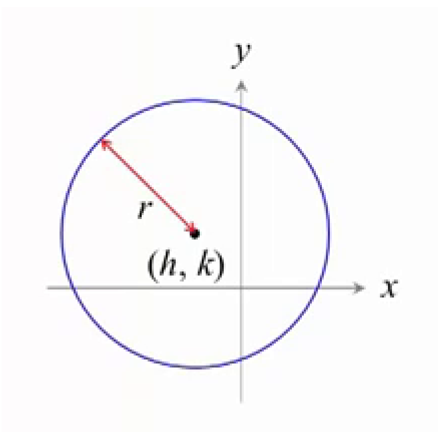
Module 3

MOHR'S CIRCLE:

Mohr's circle is a more convenient way to determine plane stress transformation.

Before understanding Mohr's Circle, let us first review the standard equation of a circle. In a given X Y co-ordinate system, if there is a circle with the center coordinates h and k and a radius r then the equation of circle is given in the standard form as:

$$(x-h)^2+(y-k)^2=r^2$$



In this equation only ***x*** and ***y*** are the variables while **h**, **k** and **r** are constants.

Now coming back to the General equation, this equation can be rearranged and transformed into equation of a circle. The derivation is as

$$\begin{cases} \sigma_{x'} - \dfrac{\sigma_x+\sigma_y}{2} = \dfrac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \\ \tau_{x'y'} = -\dfrac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \end{cases}$$

$$\begin{cases} \left(\sigma_{x'} - \dfrac{\sigma_x+\sigma_y}{2}\right)^2 = \left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2\cos^2 2\theta + (\sigma_x-\sigma_y)\tau_{xy}\sin 2\theta\cos 2\theta + \tau_{xy}^2\sin^2 2\theta \\ (\tau_{x'y'})^2 = \left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2\sin^2 2\theta - (\sigma_x-\sigma_y)\tau_{xy}\sin 2\theta\cos 2\theta + \tau_{xy}^2\cos^2 2\theta \end{cases}$$

$$\left(\sigma_{x'} - \frac{\sigma_x+\sigma_y}{2}\right)^2 + (\tau_{x'y'})^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2\underbrace{(\sin^2 2\theta+\cos^2 2\theta)}_{=1} + \tau_{xy}^2\underbrace{(\sin^2 2\theta+\cos^2 2\theta)}_{=1}$$

$$\left(\sigma_{x'} - \frac{\sigma_x+\sigma_y}{2}\right)^2 + (\tau_{x'y'})^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2$$

So the final equation is similar to the equation of a circle.

Standard equation of a circle:

$$(x-h)^2+(y-k)^2=r^2$$

$$\left(\sigma_{x'}-\frac{\sigma_x+\sigma_y}{2}\right)^2+\left(\tau_{x'y'}\right)^2=\left(\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}\right)^2$$

variables

This equation has center at

$$\left(\frac{\sigma x-\sigma y}{2},0\right)=(\sigma avg,0)$$

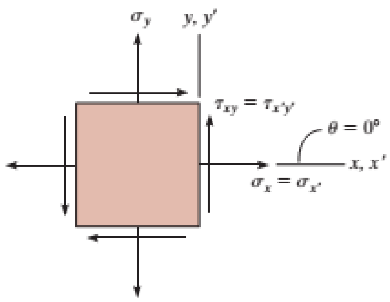
And radius at

$$R=\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

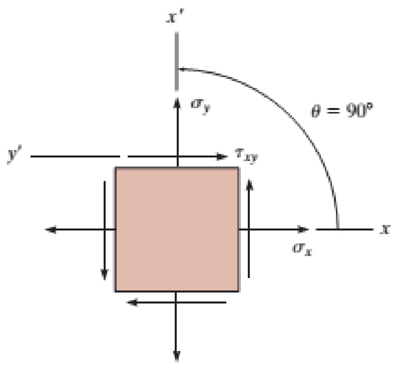
Each point on Mohr's circle represents the two stress components $\sigma_{x'}$ and $\tau_{x'y'}$ acting on the side of the element defined by the x' axis, when the axis is in a specific direction θ. For example, when x' is coincident with the x axis as shown in Fig. 9–16*a*, then $\theta = 0°$ and $\sigma_{x'} = \sigma_x$, $\tau_{x'y'} = \tau_{xy}$. We will refer to this as the "reference point" A and plot its coordinates $A(\sigma_x, \tau_{xy})$, Fig. 9–16*c*.

