Chapter 1

Matrix: An arrangement of numbers in the form of rows and columns in a square bracket is called a matrix" and is denoted by A,B,C,...

Order of the Matrix: If A is a matrix, then

order of A = ord (A)= No: of Rows $\times$ No: of Columns.

order of A = ord (A)= No: of Rows –by- No: of Columns.

Note that order of matrix is also called dimension or size

Example1. Write the number of rows and columns of following matrices and hence mention their orders.

i). $A=\begin{bmatrix} p & q \\ r & s \end{bmatrix}$ ii). $B=\begin{bmatrix} 3 & 4 & 7 \\ 5 & 6 & 8 \end{bmatrix}$

i). solution; Given $A=\begin{bmatrix} p & q \\ r & s \end{bmatrix}$

number of rows = 2
number of colums = 2
Hence order (A) = 2-by- 2

ii). solution; Given $B=\begin{bmatrix} 3 & 4 & 7 \\ 5 & 6 & 8 \end{bmatrix}$

number of rows = 2
number of colums = 3
Hence order (A) = 2-by- 3

Equal matrix: Let A and B are two matrix of the same order, are equal if their corresponding elements are equal. e.g. $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ & $B=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $A=B$ iff $a=e,\ \ b=f,\ \ c=g,\ \ d=h$

e.g. $\begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1+1 & \frac{12}{4} \\ 4+2 & \frac{10}{2} & \frac{8}{2} \end{bmatrix}$ are equal

whereas the matices

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \end{bmatrix}$ are not equal

Exercise 1.1

Q1. Which of the following are square and which are rectangular matrices?

i). $A=\begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$

Solution: Given $A=\begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$

number of rows = 2
number of colums = 2
Thus number of rows = number of columns
Therefore A is a square matrix

ii). $B=\begin{bmatrix} 6 & 3 & -1 \\ 1 & 5 & 2 \end{bmatrix}$

Solution: Given $B=\begin{bmatrix} 6 & 3 & -1 \\ 1 & 5 & 2 \end{bmatrix}$

number of rows = 2
number of colums = 3
Thus number of rows $\neq$ number of columns
Therefore B is a rectangular matrix

iii). $C=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Solution: Given $C=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

number of rows = 3
number of colums = 3
Thus number of rows = number of columns
Therefore A is a square matrix

iv). $D=[-5]$

Solution: Given $D=[-5]$

number of rows = 1
number of colums =1
Thus number of rows = number of columns
Therefore A is a square matrix

v). $E=[-3 \quad 4]$

Solution: Given $E=[-3 \quad 4]$

number of rows = 1
number of colums = 2
Thus number of rows $\neq$ number of columns
Therefore A is a rectangular matrix

vi). $F=\begin{bmatrix} -1 \\ 7 \end{bmatrix}$

Solution: Given $F=\begin{bmatrix} -1 \\ 7 \end{bmatrix}$

number of rows = 2
number of colums = 1
Thus number of rows $\neq$ number of columns
Therefore A is a rectangular matrix

Q2. List the order of the following matrices.

i). $A=\begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 2 \end{bmatrix}$

Solution: Given $A=\begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 2 \end{bmatrix}$

number of rows = 2
number of colums = 3
Hence order (A) = 2-by- 3

ii). $B=[-4]$

Solution: Given $B=[-4]$

number of rows = 1
number of colums =1
Hence order (B) = 1-by- 1

iii). $C=\begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 5 \end{bmatrix}$

Solution: Given $C=\begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 5 \end{bmatrix}$

number of rows = 2
number of colums =3
Hence order (C) = 2-by- 3

iv). $D=\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 4 & -1 \end{bmatrix}$

Solution: Given $D=\begin{bmatrix}2 & 1\\3 & 2\\4 & -1\end{bmatrix}$

number of rows = 3
number of colums = 2
Hence order (D) = 3-by- 2

v). $E=\begin{bmatrix}3 & 2\end{bmatrix}$

Solution: Given $E=\begin{bmatrix}3 & 2\end{bmatrix}$

number of rows = 1
number of colums =2
Hence order (E) = 1-by- 2

vi). $F=\begin{bmatrix}1 & 2 & 3\\6 & 5 & 9\\0 & 0 & 0\end{bmatrix}$

Solution: Given $F=\begin{bmatrix}1 & 2 & 3\\6 & 5 & 9\\0 & 0 & 0\end{bmatrix}$

number of rows = 3
number of colums = 3
Hence order (F) = 3-by- 3

Q3. If $A=\begin{bmatrix}3 & 2 & -4\\-2 & 5 & 0\\2 & 1 & 5\\-3 & 4 & 6\end{bmatrix}$ give the following elements.

i). $a_{12}=2$ ii). $a_{23}=0$

iii). $a_{32}=1$ iv). $a_{43}=6$

v). $a_{13}=-4$ vi). $a_{33}=5$

Q4. Which of the following matrices are equal?

$A=\begin{bmatrix}2 & 5\\1 & 3\end{bmatrix}, B=\begin{bmatrix}2 & 5\\4 & 3\end{bmatrix},$

$C=\begin{bmatrix}1+1 & 3+2\\4 & 2+1\end{bmatrix}, D=\begin{bmatrix}2 & 4+1\\1 & 3\end{bmatrix}$

Solution: Given $A=\begin{bmatrix}2 & 5\\1 & 3\end{bmatrix}=\begin{bmatrix}2 & 4+1\\1 & 3\end{bmatrix}=D,$

$B=\begin{bmatrix}2 & 5\\4 & 3\end{bmatrix}=\begin{bmatrix}1+1 & 3+2\\4 & 2+1\end{bmatrix}=C$

Q5.Let $A=\begin{bmatrix}2 & -3\\u & 0\end{bmatrix}$ and $B=\begin{bmatrix}v & -3\\5 & w\end{bmatrix}$ for what values of u,v and w are A and B equal

Solution: Given $A=\begin{bmatrix}2 & -3\\u & 0\end{bmatrix}$ and $B=\begin{bmatrix}v & -3\\5 & w\end{bmatrix}$ are equal so, $A=B$ then corresponding elements must be equal $\begin{bmatrix}2 & -3\\u & 0\end{bmatrix}=\begin{bmatrix}v & -3\\5 & w\end{bmatrix}$

$2=v$ $u=5$ $0=w$

Q6.If $\begin{bmatrix}x+3 & z+4 & 2y-7\\-6 & a-1 & 0\\b-3 & -21 & 0\end{bmatrix}=\begin{bmatrix}0 & 6 & 3y-2\\-6 & -3 & 2c+2\\2b+4 & -21 & 0\end{bmatrix}$

find the values of a,b,c,x,y and z

Soultion: Giventwo equal matrices so, $A=B$

$$\begin{bmatrix}x+3 & z+4 & 2y-7\\-6 & a-1 & 0\\b-3 & -21 & 0\end{bmatrix}=\begin{bmatrix}0 & 6 & 3y-2\\-6 & -3 & 2c+2\\2b+4 & -21 & 0\end{bmatrix}$$

then corresponding elements must be equal

$x+3=0$ $z+4=6$ $2y-7=3y-2$

$x=0-3$ $z=6-4$ $+2-7=3y-2y$

$x=-3$ $z=2$ $-5=y$

$b-3=2b+4$ $a-1=-3$ $0=2c+2$

$-4-3=2b-b$ $a=-3+1$ $-2c=2$

$-7=b$ $a=-2$ $c=\frac{2}{-2}=-1$

Q7. Solve the following equation for a,b,c,d

$$\begin{bmatrix}a+b & b+2c\\2c+d & 2a-d\end{bmatrix}=\begin{bmatrix}-1 & 4\\8 & 0\end{bmatrix}$$

Soultion: Giventwo equal matrices so, $A=B$

$$\begin{bmatrix}a+b & b+2c\\2c+d & 2a-d\end{bmatrix}=\begin{bmatrix}-1 & 4\\8 & 0\end{bmatrix}$$

then corresponding elements must be equal

$a+b=-1$

$b=-1-a.....(1)$ $b+2c=4......(2)$

$2c+d=8......(3)$ $2a-d=0$

$2a=d......(4)$

Putting eq (1) in eq (2)

$-1-a+2c=4$

$-a+2c=4+1$

$-a+2c=5..........(5)$

Putting eq (4) in eq (3)

$2a+2c=8................(6)$

Substracting eq (5) from eq (6)

$2a+2c=8$

$\mp a\pm 2c=\pm 5$

$3a=3$

Or $a=1$

Putting the value of a in eq (1) and eq (4)

$b=-1-1$ and $d=2(1)$

$b=-2$ $d=2$

Putting the value of d in eq (3)

$2c+2=8$

$2c=8-2$

$c=\frac{6}{2}=3$

Types of Matrices:

a). **Row matrix**

A matrix having one row is called a row matrix.

b). **Column matrix**:

A matrix having one column is called a column matrix.

c). **Rectangular Matrix**:

A matrix in which rows and columns are not equal in numbers or a matrix of order $m\times n$ if $m\neq n$

d). **Square matrix**:A matrix in which rows and column are equal in numbers or a matrix of order $m\times n$ if $m=n$

e). Null or Zero matrix: A matrix in which all the elements or entries are zero, is called a null or zero matrix denoted by O.

e.g. $O_{2\times2}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$, $O_{1\times1}=[0]$

f). Diagonal Matrix: A square matrix in which all the elements except at least one element of the diagonal are zero is called a diagonal matrix. Some elements of the diagonal matrix may be zero but not all.. e.g. $\begin{bmatrix}2&0\\0&4\end{bmatrix}$

g). Unit or Identity matrix: A scalar matrix having each element in the diagonal equal to 1 is called a Unit or Identity Matrix and is denoted by I.

i.e., $I=\begin{bmatrix}1&0\\0&1\end{bmatrix}$

h). Scalar matrix: A diagonal matrix having same/equal elements in its diagonal is called Scalar matrix.

e.g. $\begin{bmatrix}4&0\\0&4\end{bmatrix}$ and $\begin{bmatrix}-1&0\\0&-1\end{bmatrix}$ are scalar matrix.

i). Transpose of Matrix: If a matrix of order $m\times n$, then a matrix of order $n\times m$ obtained by interchanging the row and columns of A is called the transpose of A. it is denoted by A^t. i.e. $A=\begin{bmatrix}a&b\\c&d\end{bmatrix}$ then $A^t=\begin{bmatrix}a&c\\b&d\end{bmatrix}$

J). Symmetric Matrix: If a square matrix $A=A^t$ then A is said to be symmetric matrix. For example

$A=\begin{bmatrix}1&2\\2&4\end{bmatrix}\Rightarrow A^t=\begin{bmatrix}1&2\\2&4\end{bmatrix}\Rightarrow A^t=A$

J). Skew symmetric Matrix: If a square matrix $A^t=-A$ then A is said to be skew-symmetric matrix.

