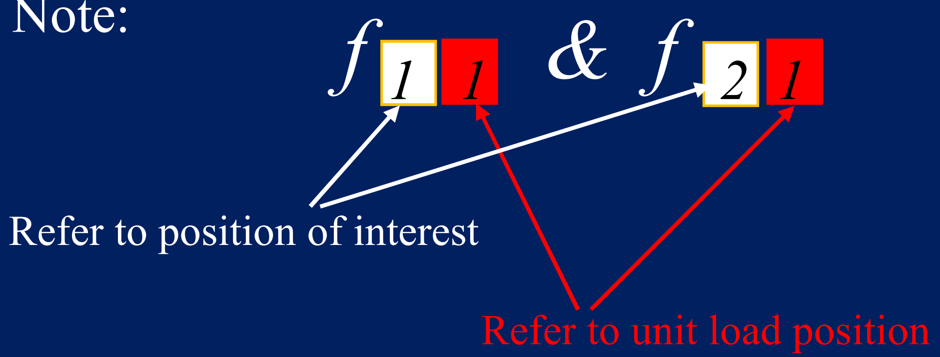
Flexibility Method for Beams Analysis

- **Step # 03 :** Compute the values of rotations/displacements in the primary structure corresponding to the redundant locations when primary structure acted upon by the UNIT actions or

  Compute the values of flexibility matrix [*f*].

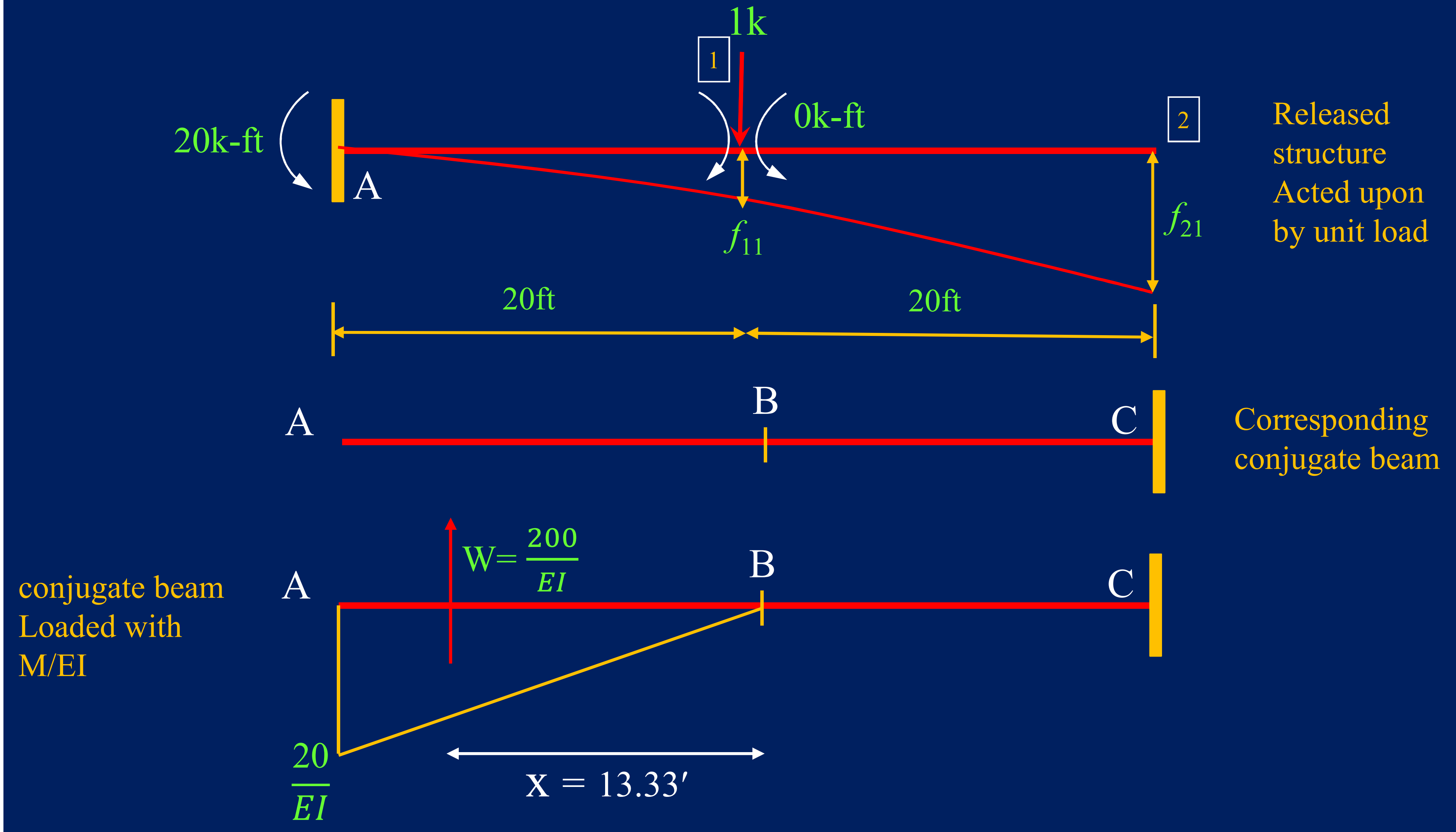
i. 1st apply a unit value of AR_1 at reference point 1 and Compute the values of flexibility coefficients (f_{11} & f_{21}).

Note:



Flexibility Method for Beams Analysis

Step # 03 (i): Contd…



Flexibility Method for Beams Analysis

Step # 03 (i): Contd…

$$W = \frac{1}{2}\left(\frac{20 + 20}{EI}\right) = \frac{200}{EI}\,\text{k}$$

$$X = \frac{2}{3}(20) = 13.33'$$

$$f_{11} = W(X)$$

$$= \frac{200}{\text{EI}}(13.33)$$

$$= \frac{2666}{EI}$$

$$f_{21} = W(X + 20)$$

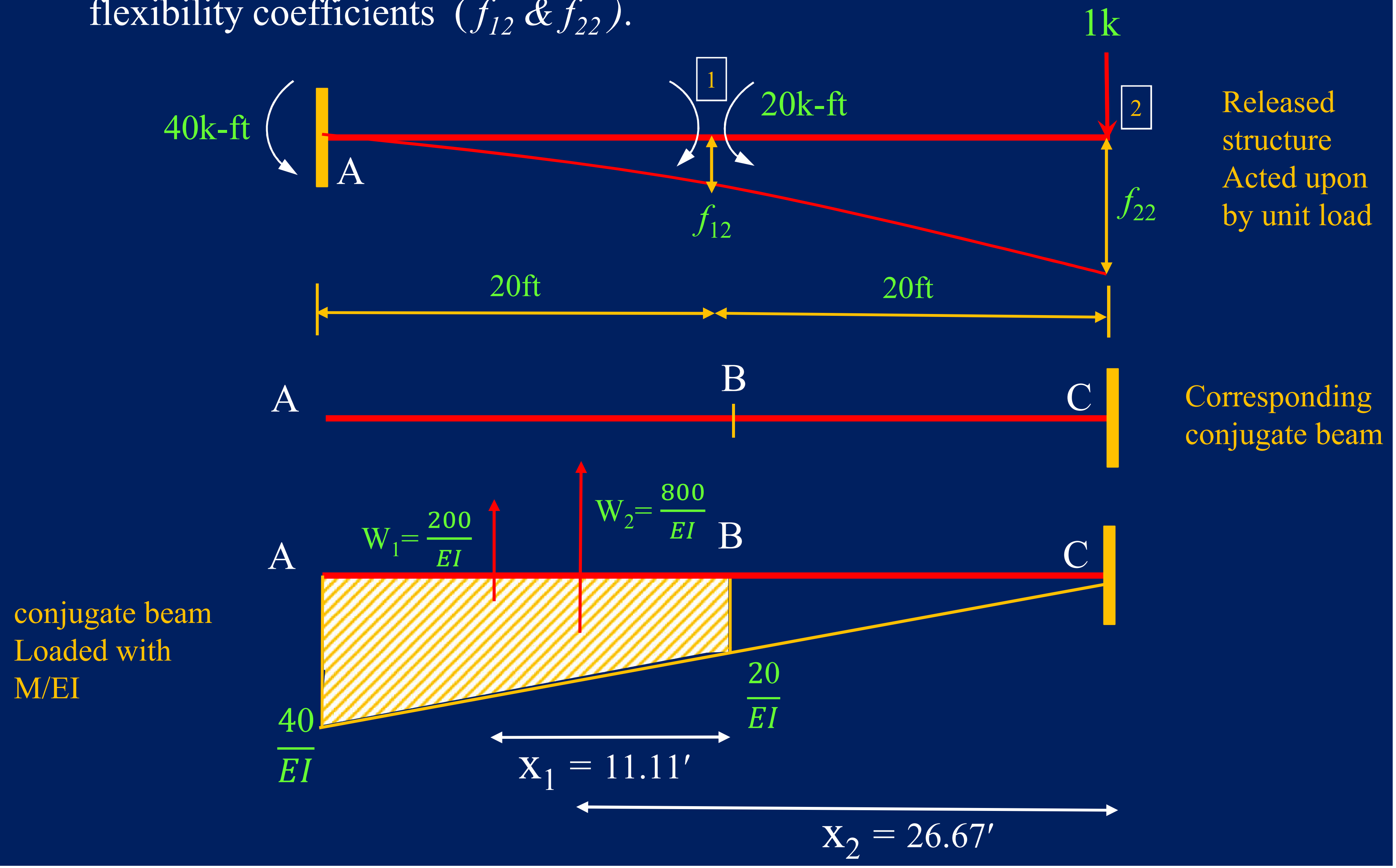
$$= \frac{200}{\text{EI}}(13.33 + 20)$$

$$= \frac{6666}{EI}$$

Note: f_{11} & f_{22} will be the moment in conjugate beam at Corresponding location

Flexibility Method for Beams Analysis

ii. Now apply a unit value of AR_2 at reference point 2 and Compute the values of flexibility coefficients $(f_{12} \& f_{22})$.



Flexibility Method for Beams Analysis

$$W_1 = \frac{1}{2}\left(\frac{20+40}{EI}\right)20 = \frac{600}{EI}\text{k}$$

$$X_1 = \frac{20}{3}\left(\frac{20+2(40)}{20+40}\right) = 11.11'$$

W_1 & X_1 refer to the trapezoidal part in $\frac{M}{EI}$ digram

$$W_2 = \frac{1}{2}\left(\frac{40*40}{EI}\right) = \frac{800}{EI}\text{k}$$

$$X_2 = \frac{2}{3}(40) = 26.67'$$

W_2 & X_2 refer to the whole triangle in $\frac{M}{EI}$ digram

$$f_{12} = W_1(X_1)$$

$$= \frac{600}{\text{EI}}(11.11)$$

$$= \frac{6666}{EI}$$

For f_{12} only the trapezium part is considered

$$f_{22} = W_2(X_2)$$

$$= \frac{200}{\text{EI}}(13.33+20)$$

$$= \frac{21336}{EI}$$

For f_{22} the whole tringle is taken.

