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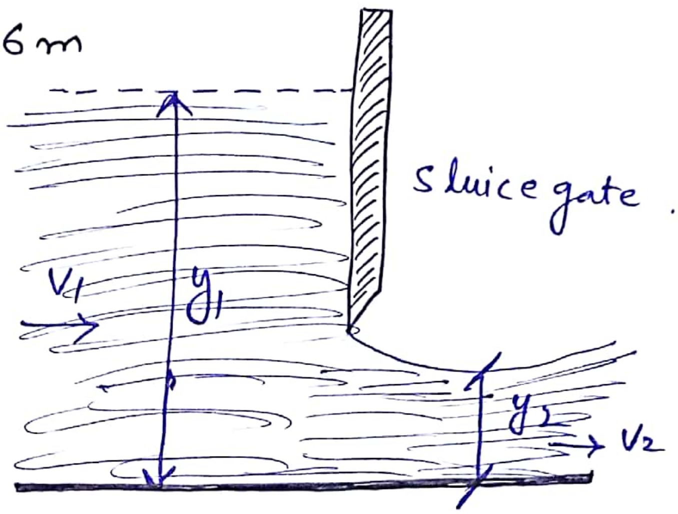
MODULE # 1(c)

Problem:-

If $y_1 = 2.5m$, $y_2 = 0.6m$
$b = 3.5m$ determine
the discharge Q
and Froud number



Solution:-

Specific energy at upstream and down stream

$$E_1 = E_2$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} \longrightarrow (1)$$

Also

$$Q = A_1V_1 = A_2V_2$$

$$b_1y_1V_1 = b_2y_2V_2 \qquad b_1 = b_2 = b$$

$$\cancel{b}y_1V_1 = \cancel{b}y_2V_2$$

$$y_1V_1 = y_2V_2$$

$$\frac{2.5V_1}{0.6} = \frac{0.6\,V_2}{0.6}$$

$$\boxed{V_2 = 4.16\,V_1} \longrightarrow (2)$$

Substituting values in eq 1

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$2.5 + \frac{V_1^2}{2g} = 0.6 + \frac{(4.16 V_1)^2}{2g}$$

$$2.5 + \frac{V_1^2}{2g} = 0.6 + \frac{17.31 V_1^2}{2g}$$

$$\frac{V_1^2}{2g} - \frac{17.31 V_1^2}{2g} = 0.6 - 2.5$$

$$\cancel{-}\, 16.31 \frac{V_1^2}{2g} = \cancel{-}\, 1.9$$

$$16.31 \frac{V_1^2}{2g} = 1.9$$

$$\sqrt{V_1^{\cancel{2}}} = \sqrt{\frac{1.9 \times 2g}{16.31}}$$

$$V_1 = \sqrt{\frac{1.9 \times 2 \times 9.81}{16.31}}$$

$$\boxed{V_1 = 1.5093 \text{ m/sec}}$$

put in eq 1 to find V_2

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$2.5 + \frac{(1.5093)^2}{2 \times 9.81} = 0.6 + \frac{V_2^2}{2 \times 9.81}$$

$$2.616 = 0.6 + \frac{V_2^2}{19.62}$$

$$2.616 - 0.6 = \frac{V_2^2}{19.62}$$

$$2.016 = \frac{V_2^2}{19.62}$$

$$2.016 \times 19.62 = V_2^2$$

$$\sqrt{39.55} = \sqrt{V_2^{\cancel{2}}}$$

$$\boxed{V_2 = 6.3\ m/sec}$$

Discharge $Q_1 = A_1 V_1$

$Q_1 = b y_1 V_1$

$Q_1 = 3.5 \times 2.5 \times 1.51$

$$\boxed{Q_1 = 13.21\ m^3/sec}$$

$Q_2 = A_2 V_2 = b y_2 V_2$

$Q_2 = 3.5 \times 0.6 \times 6.3$

$$\boxed{Q_2 = 13.22\ m^3/sec}$$

So discharge $Q_1 = Q_2 = Q = 13.21\ m^3/sec$

Froud Number:-

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{1.51}{\sqrt{9.81 \times 2.5}} \Rightarrow \boxed{Fr_1 = 0.31}$$

$$Fr_2 = \frac{V_2}{\sqrt{g y_2}} = \frac{6.3}{\sqrt{9.81 \times 0.6}} \Rightarrow \boxed{Fr_2 = 2.59}$$