For example $A=\begin{bmatrix}0&-3\\3&0\end{bmatrix}$

$A^t=\begin{bmatrix}0&3\\-3&0\end{bmatrix}=-\begin{bmatrix}0&-3\\3&0\end{bmatrix}\quad\Rightarrow A^t=-A$

Exercise 1.2

Q1. Write transpose of the following matrices.

i). $P=\begin{bmatrix}1&2\\3&1\end{bmatrix}$

Solution: $P^t=\begin{bmatrix}1&3\\2&1\end{bmatrix}$

ii). $Q=\begin{bmatrix}l&m\\n&p\end{bmatrix}$

Solution: $Q^t=\begin{bmatrix}l&n\\m&p\end{bmatrix}$

iii). $R=[6]$

Solution: $R^t=[6]$

iii). $S=\begin{bmatrix}-5&1\\-2&1\\4&4\end{bmatrix}$

Solution: $S^t=\begin{bmatrix}-5&-2&4\\1&1&4\end{bmatrix}$

iv). $T=\begin{bmatrix}6&7&8\\13&1&3\\2&4&5\end{bmatrix}$

Solution: $T^t=\begin{bmatrix}6&13&2\\7&1&4\\8&3&5\end{bmatrix}$

Q2. Find which of the following matrices are transpose of each other.

i). $A=\begin{bmatrix}a_1&a_2\\b_1&b_2\end{bmatrix}$ ii) $B=\begin{bmatrix}a_1&b_1\\a_2&b_2\end{bmatrix}$

iii). $C=\begin{bmatrix}-3&1&-1\\4&2&7\end{bmatrix}$ iv). $D=\begin{bmatrix}-3&4\\1&2\\-1&7\end{bmatrix}$

Solution: $A^t=B$ or $B^t=A$

i.e., $\begin{bmatrix}a_1&a_2\\b_1&b_2\end{bmatrix}^t=\begin{bmatrix}a_1&b_1\\a_2&b_2\end{bmatrix}$

$C^t=F$ or $F^t=C$

i.e., $\begin{bmatrix}-3&1&-1\\4&2&7\end{bmatrix}^t=\begin{bmatrix}-3&4\\1&2\\-1&7\end{bmatrix}$

Q3. Which of following matrices are symmetric.

i). $A=\begin{bmatrix}5&-7\\-1&5\end{bmatrix}$

Sol: $A^t=\begin{bmatrix}5&-1\\-7&5\end{bmatrix}\neq A\quad\Rightarrow A^t\neq A$

So by definition A is not symmetric

ii). $B=\begin{bmatrix}-1&2\\2&3\end{bmatrix}$

Sol: $B^t=\begin{bmatrix}-1&2\\2&3\end{bmatrix}=B\quad\Rightarrow B^t=B$

So by definition B is symmetric

iii). $C=\begin{bmatrix}3&4\\5&6\end{bmatrix}$

Sol: $C^t=\begin{bmatrix}3&5\\4&6\end{bmatrix}\neq C\quad\Rightarrow C^t\neq C$

So by definition C is not symmetric

iv). $D=\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}$

Sol: $D^t=\begin{bmatrix}1&4\\2&5\\3&6\end{bmatrix}\neq D\quad\Rightarrow D^t\neq D$

So by definition D is not symmetric

Q4. Find which of the following matrices are skew-symmetric matrices?

i). $A=\begin{bmatrix}0&4\\-4&0\end{bmatrix}$

Sol: $A^t=\begin{bmatrix}0&-4\\4&0\end{bmatrix}=-\begin{bmatrix}0&4\\-4&0\end{bmatrix}=-A$

$\Rightarrow A^t = -A$, So by definition A is skew-symmetric

ii). $B = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$

Sol: $B^t = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} = -B$

$\Rightarrow B^t = -B$, So by definition B is skew-symmetric

iii). $C = \begin{bmatrix} 0 & 7 \\ 7 & 0 \end{bmatrix}$

Sol: $C^t = \begin{bmatrix} 0 & 7 \\ 7 & 0 \end{bmatrix} = C \neq -C \Rightarrow C^t \neq -C$

So by definition C is not skew-symmetric

iv). $D = \begin{bmatrix} 0 & 3 & 2 \\ -3 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$

Sol: $D^t = \begin{bmatrix} 0 & -3 & -2 \\ 3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 3 & 2 \\ -3 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix} = -D$

$\Rightarrow D^t = -D$, So by definition D is skew-symmetric

Algebra of Matrix

Conformable for Addition or Subtraction

Two matrices are conformable for addition or subtraction if they are of the same order

Let $A = \begin{bmatrix} 1 & 3 \\ -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 7 \\ 10 & 13 \end{bmatrix}$ these are conformable for addition because both are same order of 2-by-2

Addition of Matrices:The sum of two matrices of the same order can be obtained by only adding their corresponding elements.

Example 2 if $A = \begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix}$

We see that A and B both are 2-by-2 matrices, so these are conformable for addition

Sol: $A + B = \begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 3+4 & 8+0 \\ 4+1 & 6-9 \end{bmatrix}$

$A + B = \begin{bmatrix} 7 & 8 \\ 5 & -3 \end{bmatrix}$

Subtraction of Matrices: Difference of two matrices of the same order can be obtained by only subtracting their corresponding elements.

Example 3 if $A = \begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix}$

We see that A and B both are 2-by-2 matrices, so these are conformable for addition

Sol: $A - B = \begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 3-7 & 8-0 \\ 4-1 & 6+9 \end{bmatrix}$

$A - B = \begin{bmatrix} -1 & 8 \\ 3 & 15 \end{bmatrix}$

Multiplication of Matrix by Real numbers.

Let A be any Matrix and $k \in R$, then matrix obtained by multiplying each element of A by k is called the scalar multiplication of A by k and is denoted by kA and k is called scalar multiple of A.

$2A = 2\begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix}\begin{bmatrix} 2\times 4 & 2\times 0 \\ 2\times 1 & 2\times -9 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & -18 \end{bmatrix}$

Commutative property w.r.t Addition If A and B are any two Matrices of Same order then $A + B = B + A$ is called commutative law under addition.

Example 4: Let $A = \begin{bmatrix} 2 & 5 \\ 4 & 7 \end{bmatrix}, B = \begin{bmatrix} -2 & 1 \\ -3 & 6 \end{bmatrix}$

Sol: Given $A = \begin{bmatrix} 2 & 5 \\ 4 & 7 \end{bmatrix}, B = \begin{bmatrix} -2 & 1 \\ -3 & 6 \end{bmatrix}$ these matrices are of the same order, SO these are conformable for addition Then $A + B = \begin{bmatrix} 2 & 5 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ -3 & 6 \end{bmatrix} = \begin{bmatrix} 2+(-2) & 5+1 \\ 4+(-3) & 7+6 \end{bmatrix}$

$A + B = \begin{bmatrix} 0 & 6 \\ 1 & 13 \end{bmatrix}$

$B + A = \begin{bmatrix} -2 & 1 \\ -3 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} -2+2 & 1+5 \\ -3+4 & 6+7 \end{bmatrix}$

$B + A = \begin{bmatrix} 0 & 6 \\ 1 & 13 \end{bmatrix}$ Hence $A + B = B + A$

Associative property of Addition:

If A ,B and C are any three Matrices of same order is associative, if $(A+B)+C = A+(B+C)$

Example 5:If $A = \begin{bmatrix} -1 & 2 \\ 4 & -3 \end{bmatrix}, B = \begin{bmatrix} 4 & -5 \\ 6 & 7 \end{bmatrix}, C = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$

then prove that $(A+B)+C = A+(B+C)$

Sol: Given $A = \begin{bmatrix} -1 & 2 \\ 4 & -3 \end{bmatrix}, B = \begin{bmatrix} 4 & -5 \\ 6 & 7 \end{bmatrix}, C = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$

these matrices are of the same order, SO these are conformable for addition, So taking LHS

$(A+B)+C = \left(\begin{bmatrix} -1 & 2 \\ 4 & -3 \end{bmatrix} + \begin{bmatrix} 4 & -5 \\ 6 & 7 \end{bmatrix}\right) + \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} -1+4 & 2+(-5) \\ 4+6 & -3+7 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 10 & 4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 3+3 & -3+(-2) \\ 10+1 & 4+0 \end{bmatrix} = \begin{bmatrix} 6 & -5 \\ 11 & 4 \end{bmatrix}$ ....(1)

$RHS = A+(B+C) = \begin{bmatrix} -1 & 2 \\ 4 & -3 \end{bmatrix} + \left(\begin{bmatrix} 4 & -5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}\right)$

$= \begin{bmatrix} -1 & 2 \\ 4 & -3 \end{bmatrix} + \begin{bmatrix} 4+3 & -5+(-2) \\ 6+1 & 7+0 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 4 & -3 \end{bmatrix} + \begin{bmatrix} 7 & -7 \\ 7 & 7 \end{bmatrix}$

$= \begin{bmatrix} -1+7 & 2+(-7) \\ 4+7 & -3+7 \end{bmatrix} = \begin{bmatrix} 6 & -5 \\ 11 & 4 \end{bmatrix}$ ....(2)

From eq (1) and eq (2)

$(A+B)+C = A+(B+C)$

Additive Identity: In real numbers zero is the additive identity i.e. the sum of real number and zero is equal to that real number $A + 0 = 0 + A = A$.

Similarly, zero matrix O is called the additive identity matrix

Example 6: $A = \begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix}$, & $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Sol: Given $A=\begin{bmatrix}3 & -1\\ 2 & 5\end{bmatrix}$ & $O=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ Now

$$A+O=\begin{bmatrix}3 & -1\\ 2 & 5\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=\begin{bmatrix}3+0 & -1+0\\ 2+0 & 5+0\end{bmatrix}$$

$$A+O=\begin{bmatrix}3 & -1\\ 2 & 5\end{bmatrix}=A$$ and

$$O+A=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}+\begin{bmatrix}3 & -1\\ 2 & 5\end{bmatrix}=\begin{bmatrix}0+3 & 0+(-1)\\ 0+2 & 0+5\end{bmatrix}$$

$$O+A=\begin{bmatrix}3 & -1\\ 2 & 5\end{bmatrix}=A$$

Hence $A+0=0+A=A$

Then Matrix O is called additive identity

Additive Inverse of Matrix: If two matrices A and B are such that their sum (A+B) is zero matrix, then A and B are called additive inverses of each other.

Example 7 Prove that $P=\begin{bmatrix}3 & 2 & -1\\ -2 & 4 & 6\end{bmatrix}$ and $Q=\begin{bmatrix}-3 & -2 & 1\\ 2 & -4 & -6\end{bmatrix}$ are additive inverse of each other.

Sol: $P=\begin{bmatrix}3 & 2 & -1\\ -2 & 4 & 6\end{bmatrix}$ and $Q=\begin{bmatrix}-3 & -2 & 1\\ 2 & -4 & -6\end{bmatrix}$

Take $P+Q=\begin{bmatrix}3 & 2 & -1\\ -2 & 4 & 6\end{bmatrix}+\begin{bmatrix}-3 & -2 & 1\\ 2 & -4 & -6\end{bmatrix}$

$$P+Q=\begin{bmatrix}3+(-3) & 2+(-2) & -1+1\\ -2+2 & 4+(-4) & 6+(-6)\end{bmatrix}$$

$$P+Q=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=O$$

Now $Q+P=\begin{bmatrix}-3 & -2 & 1\\ 2 & -4 & -6\end{bmatrix}+\begin{bmatrix}3 & 2 & -1\\ -2 & 4 & 6\end{bmatrix}$

$$Q+p=\begin{bmatrix}-3+3 & -2+1 & 1+(-1)\\ 2+(-2) & -4+4 & -6+6\end{bmatrix}$$

$Q+P=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=O$ Hence P and Q are additive inverse of each other. i.e., P+Q = Q+P = O

Exercise # 1.3

Q1. Let A & B by 2-by-3 matrices and C & D be 2-square matrices. Which of the following matrices operations are definded. For those which are defind, give the dimension of the resulting matrix.

i). A + B

Solution: both matrices are same order 2-by-3

Therefore they are conformable for addition

ii). B + D

Solution; B has order 2-by-3 and

D has order 2-by-2

Hence order of the matrices are not same

Therefore they are not conformable for additon

iii). 3A – 2C

Solution; A has order 2-by-3 and

C has order 2-by-2

Hence order of the matrices are not same

Therefore they are not conformable for additon

Note that After scalar multiplication order of the matices remains same

iv). 7C + 2D

Solution; C has order 2-by-2 and

D has order 2-by-2

Hence order of the matrices are not same

Therefore they are not conformable for additon

Note that After scalar multiplication order of the matices remains same

Q2:i). Multiply $A=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$ by 2

Solution: Given $A=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$

$$2A=2\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}=\begin{bmatrix}2\times1\\ 2\times2\\ 2\times3\end{bmatrix}=\begin{bmatrix}2\\ 4\\ 6\end{bmatrix}$$

