Hydraulic Engineering

MODULE # 1-(b)

Froud Number "Fr"

It is a dimension less numbers used in a variety of ways with open channel flow.
It can be defined as
" The ratio of inertial forces to the gravitational forces".

Mathematically $Fr = \dfrac{V}{\sqrt{gD_h}} = \dfrac{V}{\sqrt{gy}}$

Squaring Both Sides.

V = Mean Velocity of flow of water

D_h = Hydraulic depth of the Channel = y

$$Fr^2 = \frac{V^2}{gD_h}$$

$Q = AV$

$V = \dfrac{Q}{A}$

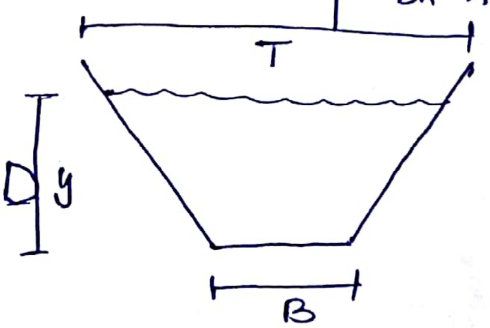
$$Fr^2 = \frac{Q^2}{A^2 g D_h}$$

$$Fr^2 = \frac{Q^2}{A^2 g} \times \frac{1}{D_h}$$

$D_h = \dfrac{\text{Area of flow}}{\text{Water Surface width}}$

$D_h = \dfrac{A}{T} \Rightarrow \dfrac{1}{D_h} = \dfrac{T}{A}$

$$Fr^2 = \frac{Q^2}{A^2 g} \times \frac{T}{A}$$



$$\sqrt{Fr^2} = \sqrt{\frac{Q^2 T}{A^3 g}}$$

$$Fr = \sqrt{\frac{Q^2 T}{A^3 g}}$$

Condition:-

$Fr < 1$	Sub-critical Flow	$y > y_c$, $E > E_{min}$
$Fr = 1$	Critical Flow	$y = y_c$, $E = E_{min}$
$Fr > 1$	Super critical Flow	$y < y_c$, $E < E_{min}$

Derive $Fr = \frac{V}{\sqrt{gy}}$

As we know that $E = y + \frac{q^2}{2gy^2}$

differentiate w-r-t "y"

$$\frac{d}{dy}(E) = \frac{d}{dy}\left(y + \frac{q^2}{2gy^2}\right)$$

$$\frac{dE}{dy} = \frac{d}{dy}(y) + \frac{d}{dy}\left(\frac{q^2}{2gy^2}\right)$$

$$\frac{dE}{dy} = 1 + \frac{q^2}{2g} \cdot \frac{d}{dy}\left(\frac{1}{y^2}\right)$$

$$\frac{dE}{dy} = 1 + \frac{q^2}{2g} \cdot \frac{d}{dy}\left(y^{-2}\right)$$

$$\frac{dE}{dy} = 1 + \frac{q^2}{2g}\left(-2\,y^{-2-1}\right)$$

$$\frac{dE}{dy} = 1 + \frac{q^2}{\cancel{2}g}\left(-\cancel{2}y^{-3}\right)$$

$$\frac{dE}{dy} = 1 + \frac{q^2}{g}(-y^{-3})$$

$$\frac{dE}{dy} = 1 - \frac{q^2 y^{-3}}{g}$$

$$\boxed{\frac{dE}{dy} = 1 - \frac{q^2}{gy^3}} \longrightarrow eq\ 1$$

As we know $Q = AV \longrightarrow (i)$

$Q = qb \longrightarrow (ii)$

equating (i) and (ii)

$AV = qb$

$\cancel{b}y\,V = q\cancel{b}$ $\quad A = by$ (Rectangular Section)

$yV = q$

$$V = \frac{q}{y} \longrightarrow 2$$

put $V = \frac{q}{y}$ in eq 1

$$\frac{dE}{dy} = 1 - \frac{q^2}{y^2} \cdot \frac{1}{gy}$$

$$\frac{dE}{dy} = 1 - V^2 \cdot \frac{1}{gy} \quad \text{OR} \quad \boxed{\frac{dE}{dy} = 1 - \frac{V^2}{gy}} \rightarrow 3$$