Hump:-

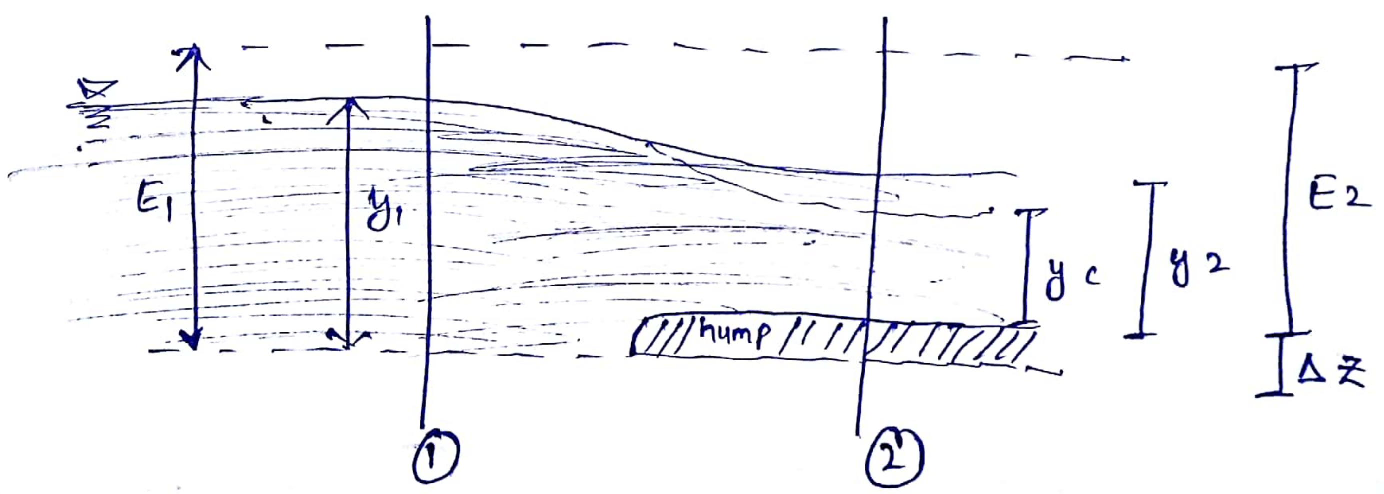
It is a structure or obstruction that is Constructed across a river or stream to raise the level of water on upstream side so that it can be diverted to Canals to meet the irrigation requirements.

Flow over Hump:-

Consider a horizontal, friction less rectangular channel of width B carrying a discharge Q at depth of y_1. let the flow be subcritical. At Section 2, a smooth hump of height ΔZ is built on the floor.

Construction of a hump cause the specific Energy at section 2 to decrease by ΔZ.

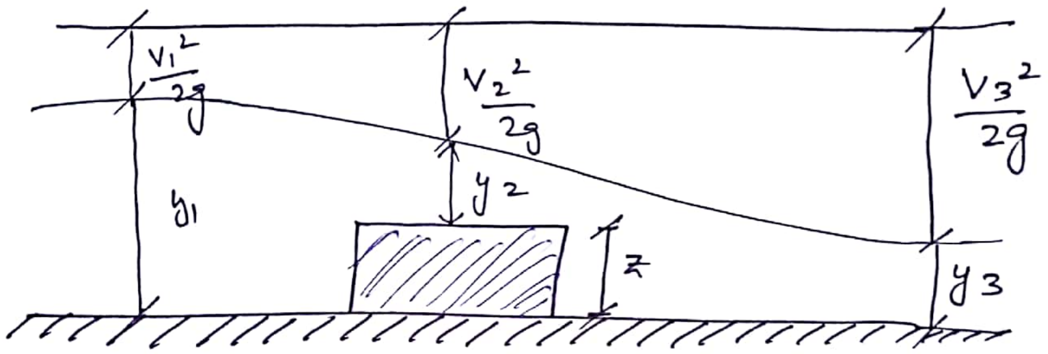
Thus the specific energies at section 1 & 2 are:



$$E_1 = y_1 + \frac{V_1^2}{2g}$$

$$E_2 = E_1 - \Delta z$$

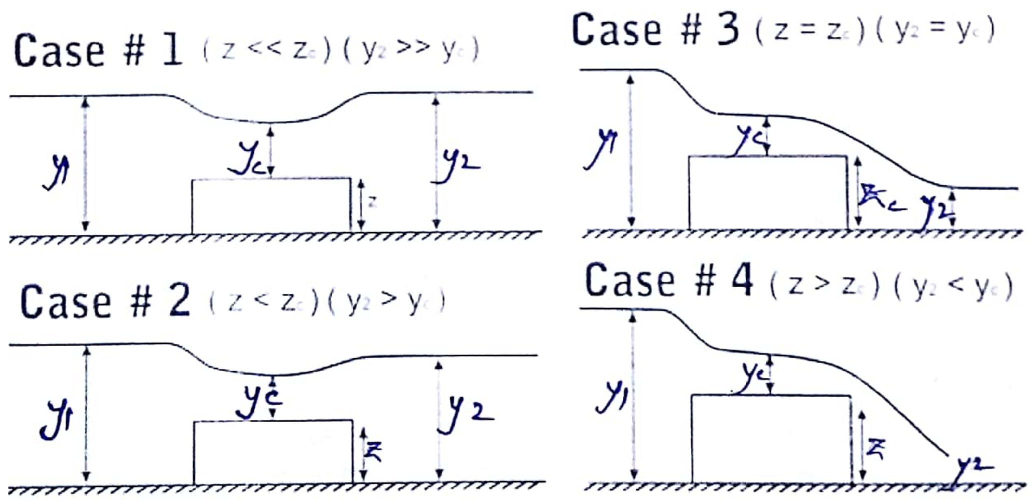
Effect of hump over depth of flow:-



Critical Hump Height:-

It is the minimum height of hump which causes the critical depth or critical flow conditions over the hump.

Hump Height and depth of flow

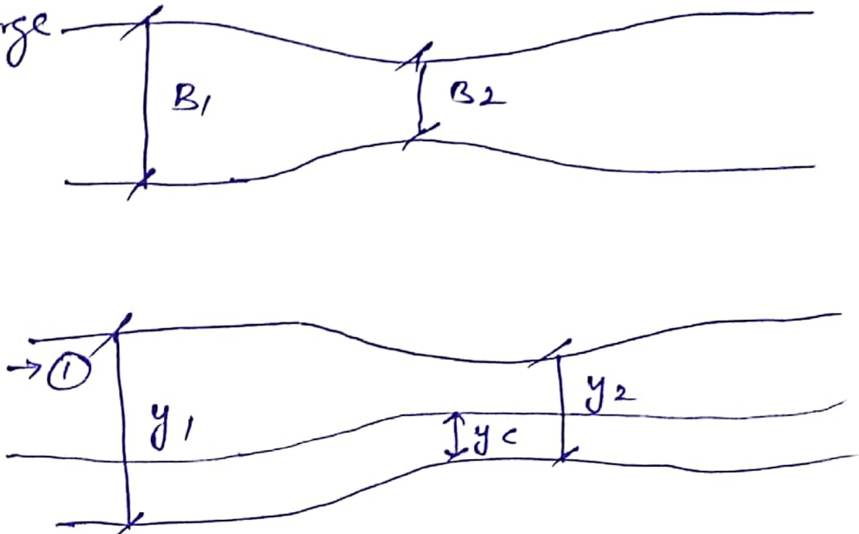


Damming Action:- If the hump height is made higher then the critical hump height, critical depth is maintained over the hump, this phenomena is known as Damming Action.