Q2:ii). Multiply $C=\begin{bmatrix}a & b & c\\ d & e & f\end{bmatrix}$ by $p\in R$

Solution: Given $C=\begin{bmatrix}a & b & c\\ d & e & f\end{bmatrix}$

$$pC=\begin{bmatrix}pa & pb & pc\\ pd & pe & pf\end{bmatrix}$$

Q3. Find a matrix X such that $4X=\begin{bmatrix}1 & 2 & 1\\ 4 & 2 & 3\\ -1 & 9 & 7\end{bmatrix}$

Solution: Given $4X=\begin{bmatrix}1 & 2 & 1\\ 4 & 2 & 3\\ -1 & 9 & 7\end{bmatrix}$

Or $X=\frac{1}{4}\begin{bmatrix}1 & 2 & 1\\ 4 & 2 & 3\\ -1 & 9 & 7\end{bmatrix}$

$$X=\begin{bmatrix}\frac{1}{4} & \frac{2}{4} & \frac{1}{4}\\ \frac{4}{4} & \frac{2}{4} & \frac{3}{4}\\ \frac{-1}{4} & \frac{9}{4} & \frac{7}{4}\end{bmatrix}$$

$$X=\begin{bmatrix}\frac{1}{4} & \frac{1}{2} & \frac{1}{4}\\ 1 & \frac{1}{2} & \frac{3}{4}\\ \frac{-1}{4} & \frac{9}{4} & \frac{7}{4}\end{bmatrix}$$

Q4. If $A=\begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix}$ & $B=\begin{bmatrix}-3 & -2\\ 1 & -5\\ 4 & 3\end{bmatrix}$, find $3A-B$

Solution: Given $A=\begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix}$ and $B=\begin{bmatrix}-3 & -2\\ 1 & -5\\ 4 & 3\end{bmatrix}$

Now $3A-B=3\begin{bmatrix}1&2\\3&4\\5&6\end{bmatrix}-\begin{bmatrix}-3&-2\\1&-5\\4&3\end{bmatrix}$

$$3A-B=\begin{bmatrix}3&6\\9&12\\15&18\end{bmatrix}-\begin{bmatrix}-3&-2\\1&-5\\4&3\end{bmatrix}$$

$$3A-B=\begin{bmatrix}3+3&6+2\\9-1&12+5\\15-4&18-3\end{bmatrix}$$

$$3A-B=\begin{bmatrix}6&8\\8&17\\11&15\end{bmatrix}$$

Q5.Given $A=\begin{bmatrix}1&2&-3\\5&0&2\\1&-1&1\end{bmatrix}$ & $B=\begin{bmatrix}3&-1&2\\4&2&5\\2&3&0\end{bmatrix}$, find the matrix C such that $A+2B=C$

Solution: $A=\begin{bmatrix}1&2&-3\\5&0&2\\1&-1&1\end{bmatrix}$ & $B=\begin{bmatrix}3&-1&2\\4&2&5\\2&3&0\end{bmatrix}$

We have to find $C=A+2B$

$$C=\begin{bmatrix}1&2&-3\\5&0&2\\1&-1&1\end{bmatrix}+2\begin{bmatrix}3&-1&2\\4&2&5\\2&3&0\end{bmatrix}$$

$$C=\begin{bmatrix}1&2&-3\\5&0&2\\1&-1&1\end{bmatrix}+\begin{bmatrix}6&-2&4\\8&4&10\\4&6&0\end{bmatrix}$$

$$C=\begin{bmatrix}1+6&2-2&-3+4\\5+8&0+4&2+10\\1+4&-1+6&1+0\end{bmatrix}$$

$$C=\begin{bmatrix}7&0&1\\13&4&12\\5&5&1\end{bmatrix}$$

Q6. If $A=\begin{bmatrix}2&-2\\4&2\\-5&1\end{bmatrix}$ and $B=\begin{bmatrix}8&0\\4&-2\\3&6\end{bmatrix}$, then find the matrix X such that $2A+3X=5B$

Solution; Given $A=\begin{bmatrix}2&-2\\4&2\\-5&1\end{bmatrix}$ & $B=\begin{bmatrix}8&0\\4&-2\\3&6\end{bmatrix}$

Given that $2A+3X=5B$

$$3X=5B-2A$$

$X=\frac{1}{3}\{5B-2A\}$ putting the values of A and B

$$X=\frac{1}{3}\left\{5\begin{bmatrix}8&0\\4&-2\\3&6\end{bmatrix}-2\begin{bmatrix}2&-2\\4&2\\-5&1\end{bmatrix}\right\}$$

$$X=\frac{1}{3}\begin{bmatrix}40-4&0+4\\20-8&-10-4\\15+10&30-2\end{bmatrix}$$

$$X=\frac{1}{3}\begin{bmatrix}36&4\\12&-14\\25&28\end{bmatrix}=\begin{bmatrix}12&\frac{4}{3}\\4&\frac{-14}{3}\\\frac{25}{3}&\frac{28}{3}\end{bmatrix}$$

Q7. Find x,y,z and w if

$$3\begin{bmatrix}x&y\\z&w\end{bmatrix}=\begin{bmatrix}x&6\\-1&2w\end{bmatrix}+\begin{bmatrix}4&x+y\\3+w&3\end{bmatrix}$$

Sol: $3\begin{bmatrix}x&y\\z&w\end{bmatrix}=\begin{bmatrix}x&6\\-1&2w\end{bmatrix}+\begin{bmatrix}4&x+y\\3+w&3\end{bmatrix}$

$$\begin{bmatrix}3x&3y\\3z&3w\end{bmatrix}=\begin{bmatrix}x+4&6+x+y\\-1+3+w&2w+3\end{bmatrix}$$

By definition of equal matrices their corresponding elements must be equal

$3x=x+4$ $3y=6+x+y$

$3x-x=4$ $3y-y=6+2$ *putting* $x=2$

$2x=4$ $2y=8$

$x=2$ $y=4$

$3w=2w+3$ $3z=-1+3+w$

$3w-2w=3$ $3z=2+3$ *putting* $w=3$

$w=3$ $3z=5$

$z=\frac{5}{3}$

Q8. Find X & Y if $X+Y=\begin{bmatrix}5&2\\0&9\end{bmatrix}$ & $X-Y=\begin{bmatrix}3&6\\0&-1\end{bmatrix}$

Solution: $X+Y=\begin{bmatrix}5&2\\0&9\end{bmatrix}$ ....(1)

and $X-Y=\begin{bmatrix}3&6\\0&-1\end{bmatrix}$ ....(2)

adding eq (1) and (2)

$$2X=\begin{bmatrix}5&2\\0&9\end{bmatrix}+\begin{bmatrix}3&6\\0&-1\end{bmatrix}$$

$$2X=\frac{1}{2}\begin{bmatrix}5+3&2+6\\0+0&9-1\end{bmatrix}$$

$$X=\frac{1}{2}\begin{bmatrix}8&8\\0&8\end{bmatrix}=\begin{bmatrix}4&4\\0&4\end{bmatrix}$$

Putting the value of x in eq (1)

$$\begin{bmatrix}4&4\\0&4\end{bmatrix}+Y=\begin{bmatrix}5&2\\0&9\end{bmatrix}$$

$$Y=\begin{bmatrix}5&2\\0&9\end{bmatrix}-\begin{bmatrix}4&4\\0&4\end{bmatrix}$$

$$Y=\begin{bmatrix}5-4&2-4\\0-0&9-4\end{bmatrix}$$

$$Y=\begin{bmatrix}1&-2\\0&5\end{bmatrix}$$

Q9:i).Let $A=\begin{bmatrix}2&-3\\4&5\end{bmatrix}$, if $c=2$ and $d=-4$ then verify that $(c+d)A=cA+dA$

Solution: Given $A=\begin{bmatrix}2 & -3\\4 & 5\end{bmatrix}$ and $c=2$, $d=-4$

Taking LHS $(c+d)A=(2-4)\begin{bmatrix}2 & -3\\4 & 5\end{bmatrix}$

$(c+d)A=(-2)\begin{bmatrix}2 & -3\\4 & 5\end{bmatrix}$

$(c+d)A=\begin{bmatrix}(-2)(2) & (-2)(-3)\\(-2)(4) & (-2)(5)\end{bmatrix}$

$(c+d)A=\begin{bmatrix}-4 & 6\\-8 & -10\end{bmatrix}$..................(1)

Now RHS $cA+dA=2\begin{bmatrix}2 & -3\\4 & 5\end{bmatrix}+(-4)\begin{bmatrix}2 & -3\\4 & 5\end{bmatrix}$

$cA+dA=\begin{bmatrix}4 & -6\\8 & 10\end{bmatrix}+\begin{bmatrix}-8 & 12\\-16 & -20\end{bmatrix}$

$cA+dA=\begin{bmatrix}4-8 & -6+12\\8-16 & 10-20\end{bmatrix}$

$cA+dA=\begin{bmatrix}-4 & 6\\-8 & -10\end{bmatrix}$................(2)

From eq (1) and eq (2) we get $(c+d)A=cA+dA$

Q9:ii). Let $A=\begin{bmatrix}2 & -3\\4 & 5\end{bmatrix}$, $B=\begin{bmatrix}2 & 5\\-1 & 3\end{bmatrix}$ and if $c=2$ then verify that $c(A+B)=cA+cB$

Solution: Since $A=\begin{bmatrix}2 & -3\\4 & 5\end{bmatrix}$, $B=\begin{bmatrix}2 & 5\\-1 & 3\end{bmatrix}$, $c=2$

Taking LHS $c(A+B)=2\left(\begin{bmatrix}2 & -3\\4 & 5\end{bmatrix}+\begin{bmatrix}2 & 5\\-1 & 3\end{bmatrix}\right)$

$c(A+B)=2\begin{bmatrix}2+2 & -3+5\\4-1 & 5+3\end{bmatrix}$

$c(A+B)=2\begin{bmatrix}4 & 2\\3 & 8\end{bmatrix}$

$c(A+B)=\begin{bmatrix}8 & 4\\6 & 16\end{bmatrix}$..................(1)

Now RHS $cA+cB=2\begin{bmatrix}2 & -3\\4 & 5\end{bmatrix}+2\begin{bmatrix}2 & 5\\-1 & 3\end{bmatrix}$

$cA+cB=\begin{bmatrix}4 & -6\\8 & 10\end{bmatrix}+\begin{bmatrix}4 & 10\\-2 & 6\end{bmatrix}$

$cA+cB=\begin{bmatrix}4+4 & -6+10\\8-2 & 10+6\end{bmatrix}$

$cA+cB=\begin{bmatrix}8 & 4\\6 & 16\end{bmatrix}$................(2)

From eq (1) and eq (2) we get $c(A+B)=cA+cB$

Q9:iii). Let $A=\begin{bmatrix}2 & -3\\4 & 5\end{bmatrix}$,if $c=2$ and $d=-4$ then verify that $cd(A)=c(dA)$

Solution: Given $A=\begin{bmatrix}2 & -3\\4 & 5\end{bmatrix}$ & $c=2, d=-4$

Taking LHS $cd(A)=(2)(-4)\begin{bmatrix}2 & -3\\4 & 5\end{bmatrix}$

$cd(A)=-8\begin{bmatrix}2 & -3\\4 & 5\end{bmatrix}$

$cd(A)=\begin{bmatrix}-16 & 24\\-32 & -40\end{bmatrix}$...........(1)

Now taking RHS $c(dA)=(2)\left((-4)\begin{bmatrix}2 & -3\\4 & 5\end{bmatrix}\right)$

$c(dA)=(2)\begin{bmatrix}-8 & 12\\-16 & -20\end{bmatrix}$

$c(dA)=\begin{bmatrix}-16 & 24\\-32 & -40\end{bmatrix}$.......................(2)