Condition:- When there is a Critical flow

$E = E_{min}$ so $\frac{dE}{dy} = 0$

After apply condition on eq (3)

$$0 = 1 - \frac{V^2}{gy}$$

$$\frac{V^2}{gy} = 1$$

$$\sqrt{V^{\cancel{2}}} = \sqrt{gy}$$

$$V = \sqrt{gy}$$

$$\frac{V}{\sqrt{gy}} = 1$$

At critical flow

$Fr = 1$

$$\boxed{Fr = \frac{V}{\sqrt{gy}}}$$

Hence proved.

Critical depth y_c for Rectangular Channel

Critical depth y_c is defined as the depth of flow of liquid at which the specific energy is minimum E_{min}

$$E = y + \frac{q^2}{2gy^2}$$

Differentiating both side wrt y

$$\frac{d(E)}{dy} = \frac{d}{dy}(y) + \frac{d}{dy}\left(\frac{q^2}{2gy^2}\right)$$

At Critical depth

$E = E_{min}$ so $\frac{dE}{dy} = 0$.

$$0 = \frac{d}{dy}(y) + \frac{d}{dy}\left(\frac{q^2}{2gy^2}\right)$$

$$0 = 1 + \frac{q^2}{2g} \cdot \frac{d}{dy}\left(\frac{1}{y^2}\right)$$

$$0 = 1 + \frac{q^2}{2g} \cdot \frac{d}{dy}\left(y^{-2}\right)$$

$$0 = 1 + \frac{q^2}{2g} \cdot -2\left(y^{-2-1}\right)$$

$$0 = 1 - \frac{q^2}{\cancel{2}g} \times \cancel{2}y^{-3}$$

$$0 = 1 - \frac{q^2}{gy^3}$$

$$\frac{q^2}{gy^3} = 1$$

$$\frac{q^2}{g} = y^3$$

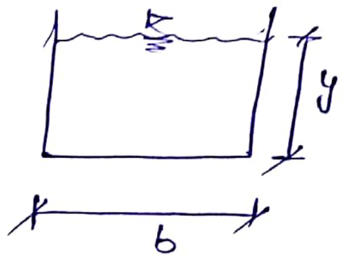
$$\boxed{y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}}$$ proved.

Problem:-

What is Smallest energy that Can be associated with $q = 10 \frac{ft^2}{sec}$

Solution:-

let Suppose the channel is rectangular



Condition for Smallest Energy

$y = y_c$, $E = E_{min}$, $Fr = 1$

As we know that

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$g = 9.8 \, m/sec^2$$

$$g = 9.8 \times 3.28$$

$$\therefore g = 32.17 \frac{ft}{sec^2}$$

$$y_c = \left(\frac{10^2}{32.17}\right)^{1/3}$$

$$\boxed{y_c = 1.458 \, ft}$$

Also

$$Q = AV \rightarrow (1)$$

$$Q = qb \rightarrow (2)$$

equating (1) & (2)

$$AV = qb$$

$$\cancel{b}yV = q\cancel{b}$$

$$yV = q$$

$$V = \frac{q}{y_c} = \frac{10}{1.458} \Rightarrow \boxed{V = 6.86 \frac{ft}{sec}}$$