Constrictions:-

When the width of the channel is reduced while the bed of the channel remains Constant, the discharge per unit width increase. If the losses are negligible the Specific energy remains Constant

let Suppose discharge is Same



$Q_1 = Q_2$.

$A_1V_1 = A_2V_2$

$B_1y_1V_1 = B_2y_2V_2 \rightarrow (1)$

Also

When the losses are negligible

$$E_1 = E_2.$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} \rightarrow (2)$$

using eq (2) to find V_1^2

$$y_1 - y_2 = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$y_1 - y_2 = \frac{V_2^2 - V_1^2}{2g}$$

$$V_2^2 - V_1^2 = (y_1 - y_2)2g$$

$$-V_1^2 = (y_1 - y_2)2g - V_2^2$$

$$\boxed{V_1^2 = -(y_1 - y_2)2g + V_2^2} \rightarrow \text{eq } 3$$

Now using eq 1.

$$B_1 y_1 V_1 = B_2 y_2 V_2$$

$$V_1 = \frac{B_2 y_2 V_2}{B_1 y_1}$$

Squaring both side.

$$\boxed{V_1^2 = \frac{B_2^2 y_2^2 V_2^2}{B_1^2 y_1^2}} \rightarrow \text{eq } 4$$

Equating eq 3 & 4.

$$\frac{B_2^2 y_2^2 V_2^2}{B_1^2 y_1^2} = -(y_1 - y_2)2g + V_2^2$$

$$(y_1 - y_2)\,2g = V_2^2 - \frac{B_2^2 y_2^2 V_2^2}{B_1^2 y_1^2}$$

$$(y_1 - y_2)\,2g = V_2^2\left[1 - \frac{B_2^2 y_2^2}{B_1^2 y_1^2}\right]$$

$$V_2^2 = \frac{(y_1 - y_2)\,2g}{1 - \left(\frac{B_2 y_2}{B_1 y_1}\right)^2}$$

Taking "$\sqrt{\ }$" b/s

$$V_2 = \sqrt{\frac{(y_1 - y_2)\, 2g}{1 - \left(\frac{B_2 y_2}{B_1 y_1}\right)^2}}$$

As

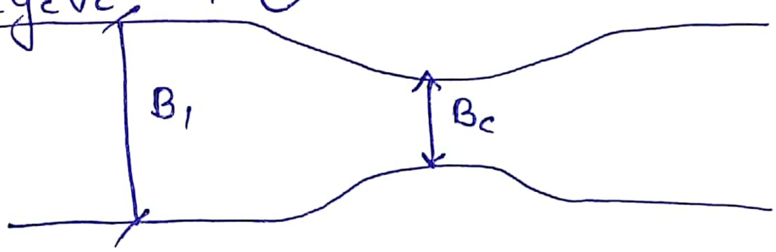
$$Q_2 = B_2 y_2 V_2$$

$$\boxed{Q_2 = B_2 y_2 \sqrt{\frac{(y_1 - y_2)\, 2g}{1 - \left(\frac{B_2 y_2}{B_1 y_1}\right)^2}}}$$

When the flow is critical.

$$Q_c = A V_c$$

Discharge $= Q_c = B_c y_c V_c \rightarrow (1)$



As $E_{min} = y_c + \frac{V_c^2}{2g}$.

$$E_{min} - y_c = \frac{V_c^2}{2g}.$$

$$\sqrt{(E_{min} - y_c)\, 2g} = \sqrt{V_c^2}$$

$$V_c = \sqrt{(E_{min} - y_c)\, 2g}$$

Put V_c in eq (1)

$$Q = B_c y_c V_c$$

$$\boxed{Q = B_c y_c \sqrt{(E_{min} - y_c) 2g}}$$

Hydraulic Jump :-

A hydraulic Jump is defined as a rise in level of water in an open channel.