From eq (1) and eq (2) we get $cd(A)=c(dA)$

Q10:i).Let $A=\begin{bmatrix}-1 & 2 & 3\\4 & 2 & 0\\-3 & 2 & 5\end{bmatrix}$, $B=\begin{bmatrix}3 & -1 & 2\\-5 & 3 & 4\\-3 & -4 & 0\end{bmatrix}$ compute if Possible $A+2B$

Solution: since $A=\begin{bmatrix}-1 & 2 & 3\\4 & 2 & 0\\-3 & 2 & 5\end{bmatrix}$, $B=\begin{bmatrix}3 & -1 & 2\\-5 & 3 & 4\\-3 & -4 & 0\end{bmatrix}$

Now $A+2B=\begin{bmatrix}-1 & 2 & 3\\4 & 2 & 0\\-3 & 2 & 5\end{bmatrix}+2\begin{bmatrix}3 & -1 & 2\\-5 & 3 & 4\\-3 & -4 & 0\end{bmatrix}$

$A+2B=\begin{bmatrix}-1 & 2 & 3\\4 & 2 & 0\\-3 & 2 & 5\end{bmatrix}+\begin{bmatrix}6 & -2 & 4\\-10 & 6 & 8\\-6 & -8 & 0\end{bmatrix}$

$A+2B=\begin{bmatrix}-1+6 & 2-2 & 3+4\\4-10 & 2+6 & 0+8\\-3-6 & 2-8 & 5+0\end{bmatrix}$

$A+2B=\begin{bmatrix}5 & 0 & 7\\-6 & 8 & 8\\-9 & -6 & 5\end{bmatrix}$

Q10:ii). Let $A=\begin{bmatrix}-1 & 2 & 3\\4 & 2 & 0\\-3 & 2 & 5\end{bmatrix}$, $B=\begin{bmatrix}3 & -1 & 2\\-5 & 3 & 4\\-3 & -4 & 0\end{bmatrix}$ compute if Possible $3A-4B$

Sol:since $A=\begin{bmatrix}-1 & 2 & 3\\4 & 2 & 0\\-3 & 2 & 5\end{bmatrix}$, $B=\begin{bmatrix}3 & -1 & 2\\-5 & 3 & 4\\-3 & -4 & 0\end{bmatrix}$

Now $3A-4B=3\begin{bmatrix}-1 & 2 & 3\\4 & 2 & 0\\-3 & 2 & 5\end{bmatrix}-4\begin{bmatrix}3 & -1 & 2\\-5 & 3 & 4\\-3 & -4 & 0\end{bmatrix}$

$$3A-4B=\begin{bmatrix}-3&6&9\\12&6&0\\-9&6&15\end{bmatrix}-\begin{bmatrix}12&-4&8\\-20&12&16\\-12&-16&0\end{bmatrix}$$

$$3A-4B=\begin{bmatrix}-3-12&6+4&9-8\\12+20&6-12&0-16\\-9+12&6+16&15-0\end{bmatrix}$$

$$3A-4B=\begin{bmatrix}-15&10&1\\32&-6&-16\\3&22&15\end{bmatrix}$$

Q10:iii). Let $A=\begin{bmatrix}-1&2&3\\4&2&0\\-3&2&5\end{bmatrix},B=\begin{bmatrix}3&-1&2\\-5&3&4\\-3&-4&0\end{bmatrix}$ and $C=\begin{bmatrix}2&-3&6\\0&4&-1\\-5&1&3\end{bmatrix}$ compute if Possible $(A+B)-C$

Sol: $A=\begin{bmatrix}-1&2&3\\4&2&0\\-3&2&5\end{bmatrix},B=\begin{bmatrix}3&-1&2\\-5&3&4\\-3&-4&0\end{bmatrix}C=\begin{bmatrix}2&-3&6\\0&4&-1\\-5&1&3\end{bmatrix}$

Now

$$(A+B)-C=\left(\begin{bmatrix}-1&2&3\\4&2&0\\-3&2&5\end{bmatrix}+\begin{bmatrix}3&-1&2\\-5&3&4\\-3&-4&0\end{bmatrix}\right)-\begin{bmatrix}2&-3&6\\0&4&-1\\-5&1&3\end{bmatrix}$$

$$(A+B)-C=\begin{bmatrix}-1+3&2-1&3+2\\4-5&2+3&0+4\\-3-3&2-4&5+0\end{bmatrix}-\begin{bmatrix}2&-3&6\\0&4&-1\\-5&1&3\end{bmatrix}$$

$$(A+B)-C=\begin{bmatrix}2&1&5\\-1&5&4\\-6&-2&5\end{bmatrix}-\begin{bmatrix}2&-3&6\\0&4&-1\\-5&1&3\end{bmatrix}$$

$$(A+B)-C=\begin{bmatrix}2-2&1+3&5-6\\-1-0&5-4&4+1\\-6+5&-2-1&5-3\end{bmatrix}$$

$$(A+B)-C=\begin{bmatrix}0&4&-1\\-1&1&5\\-1&-3&2\end{bmatrix}$$

Q10:iv). Let $A=\begin{bmatrix}-1&2&3\\4&2&0\\-3&2&5\end{bmatrix},B=\begin{bmatrix}3&-1&2\\-5&3&4\\-3&-4&0\end{bmatrix}$ and $C=\begin{bmatrix}2&-3&6\\0&4&-1\\-5&1&3\end{bmatrix}$ compute if Possible $A+(B+C)$

Sol: $A=\begin{bmatrix}-1&2&3\\4&2&0\\-3&2&5\end{bmatrix},B=\begin{bmatrix}3&-1&2\\-5&3&4\\-3&-4&0\end{bmatrix}C=\begin{bmatrix}2&-3&6\\0&4&-1\\-5&1&3\end{bmatrix}$

Now

$$(A+B)+C=\left(\begin{bmatrix}-1&2&3\\4&2&0\\-3&2&5\end{bmatrix}+\begin{bmatrix}3&-1&2\\-5&3&4\\-3&-4&0\end{bmatrix}\right)+\begin{bmatrix}2&-3&6\\0&4&-1\\-5&1&3\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}-1+3&2-1&3+2\\4-5&2+3&0+4\\-3-3&2-4&5+0\end{bmatrix}+\begin{bmatrix}2&-3&6\\0&4&-1\\-5&1&3\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}2&1&5\\-1&5&4\\-6&-2&5\end{bmatrix}+\begin{bmatrix}2&-3&6\\0&4&-1\\-5&1&3\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}2+2&1-3&5+6\\-1+0&5+4&4-1\\-6-5&-2+1&5+3\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}4&-2&11\\-1&9&3\\-11&-1&8\end{bmatrix}$$

Q11. Prove that in the following matrices commutative law of addition holds.

i). $A=\begin{bmatrix}7&1\\2&4\end{bmatrix},B=\begin{bmatrix}1&1\\2&2\end{bmatrix}$

Solution: LHS $A+B=\begin{bmatrix}7&1\\2&4\end{bmatrix}+\begin{bmatrix}1&1\\2&2\end{bmatrix}$

$$A+B=\begin{bmatrix}7+1&1+1\\2+2&4+2\end{bmatrix}=\begin{bmatrix}8&2\\4&6\end{bmatrix}\dots\dots\dots\dots(1)$$

Now RHS $B+A=\begin{bmatrix}1&1\\2&2\end{bmatrix}+\begin{bmatrix}7&1\\2&4\end{bmatrix}$

$$B+A=\begin{bmatrix}1+7&1+1\\2+2&2+4\end{bmatrix}$$

$$B+A=\begin{bmatrix}8&2\\4&6\end{bmatrix}\dots\dots\dots\dots(2)$$

From eq (1) and eq (2)

LHS=RHS Hence proved

ii). $A=\begin{bmatrix}-3&4&-5\\2&3&1\end{bmatrix},B=\begin{bmatrix}-3&-4&5\\1&2&3\end{bmatrix}$

Solution: LHS $A+B=\begin{bmatrix}-3&4&-5\\2&3&1\end{bmatrix}+\begin{bmatrix}-3&-4&5\\1&2&3\end{bmatrix}$

$$A+B=\begin{bmatrix}-3+(-3)&4+(-4)&-5+5\\2+1&3+2&1+3\end{bmatrix}$$

$$A+B=\begin{bmatrix}-6&0&0\\3&5&4\end{bmatrix}\dots\dots\dots\dots(1)$$

RHS $B+A=\begin{bmatrix}-3&-4&5\\1&2&3\end{bmatrix}+\begin{bmatrix}-3&4&-5\\2&3&1\end{bmatrix}$

$$B+A=\begin{bmatrix}-3+(-3)&-4+4&5+(-5)\\1+2&2+3&3+1\end{bmatrix}$$

$$B+A=\begin{bmatrix}-6&0&0\\3&5&4\end{bmatrix}\dots\dots\dots\dots(2)$$

From eq (1) and eq(2)

LHS=RHS Hence proved

Q12:i). Verify that $A+(B+C)=(A+B)+C$

where $A=\begin{bmatrix}2&-3\\4&1\end{bmatrix},B=\begin{bmatrix}5&-2\\3&6\end{bmatrix},C=\begin{bmatrix}1&7\\-6&-3\end{bmatrix}$

Solution: LHS

$$A+(B+C)=\begin{bmatrix}2&-3\\4&1\end{bmatrix}+\left(\begin{bmatrix}5&-2\\3&6\end{bmatrix}+\begin{bmatrix}1&7\\-6&-3\end{bmatrix}\right)$$

$$A+(B+C)=\begin{bmatrix}2&-3\\4&1\end{bmatrix}+\begin{bmatrix}5+1&-2+7\\3+(-6)&6+(-3)\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}2&-3\\4&1\end{bmatrix}+\begin{bmatrix}6&5\\-3&3\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}2+6&-3+5\\4+(-3)&1+3\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}8&2\\1&4\end{bmatrix}\text{.............................(1)}$$

RHS $(A+B)+C=\left(\begin{bmatrix}2&-3\\4&1\end{bmatrix}+\begin{bmatrix}5&-2\\3&6\end{bmatrix}\right)+\begin{bmatrix}1&7\\-6&-3\end{bmatrix}$

$$(A+B)+C=\begin{bmatrix}2+5&-3+(-2)\\4+3&1+6\end{bmatrix}+\begin{bmatrix}1&7\\-6&-3\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}7&-5\\7&7\end{bmatrix}+\begin{bmatrix}1&7\\-6&-3\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}7+1&-5+7\\7+(-6)&7+(-3)\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}8&2\\1&4\end{bmatrix}\text{.............................(2)}$$

From eq (1) and eq(2)

LHS=RHS Hence proved

Q12:ii). Verify that $A+(B+C)=(A+B)+C$

where $A=\begin{bmatrix}a&b&c\\3&4&5\end{bmatrix},B=\begin{bmatrix}1&2&3\\-2&1&4\end{bmatrix},C=\begin{bmatrix}2&1&-1\\3&1&-2\end{bmatrix}$

Sol: LHS $A+(B+C)=\begin{bmatrix}a&b&c\\3&4&5\end{bmatrix}+\left(\begin{bmatrix}1&2&3\\-2&1&4\end{bmatrix}+\begin{bmatrix}2&1&-1\\3&1&-2\end{bmatrix}\right)$

$$A+(B+C)=\begin{bmatrix}a&b&c\\3&4&5\end{bmatrix}+\begin{bmatrix}1+2&2+1&3+(-1)\\-2+3&1+1&4+(-2)\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}a&b&c\\3&4&5\end{bmatrix}+\begin{bmatrix}3&3&2\\1&2&2\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}a+3&b+3&c+2\\3+1&4+2&5+2\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}a+3&b+3&c+2\\4&6&7\end{bmatrix}\text{...........(1)}$$

RHS $(A+B)+C=\left(\begin{bmatrix}a&b&c\\3&4&5\end{bmatrix}+\begin{bmatrix}1&2&3\\-2&1&4\end{bmatrix}\right)+\begin{bmatrix}2&1&-1\\3&1&-2\end{bmatrix}$

$$(A+B)+C=\begin{bmatrix}a+1&b+2&c+3\\3+(-2)&4+1&5+4\end{bmatrix}+\begin{bmatrix}2&1&-1\\3&1&-2\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}a+1&b+2&c+3\\1&5&9\end{bmatrix}+\begin{bmatrix}2&1&-1\\3&1&-2\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}a+1+2&b+2+1&c+3+(-1)\\1+3&5+1&9+(-2)\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}a+3&b+3&c+2\\4&6&7\end{bmatrix}\text{..........(2)}$$

From eq (1) and eq (2)

LHS=RHS Hence proved

Q13i). Find additive inverse of $A=\begin{bmatrix}3&4\\6&2\end{bmatrix}$

Solution: Suppose that B is the additive inverse of A then by definition of additive inverse

$A+B=0 \quad \Rightarrow B=-A$

Then $-A=-\begin{bmatrix}3&4\\6&2\end{bmatrix}=\begin{bmatrix}-3&-4\\-6&-2\end{bmatrix}$

Q13ii). Find additive inverse of $B=\begin{bmatrix}a&-a&b\\-c&a&-b\\l&m&n\end{bmatrix}$

Solution: Suppose that A is the additive inverse of B then by definition of additive inverse

$A+B=0 \quad \Rightarrow A=-B$

Then $-B=-\begin{bmatrix}a&-a&b\\-c&a&-b\\l&m&n\end{bmatrix}=\begin{bmatrix}-a&a&-b\\c&-a&b\\-l&-m&-n\end{bmatrix}$

Q14:i). Show that

$A=\begin{bmatrix}1&-2&3\end{bmatrix},B=\begin{bmatrix}-1&2&-3\end{bmatrix}$ are additive inverse of each other.