Flexibility Method for Beams Analysis

$$f_{11} = \frac{2666}{EI} \qquad f_{12} = \frac{6666}{EI}$$

$$f_{21} = \frac{6666}{EI} \qquad f_{22} = \frac{21336}{EI}$$

$$[f] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$$

$$[f] = \frac{1}{EI}\begin{bmatrix} 2666 & 6666 \\ 6666 & 21336 \end{bmatrix}$$

Flexibility coefficient matrix

Flexibility Method for Beams Analysis

Step # 04: Apply compatibility and principle of superposition at the location of the removed redundants and solve the matrices to compute the values of redundant actions AR. As we know

$$[DRS] = [DRL] + [f] \bullet [AR]$$

From this

$$[AR] = [f]^{-1} \bullet [DRS - DRL]$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

Flexibility Method for Beams Analysis

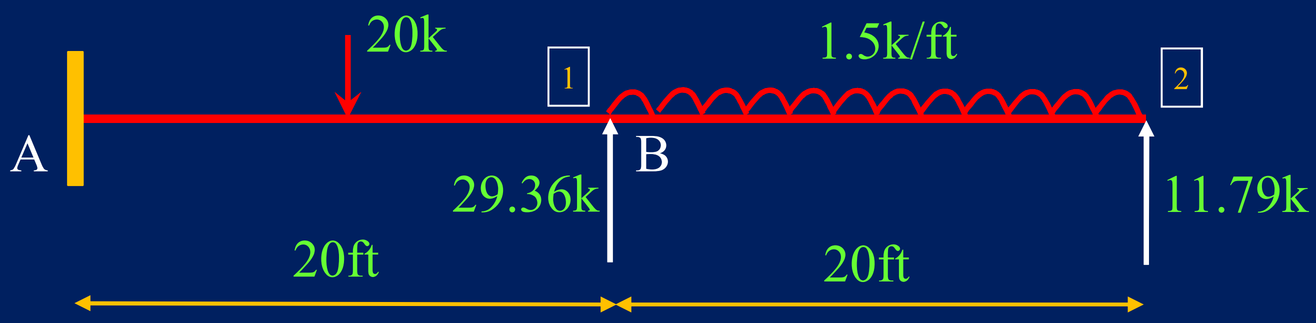
$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = EI \begin{bmatrix} 0.0017 & -0.00053 \\ -0.00053 & 0.00021 \end{bmatrix} \begin{bmatrix} 0 - 156685 \\ 0 - 446685 \end{bmatrix} \times \frac{1}{EI}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} -29.36 \\ -11.76 \end{bmatrix}$$

-ive sign shows that our assumed redundant actions direction is wrong

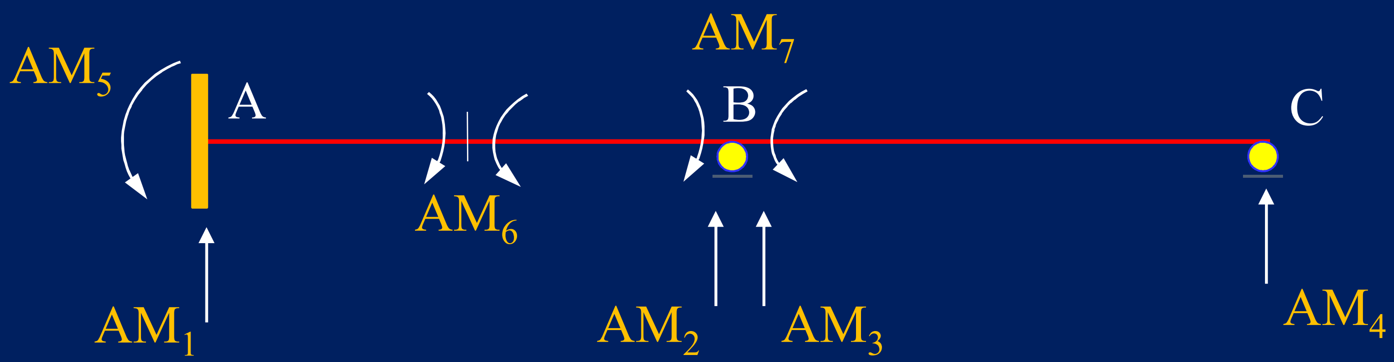
So Final determinate structure



Flexibility Method for Beams Analysis

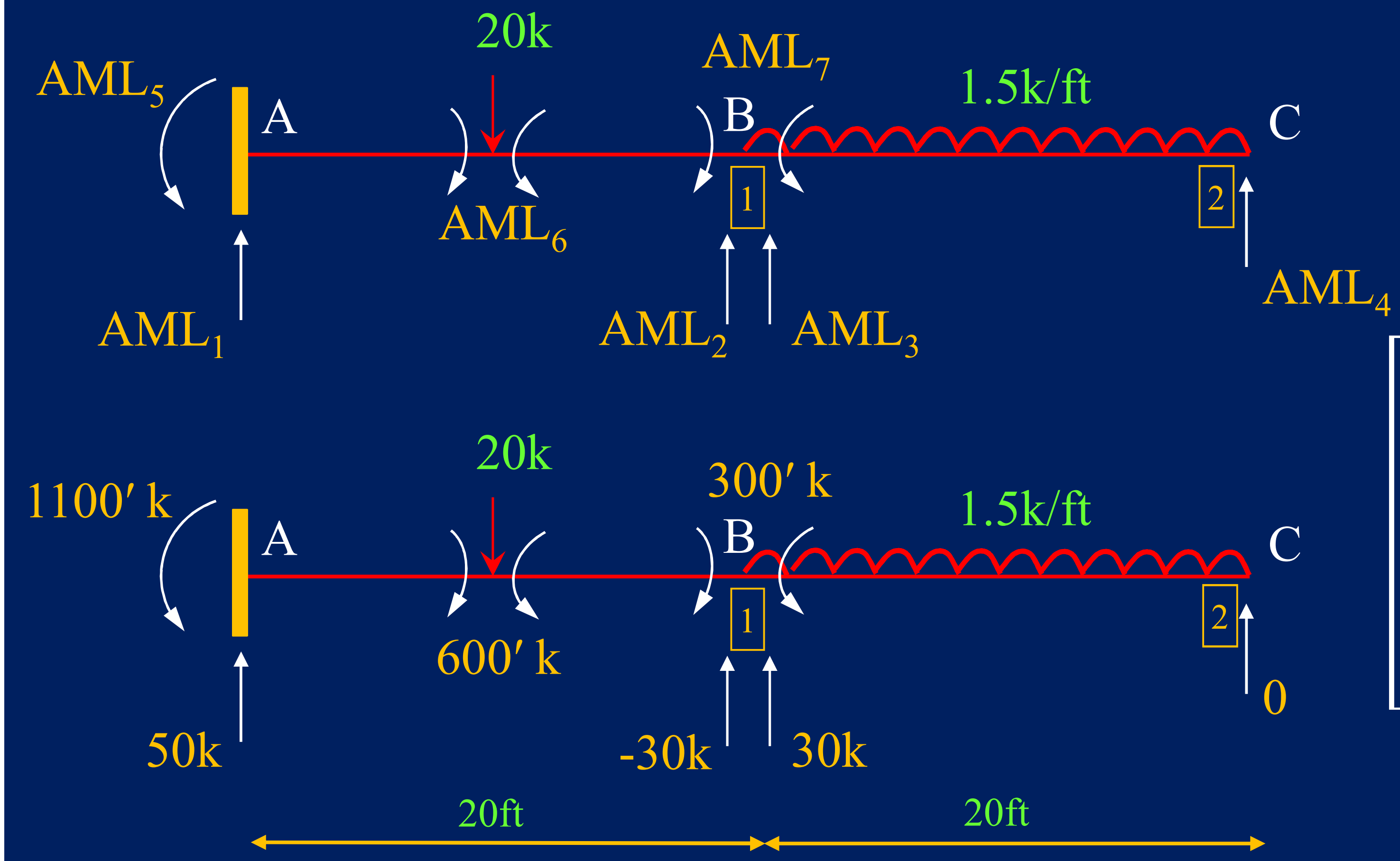
Step # 05: Compute the member end actions. As we know that

$$[AM] = [AML] + [AMR][AR]$$



Flexibility Method for Beams Analysis

a) Compute AML values.