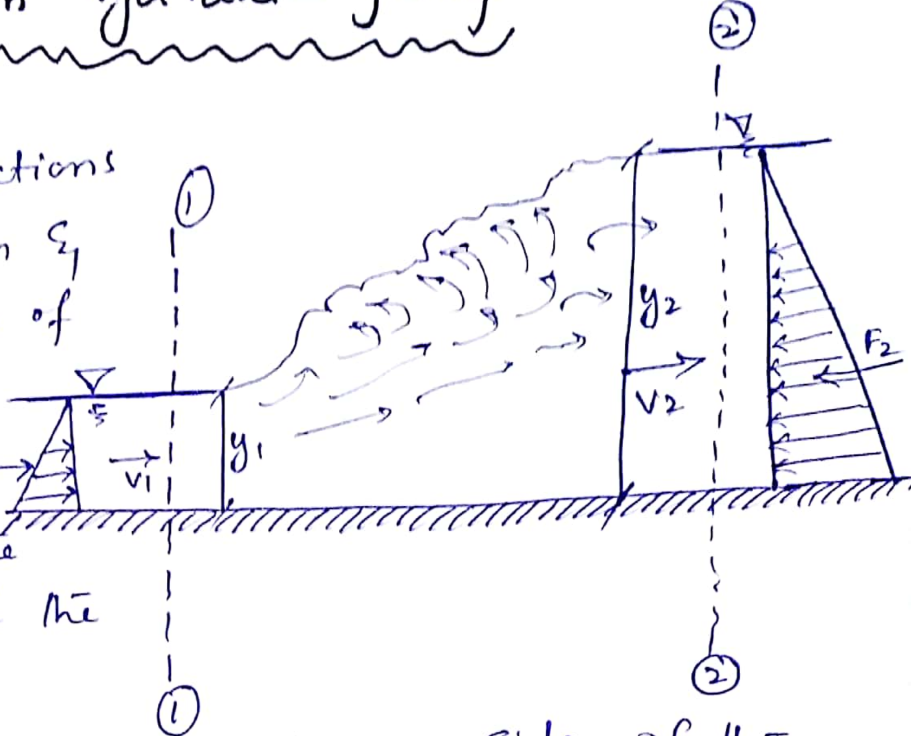
A hydraulic jump occurs when a liquid at a high velocity discharges into a zone that has a lower velocity.

A hydraulic Jump occurs when the flow goes from Super Critical flow ($F_r > 1$) to Subcritical flow ($F_r < 1$) or from unstable flow to a Stable flow.

There are two possible depths h_1 & h_2. The depth h_1 is smaller than critical depth and h_2 is greater than the critical depth.

Water rise in hydraulic Jump:-

Consider two sections on the upstream & downstream side of a Jump



1-1 = Section on the upstream side of the hydraulic Jump

2-2 - Section on the downstream side of the hydraulic Jump.

y_1 = depth of flow at Section 1-1

V_2 = Flow velocity at Section 1-1

y_2, V_2 = Corresponding value at Section 2-2

q = discharge per unit width.

$Q = qb$

$- q = \frac{Q}{b}$

Q = Total discharge

b = width of channel and hydraulic Jump.

$- Q = AV$

$- Q = qb$

$AV = qb$

$\cancel{b}yV = q\cancel{b}$

$\boxed{q = yV}$

$q = y_1V_1 = y_2V_2$

Force on section 1-1.

$$F_1 = \frac{\gamma y_1^2}{2}$$

Force on section 2-2

$$F_2 = \frac{\gamma y_2^2}{2}$$

$$F = F_1 - F_2 = \frac{\gamma y_1^2}{2} - \frac{\gamma y_2^2}{2}$$

$$\boxed{F = \frac{\gamma}{2}\left(y_1^2 - y_2^2\right)} \longrightarrow ①$$

This force is responsible for change of velocity from V_1 to V_2.