Solution: $A+B=\begin{bmatrix}1&-2&3\end{bmatrix}+\begin{bmatrix}-1&2&-3\end{bmatrix}$

$$A+B=\begin{bmatrix}1+(-1)&-2+2&3+(-3)\end{bmatrix}$$

$$A+B=\begin{bmatrix}0&0&0\end{bmatrix}=O$$

Similarly $B+A=\begin{bmatrix}-1&2&-3\end{bmatrix}+\begin{bmatrix}1&-2&3\end{bmatrix}$

$$B+A=\begin{bmatrix}-1+1&2+(-2)&-3+3\end{bmatrix}$$

$$B+A=\begin{bmatrix}0&0&0\end{bmatrix}=O$$

Hence By definition A and B are additive inverse of each other. i.e., $A+B=B+A=O$

Q14:ii). Show that $C=\begin{bmatrix}a&-b\\-c&d\end{bmatrix},D=\begin{bmatrix}-a&b\\c&-d\end{bmatrix}$ are additive inverse of each other.

Solution: $C+D=\begin{bmatrix}a&-b\\-c&d\end{bmatrix}+\begin{bmatrix}-a&b\\c&-d\end{bmatrix}$

$$C+D=\begin{bmatrix}a+(-a)&-b+b\\-c+c&d+(-d)\end{bmatrix}$$

$$C+D=\begin{bmatrix}0&0\\0&0\end{bmatrix}=O$$

Similarly $D+C=\begin{bmatrix}-a&b\\c&-d\end{bmatrix}+\begin{bmatrix}a&-b\\-c&d\end{bmatrix}$

$$D+C=\begin{bmatrix}-a+a&b+(-b)\\c+(-c)&-d+d\end{bmatrix}$$

$$D+C=\begin{bmatrix}0&0\\0&0\end{bmatrix}=O$$

Hence By definition C and D are additive inverse of each other. i.e., $C+D=D+C=O$

Q14:iii). Show that

$E=\begin{bmatrix}1&-2&-4\\2&1&3\\-3&4&-2\end{bmatrix},F=\begin{bmatrix}-1&2&4\\-2&-1&-3\\3&-4&2\end{bmatrix}$ are additive inverse of each other.

Sol: $E+F=\begin{bmatrix}1 & -2 & -4\\ 2 & 1 & 3\\ -3 & 4 & -2\end{bmatrix}+\begin{bmatrix}-1 & 2 & 4\\ -2 & -1 & -3\\ 3 & -4 & 2\end{bmatrix}$

$$E+F=\begin{bmatrix}1+(-1) & -2+2 & -4+4\\ 2+(-2) & 1+(-1) & 3+(-3)\\ -3+3 & 4+(-4) & -2+2\end{bmatrix}$$

$$E+F=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=O$$

Similarly $F+E=\begin{bmatrix}-1 & 2 & 4\\ -2 & -1 & -3\\ 3 & -4 & 2\end{bmatrix}+\begin{bmatrix}1 & -2 & -4\\ 2 & 1 & 3\\ -3 & 4 & -2\end{bmatrix}$

$$F+E=\begin{bmatrix}-1+1 & 2+(-2) & 4+(-4)\\ -2+2 & -1+1 & -3+3\\ 3+(3) & -4+4 & 2+(-2)\end{bmatrix}$$

$$F+E=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=O$$

Hence By definition E and F are additive inverse of each other. i.e., $E+F=F+E=O$

Multiplication of Matrices:To multiply these two matrices, we start with first row of the matrix A and multiply its each element with the corresponding elements of the first column of the matrix B and add the products.

Note that

1. The product of the matrices A and B is possible only when the numbers of columns of a matrix A is equal to the numbers of the rows of the matrix B.
2. The number of rows in the product AB is equal to the number of rows in the matrix A and the number of the columns in matrix B.
3. Product of A and B is written as $A\times B$ or simply AB.
4. In general matrices do not posses commutative property of multiplication. i.e. $AB\neq BA$

Example 9:i) If $A=\begin{bmatrix}2 & 3\\ 1 & 4\end{bmatrix}, B=\begin{bmatrix}3\\ 5\end{bmatrix}$,then Is it possible to find AB and BA

Solution: The number of columns of A= The numbers of rows of B, So the product AB is conformable for multiplication.

Similarly, The number of columns of B $\neq$ The numbers of rows of A, So the product BA is not conformable for multiplication.

Example 9:i) If $A=\begin{bmatrix}2 & 3\\ 1 & 4\end{bmatrix}, B=\begin{bmatrix}3\\ 5\end{bmatrix}$,then find possible product.

Sol: Given $A=\begin{bmatrix}2 & 3\\ 1 & 4\end{bmatrix}, B=\begin{bmatrix}3\\ 5\end{bmatrix}$ Now

$$AB=\begin{bmatrix}2 & 3\\ 1 & 4\end{bmatrix}\begin{bmatrix}3\\ 5\end{bmatrix}=\begin{bmatrix}2\times3+3\times5\\ 1\times3+4\times5\end{bmatrix}$$

$$AB=\begin{bmatrix}6+15\\ 3+20\end{bmatrix}=\begin{bmatrix}21\\ 23\end{bmatrix}$$

Note That Commutative law of multiplication does not hold

Example 10: Let $A=\begin{bmatrix}6 & 3\\ 2 & 5\end{bmatrix}$ $B=\begin{bmatrix}-3 & 2\\ 1 & 5\end{bmatrix}$

Determine whether $AB=BA$

Solution: Given $A=\begin{bmatrix}6 & 3\\ 2 & 5\end{bmatrix}$ $B=\begin{bmatrix}-3 & 2\\ 1 & 5\end{bmatrix}$

$$AB=\begin{bmatrix}6 & 3\\ 2 & 5\end{bmatrix}\begin{bmatrix}-3 & 2\\ 1 & 5\end{bmatrix}$$

$$AB=\begin{bmatrix}(6)(-3)+(3)(1) & (6)(2)+(3)(5)\\ (2)(-3)+(5)(1) & (2)(2)+(5)(5)\end{bmatrix}$$

$$AB=\begin{bmatrix}-18+3 & 12+15\\ -6+5 & 4+25\end{bmatrix}$$

$$AB=\begin{bmatrix}-15 & 27\\ -1 & 29\end{bmatrix}\text{....................}(1)$$

Now $BA=\begin{bmatrix}-3 & 2\\ 1 & 5\end{bmatrix}\begin{bmatrix}6 & 3\\ 2 & 5\end{bmatrix}$

$$BA=\begin{bmatrix}(-3)(6)+(2)(2) & (-3)(3)+(2)(5)\\ (1)(6)+(5)(2) & (1)(3)+(5)(5)\end{bmatrix}$$

$$BA=\begin{bmatrix}-18+4 & -9+10\\ 6+10 & 3+25\end{bmatrix}$$

$$BA=\begin{bmatrix}-14 & 1\\ 16 & 28\end{bmatrix}\text{...........................}(2)$$

Form eq (1) and eq (2) we get $AB\neq BA$

Example 11: Let $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ & $B=\begin{bmatrix}2 & 2\\ 3 & 5\end{bmatrix}$

Show that $AB=BA$

Solution: Given $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ & $B=\begin{bmatrix}2 & 2\\ 3 & 5\end{bmatrix}$

Taking LHS $AB=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}2 & 2\\ 3 & 5\end{bmatrix}$

$$AB=\begin{bmatrix}(1)(2)+(2)(3) & (1)(2)+(2)(5)\\ (3)(2)+(4)(3) & (3)(2)+(4)(5)\end{bmatrix}$$

$$AB=\begin{bmatrix}2+6 & 2+10\\ 6+12 & 6+20\end{bmatrix}$$

$$AB=\begin{bmatrix}8 & 12\\ 18 & 26\end{bmatrix}\text{..................}(1)$$

Now taking RHS $BA=\begin{bmatrix}2 & 2\\ 3 & 5\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$

$$BA=\begin{bmatrix}(2)(1)+(2)(3) & (2)(2)+(2)(4)\\ (3)(1)+(5)(3) & (3)(2)+(5)(4)\end{bmatrix}$$

$$BA=\begin{bmatrix}2+6 & 4+8\\ 3+15 & 6+20\end{bmatrix}$$

$$BA=\begin{bmatrix}8 & 12\\ 18 & 26\end{bmatrix}\ldots\ldots\ldots\ldots\ldots(2)$$

Form eq (1) and eq (2) we get $AB=BA$

Associative Law under multiplication If A ,B and C are any three Matrices are conformable for multiplication then property $(AB)C=A(BC)$ is called associative law of matrices under multiplication.

Example 12: $A=\begin{bmatrix}1\\ 2\end{bmatrix},B=\begin{bmatrix}3 & 2\end{bmatrix},C=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$

then verify that $A(BC)=(AB)C$

Solution; LHS $A(BC)=\begin{bmatrix}1\\ 2\end{bmatrix}\left(\begin{bmatrix}3 & 2\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\right)$

$$A(BC)=\begin{bmatrix}1\\ 2\end{bmatrix}\begin{bmatrix}(3)(1)+(2)(3) & (3)(2)+(2)(4)\end{bmatrix}$$

$$A(BC)=\begin{bmatrix}1\\ 2\end{bmatrix}\begin{bmatrix}3+6 & 6+8\end{bmatrix}$$

$$A(BC)=\begin{bmatrix}1\\ 2\end{bmatrix}\begin{bmatrix}9 & 14\end{bmatrix}=\begin{bmatrix}1\times 9 & 1\times 14\\ 2\times 9 & 2\times 14\end{bmatrix}$$

$$A(BC)=\begin{bmatrix}9 & 14\\ 18 & 28\end{bmatrix}$$

Now RHS $(AB)C=\left(\begin{bmatrix}1\\ 2\end{bmatrix}\begin{bmatrix}3 & 2\end{bmatrix}\right)\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$

$$(AB)C=\begin{bmatrix}1\times 3 & 1\times 2\\ 2\times 3 & 2\times 2\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\cdots(1)$$

$$(AB)C=\begin{bmatrix}3 & 2\\ 6 & 4\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}=\begin{bmatrix}3\times 1+2\times 3 & 3\times 2+2\times 4\\ 6\times 1+4\times 3 & 6\times 2+4\times 4\end{bmatrix}$$

$$(AB)C=\begin{bmatrix}3+6 & 6+8\\ 6+12 & 12+16\end{bmatrix}=\begin{bmatrix}9 & 14\\ 18 & 28\end{bmatrix}\ldots\ldots\ldots\ldots(2)$$

From equation (1) and (2) we get

$$A(BC)=(AB)C$$

Distributive Law of Multiplication over Addition

If A ,B and C are any three Matrices, then

$$A(B+C)=AB+AC$$

$$(A+B)C=AC+BC$$

Example 13: If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix},B=\begin{bmatrix}5 & 3\\ 2 & 4\end{bmatrix}C=\begin{bmatrix}6 & 2\\ 5 & 1\end{bmatrix}$

Verify the distributive law of multiplication over addition.