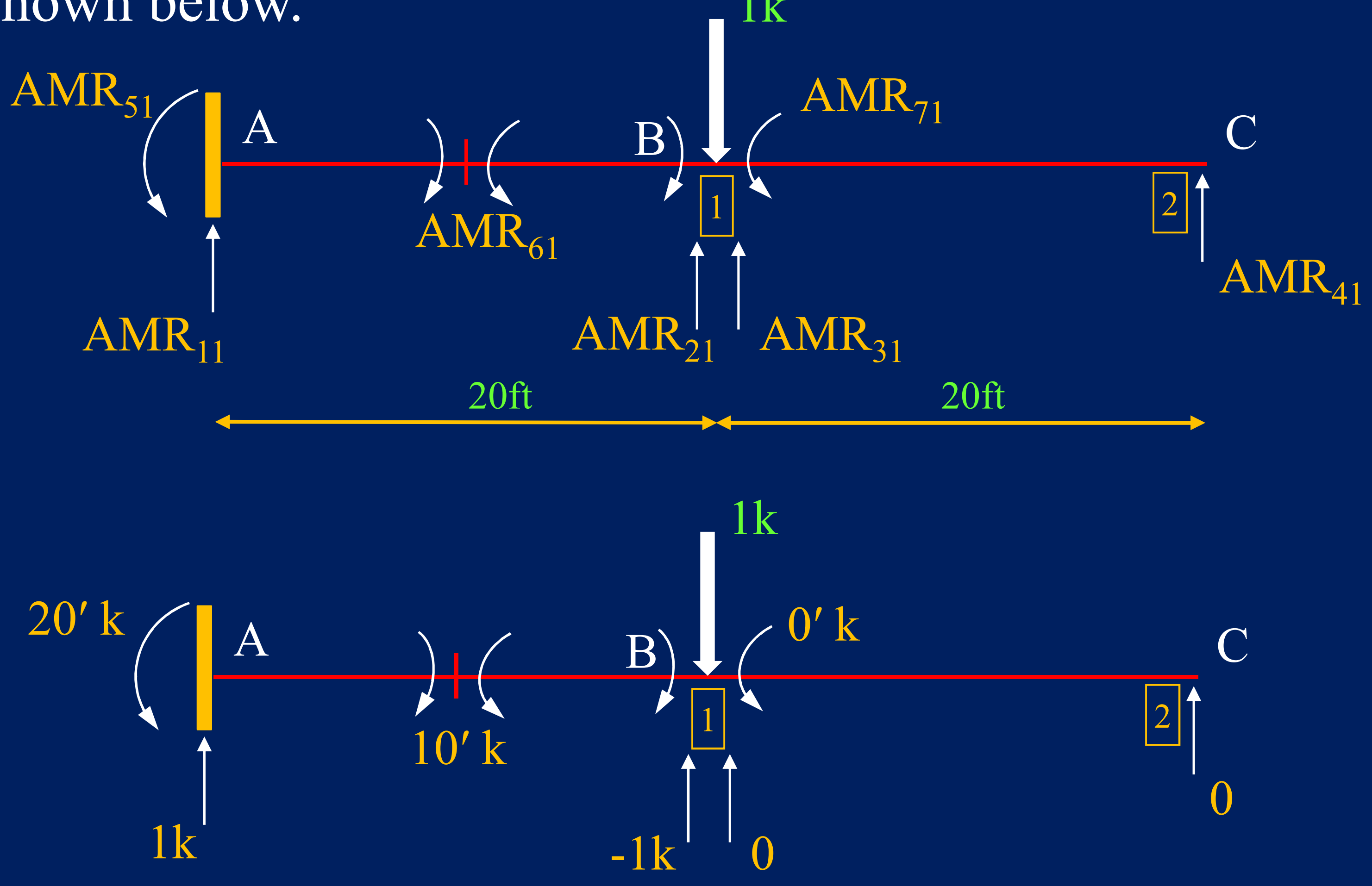


$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \\ AML_7 \end{bmatrix} = \begin{bmatrix} 50 \\ -30 \\ 30 \\ 0 \\ 1100 \\ 600 \\ 300 \end{bmatrix}$$

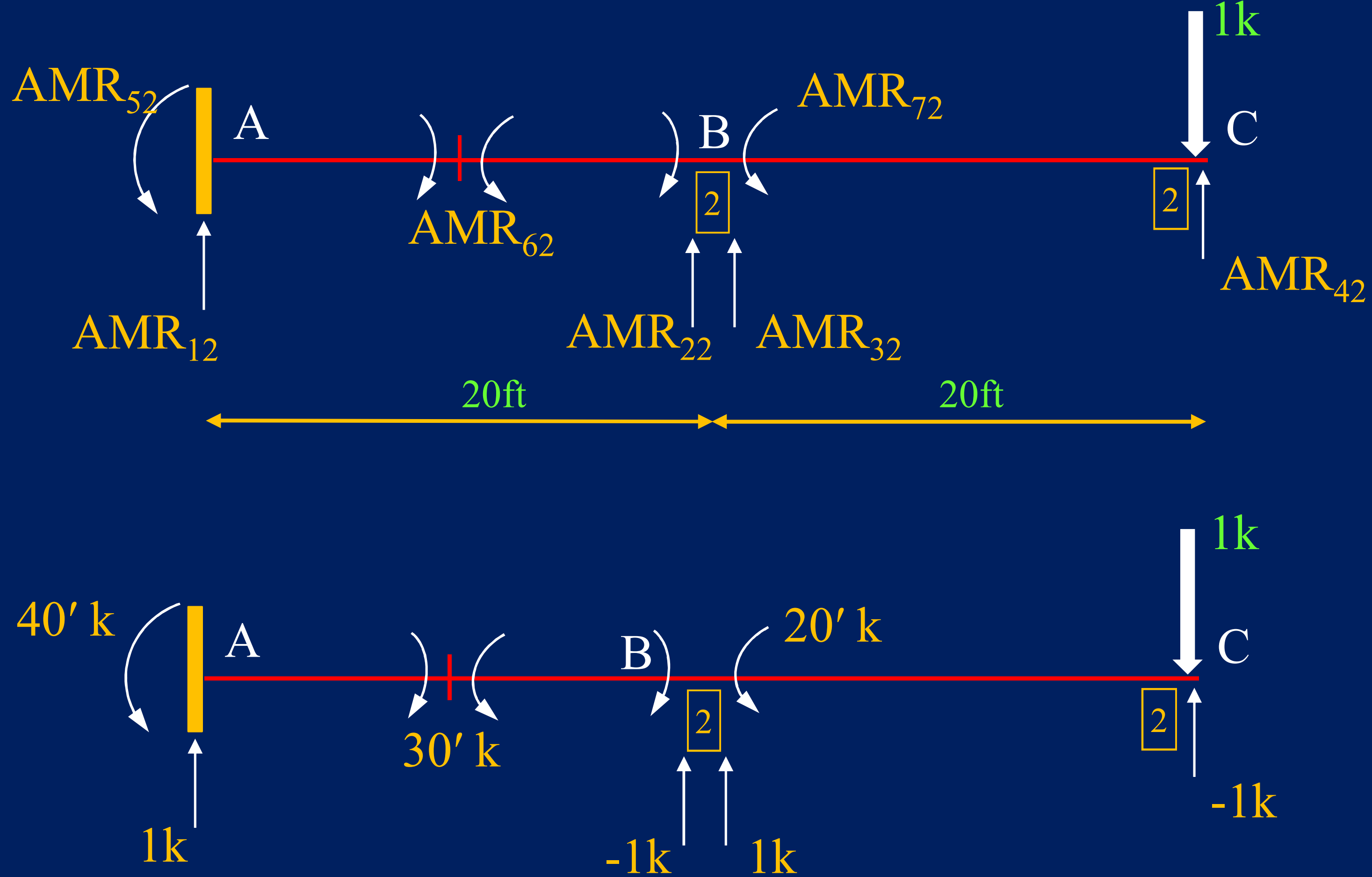
Flexibility Method for Beams Analysis

b) Compute the AMR values.

1st apply a unit action at redundant location 1 and then at 2 as shown below.



Flexibility Method for Beams Analysis



Flexibility Method for Beams Analysis

So the AMR values are

$$[AMR]_{7*2} = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \\ AMR_{71} & AMR_{72} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 0 & 1 \\ 0 & -1 \\ 20 & 40 \\ 10 & 30 \\ 0 & 20 \end{bmatrix}$$

Flexibility Method for Beams Analysis

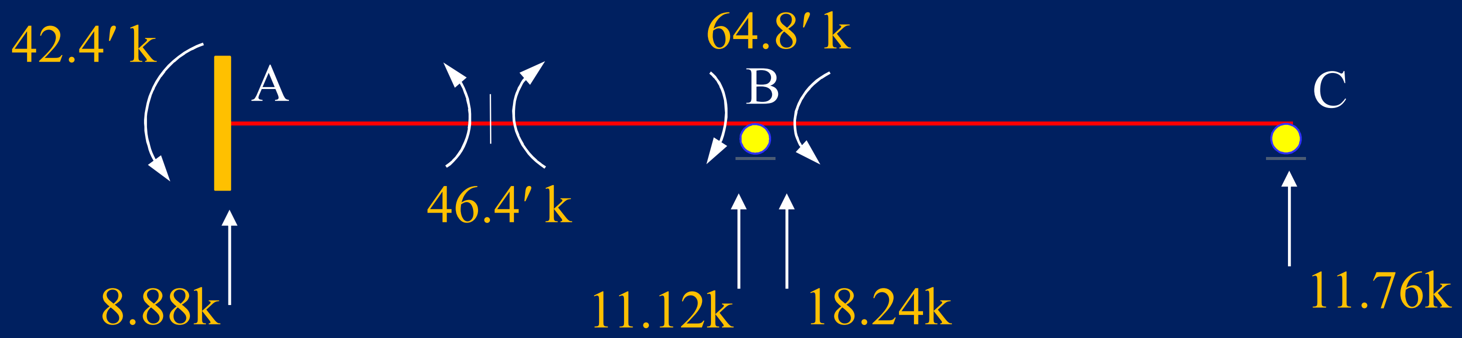
Now member end actions will be computed as given below

$$[AM] = [AML] + [AMR][AR]$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \\ AM_7 \end{bmatrix} = \begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \\ AML_7 \end{bmatrix} + \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \\ AMR_{71} & AMR_{72} \end{bmatrix} \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix}$$