Now consider rotating the x' axis 90° counterclockwise, Fig. 9–16*b*. Then $\sigma_{x'} = \sigma_y$, $\tau_{x'y'} = -\tau_{xy}$. These values are the coordinates of point $G(\sigma_y, -\tau_{xy})$ on the circle, Fig. 9–16*c*. Hence, the radial line CG is 180° counterclockwise from the "reference line" CA. In other words, a rotation θ of the x' axis on the element will correspond to a rotation 2θ on the circle in the *same direction*.*

Once constructed, Mohr's circle can be used to determine the principal stresses, the maximum in-plane shear stress and associated average normal stress, or the stress on any arbitrary plane.



(a)



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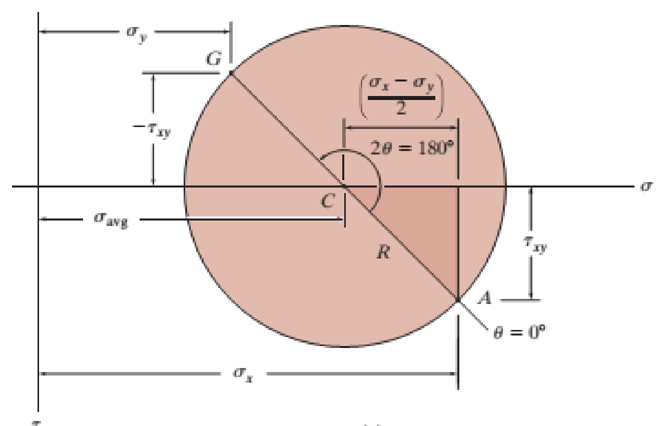


(c)

Fig. 9–16

The following steps are required to draw and use Mohr's circle.

Construction of the Circle.

- Establish a coordinate system such that the horizontal axis represents the normal stress σ, with *positive to the right*, and the vertical axis represents the shear stress τ, with *positive downwards*, Fig. 9–17*a*.*
- Using the positive sign convention for σ_x, σ_y, τ_{xy}, as shown in Fig. 9–17*b*, plot the center of the circle *C*, which is located on the σ axis at a distance $\sigma_{avg} = (\sigma_x + \sigma_y)/2$ from the origin, Fig. 9–17*a*.
- Plot the "reference point" *A* having coordinates $A(\sigma_x, \tau_{xy})$. This point represents the normal and shear stress components on the element's right-hand vertical face, and since the x' axis coincides with the x axis, this represents $\theta = 0°$, Fig. 9–17*a*.
- Connect point *A* with the center *C* of the circle and determine *CA* by trigonometry. This distance represents the radius *R* of the circle, Fig. 9–17*a*.
- Once *R* has been determined, sketch the circle.

Principal Stress.

- The principal stresses σ_1 and σ_2 ($\sigma_1 \geq \sigma_2$) are the coordinates of points *B* and *D* where the circle intersects the σ axis, i.e., where $\tau = 0$, Fig. 9–17*a*.
- These stresses act on planes defined by angles θ_{p_1} and θ_{p_2}, Fig. 9–17*c*. They are represented on the circle by angles $2\theta_{p_1}$ (shown) and $2\theta_{p_2}$ (not shown) and are measured *from* the radial reference line *CA to* lines *CB* and *CD*, respectively.
- Using trigonometry, only one of these angles needs to be calculated from the circle, since θ_{p_1} and θ_{p_2} are 90° apart. Remember that the direction of rotation $2\theta_p$ on the circle (here it happens to be counterclockwise) represents the *same* direction of rotation θ_p from the reference axis ($+x$) to the principal plane ($+x'$), Fig. 9–17*c*.*

Maximum In-Plane Shear Stress.

- The average normal stress and maximum in-plane shear stress components are determined from the circle as the coordinates of either point *E* or *F*, Fig. 9–17*a*.
- In this case the angles θ_{s_1} and θ_{s_2} give the orientation of the planes that contain these components, Fig. 9–17*d*. The angle $2\theta_{s_1}$ is shown in Fig. 9–17*a* and can be determined using trigonometry. Here the rotation happens to be clockwise, from *CA* to *CE*, and so θ_{s_1} must be clockwise on the element, Fig. 9–17*d*.*

sis:

Stresses on Arbitrary Plane.

