1/1 MECHANICS

Mechanics is the physical science which deals with the effects of forces on objects. No other subject plays a greater role in engineering analysis than mechanics. Although the principles of mechanics are few, they have wide application in engineering. The principles of mechanics are central to research and development in the fields of vibrations, stability and strength of structures and machines, robotics, rocket and spacecraft design, automatic control, engine performance, fluid flow, electrical machines and apparatus, and molecular, atomic, and subatomic behavior. A thorough understanding of this subject is an essential prerequisite for work in these and many other fields.

1/2 BASIC CONCEPTS

The following concepts and definitions are basic to the study of mechanics, and they should be understood at the outset.

Space is the geometric region occupied by bodies whose positions are described by linear and angular measurements relative to a coordinate system. For three-dimensional problems, three independent coordinates are needed. For two-dimensional problems, only two coordinates are required.

Time is the measure of the succession of events and is a basic quantity in dynamics. Time is not directly involved in the analysis of statics problems.

Mass is a measure of the inertia of a body, which is its resistance to a change of velocity. Mass can also be thought of as the quantity of matter in a body. The mass of a body affects the gravitational attraction force between it and other bodies. This force appears in many applications in statics.

Force is the action of one body on another. A force tends to move a body in the direction of its action. The action of a force is characterized by its *magnitude*, by the *direction* of its action, and by its *point of application*. Thus force is a vector quantity, and its properties are discussed in detail in Chapter 2.

A ***particle*** is a body of negligible dimensions. In the mathematical sense, a particle is a body whose dimensions are considered to be near zero so that we may analyze it as a mass concentrated at a point. We often choose a particle as a differential element of a body. We may treat a body as a particle when its dimensions are irrelevant to the description of its position or the action of forces applied to it.

Rigid body. A body is considered rigid when the change in distance between any two of its points is negligible for the purpose at hand. For instance, the calculation of the tension in the cable which supports the boom of a mobile crane under load is essentially unaffected by the small internal deformations in the structural members of the boom. For the purpose, then, of determining the external forces which act on the boom, we may treat it as a rigid body. Statics deals primarily with the calculation of external forces which act on rigid bodies in equilibrium. Deter-

1/3 SCALARS AND VECTORS

We use two kinds of quantities in mechanics—scalars and vectors. *Scalar quantities* are those with which only a magnitude is associated. Examples of scalar quantities are time, volume, density, speed, energy, and mass. *Vector quantities*, on the other hand, possess direction as well as magnitude, and must obey the parallelogram law of addition as described later in this article. Examples of vector quantities are displacement, velocity, acceleration, force, moment, and momentum. Speed is a scalar. It is the magnitude of velocity, which is a vector. Thus velocity is specified by a direction as well as a speed.

Vectors representing physical quantities can be classified as free, sliding, or fixed.

A ***free vector*** is one whose action is not confined to or associated with a unique line in space. For example, if a body moves without rotation, then the movement or displacement of any point in the body may be taken as a vector. This vector describes equally well the direction and magnitude of the displacement of every point in the body. Thus, we may represent the displacement of such a body by a free vector.

A ***sliding vector*** has a unique line of action in space but not a unique point of application. For example, when an external force acts on a rigid body, the force can be applied at any point along its line of action without changing its effect on the body as a whole,* and thus it is a sliding vector.

A ***fixed vector*** is one for which a unique point of application is specified. The action of a force on a deformable or nonrigid body must be specified by a fixed vector at the point of application of the force. In this instance the forces and deformations within the body depend on the point of application of the force, as well as on its magnitude and line of action.

1/4 Newton's Laws

Sir Isaac Newton was the first to state correctly the basic laws governing the motion of a particle and to demonstrate their validity.* Slightly reworded with modern terminology, these laws are:

Law I. A particle remains at rest or continues to move with *uniform velocity* (in a straight line with a constant speed) if there is no unbalanced force acting on it.

Law II. The acceleration of a particle is proportional to the vector sum of forces acting on it, and is in the direction of this vector sum.

Law III. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and *collinear* (they lie on the same line).

The correctness of these laws has been verified by innumerable accurate physical measurements. Newton's second law forms the basis for most of the analysis in dynamics. As applied to a particle of mass m, it may be stated as

$$\mathbf{F} = m\mathbf{a} \tag{1/1}$$

where $\mathbf{F}$ is the vector sum of forces acting on the particle and $\mathbf{a}$ is the resulting acceleration. This equation is a *vector* equation because the direction of $\mathbf{F}$ must agree with the direction of $\mathbf{a}$, and the magnitudes of $\mathbf{F}$ and $m\mathbf{a}$ must be equal.

Newton's first law contains the principle of the equilibrium of forces, which is the main topic of concern in statics. This law is actually a consequence of the second law, since there is no acceleration when the force is zero, and the particle either is at rest or is moving with a uniform velocity. The first law adds nothing new to the description of motion but is included here because it was part of Newton's classical statements.

The third law is basic to our understanding of force. It states that forces always occur in pairs of equal and opposite forces. Thus, the downward force exerted on the desk by the pencil is accompanied by an upward force of equal magnitude exerted on the pencil by the desk. This principle holds for all forces, variable or constant, regardless of their source, and holds at every instant of time during which the forces are applied. Lack of careful attention to this basic law is the cause of frequent error by the beginner.