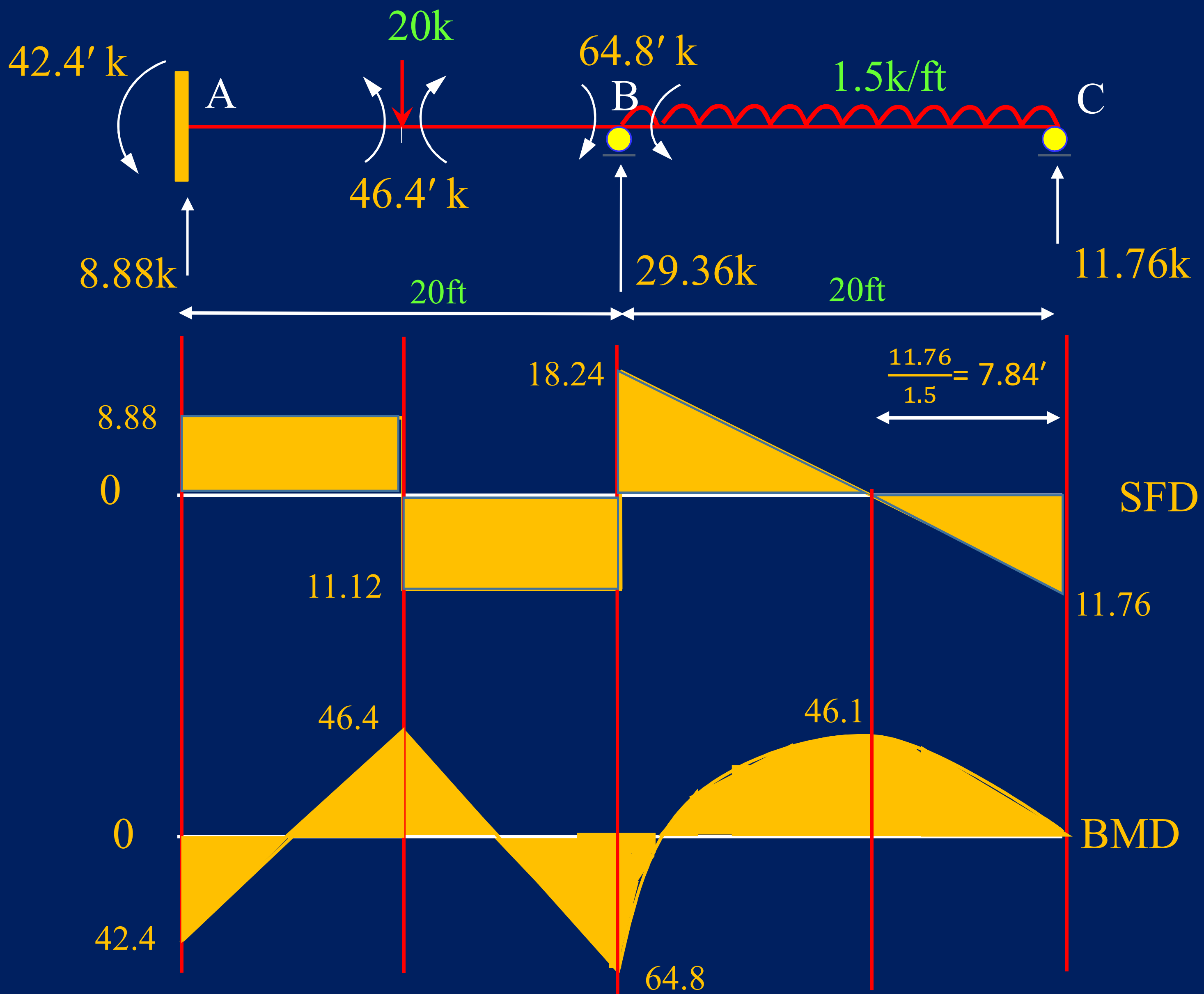
Flexibility Method for Beams Analysis

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \\ AM_7 \end{bmatrix} = \begin{bmatrix} 50 \\ -30 \\ 30 \\ 0 \\ 1100 \\ 600 \\ 300 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 0 & 1 \\ 0 & -1 \\ 20 & 40 \\ 10 & 30 \\ 0 & 20 \end{bmatrix} \begin{bmatrix} -29.36 \\ -11.76 \end{bmatrix} = \begin{bmatrix} 8.88\ \text{k} \\ 11.12\ \text{k} \\ 18.24\ \text{k} \\ 11.76\ \text{k} \\ 42.4'\ \text{k} \\ -46.4'\ \text{k} \\ 64.8'\ \text{k} \end{bmatrix}$$



Complete analyzed structure. Shear force and bending moment diagrams are given on next slide.

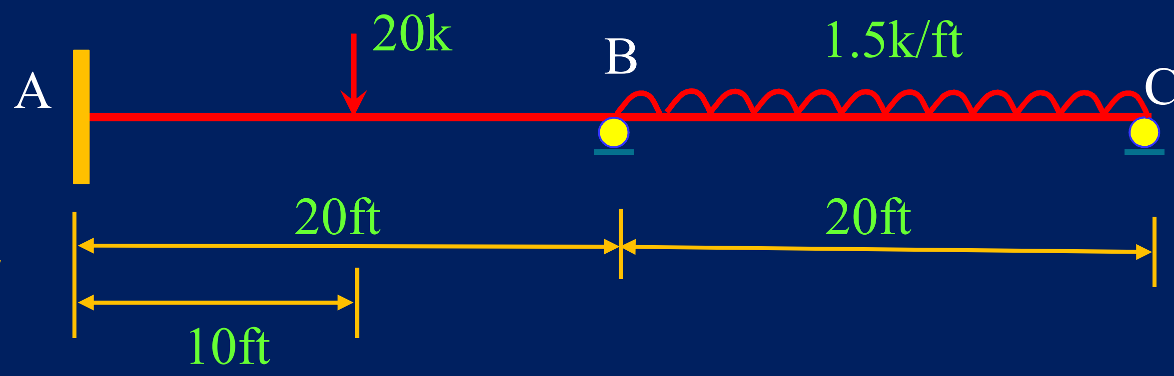
Flexibility Method for Beams Analysis

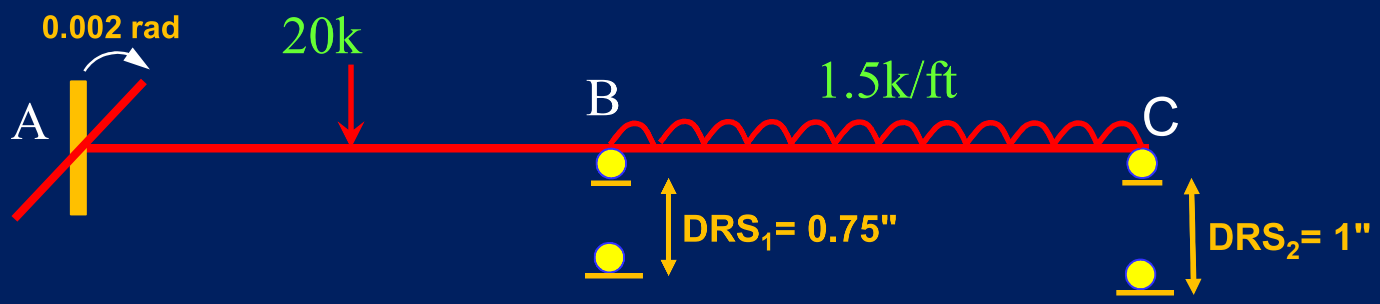


Flexibility Method for Beams Analysis

Problem 02: Analyze the given beam using flexibility method, if support A rotates by 0.002 rad clockwise, support B settles down by 0.75 in & support C settles down by 1 in.

Take
E = 30000 ksi
I = 800 in^4
EI = 166666.6 k-ft^2



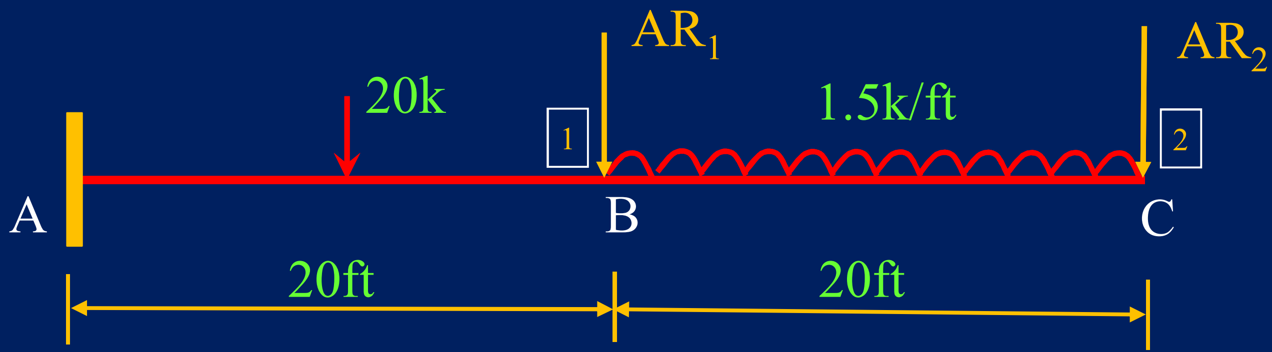


S.I = 2 degree So two redundant actions should be chosen.

Flexibility Method for Beams Analysis

- **Step # 01:** Select the redundant actions and assign coordinates at those locations.

  Vertical reactions at B & C are taken as redundants.



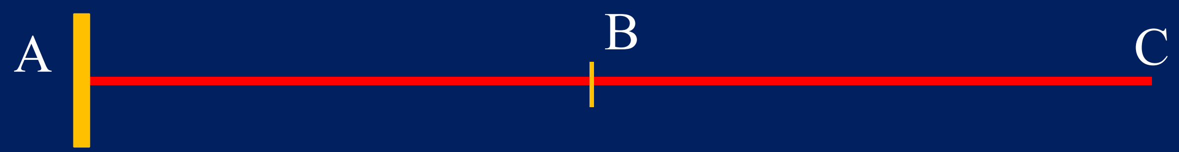
$$[AR]_{2*1} = \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[DRS]_{2*1} = \begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0.75'' \\ 1'' \end{bmatrix} = \begin{bmatrix} 0.0625' \\ 0.0833' \end{bmatrix}$$

- DRS_1 & DRS_2 are the initial support settlement corresponding to the redundant actions 1 & 2.

Flexibility Method for Beams Analysis

- Remove all the loads and redundant constraints to get the primary structure.



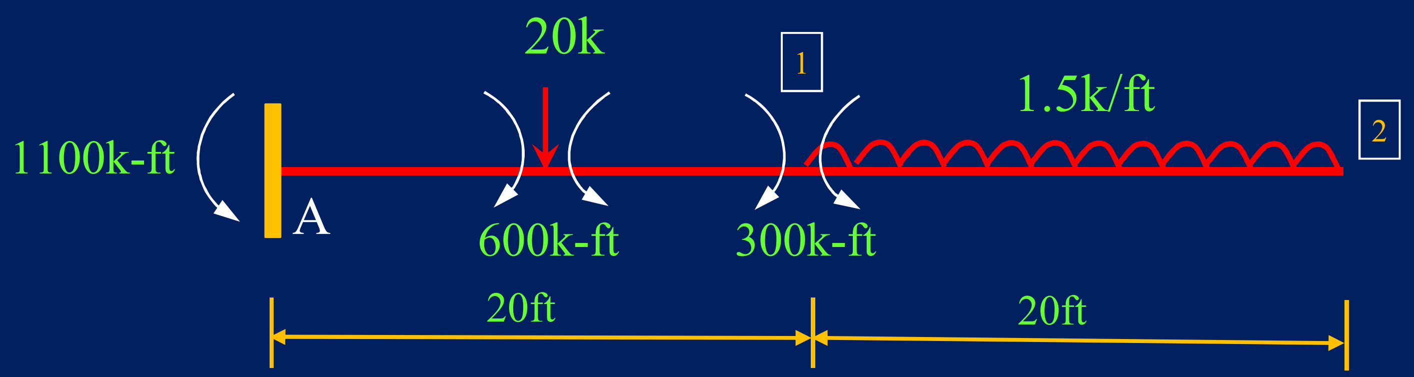
Basic determinate structure (BDS) or released structure

Note: when the support settlement and redundant action are in opposite direction then DRS will have –ve sign.