Sol: i). first we verify that $A(B+C)=AB+AC$

Take LHS $A(B+C)=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\left(\begin{bmatrix}5 & 3\\ 2 & 4\end{bmatrix}+\begin{bmatrix}6 & 2\\ 5 & 1\end{bmatrix}\right)$

$$A(B+C)=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}5+6 & 3+2\\ 2+5 & 4+1\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}11 & 5\\ 7 & 5\end{bmatrix}=\begin{bmatrix}1\times 11+2\times 7 & 1\times 5+2\times 5\\ 3\times 11+4\times 7 & 3\times 5+4\times 5\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}11+14 & 5+10\\ 33+28 & 15+20\end{bmatrix}=\begin{bmatrix}25 & 15\\ 61 & 35\end{bmatrix}\cdots(1)$$

RHS $AB+AC=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & 3\\ 2 & 4\end{bmatrix}+\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}6 & 2\\ 5 & 1\end{bmatrix}$

$$=\begin{bmatrix}1\times 5+2\times 2 & 1\times 3+2\times 4\\ 3\times 5+4\times 2 & 3\times 3+4\times 4\end{bmatrix}+\begin{bmatrix}1\times 6+2\times 5 & 1\times 2+2\times 1\\ 3\times 6+4\times 5 & 3\times 2+4\times 1\end{bmatrix}$$

$$=\begin{bmatrix}5+4 & 3+8\\ 15+8 & 9+16\end{bmatrix}+\begin{bmatrix}6+10 & 2+2\\ 18+20 & 6+4\end{bmatrix}$$

$$=\begin{bmatrix}9 & 11\\ 23 & 25\end{bmatrix}+\begin{bmatrix}16 & 4\\ 38 & 10\end{bmatrix}=\begin{bmatrix}9+16 & 11+4\\ 23+38 & 25+10\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}25 & 15\\ 61 & 35\end{bmatrix}\cdots(2)$$

From eq (1) & eq (2) we get $A(B+C)=AB+AC$

Multiplicative identity of a matrix

Let Givena matrix I and a matrix A. so $AI=IA=A$

Example 14: Let $A=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix},I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Then we see that $IA=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}$

$$IA=\begin{bmatrix}(1)(1)+(0)(4) & (1)(2)+(0)(5) & (1)(3)+(0)(6)\\ (0)(1)+(1)(4) & (0)(2)+(1)(5) & (0)(3)+(1)(6)\end{bmatrix}$$

$$IA=\begin{bmatrix}1+0 & 2+0 & 3+0\\ 0+4 & 0+5 & 0+6\end{bmatrix}$$

$IA=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=A$ But AI is not defined because number of columns in A $\neq$ number of rows in I

Example 15 ; if $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},A=\begin{bmatrix}9 & -3\\ -4 & 5\end{bmatrix}$

Solution; $IA=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}9 & -3\\ -4 & 5\end{bmatrix}$

$$IA=\begin{bmatrix}1\times 9+0\times(-4) & 1\times(-3)+0\times 5\\ 0\times 9+1\times(-4) & 0\times(-3)+1\times 5\end{bmatrix}$$

$$IA=\begin{bmatrix}9 & -3\\ -4 & 5\end{bmatrix}\cdots(1)$$

Now $AI=\begin{bmatrix}9 & -3\\ -4 & 5\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$$AI=\begin{bmatrix}9\times 1+(-3)\times 0 & 9\times 0+(-3)\times 1\\ -4\times 1+5\times 0 & -4\times 0+5\times 1\end{bmatrix}$$

$$AI=\begin{bmatrix}9 & -3\\ -4 & 5\end{bmatrix}\cdots(2)$$

From equation (1) and (2) we get $IA=AI=A$

Transpose of a matrix A matrix which is obtained by interchanging all the rows and columns of given matrix is called its transpose and it is denoted by A^t

Example 15: if $A=\begin{bmatrix}3 & 4 & 5\\ 2 & 4 & 6\end{bmatrix}$ then $A^t=\begin{bmatrix}3 & 2\\ 4 & 4\\ 5 & 6\end{bmatrix}$

Exp 16: if $A=\begin{bmatrix}3 & -2\\ 1 & 4\end{bmatrix},B=\begin{bmatrix}2 & -5\\ 6 & -7\end{bmatrix}$ Show tha $(AB)^t=B^tA^t$

Solution: $A=\begin{bmatrix}3 & -2\\ 1 & 4\end{bmatrix}\Rightarrow A^t=\begin{bmatrix}3 & 1\\ -2 & 4\end{bmatrix}$

$$B=\begin{bmatrix}2&-5\\6&-7\end{bmatrix}\quad\Rightarrow B^t=\begin{bmatrix}2&6\\-5&-7\end{bmatrix}$$

Take LHS $(AB)^t=\left(\begin{bmatrix}3&-2\\1&4\end{bmatrix}\begin{bmatrix}2&-5\\6&-7\end{bmatrix}\right)^t$

$$(AB)^t=\begin{bmatrix}(3)(2)+(-2)(6)&(3)(-5)+(-2)(-7)\\(1)(2)+(4)(6)&(1)(-5)+(4)(-7)\end{bmatrix}^t$$

$$(AB)^t=\begin{bmatrix}6-12&-15+14\\2+24&-5-28\end{bmatrix}^t$$

$$(AB)^t=\begin{bmatrix}-6&-1\\26&-33\end{bmatrix}^t=\begin{bmatrix}-6&26\\-1&-33\end{bmatrix}\cdots(1)$$

Now RHS $B^tA^t=\begin{bmatrix}2&6\\-5&-7\end{bmatrix}\begin{bmatrix}3&1\\-2&4\end{bmatrix}$

$$B^tA^t=\begin{bmatrix}(2)(3)+(6)(-2)&(2)(1)+(6)(4)\\(-5)(3)+(-7)(-2)&(-5)(1)+(-7)(4)\end{bmatrix}$$

$$B^tA^t=\begin{bmatrix}6-12&2+24\\-15+14&-5-28\end{bmatrix}=\begin{bmatrix}-6&26\\-1&-33\end{bmatrix}\cdots(2)$$

From equation (1) and (2) we get $(AB)^t=B^tA^t$

Exercise 1.4

Q1: Show that which of the following matrices are conformable for multiplication.

$$A=\begin{bmatrix}a\\b\end{bmatrix},B=\begin{bmatrix}p&q\end{bmatrix},C=\begin{bmatrix}1&-1\\-2&1\end{bmatrix},D=\begin{bmatrix}p&r&s\end{bmatrix}$$

Solution: i): The number of columns of A is 1= The numbers of rows of B is 1, So the product AB is conformable for multiplication.
ii): The number of columns of A is 1= The numbers of rows of D is 1,
So the product AD is conformable for multiplication.
iii): The number of columns of B is 2= The numbers of rows of C is 2, So the product BC is conformable for multiplication.
iv): The number of columns of C is 2= The numbers of rows of A is 2, So the product CA is conformable for multiplication.

Q2.i) If $A=\begin{bmatrix}-1&0\\2&1\end{bmatrix},B=\begin{bmatrix}3\\-2\end{bmatrix}$ Is it possible to find AB

Sol: i). The number of columns of A is 2= The numbers of rows of B is 2,
So the product AB is conformable for multiplication.

Q2.ii) If $A=\begin{bmatrix}-1&0\\2&1\end{bmatrix},B=\begin{bmatrix}3\\-2\end{bmatrix}$ Is it possible to find BA

Solution: The number of columns of B is 1 $\neq$ The numbers of rows of A is 2,
So the product BA is not conformable for multiplication.

Q2.iii) If $A=\begin{bmatrix}-1&0\\2&1\end{bmatrix},B=\begin{bmatrix}3\\-2\end{bmatrix}$ Find possible product

Solution: iii). $AB=\begin{bmatrix}-1&0\\2&1\end{bmatrix}\begin{bmatrix}3\\-2\end{bmatrix}$

$$AB=\begin{bmatrix}(-1)(3)+(0)(-2)\\(2)(3)+(1)(-2)\end{bmatrix}=\begin{bmatrix}-3+0\\6-2\end{bmatrix}=\begin{bmatrix}-3\\4\end{bmatrix}$$

Q3.i) Given that $A=\begin{bmatrix}4&1\\3&1\end{bmatrix},B=\begin{bmatrix}1&-1\\-3&4\end{bmatrix}$ Find AB

Solution; $AB=\begin{bmatrix}4&1\\3&1\end{bmatrix}\begin{bmatrix}1&-1\\-3&4\end{bmatrix}$

$$AB=\begin{bmatrix}(4)(1)+(1)(-3)&(4)(-1)+(1)(4)\\(3)(1)+(1)(-3)&(3)(-1)+(1)(4)\end{bmatrix}$$

$$AB=\begin{bmatrix}4-3&-4+4\\3-3&-3+4\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

Q3: ii). Given $C=\begin{bmatrix}3&4\\1&2\end{bmatrix},D=\begin{bmatrix}1&-2\\\frac{-1}{2}&\frac{2}{3}\end{bmatrix}$ Find CD

Solution; $CD=\begin{bmatrix}3&4\\1&2\end{bmatrix}\begin{bmatrix}1&-2\\\frac{-1}{2}&\frac{2}{3}\end{bmatrix}$

$$CD=\begin{bmatrix}(3)(1)+(4)(\frac{-1}{2})&(3)(-2)+(4)(\frac{2}{3})\\(1)(1)+(2)(\frac{-1}{2})&(1)(-2)+(2)(\frac{2}{3})\end{bmatrix}$$

$$CD=\begin{bmatrix}3-2&-6+\frac{8}{3}\\1-1&-2+\frac{4}{3}\end{bmatrix}=\begin{bmatrix}1&\frac{-10}{3}\\0&\frac{-2}{3}\end{bmatrix}$$

Q4:i). Let $A=\begin{bmatrix}2&1\\3&0\\-1&4\end{bmatrix}$ and $B=\begin{bmatrix}1&0\\2&1\end{bmatrix}$ Find AB

Solution: Given $A=\begin{bmatrix}2&1\\3&0\\-1&4\end{bmatrix}$ and $B=\begin{bmatrix}1&0\\2&1\end{bmatrix}$

Now $AB=\begin{bmatrix}2&1\\3&0\\-1&4\end{bmatrix}\begin{bmatrix}1&0\\2&1\end{bmatrix}$

$$AB=\begin{bmatrix}(2)(1)+(1)(2)&(2)(0)+(1)(1)\\(3)(1)+(0)(2)&(3)(0)+(0)(1)\\(-1)(1)+(4)(2)&(-1)(0)+(4)(1)\end{bmatrix}$$

$$AB=\begin{bmatrix}2+2&0+1\\3+0&0+0\\-1+8&-0+4\end{bmatrix}$$

$$AB=\begin{bmatrix}4&1\\3&0\\7&4\end{bmatrix}$$

Q4: ii). Let $A=\begin{bmatrix}2&1\\3&0\\-1&4\end{bmatrix}$ & $B=\begin{bmatrix}1&0\\2&1\end{bmatrix}$ Does BA exists?