1/6 Law of Gravitation

In statics as well as dynamics we often need to compute the weight of a body, which is the gravitational force acting on it. This computation depends on the *law of gravitation*, which was also formulated by Newton. The law of gravitation is expressed by the equation

$$F = G\frac{m_1 m_2}{r^2} \tag{1/2}$$

where F = the mutual force of attraction between two particles
G = a universal constant known as the *constant of gravitation*
m_1, m_2 = the masses of the two particles
r = the distance between the centers of the particles

The mutual forces F obey the law of action and reaction, since they are equal and opposite and are directed along the line joining the centers of the particles, as shown in Fig. 1/7. By experiment the gravitational constant is found to be $G = 6.673(10^{-11})\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$.

Sample Problem 1/1

Determine the weight in newtons of a car whose mass is 1400 kg. Convert the mass of the car to slugs and then determine its weight in pounds.

Solution. From relationship 1/3, we have

① $$W = mg = 1400(9.81) = 13\,730 \text{ N} \qquad \textit{Ans.}$$

From the table of conversion factors inside the front cover of the textbook, we see that 1 slug is equal to 14.594 kg. Thus, the mass of the car in slugs is

② $$m = 1400 \text{ kg}\left[\frac{1 \text{ slug}}{14.594 \text{ kg}}\right] = 95.9 \text{ slugs} \qquad \textit{Ans.}$$

Finally, its weight in pounds is

③ $$W = mg = (95.9)(32.2) = 3090 \text{ lb} \qquad \textit{Ans.}$$

As another route to the last result, we can convert from kg to lbm. Again using the table inside the front cover, we have

$$m = 1400 \text{ kg}\left[\frac{1 \text{ lbm}}{0.45359 \text{ kg}}\right] = 3090 \text{ lbm}$$

Principle of Transmissibility

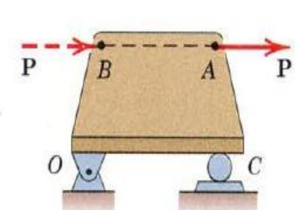

Figure 2/2

When dealing with the mechanics of a rigid body, we ignore deformations in the body and concern ourselves with only the net external effects of external forces. In such cases, experience shows us that it is not necessary to restrict the action of an applied force to a given point. For example, the force **P** acting on the rigid plate in Fig. 2/2 may be applied at A or at B or at any other point on its line of action, and the net external effects of **P** on the bracket will not change. The external effects are the force exerted on the plate by the bearing support at O and the force exerted on the plate by the roller support at C.

This conclusion is summarized by the *principle of transmissibility*, which states that a force may be applied at any point on its given line of action without altering the resultant effects of the force *external* to the *rigid* body on which it acts. Thus, whenever we are interested in only the resultant external effects of a force, the force may be treated as a *sliding* vector, and we need specify only the *magnitude*, *direction*, and *line of action* of the force, and not its *point of application*. Because this book deals essentially with the mechanics of rigid bodies, we will treat almost all forces as sliding vectors for the rigid body on which they act.

Action and Reaction

According to Newton's third law, the *action* of a force is always accompanied by an *equal* and *opposite reaction*. It is essential to distinguish between the action and the reaction in a pair of forces. To do so, we first *isolate* the body in question and then identify the force exerted *on* that body (not the force exerted *by* the body). It is very easy to mistakenly use the wrong force of the pair unless we distinguish carefully between action and reaction.

Concurrent Forces

Two or more forces are said to be *concurrent at a point* if their lines of action intersect at that point. The forces $\mathbf{F}_1$ and $\mathbf{F}_2$ shown in Fig. 2/3*a* have a common point of application and are concurrent at the point *A*. Thus, they can be added using the parallelogram law in their common plane to obtain their sum or *resultant* $\mathbf{R}$, as shown in Fig. 2/3*a*. The resultant lies in the same plane as $\mathbf{F}_1$ and $\mathbf{F}_2$.

2/5 COUPLE

The moment produced by two equal, opposite, and noncollinear forces is called a *couple*. Couples have certain unique properties and have important applications in mechanics.

Consider the action of two equal and opposite forces $\mathbf{F}$ and $-\mathbf{F}$ a distance d apart, as shown in Fig. 2/10*a*. These two forces cannot be combined into a single force because their sum in every direction is zero. Their only effect is to produce a tendency of rotation. The combined moment of the two forces about an axis normal to their plane and passing through any point such as O in their plane is the couple $\mathbf{M}$. This couple has a magnitude

$$M = F(a + d) - Fa$$

or

$$M = Fd$$

Its direction is counterclockwise when viewed from above for the case illustrated. Note especially that the magnitude of the couple is independent of the distance a which locates the forces with respect to the moment center O. It follows that the moment of a couple has the same value for all moment centers.

Force–Couple Systems

The effect of a force acting on a body is the tendency to push or pull the body in the direction of the force, and to rotate the body about any fixed axis which does not intersect the line of the force. We can represent this dual effect more easily by replacing the given force by an equal parallel force and a couple to compensate for the change in the moment of the force.

The replacement of a force by a force and a couple is illustrated in Fig. 2/12, where the given force $\mathbf{F}$ acting at point A is replaced by an equal force $\mathbf{F}$ at some point B and the counterclockwise couple $M = Fd$. The transfer is seen in the middle figure, where the equal and opposite forces $\mathbf{F}$ and $-\mathbf{F}$ are added at point B without introducing any net external effects on the body. We now see that the original force at A and the equal and opposite one at B constitute the couple $M = Fd$, which is counterclockwise for the sample chosen, as shown in the right-hand part of the figure. Thus, we have replaced the original force at A by the same force acting at a different point B and a couple, without altering the external effects of the original force on the body. The combination of the force and couple in the right-hand part of Fig. 2/12 is referred to as a *force–couple system*.