Flexibility Method for Beams Analysis

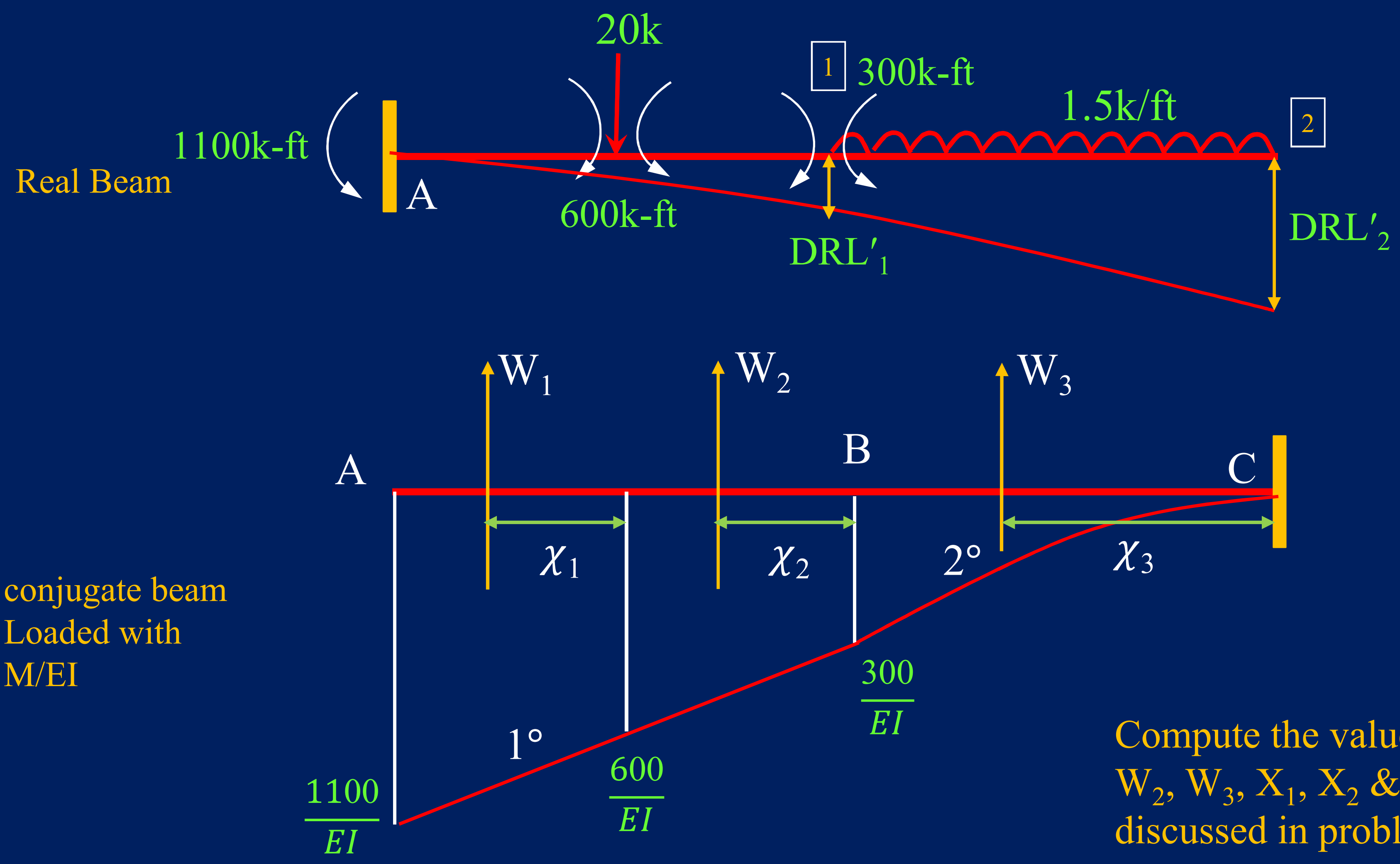
- **Step # 02 :** Compute the values of [DRL].

a) Due to direct loads [DRL′]



1st Compute the moments at different locations in the released structure

Flexibility Method for Beams Analysis



Compute the values of W_1, W_2, W_3, X_1, X_2 & X_3 as discussed in problem 1.

Flexibility Method for Beams Analysis

$$[DRL'] = \begin{bmatrix} DRL'1 \\ DRL'2 \end{bmatrix} = \begin{bmatrix} 156685 \\ 446685 \end{bmatrix} \times \frac{1}{EI}$$

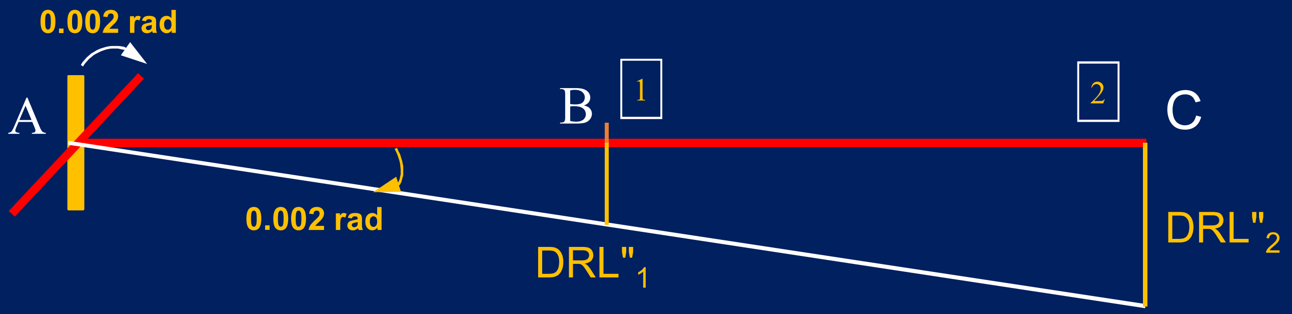
Check problem 1 for details calculations

$$[DRL'] = \begin{bmatrix} DRL'1 \\ DRL'2 \end{bmatrix} = \begin{bmatrix} 156685 \\ 446685 \end{bmatrix} \times \frac{1}{166666.67}$$

$$[DRL'] = \begin{bmatrix} DRL'1 \\ DRL'2 \end{bmatrix} = \begin{bmatrix} 0.94' \\ 2.68' \end{bmatrix}$$

Flexibility Method for Beams Analysis

b) Due to indirect loads [DRL″]



As we know that for smaller angles

$\tan\theta = \theta$

$DRL'_1 = 0.002 * 20 = 0.04$ ft

$DRL''_2 = 0.002 * 40 = 0.08$ ft

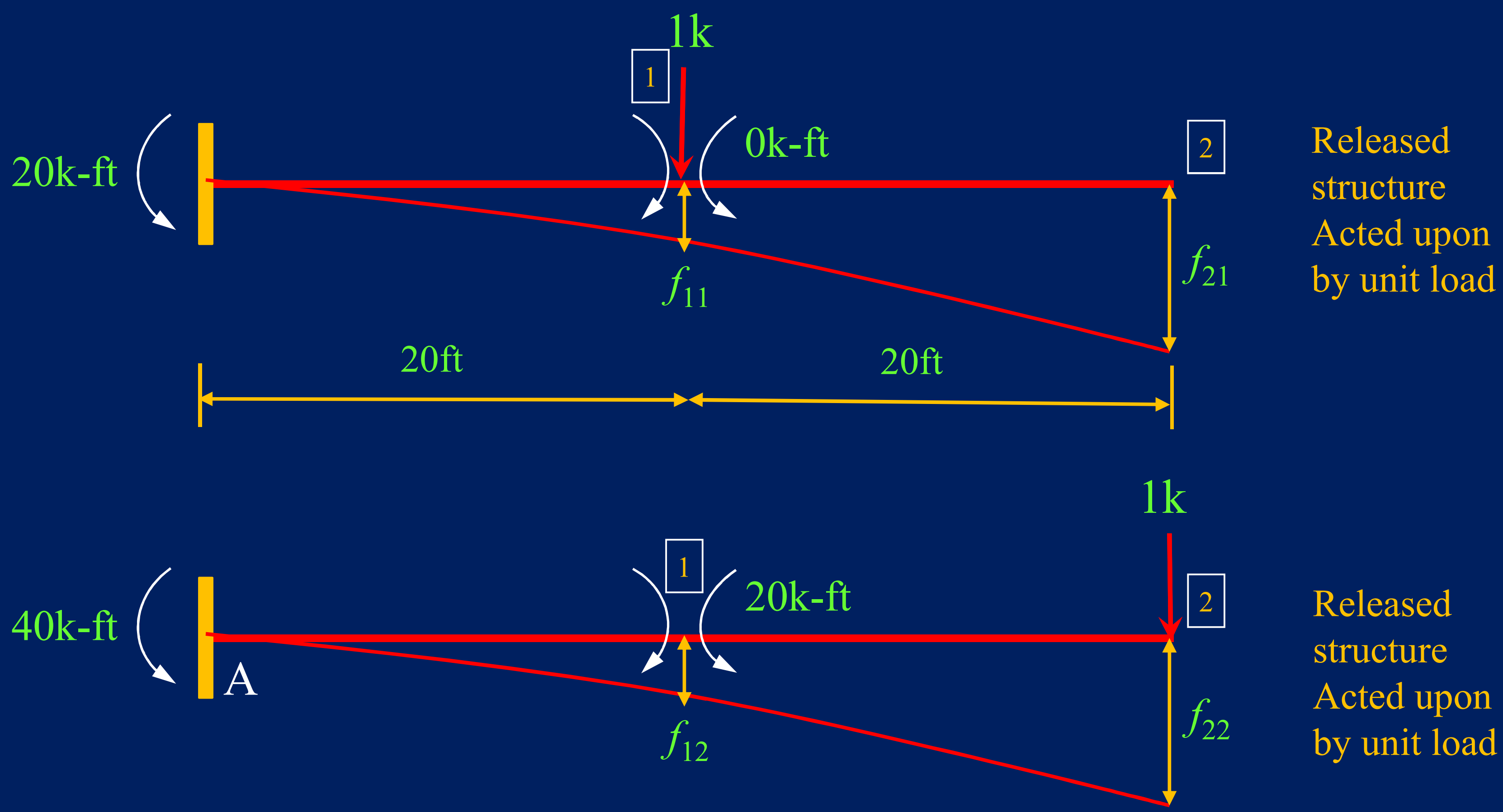
$$\Rightarrow [DRL''] = \begin{bmatrix} DRL''_1 \\ DRL''_2 \end{bmatrix} = \begin{bmatrix} 0.04' \\ 0.08' \end{bmatrix}$$

So final DRL will be

$$[DRL] = \begin{bmatrix} DRL'1 \\ DRL'2 \end{bmatrix} + \begin{bmatrix} DRL''1 \\ DRL''2 \end{bmatrix} = \begin{bmatrix} 0.94' \\ 2.68' \end{bmatrix} + \begin{bmatrix} 0.04' \\ 0.08' \end{bmatrix} = \begin{bmatrix} 0.98' \\ 2.76' \end{bmatrix}$$

Flexibility Method for Beams Analysis

- **Step # 03 :** Compute the values of flexibility matrix [f].