- The normal and shear stress components $\sigma_{x'}$ and $\tau_{x'y'}$ acting on a specified plane or x' axis, defined by the angle θ, Fig. 9–17*e*, can be obtained from the circle using trigonometry to determine the coordinates of point *P*, Fig. 9–17*a*.
- To locate *P*, the known angle θ (in this case counterclockwise), Fig. 9–17*e*, must be measured on the circle in the *same direction* 2θ (counterclockwise), *from* the radial reference line *CA to* the radial line *CP*, Fig. 9–17*a*.*

*If the τ axis were constructed *positive upwards*, then the angle 2θ on the circle would be measured in the *opposite direction* to the orientation θ of the x' axis.

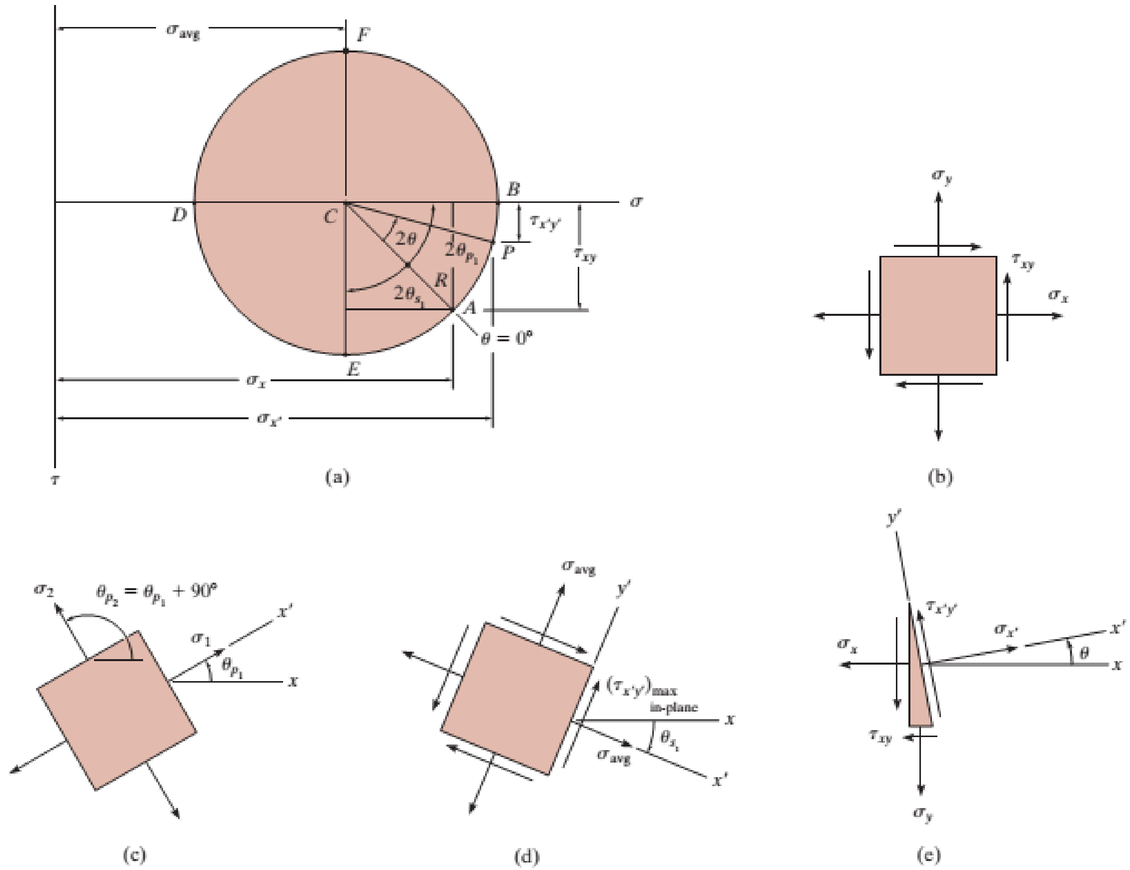
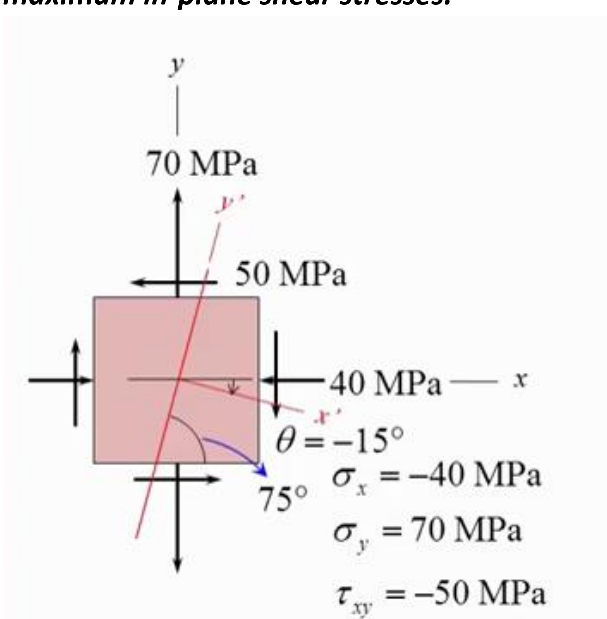