Force = mass of water × change of velocity.

$$\boxed{F = \frac{\gamma q}{g}\left(V_2 - V_1\right)} \longrightarrow ②$$

Equating eq ① & eq ②.

$$\frac{\cancel{\gamma}}{2}\left(y_1^2 - y_2^2\right) = \frac{\cancel{\gamma} q}{g}\left(V_2 - V_1\right)$$

$$\left(y_1^2 - y_2^2\right) = \frac{2q}{g}\left(\frac{q}{y_2} - \frac{q}{y_1}\right)$$

$$\left(y_1^2 - y_2^2\right) = \frac{2q \cdot q}{g}\left(\frac{1}{y_2} - \frac{1}{y_1}\right)$$

$$\left(y_1^2 - y_2^2\right) = \frac{2q^2}{g}\left(\frac{y_1 - y_2}{y_1 y_2}\right)$$

$$(y_1 - y_2)(y_1 + y_2) = \frac{2q^2}{g}\left(\frac{y_1 - y_2}{y_1 y_2}\right)$$

$$\cancel{(y_1-y_2)}(y_1+y_2) = \frac{2q^2}{gy_1y_2}\cancel{(y_1-y_2)}$$

$$y_1+y_2 = \frac{2q^2}{gy_1y_2}$$

$$(y_1+y_2)y_2 = \frac{2q^2}{gy_1}$$

$$y_1y_2 + y_2^2 = \frac{2q^2}{gy_1}$$

$$y_2^2 + y_1y_2 - \frac{2q^2}{gy_1} = 0.$$

Solving the above quadratic equation

$$h_2 = -\frac{h_1}{2} \pm \sqrt{\frac{h_1^2}{4} + \frac{2q^2}{gh_1}}$$

Take only + sign and Substituting $q_1 = y_1 v_1$

$$h_2 = -\frac{h_1}{2} + \sqrt{\frac{h_1^2}{4} + \frac{2y_1v_1}{g}}$$

Problem:- A discharge of 1000 Liter/sec flows along a rectangular channel, 1.5 m wide. What would be the critical depth in the channel? If a standing wave is to be formed at point, where the upstream depth is 180mm What would be the rise in the water level?

Given Data:-

Discharge : $Q = 1000$ litre/sec $= 1\ m^3/sec$

Channel width : $b = 1.5m$

upstream depth : $y_1 = 180mm$

Solution:-

Discharge per unit width $= q = \frac{Q}{b} = \frac{1}{1.5}$

$$\boxed{q = 0.67\ m^2/sec}$$

Critical depth in the channel

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{(0.67)^2}{9.81}\right)^{1/3}$$

$$\boxed{y_c = 0.358m}$$

$$y_2 = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{gy_1}}$$

$$y_2 = -\frac{1.5}{2} + \sqrt{\frac{(1.5)^2}{4} + \frac{2 \times (0.67)^2}{9.81 \times 180}}$$

$$y_2 = 0.63m$$

$$\boxed{y_2 = 630mm}$$

Rise in water level $\Delta y = h_2 - h_1 = 630 - 180$

$$\boxed{\Delta y = 450mm}$$

Energy Loss due to hydraulic Jump:-

The loss of energy head due to the occurance of the hydraulic Jump is the difference between the specific energy heads at Sections 1-1 and Section 2-2

$$\Delta E = E_1 - E_2$$

$$\Delta E = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$

Problem:-

A rectangular channel, 6m wide, discharges 1200 litre/sec of water into a 6m wide apron, with zero slope, with a mean Velocity of 6m/sec - What is the height of the Jump? How much power is absorbed in the jump?

Given data:-

Channel width = $b = 6m$.

discharge = $Q = 1200 \frac{Litre}{sec}$

$Q = 1.2 \frac{m^3}{sec}$

Mean Velocity : $V = 6 m/sec$.