$$E = y + \frac{v^2}{2g}.$$

$$E_{min} = y_c + \frac{v^2}{2g}$$

$$E_{min} = 1.458 + \frac{(6.86)^2}{2(32.17)}$$

$$E_{min} = 1.458 + 0.73$$

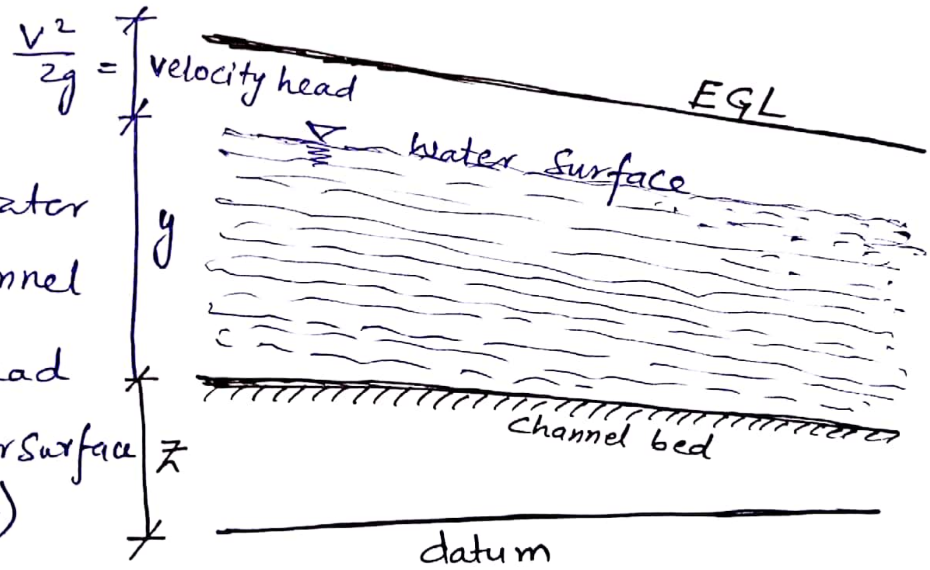
$$\boxed{E_{min} = 2.21\ ft}$$

Dynamic Equation of Gradually Varied Flow:-

Z = height from datum to Channel bed.

y = depth of water in open channel

$\frac{v^2}{2g}$ = Velocity head (from watersurface to EGL)



EGL = Energy Grade line.

The total head "H" at any Section is Given by

$$H = z + y + \frac{v^2}{2g} \rightarrow eq\ ①$$

Differentiating (1) with respect to horizontal distance on both sides.

$$\frac{d}{dx}(H) = \frac{d}{dx}(z) + \frac{d}{dx}(y) + \frac{d}{dx}\left(\frac{V^2}{2g}\right) \rightarrow eq\ (2)$$

(a) $\frac{dH}{dx}$ = The slope of the energy line $= -S_f$

(b) $\frac{dz}{dx}$ = The channel bed slope $= -S_o$

put (a) and (b) in eq (2)

$$-S_f = -S_o + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right)$$

$$S_o - S_f = \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right) \rightarrow eq\ (3)$$

Multiplying and dividing dy with Velocity Term in eq (3).

$$S_o - S_f = \frac{dy}{dx} + \frac{d}{dx} \times \frac{dy}{dy}\left(\frac{V^2}{2g}\right)$$

$$S_o - S_f = \frac{dy}{dx}\left[1 + \frac{d}{dy}\left(\frac{V^2}{2g}\right)\right]$$

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 + \frac{d}{dy}\left(\frac{V^2}{2g}\right)} \rightarrow eq\ (x)$$

The above eq (x) is known as the dynamic equation of gradually varied flow.

Surface Profiles and Back Water Curves

Classification of Channel Bed Slopes :-

The slope of the channel bed is very important in determining the characteristics of the flow.

let

S_o = The slope of the channel bed

S_c = Critical slope

y_n = The normal depth of water

The slope of the channel bed can be classified as :-

① **Critical slope** :- The bottom slope of the channel is equal to critical slope

$$S_o = S_c \quad \text{or} \quad y_n = y_c$$

② **Mild slop** :- The bottom slope of the channel is less than the critical slope.

$$S_o < S_c$$

③ **Steep slop** :- The bottom slope of the channel is greater than the Critical slope

$$S_o > S_c$$

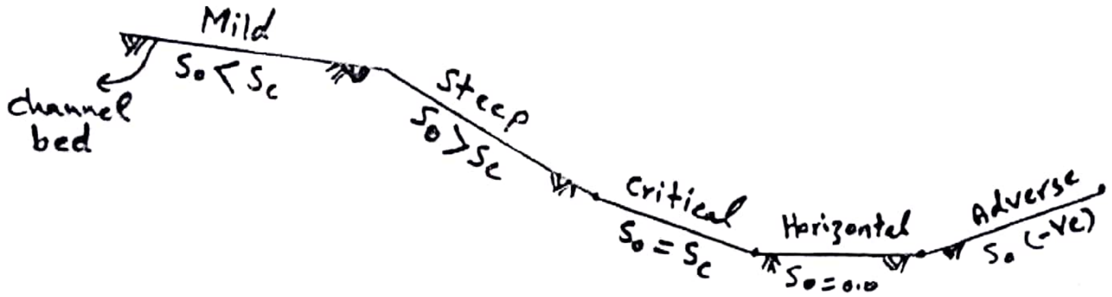
④ **Horizontal slop** :- The bottom slope of the channel is equal to zero