Fig. 9–17

Example: Determine the stresses for the rotated orientation, the principle stresses and the maximum in-plane shear stresses.



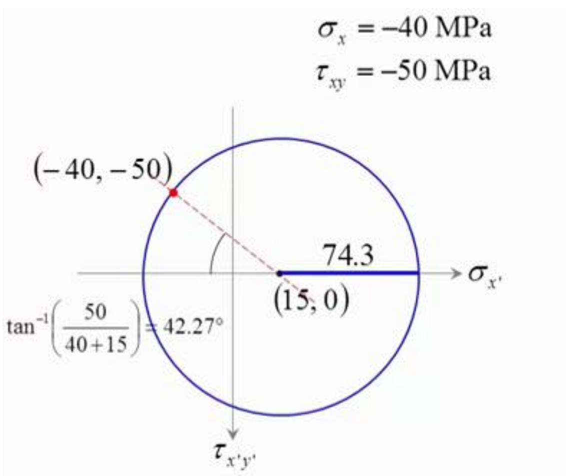
We need to determine the new state of stress associated with new orientation as shown. Also we need to determine the principle normal stresses and maximum in-plane shear stresses. But this time we are going to solve this problem using Mohr's circle. We need to first calculate the Avg normal shear stress.

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = 15 \text{ MPa}$$

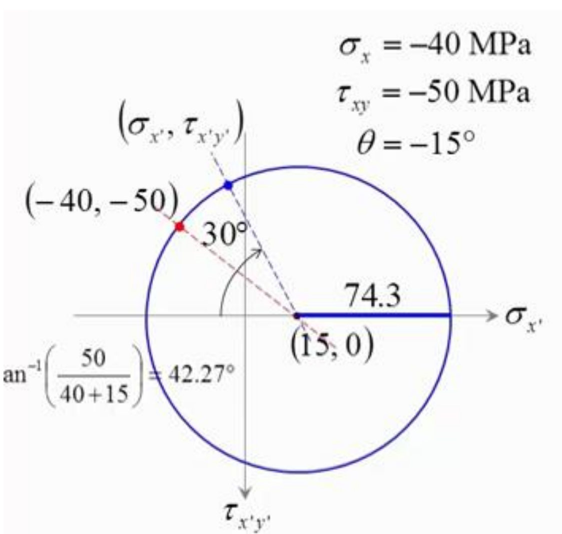
We also need to find the radius of Mohr's Circle.

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 74.3 \text{ MPa}$$

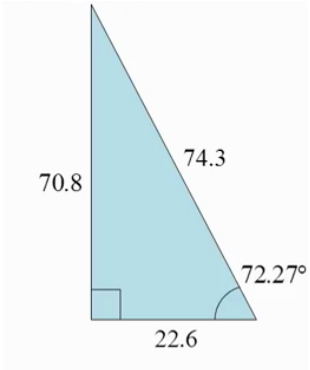
So in a coordinate system draw a circle with center at (15,0) .The R=74.3.Also plot the original values of stresses