Solution:-

$$Q = qb$$

$$q = \frac{Q}{b} = \frac{1.2}{6} \Rightarrow \boxed{q = 0.2 m^2/sec}$$

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{(0.2)^2}{9.81}\right)^{\frac{1}{3}}$$

$$\boxed{y_c = 0.16 m}$$

$q = yV$

$$V_c = \frac{q}{y_c} = \frac{0.2}{0.16}$$

$$\boxed{V_c = 1.25 \, m/sec}$$

$V_1 > V_c$ Super critical flow.

Depth of water on the upstream side of the Jump

$$y_1 = \frac{Q}{V_1 \times b} = \frac{1.2}{1.25 \times 6}$$

$Q = AV$

$Q = byV$

$y = \frac{Q}{Vb}$

$$\boxed{y_1 = 0.033 m}$$

$$y_2 = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2 y_1 V_1^2}{g}}$$

$$y_2 = -\frac{0.033}{2} + \sqrt{\frac{(0.033)^2}{4} + \frac{2 \times 0.033 \times 1.25^2}{9.81}}$$

$$\boxed{y_2 = 0.476 m}$$

$$\Delta y = y_2 - y_1$$

$$\Delta y = 0.476 - 0.033$$

$$\boxed{\Delta y = 0.443 m}$$

$$\Delta E = E_1 - E_2$$

As we know that

$$Q_1 = Q_2$$

$$A_1 V_1 = A_1 V_2$$

$$b_1 y_1 V_1 = b_2 y_2 V_2 \qquad b_1 = b_2 = b$$

$$\cancel{b} y_1 V_1 = \cancel{b} y_2 V_2$$

$$V_2 = \frac{y_1 V_1}{y_2} = \frac{0.033 \times 6}{0.476}$$

$$\boxed{V_2 = 0.42 m/sec}$$

$$\Delta E = E_1 - E_2 = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$

$$E_1 - E_2 = \left(0.033 + \frac{6^2}{2 \times 9.81}\right) - \left(0.476 + \frac{0.42^2}{2 \times 9.81}\right)$$

$$\boxed{E_1 - E_2 = 1.384 m}$$

* Dissipation of power in hydraulic Jump

$$\Delta P = \rho g Q (E_1 - E_2) = 1000 \times 9.81 \times 1.2 (1.384)$$

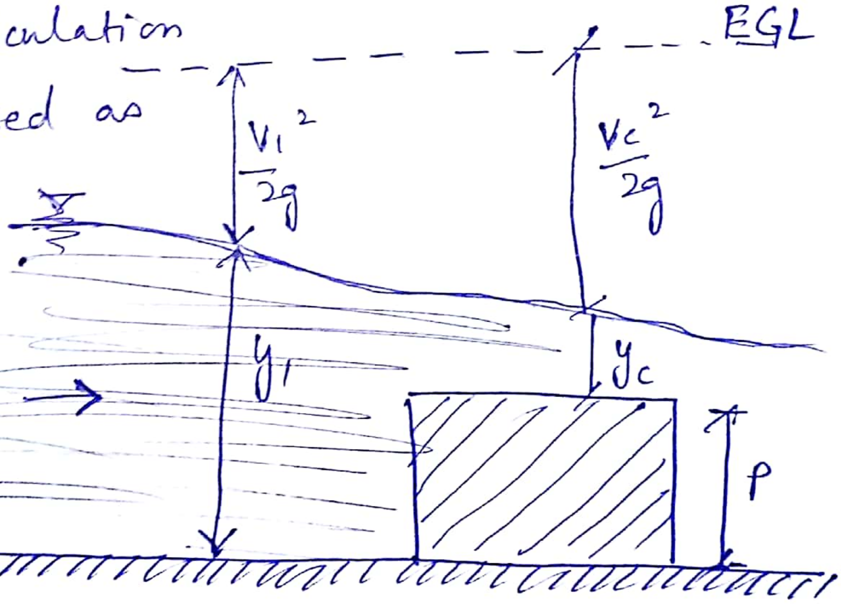
$$\boxed{\Delta P = 16327.76 W}$$

Broad Crested weirs

Broad crested weirs are structures that are generally constructed from reinforced concrete and which usually span the full width of the channel.