Flexibility Method for Beams Analysis

$$f_{11} = \frac{2666}{EI} \qquad f_{12} = \frac{6666}{EI}$$

$$f_{21} = \frac{6666}{EI} \qquad f_{22} = \frac{21336}{EI}$$

For detail calculations see problem 1.

$$[f] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$$

$$[f] = \frac{1}{EI}\begin{bmatrix} 2666 & 6666 \\ 6666 & 21336 \end{bmatrix}$$

Flexibility coefficient matrix

$$[f] = \frac{1}{166666.67}\begin{bmatrix} 2666 & 6666 \\ 6666 & 21336 \end{bmatrix}$$

$$[f] = \begin{bmatrix} 0.016 & 0.04 \\ 0.04 & 0.128 \end{bmatrix}$$

Flexibility Method for Beams Analysis

Step # 04: Compute the values of redundant actions AR. As we know that

$$[DRS] = [DRL] + [f] \bullet [AR]$$

$$[AR] = [f]^{-1} \bullet [DRS - DRL]$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

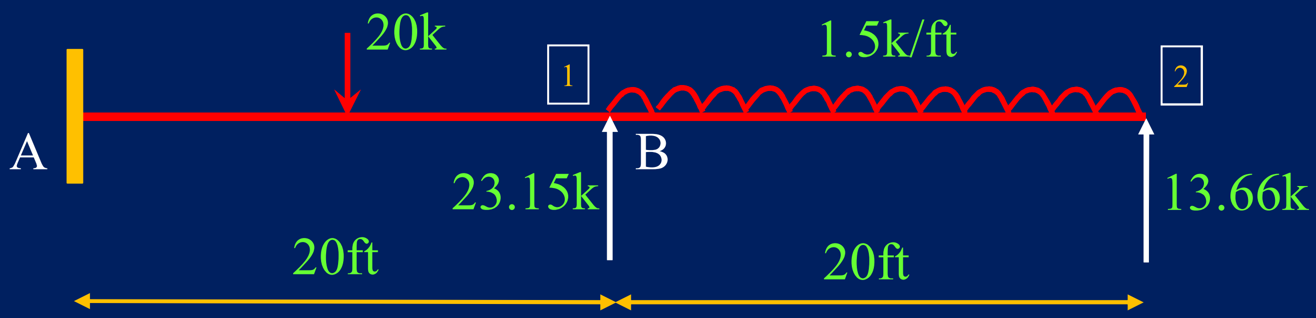
$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 0.016 & 0.04 \\ 0.04 & 0.128 \end{bmatrix}^{-1} \begin{bmatrix} 0.0625 - 0.98 \\ 00833 - 2.76 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} -23.15 \\ -13.66 \end{bmatrix}$$

-ive sign shows that our assumed redundant actions directions are wrong

Flexibility Method for Beams Analysis

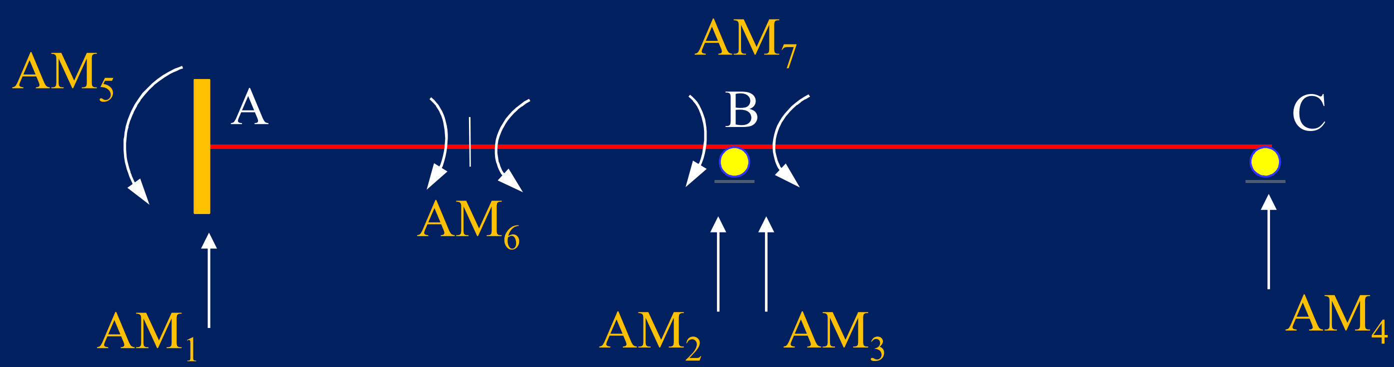
Final determinate structure



Now we can apply equilibrium equations to further solve the structure.
Or we can compute member end actions using matrix approach as given in the coming slides

Flexibility Method for Beams Analysis

Step # 05: Compute the member end actions. As we know that



a). Compute the AML values

$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \\ AML_7 \end{bmatrix} = \begin{bmatrix} 50 \\ -30 \\ 30 \\ 0 \\ 1100 \\ 600 \\ 300 \end{bmatrix}$$

For detail calculation See problem 1.

Flexibility Method for Beams Analysis

b). Compute the AMR values

$$[AMR]_{7*2} = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \\ AMR_{71} & AMR_{72} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 0 & 1 \\ 0 & -1 \\ 20 & 40 \\ 10 & 30 \\ 0 & 20 \end{bmatrix}$$

For detail calculation see problem 1 in this module.

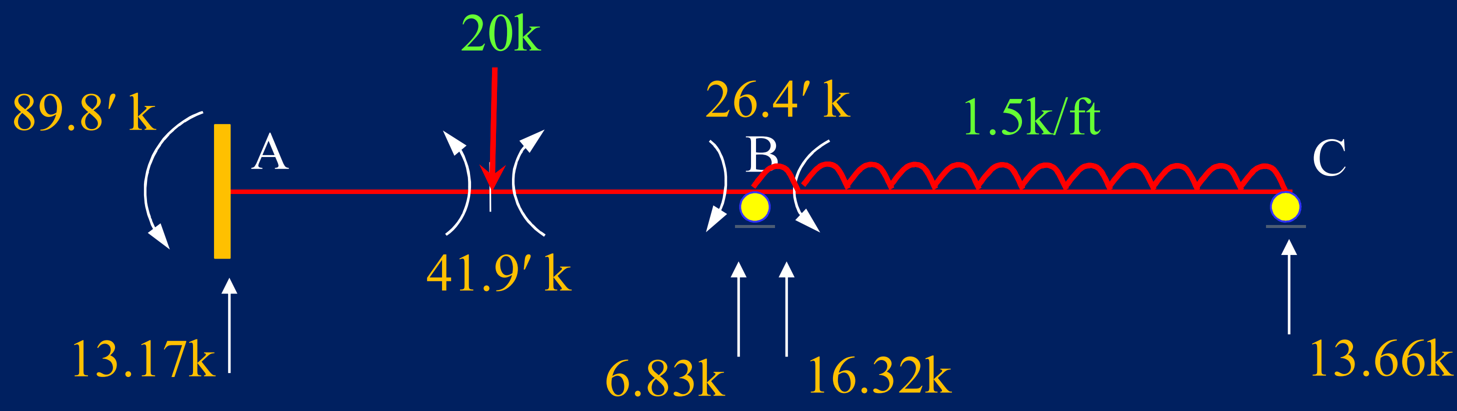
Flexibility Method for Beams Analysis

Now

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \\ AM_7 \end{bmatrix} = \begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \\ AML_7 \end{bmatrix} + \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \\ AMR_{71} & AMR_{72} \end{bmatrix} \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \\ AM_7 \end{bmatrix} = \begin{bmatrix} 50 \\ -30 \\ 30 \\ 0 \\ 1100 \\ 600 \\ 300 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 0 & 1 \\ 0 & -1 \\ 20 & 40 \\ 10 & 30 \\ 0 & 20 \end{bmatrix} \begin{bmatrix} -23.15 \\ -13.66 \end{bmatrix} = \begin{bmatrix} 13.17\ \text{k} \\ 6.83\ \text{k} \\ 16.32\ \text{k} \\ 13.68\ \text{k} \\ 89.80'\ \text{k} \\ -41.90'\ \text{k} \\ 26.40'\ \text{k} \end{bmatrix}$$

Flexibility Method for Beams Analysis



Final analyzed structure

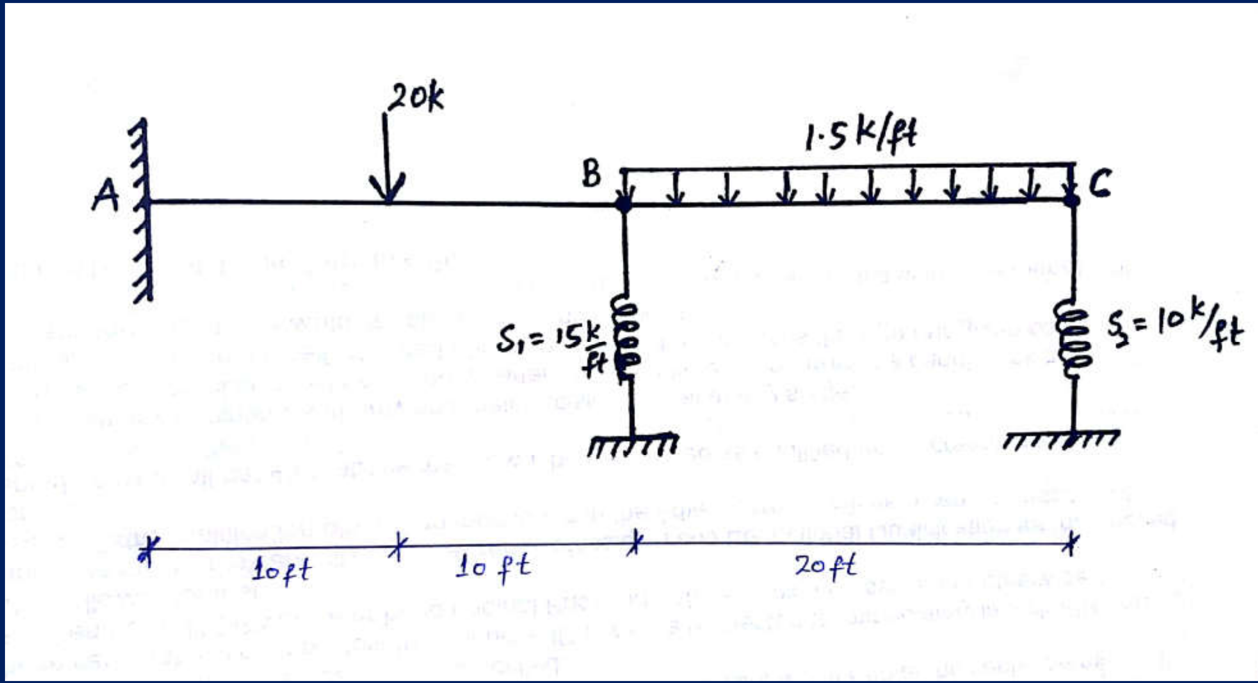
- **Class Activity:** Draw shear force and bending moment diagram for the above given beam .