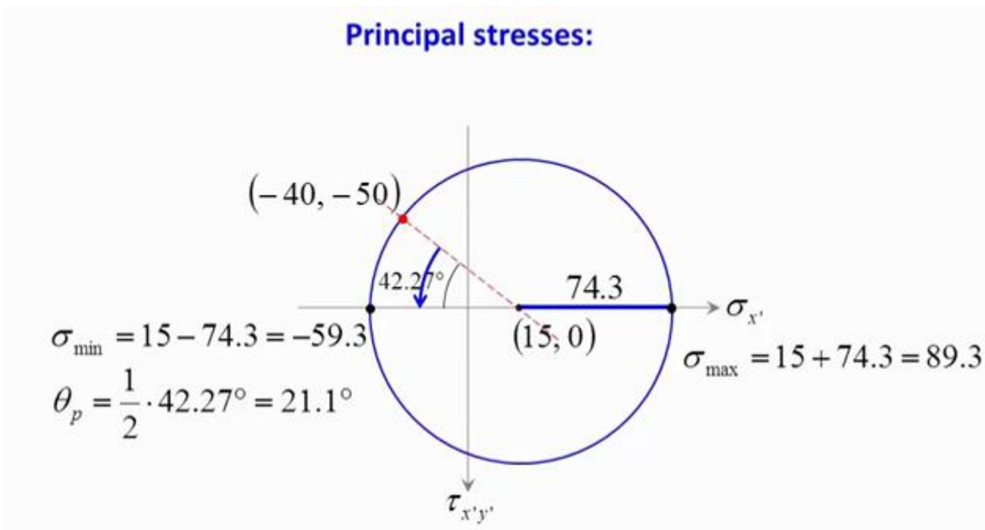
For the new orientation θ=-15

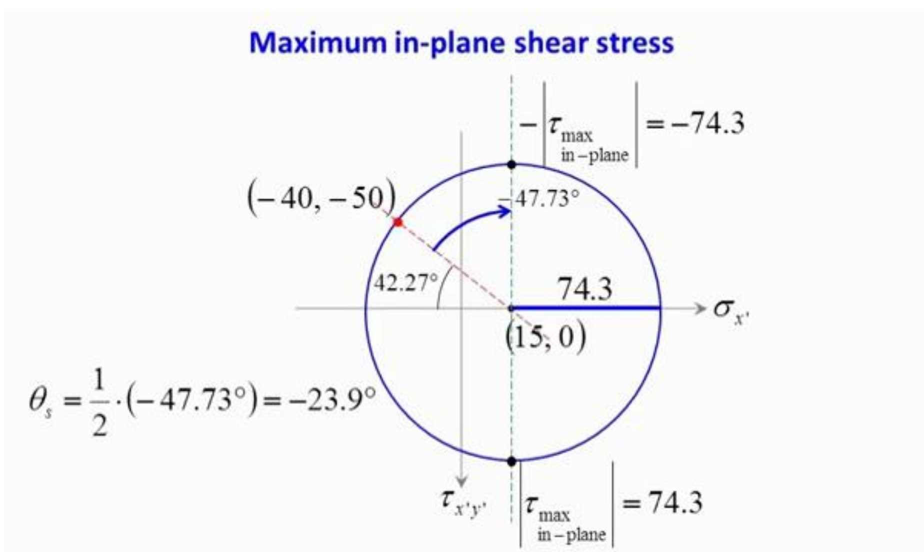


The coordinates can be found by simple trigonometry

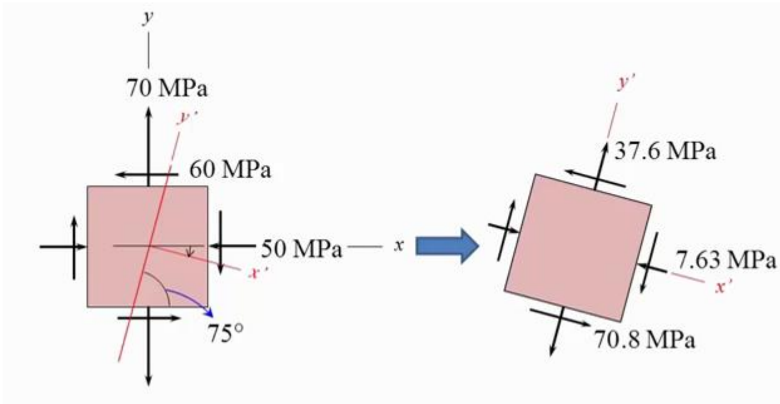


$$\sigma_{x'} = -(22.6 - 15) = -7.6 \text{ (MPa)} \quad \tau_{x'y'} = -70.8 \text{(MPa)}$$





These result can be visualized. The state of stress at new orientation is (theta =-15)



The orientation of principal stresses and maximum in plane shear stresses is as.