Solution: Given $A=\begin{bmatrix}2&1\\3&0\\-1&4\end{bmatrix}$ and $B=\begin{bmatrix}1&0\\2&1\end{bmatrix}$

Number of columns of B $\neq$ number of rows of A
Therefore the product BA is not possible

Q5.If $A=\begin{bmatrix}1&1\\0&0\end{bmatrix},B=\begin{bmatrix}0&-1\\0&0\end{bmatrix}$, show that AB $\neq$ BA

Solution: Given $A=\begin{bmatrix}1&1\\0&0\end{bmatrix}, B=\begin{bmatrix}0&-1\\0&0\end{bmatrix}$

Now $AB=\begin{bmatrix}1&1\\0&0\end{bmatrix}\begin{bmatrix}0&-1\\0&0\end{bmatrix}$

$AB=\begin{bmatrix}(1)(0)+(1)(0)&(1)(-1)+(1)(0)\\(0)(0)+(0)(0)&(0)(-1)+(0)(0)\end{bmatrix}$

$AB=\begin{bmatrix}0+0&-1+0\\0+0&0+0\end{bmatrix}=\begin{bmatrix}0&-1\\0&0\end{bmatrix}$............(1)

Now $BA=\begin{bmatrix}0&-1\\0&0\end{bmatrix}\begin{bmatrix}1&1\\0&0\end{bmatrix}$

$BA=\begin{bmatrix}(0)(1)+(-1)(0)&(0)(1)+(-1)(0)\\(0)(1)+(0)(0)&(0)(1)+(0)(0)\end{bmatrix}$

$BA=\begin{bmatrix}0-0&0-0\\0+0&0+0\end{bmatrix}$

$BA=\begin{bmatrix}0&0\\0&0\end{bmatrix}$...................(2)

Form eq (1) and (2) we get AB ≠ BA

Q6. If $A=\begin{bmatrix}1&1\\0&0\end{bmatrix}$, then find AxA

Solution; Given $A=\begin{bmatrix}1&1\\0&0\end{bmatrix}$

Now $A\times A=\begin{bmatrix}1&1\\0&0\end{bmatrix}\begin{bmatrix}1&1\\0&0\end{bmatrix}$

$A\times A=\begin{bmatrix}(1)(1)+(1)(0)&(1)(1)+(1)(0)\\(0)(1)+(0)(0)&(0)(1)+(0)(0)\end{bmatrix}$

$A\times A=\begin{bmatrix}1+0&1+0\\0+0&0+0\end{bmatrix}$

$A\times A=\begin{bmatrix}1&1\\0&0\end{bmatrix}$

Q7. If $A=\begin{bmatrix}-2&3\\2&-1\end{bmatrix}, B=\begin{bmatrix}1&-1\\2&4\end{bmatrix}$, is $AB=BA$

Solution: Given $A=\begin{bmatrix}-2&3\\2&-1\end{bmatrix}, B=\begin{bmatrix}1&-1\\2&4\end{bmatrix}$

Now $AB=\begin{bmatrix}-2&3\\2&-1\end{bmatrix}\begin{bmatrix}1&-1\\2&4\end{bmatrix}$

$AB=\begin{bmatrix}(-2)(1)+(3)(2)&(-2)(-1)+(3)(4)\\(2)(1)+(-1)(2)&(2)(-1)+(-1)(4)\end{bmatrix}$

$AB=\begin{bmatrix}-2+6&2+12\\2-2&-2-4\end{bmatrix}=\begin{bmatrix}4&14\\0&-6\end{bmatrix}\cdots(1)$

Now $BA=\begin{bmatrix}1&-1\\2&4\end{bmatrix}\begin{bmatrix}-2&3\\2&-1\end{bmatrix}$

$BA=\begin{bmatrix}(1)(-2)+(-1)(2)&(1)(3)+(-1)(-1)\\(2)(-2)+(4)(2)&(2)(3)+(4)(-1)\end{bmatrix}$

$BA=\begin{bmatrix}-2-2&3+1\\-4+8&6-4\end{bmatrix}=\begin{bmatrix}-4&4\\4&2\end{bmatrix}\cdots(2)$

From equation (1) and (2) we get $AB\neq BA$

Q8.i). If $A=\begin{bmatrix}-1\\1\end{bmatrix}, B=\begin{bmatrix}2&-2\end{bmatrix}, C=\begin{bmatrix}3&1\\-1&2\end{bmatrix}$, then find (AB)C & A(BC)

Sol: Given $A=\begin{bmatrix}-1\\1\end{bmatrix}, B=\begin{bmatrix}2&-2\end{bmatrix}, C=\begin{bmatrix}3&1\\-1&2\end{bmatrix}$

Take $(AB)C=\left(\begin{bmatrix}-1\\1\end{bmatrix}\begin{bmatrix}2&-2\end{bmatrix}\right)\begin{bmatrix}3&1\\-1&2\end{bmatrix}$

$(AB)C=\begin{bmatrix}(-1)(2)&(-1)(-2)\\(1)(2)&(1)(-2)\end{bmatrix}\begin{bmatrix}3&1\\-1&2\end{bmatrix}$

$(AB)C=\begin{bmatrix}-2&2\\2&-2\end{bmatrix}\begin{bmatrix}3&1\\-1&2\end{bmatrix}$

$(AB)C=\begin{bmatrix}(-2)(3)+(2)(-1)&(-2)(1)+(2)(2)\\(2)(3)+(-2)(-1)&(2)(1)+(-2)(2)\end{bmatrix}$

$(AB)C=\begin{bmatrix}-6-2&-2+4\\6+2&2-4\end{bmatrix}$

$(AB)C=\begin{bmatrix}-8&2\\8&-2\end{bmatrix}\cdots(1)$

Now Take $A(BC)=\begin{bmatrix}-1\\1\end{bmatrix}\left(\begin{bmatrix}2&-2\end{bmatrix}\begin{bmatrix}3&1\\-1&2\end{bmatrix}\right)$

$A(BC)=\begin{bmatrix}-1\\1\end{bmatrix}\begin{bmatrix}(2)(3)+(-2)(-1)&(2)(1)+(-2)(2)\end{bmatrix}$

$A(BC)=\begin{bmatrix}-1\\1\end{bmatrix}\begin{bmatrix}6+2&2-4\end{bmatrix}$

$A(BC)=\begin{bmatrix}-1\\1\end{bmatrix}\begin{bmatrix}8&-2\end{bmatrix}$

$A(BC)=\begin{bmatrix}(-1)(8)&(-1)(-2)\\(1)(8)&(1)(-2)\end{bmatrix}$

$A(BC)=\begin{bmatrix}-8&2\\8&-2\end{bmatrix}\cdots(2)$

Q8:ii). If $A=\begin{bmatrix}-1\\1\end{bmatrix}, B=\begin{bmatrix}2&-2\end{bmatrix}, C=\begin{bmatrix}3&1\\-1&2\end{bmatrix}$, then Determine whether $(AB)C=A(BC)$

Solution: From equation (1) and (2) we get

$(AB)C=A(BC)=\begin{bmatrix}-8&2\\8&-2\end{bmatrix}$

Q8iii). Interpret which law of multiplication this result shows

Answer: Associative Law of Multiplication

Q9:i). Verify that $A(B+C)=AB+AC$ where

$A=\begin{bmatrix}1&-2\\3&-1\end{bmatrix}, B=\begin{bmatrix}1&0\\0&2\end{bmatrix}, C=\begin{bmatrix}3&-1\\0&2\end{bmatrix}$

Sol: $A=\begin{bmatrix}1&-2\\3&-1\end{bmatrix}, B=\begin{bmatrix}1&0\\0&2\end{bmatrix}, C=\begin{bmatrix}3&-1\\0&2\end{bmatrix}$

we verify that $A(B+C)=AB+AC$

Take LHS $A(B+C)=\begin{bmatrix}1&-2\\3&-1\end{bmatrix}\left(\begin{bmatrix}1&0\\0&2\end{bmatrix}+\begin{bmatrix}3&-1\\0&2\end{bmatrix}\right)$

$A(B+C)=\begin{bmatrix}1&-2\\3&-1\end{bmatrix}\begin{bmatrix}1+3&0+(-1)\\0+0&2+2\end{bmatrix}$

$$A(B+C)=\begin{bmatrix}1 & -2\\ 3 & -1\end{bmatrix}\begin{bmatrix}4 & -1\\ 0 & 4\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}(1)(4)+(-2)(0) & (1)(-1)+(-2)(4)\\ (3)(4)+(-1)(0) & (3)(-1)+(-1)(4)\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}4+0 & -1-8\\ 12+0 & -3-4\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}4 & -9\\ 12 & -7\end{bmatrix}\cdots(1)$$

Now RHS $AB+AC=\begin{bmatrix}1 & -2\\ 3 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}+\begin{bmatrix}1 & -2\\ 3 & -1\end{bmatrix}\begin{bmatrix}3 & -1\\ 0 & 2\end{bmatrix}$

$$=\begin{bmatrix}(1)(1)+(-2)(0) & (1)(0)+(-2)(2)\\ (3)(1)+(-1)(0) & (3)(0)+(-1)(2)\end{bmatrix}+\begin{bmatrix}(1)(3)+(-2)(0) & (1)(-1)+(-2)(2)\\ (3)(3)+(-1)(0) & (3)(-1)+(-1)(2)\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}1+0 & 0-4\\ 3+0 & 0-2\end{bmatrix}+\begin{bmatrix}3+0 & -1-4\\ 9+0 & -3-2\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}1 & -4\\ 3 & -2\end{bmatrix}+\begin{bmatrix}3 & -5\\ 9 & -5\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}1+3 & -4-5\\ 3+9 & -2-5\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}4 & -9\\ 12 & -7\end{bmatrix}\cdots(2)$$

From equation (1) and (2) $A(B+C)=AB+AC$

Q9:ii). Verify that $A(B+C)=AB+AC$ where

$A=\begin{bmatrix}3 & -1\\ 0 & 2\end{bmatrix},B=\begin{bmatrix}1\\ 2\end{bmatrix},C=\begin{bmatrix}-1\\ 1\end{bmatrix}$

Solution:since $A=\begin{bmatrix}3 & -1\\ 0 & 2\end{bmatrix},B=\begin{bmatrix}1\\ 2\end{bmatrix},C=\begin{bmatrix}-1\\ 1\end{bmatrix}$

we verify that $A(B+C)=AB+AC$

Take LHS $A(B+C)=\begin{bmatrix}3 & -1\\ 0 & 2\end{bmatrix}\left(\begin{bmatrix}1\\ 2\end{bmatrix}+\begin{bmatrix}-1\\ 1\end{bmatrix}\right)$

$$A(B+C)=\begin{bmatrix}3 & -1\\ 0 & 2\end{bmatrix}\begin{bmatrix}1+(-1)\\ 2+1\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}3 & -1\\ 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 3\end{bmatrix}$$

$$=\begin{bmatrix}(3)(0)+(-1)(3)\\ (0)(0)+(2)(3)\end{bmatrix}=\begin{bmatrix}0-3\\ 0+6\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}-3\\ 6\end{bmatrix}\cdots(1)$$

RHS $AB+AC=\begin{bmatrix}3 & -1\\ 0 & 2\end{bmatrix}\begin{bmatrix}1\\ 2\end{bmatrix}+\begin{bmatrix}3 & -1\\ 0 & 2\end{bmatrix}\begin{bmatrix}-1\\ 1\end{bmatrix}$

$$=\begin{bmatrix}(3)(1)+(-1)(2)\\ (0)(1)+(2)(2)\end{bmatrix}+\begin{bmatrix}(3)(-1)+(-1)(1)\\ (0)(-1)+(2)(1)\end{bmatrix}$$

$$=\begin{bmatrix}3-2\\ 0+4\end{bmatrix}+\begin{bmatrix}-3-1\\ 0+2\end{bmatrix}=\begin{bmatrix}1\\ 4\end{bmatrix}+\begin{bmatrix}-4\\ 2\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}1+(-4)\\ 4+2\end{bmatrix}=\begin{bmatrix}-3\\ 6\end{bmatrix}\cdots(2)$$