Flexibility Method for Beams Analysis

Problem 03: Analyze the given beam using flexibility method.

The supports at B & C are spring supports having stiffness's S_1= 15 k/ft & S_2=10 k/ft as shown in fig.

Take
E = 30000 ksi
I = 800 in^4
EI = 166666.6 k-ft^2

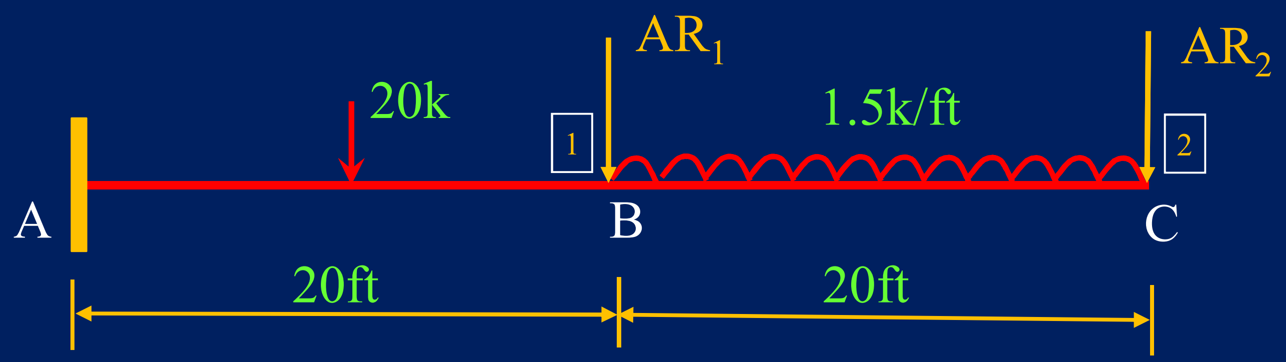


S.I = 2 degree So two redundant actions should be chosen.

Flexibility Method for Beams Analysis

- **Step # 01:** Select the redundant actions and assign coordinates at those locations.

  Vertical reactions at B & C are taken as redundants.



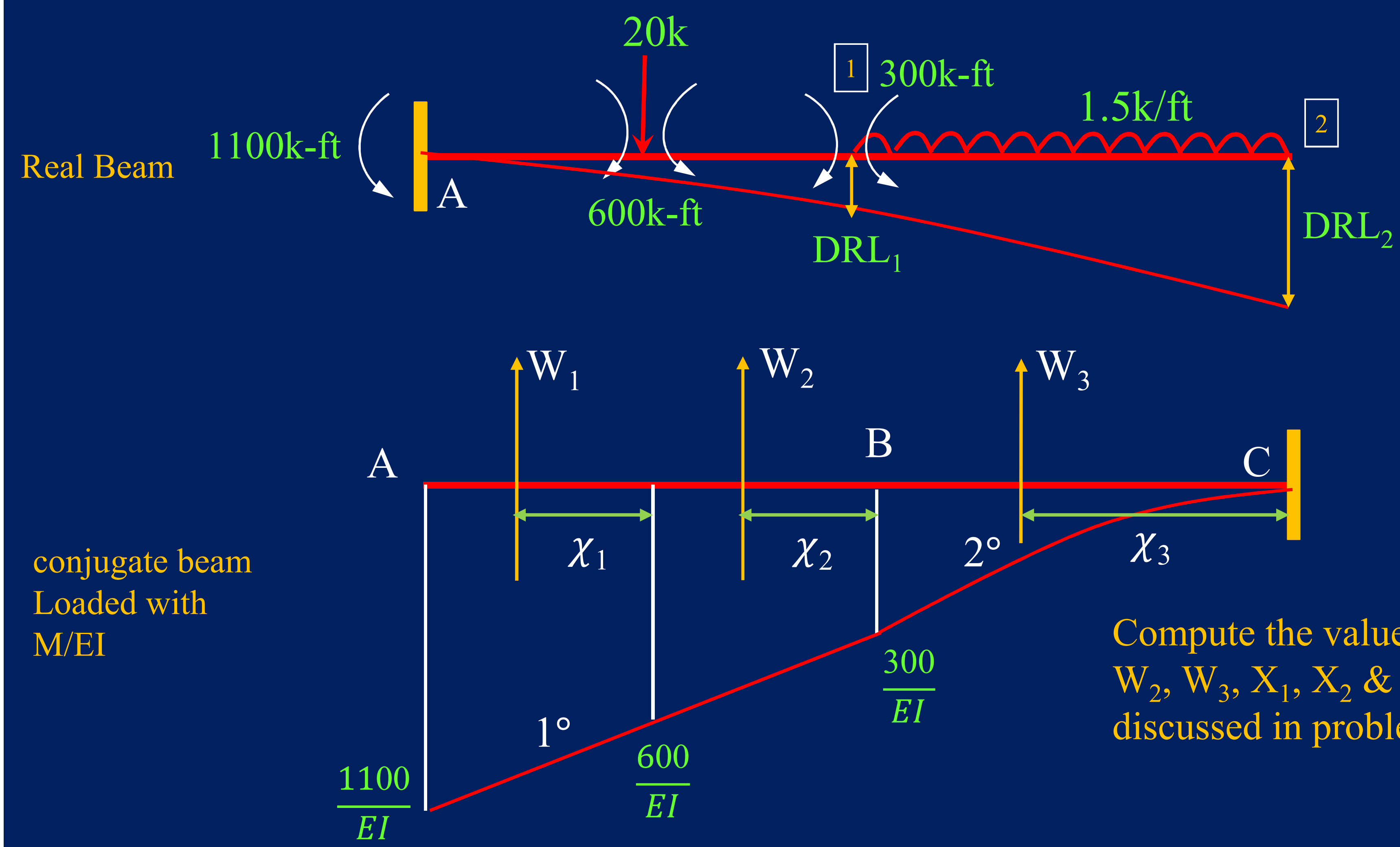
$$[AR]_{2*1} = \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[DRS]_{2*1} = \begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- DRS_1 & DRS_2 are the initial support settlement corresponding to the redundant actions 1 & 2.

Flexibility Method for Beams Analysis

- **Step # 02 :** Compute the values of [DRL].



Compute the values of W_1, W_2, W_3, X_1, X_2 & X_3 as discussed in problem 1.

Flexibility Method for Beams Analysis

$$DRL_1 = \frac{156685}{\mathrm{EI}}$$

$$DRL_2 = \frac{446685}{\mathrm{EI}}$$

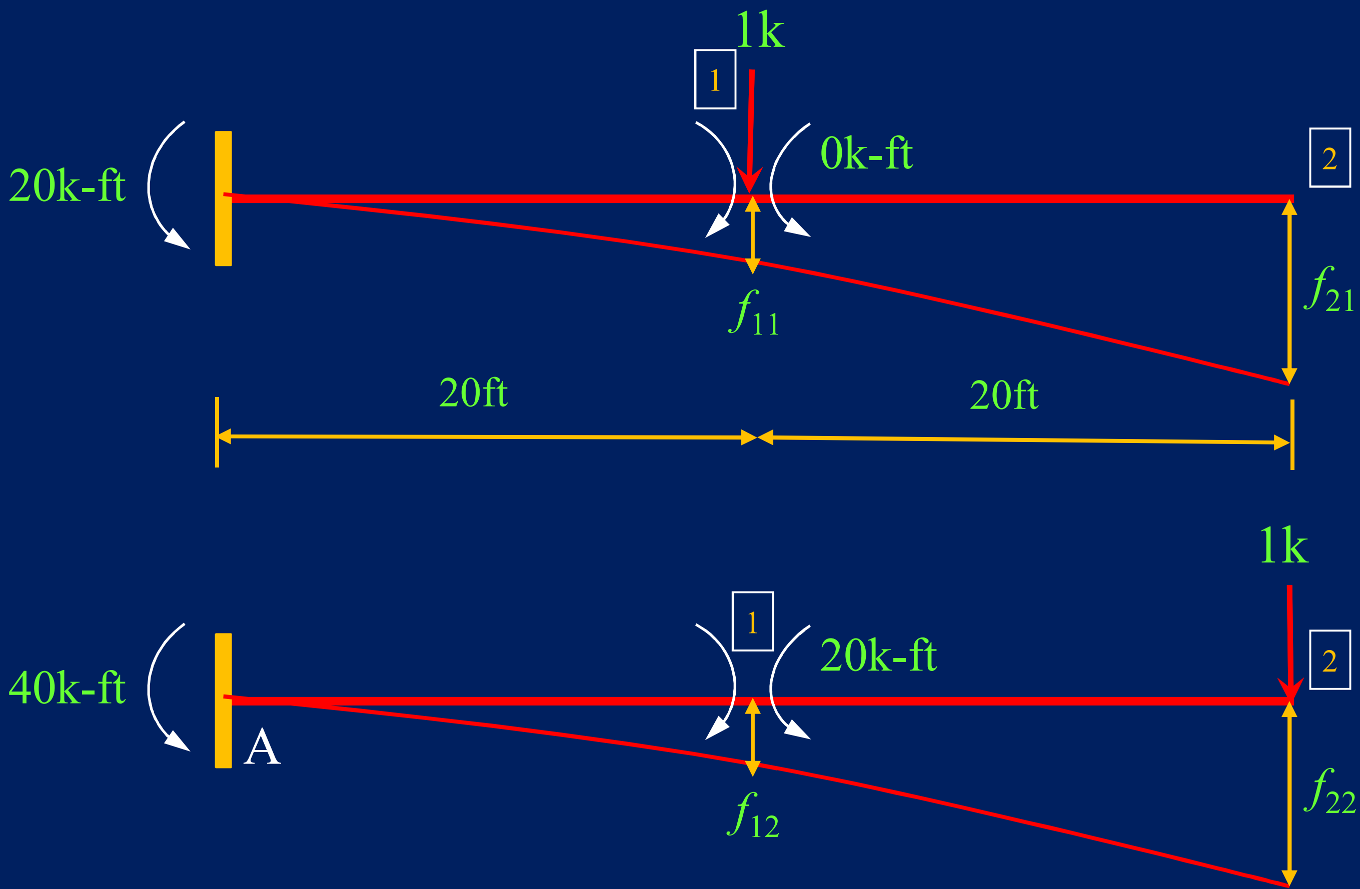
$$[DRL] = \begin{bmatrix} DRL_1 \\ DRL_2 \end{bmatrix} = \begin{bmatrix} 156685 \\ 446685 \end{bmatrix} \times \frac{1}{EI}$$

$$= \begin{bmatrix} 156685 \\ 446685 \end{bmatrix} \times \frac{1}{166666.6}$$

$$= \begin{bmatrix} 0.94 \\ 2.68 \end{bmatrix}$$

Flexibility Method for Beams Analysis

- **Step # 03 :** Compute the values of flexibility matrix [f]. Also as there are spring supports at redundant location 1& 2 so add the effect of spring with flexibility matrix.