$$S_o = 0$$

(5) Adverse slope :-

The slope opposite to the direction of flow.

S_o = negative



Classification of flow Profiles (Water Surface Profile)

* The Surface Curves of water are called flow profiles (or water Surface profiles)

* The shape of water Surface profiles is mainly determined by the slope of the Channel bed S_o.

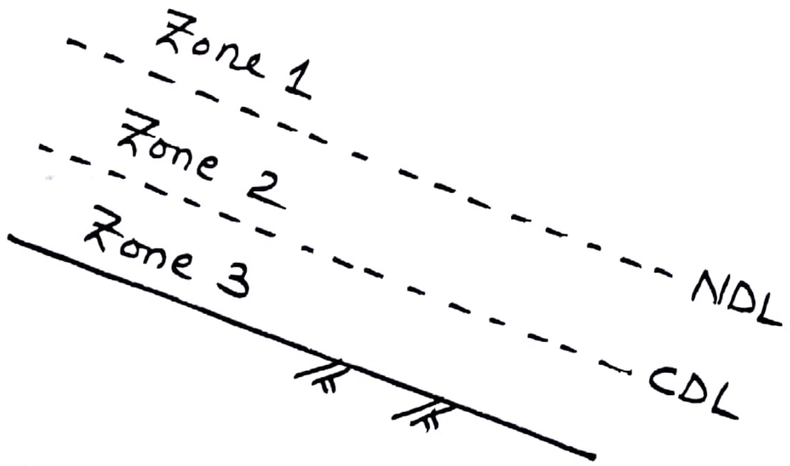
For a given discharge, the normal depth y_n and the critical depth y_c may be Calculated.

The following steps are followed to classify the flow profiles:-

1- A line parallel to the channel bottom with a height of y_n is drawn and is designated as the normal depth line (N.D.L)

2- A line parallel to the channel bottom with a height y_c is drawn and is designated as the critical depth line (C.D.L)

3- The vertical space in a longitudinal section is divided into 3 zones using the two lines drawn in step 1 and 2



4- Depending upon the zone and the slope of the bed channel, the water profiles are classified into 13 types as follows:-