* They are used to measure the discharge of rivers.
* Used in medium to large size rivers and cannals.

The discharge calculation can be summarised as



$Q = CbH^n$

Q = Volumetric Flow (discharge)

C = Flow coefficient for the structure (About 0.62).

b = width of the crest.

H = Height of head of water

n = varies with structure ($\frac{3}{2}$ for horizontal weir)

Problem:-

Water flows along a rectangular channel at a depth of 1.3m when the discharge is 8.74 m^3/sec. The channel width B is 5.5m, Ignoring energy loss, what is the minimum height (P) of a broad Crested weir if it is to function with critical depth on the crest?

Given data:-

$y = 1.3m$

$b = 5.5m$

$Q = 8.74 m^3/sec$.

Required data:- $P =$ weir height $= ?$

Solution:-

$$V_1 = \frac{Q}{A} = \frac{Q}{by}$$

$$V_1 = \frac{8.74}{1.3 \times 5.5}$$

$$\boxed{V_1 = 1.222 m/sec}$$

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{Q^2}{b^2 g}\right)^{\frac{1}{3}}$$

$Q = qb$

$q = \frac{Q}{b}$

$$y_c = \left(\frac{(8.74)^2}{(5.5)^2 \times 9.81}\right)^{\frac{1}{3}}$$

$$\boxed{y_c = 0.636m}$$

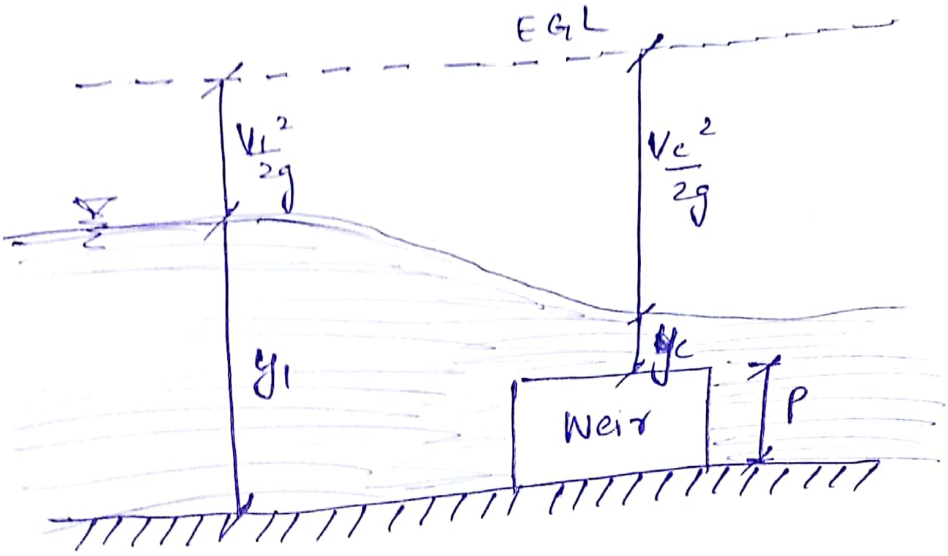
Also

$$V = \sqrt{gy}$$

$$V_c = \sqrt{g y_c}$$

$$V_c = \sqrt{9.81 \times 0.636}$$

$$\boxed{V_c = 2.498 \, m/sec}$$



$$\frac{V_1^2}{2g} + y_1 = \frac{V_c^2}{2g} + y_c + P$$

$$\frac{(1.222)^2}{19.62} + 1.3 = \frac{(2.498)^2}{19.62} + 0.636 + P$$

$$0.0761 + 1.3 = 0.318 + 0.636 + P$$

$$\boxed{P = 0.422 m}$$

Thus the weir should have a height of 0.422m measured from the bed level.

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