Released structure Acted upon by unit load

Released structure Acted upon by unit load

Flexibility Method for Beams Analysis

$$[f] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$$

For detail calculations see problem 1.

$$[f] = \frac{1}{EI}\begin{bmatrix} 2666 & 6666 \\ 6666 & 21336 \end{bmatrix}$$

$$[f] = \frac{1}{166666.67}\begin{bmatrix} 2666 & 6666 \\ 6666 & 21336 \end{bmatrix} = \begin{bmatrix} 0.016 & 0.04 \\ 0.04 & 0.128 \end{bmatrix}$$

Now add the effect of spring with Flexibility matrix , as we know that

Stiffness = 1/ Flexibility

$$[f] = \begin{bmatrix} 0.016 & 0.04 \\ 0.04 & 0.128 \end{bmatrix} + \begin{bmatrix} \frac{1}{15} & 0 \\ 0 & \frac{1}{10} \end{bmatrix} = \begin{bmatrix} 0.083 & 0.04 \\ 0.04 & 0.228 \end{bmatrix}$$

Flexibility coefficient matrix

Flexibility Method for Beams Analysis

Step # 04: Compute the values of redundant actions AR. As we know that

$$[DRS] = [DRL] + [f] \bullet [AR]$$

$$[AR] = [f]^{-1} \bullet [DRS - DRL]$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} DRS_1 - DRL_1 \\ DRS_2 - DRL_2 \end{bmatrix}$$

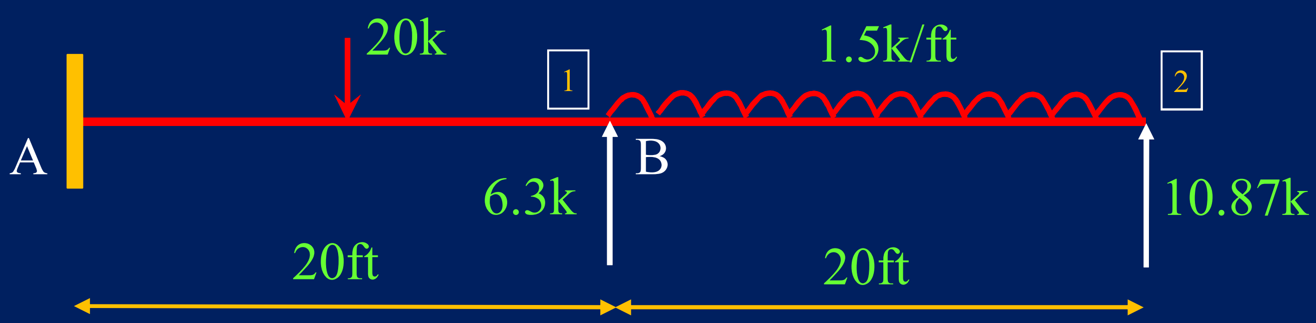
$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 0.016 & 0.04 \\ 0.04 & 0.128 \end{bmatrix}^{-1} \begin{bmatrix} 0.0625 - 0.98 \\ 00833 - 2.76 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} -6.30 \\ -10.87 \end{bmatrix}$$

-ive sign shows that our assumed redundant actions directions are wrong

Flexibility Method for Beams Analysis

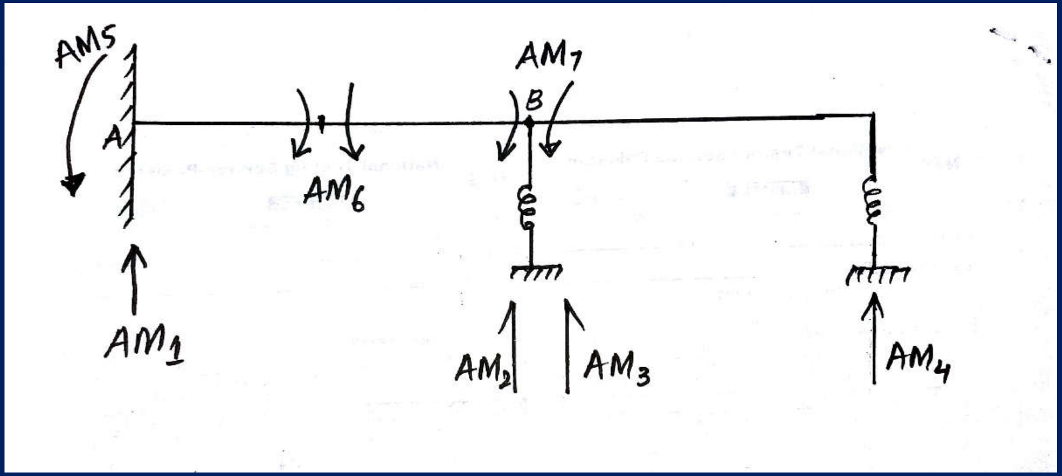
Final determinate structure



Now we can apply equilibrium equations to further solve the structure. Or we can compute member end actions using matrix approach as given in the coming slides

Flexibility Method for Beams Analysis

Step # 05: Compute the member end actions. As we know that



a). Compute the AML values

$$\begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \\ AML_7 \end{bmatrix} = \begin{bmatrix} 50 \\ -30 \\ 30 \\ 0 \\ 1100 \\ 600 \\ 300 \end{bmatrix}$$

For detail calculation See problem 1.

Flexibility Method for Beams Analysis

b). Compute the AMR values

$$[AMR]_{7\,*\,2} = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \\ AMR_{71} & AMR_{72} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 0 & 1 \\ 0 & -1 \\ 20 & 40 \\ 10 & 30 \\ 0 & 20 \end{bmatrix}$$

For detail calculation see problem 1 in this module.

Flexibility Method for Beams Analysis

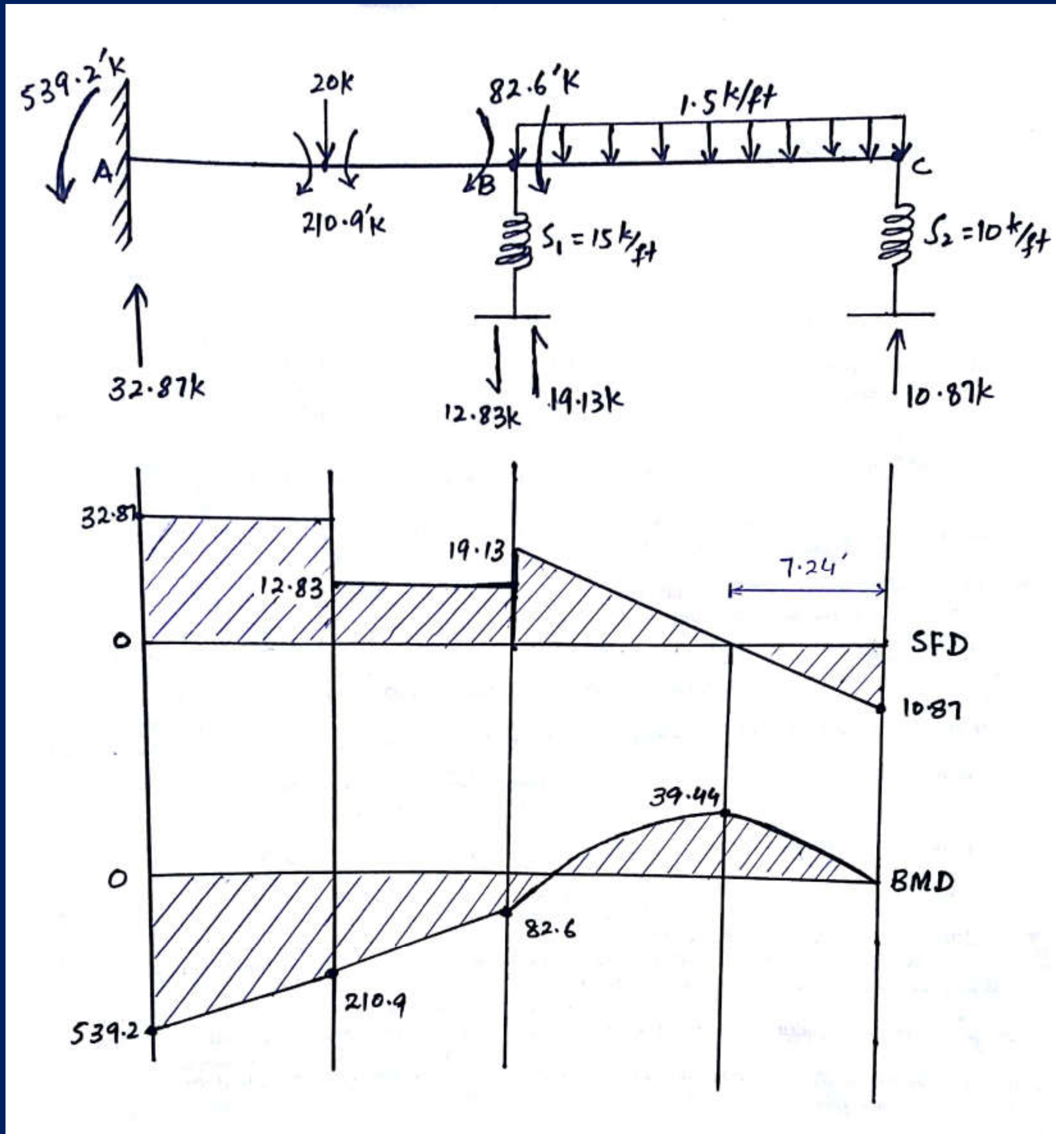
Now

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \\ AM_7 \end{bmatrix} = \begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \\ AML_7 \end{bmatrix} + \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \\ AMR_{71} & AMR_{72} \end{bmatrix} \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \\ AM_7 \end{bmatrix} = \begin{bmatrix} 50 \\ -30 \\ 30 \\ 0 \\ 1100 \\ 600 \\ 300 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 0 & 1 \\ 0 & -1 \\ 20 & 40 \\ 10 & 30 \\ 0 & 20 \end{bmatrix} \begin{bmatrix} -6.30 \\ -10.87 \end{bmatrix} = \begin{bmatrix} 32.87\ \text{k} \\ -12.83\ \text{k} \\ 19.13\ \text{k} \\ 10.87\ \text{k} \\ 539.2'\ \text{k} \\ 210.9'\ \text{k} \\ 82.6'\ \text{k} \end{bmatrix}$$

Flexibility Method for Beams Analysis

Now Shear force and Bending moment diagram



References

- Structural Analysis by R. C. Hibbeler
- Matrix structural analysis by William Mc Guire
- Matrix analysis of frame structures by William Weaver
- Online Civil Engineering blogs