(a) Mild slope curves M_1, M_2, M_3

(b) Steep slope curves S_1, S_2, S_3

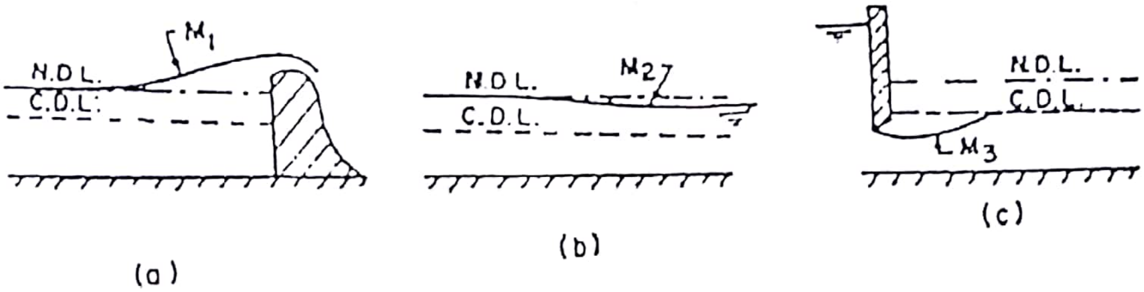
(c) Critical slope curves C_1, C_2, C_3

(d) Horizontal slope curves H_2, H_3

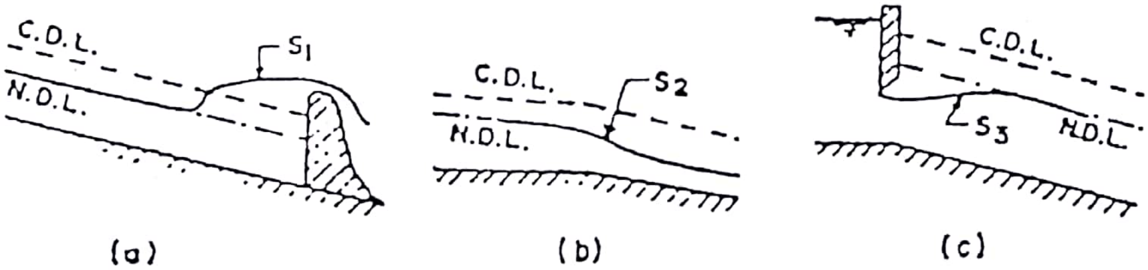
(e) Adverse slope curves A_2, A_3

In all these curves the letter indicates the slope type and the Subscript indicates the zone.

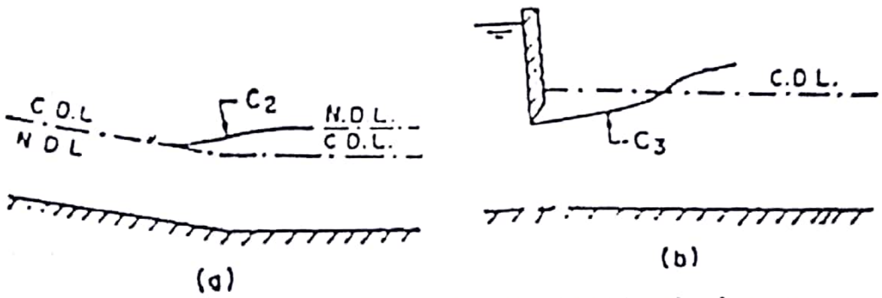
Example : S_2 Curve occurs in the zone 2 of the steep slope.



Flow Profiles in Mild slope



Flow Profiles in Steep slope



Flow Profiles in Critical slope

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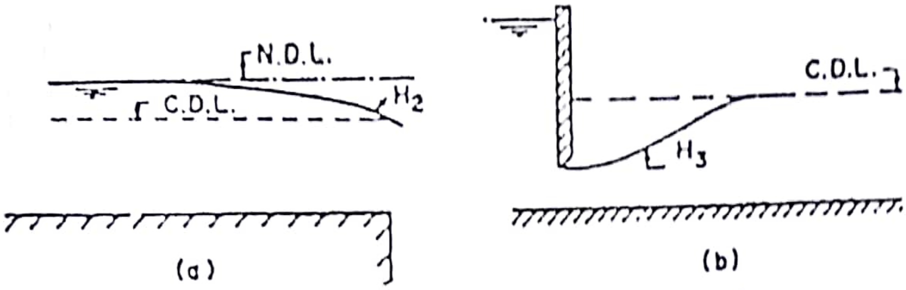
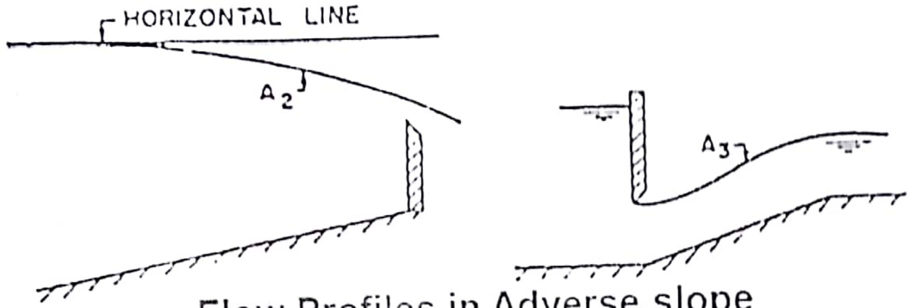


Flow Profiles in Horizontal slope



Flow Profiles in Adverse slope

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