From equation (1) and (2) $A(B+C)=AB+AC$

Q10: i). Let $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},A=\begin{bmatrix}5 & -3\\ 4 & 6\end{bmatrix}$ Find AI

Solution: Take $AI=\begin{bmatrix}5 & -3\\ 4 & 6\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$$AI=\begin{bmatrix}(5)(1)+(-3)(0) & (5)(0)+(-3)(1)\\ (4)(1)+(6)(0) & (4)(0)+(6)(1)\end{bmatrix}$$

$$AI=\begin{bmatrix}5+0 & 0-3\\ 4+0 & 0+6\end{bmatrix}=\begin{bmatrix}5 & -3\\ 4 & 6\end{bmatrix}$$

Q10: ii). Let $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $B=\begin{bmatrix}-7 & 3\\ 2 & 8\end{bmatrix}$ Find BI

Solution: $BI=\begin{bmatrix}-7 & 3\\ 2 & 8\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$$BI=\begin{bmatrix}(-7)(1)+(3)(0) & (-7)(0)+(3)(1)\\ (2)(1)+(8)(0) & (2)(0)+(8)(1)\end{bmatrix}$$

$$BI=\begin{bmatrix}-7+0 & 0+3\\ 2+0 & 0+8\end{bmatrix}=\begin{bmatrix}-7 & 3\\ 2 & 8\end{bmatrix}$$

Q11: i). Let $A=\begin{bmatrix}3 & 2 & 1\end{bmatrix},B=\begin{bmatrix}-3 & 4 & 2\end{bmatrix}$,

prove that $(A+B)^t=A^t+B^t$ & $(A-B)^t=A^t-B^t$

Solution; First we prove that $(A+B)^t=A^t+B^t$

Take LHS $(A+B)^t=\left(\begin{bmatrix}3 & 2 & 1\end{bmatrix}+\begin{bmatrix}-3 & 4 & 2\end{bmatrix}\right)^t$

$$=\begin{bmatrix}3+(-3) & 2+4 & 1+2\end{bmatrix}^t$$

$$=\begin{bmatrix}0 & 6 & 3\end{bmatrix}^t$$

$$(A+B)^t=\begin{bmatrix}0\\ 6\\ 3\end{bmatrix}\cdots(1)$$

Now RHS $A^t+B^t=\begin{bmatrix}3 & 2 & 1\end{bmatrix}^t+\begin{bmatrix}-3 & 4 & 2\end{bmatrix}^t$

$$A^t+B^t=\begin{bmatrix}3\\ 2\\ 1\end{bmatrix}+\begin{bmatrix}-3\\ 4\\ 2\end{bmatrix}=\begin{bmatrix}3+(-3)\\ 2+4\\ 1+2\end{bmatrix}$$

$$A^t+B^t=\begin{bmatrix}0\\ 6\\ 3\end{bmatrix}\cdots(2)$$

Form equations (1) and (2) $(A+B)^t=A^t+B^t$

Now we will prove that $(A-B)^t=A^t-B^t$

Take LHS $(A-B)^t=\left(\begin{bmatrix}3 & 2 & 1\end{bmatrix}-\begin{bmatrix}-3 & 4 & 2\end{bmatrix}\right)^t$

$$=\begin{bmatrix}3-(-3) & 2-4 & 1-2\end{bmatrix}^t$$

$$=\begin{bmatrix}6 & -2 & -1\end{bmatrix}^t$$

$$(A-B)^t=\begin{bmatrix}6\\ -2\\ -1\end{bmatrix}\text{.....................}(3)$$

Now RHS $A^t-B^t=\begin{bmatrix}3 & 2 & 1\end{bmatrix}^t-\begin{bmatrix}-3 & 4 & 2\end{bmatrix}^t$

$$A^t-B^t=\begin{bmatrix}3\\ 2\\ 1\end{bmatrix}-\begin{bmatrix}-3\\ 4\\ 2\end{bmatrix}=\begin{bmatrix}3-(-3)\\ 2-4\\ 1-2\end{bmatrix}$$

$$A^t - B^t = \begin{bmatrix} 6 \\ -2 \\ -1 \end{bmatrix} \text{.....................(4)}$$

Form equations (3) and (4) $(A-B)^t = A^t - B^t$

Q11: ii). $C = \begin{bmatrix} 7 & -3 \\ 2 & -1 \end{bmatrix}, D = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, prove that

$(C+D)^t = C^t + D^t$ and $(C-D)^t = C^t - D^t$

Solution; First we prove that $(C+D)^t = C^t + D^t$

Take LHS $(C+D)^t = \left(\begin{bmatrix} 7 & -3 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\right)^t$

$$(C+D)^t = \begin{bmatrix} 7+1 & -3+1 \\ 2+2 & -1+2 \end{bmatrix}^t$$

$$(C+D)^t = \begin{bmatrix} 8 & -2 \\ 4 & 1 \end{bmatrix}^t = \begin{bmatrix} 8 & 4 \\ -2 & 1 \end{bmatrix} \cdots (1)$$

Now RHS $C^t + D^t = \begin{bmatrix} 7 & -3 \\ 2 & -1 \end{bmatrix}^t + \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}^t$

$$C^t + D^t = \begin{bmatrix} 7 & 2 \\ -3 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

$$C^t + D^t = \begin{bmatrix} 7+1 & 2+2 \\ -3+1 & -1+2 \end{bmatrix}$$

$$C^t + D^t = \begin{bmatrix} 8 & 4 \\ -2 & 1 \end{bmatrix} \cdots (2)$$

Form equations (1) and (2) $(C+D)^t = C^t + D^t$

Now we will prove that $(C-D)^t = C^t - D^t$

Take LHS $(C-D)^t = \left(\begin{bmatrix} 7 & -3 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\right)^t$

$$(C+D)^t = \begin{bmatrix} 7-1 & -3-1 \\ 2-2 & -1-2 \end{bmatrix}^t$$

$$= \begin{bmatrix} 6 & -4 \\ 0 & -3 \end{bmatrix}^t$$

$$= \begin{bmatrix} 6 & 0 \\ -4 & -3 \end{bmatrix} \cdots (3)$$

Now RHS $C^t - D^t = \begin{bmatrix} 7 & -3 \\ 2 & -1 \end{bmatrix}^t - \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}^t$

$$C^t - D^t = \begin{bmatrix} 7 & 2 \\ -3 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

$$C^t - D^t = \begin{bmatrix} 7-1 & 2-2 \\ -3-1 & -1-2 \end{bmatrix}$$

$$C^t - D^t = \begin{bmatrix} 6 & 0 \\ -4 & -3 \end{bmatrix} \cdots (4)$$

Form equations (3) and (4) $(C-D)^t = C^t - D^t$

Q12. If $A = \begin{bmatrix} 2 & 5 \\ -3 & 4 \end{bmatrix}, B = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$, show $(AB)^t = B^t A^t$

Solution; LHS $(AB)^t = \left(\begin{bmatrix} 2 & 5 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}\right)^t$

$$= \left(\begin{bmatrix} (2)(-1)+(5)(2) & (2)(1)+(5)(3) \\ (-3)(-1)+(4)(2) & (-3)(1)+(4)(3) \end{bmatrix}\right)^t$$

$$(AB)^t = \begin{bmatrix} -2+10 & 2+15 \\ 3+8 & -3+12 \end{bmatrix}^t = \begin{bmatrix} 8 & 17 \\ 11 & 9 \end{bmatrix}^t$$

$$(AB)^t = \begin{bmatrix} 8 & 11 \\ 17 & 9 \end{bmatrix} \text{..................(1)}$$

Now Take RHS $B^t A^t = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}^t \begin{bmatrix} 2 & 5 \\ -3 & 4 \end{bmatrix}^t$

$$B^t A^t = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} (-1)(2)+(2)(5) & (-1)(-3)+(2)(4) \\ (1)(2)+(3)(5) & (1)(-3)+(3)(4) \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} -2+10 & 3+8 \\ 2+15 & -3+12 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} 8 & 11 \\ 17 & 9 \end{bmatrix} \cdots (2)$$

Form equations (1) and (2) we get $(AB)^t = B^t A^t$

Q12 ii). If $C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, show that $(C^t)^t = C$

Solution: Given $C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then

$$C^t = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^t = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

again taking transpose

$$(C^t)^t = \begin{bmatrix} a & c \\ b & d \end{bmatrix}^t = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$(C^t)^t = C$$

Q12 iii). If $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 6 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 7 \\ -8 & 4 \\ 0 & 1 \end{bmatrix}$

show that $(AB)^t = B^t A^t$

Solution: Given $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 6 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 7 \\ -8 & 4 \\ 0 & 1 \end{bmatrix}$

First we find $AB = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 6 \end{bmatrix}\begin{bmatrix} 1 & 7 \\ -8 & 4 \\ 0 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} (1)(1)+(0)(-8)+(-1)(0) & (1)(7)+(0)(4)+(-1)(1) \\ (2)(1)+(0)(-8)+(6)(0) & (2)(7)+(0)(4)+(6)(1) \end{bmatrix}$$

$$AB = \begin{bmatrix} 1-0-0 & 7+0-1 \\ 2-0+0 & 14+0+6 \end{bmatrix}$$

$$AB=\begin{bmatrix}1 & 6\\ 2 & 20\end{bmatrix}$$

Taking transpose on both sides

$$(AB)^t=\begin{bmatrix}1 & 6\\ 2 & 20\end{bmatrix}^t$$

$$(AB)^t=\begin{bmatrix}1 & 2\\ 6 & 20\end{bmatrix}\ldots\ldots\ldots\ldots\ldots(1)$$

Now $B^tA^t=\begin{bmatrix}1 & 7\\ -8 & 4\\ 0 & 1\end{bmatrix}^t\begin{bmatrix}1 & 0 & -1\\ 2 & 0 & 6\end{bmatrix}^t$

$$B^tA^t=\begin{bmatrix}1 & -8 & 0\\ 7 & 4 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & 0\\ -1 & 6\end{bmatrix}$$

$$B^tA^t=\begin{bmatrix}(1)(1)+(-8)(0)+(0)(-1) & (1)(2)+(-8)(0)+(0)(6)\\ (7)(1)+(4)(0)+(1)(-1) & (7)(2)+(4)(0)+(1)(6)\end{bmatrix}$$

$$B^tA^t=\begin{bmatrix}1-0-0 & 2-0+0\\ 7+0-1 & 14+0+6\end{bmatrix}$$

$$B^tA^t=\begin{bmatrix}1 & 2\\ 6 & 20\end{bmatrix}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Form equations (1) and (2) we get $(AB)^t=B^tA^t$

Determinant of a matrix:

If a square matrix A of order 2×2, the determinant of A is denoted by det A or $|A|$ and is defined as if

$$A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\text{, then } |A|=\begin{vmatrix}a & b\\ c & d\end{vmatrix}=ad-bc$$

Example 17: Find the determinant of the matrix $A=\begin{bmatrix}7 & 5\\ 7 & -12\end{bmatrix}$ and evaluate it.

Solution: if $A=\begin{bmatrix}7 & 5\\ 7 & -12\end{bmatrix}$, then

$$|A|=\begin{vmatrix}7 & 5\\ 7 & -12\end{vmatrix}$$

$$|A|=(7)(-12)-(5)(7)$$

$$|A|=-84-35=-119$$

Singular Matrix: A square matrix A is called a singular matrix if $|A|=0$

Non-Singular Matrix: A square matrix A is called a Non-singular matrix if $|A|\neq 0$

Example 18: find whether $A=\begin{bmatrix}4 & -2\\ -2 & 1\end{bmatrix}$ is a singular matrix

Solution: Given $A=\begin{bmatrix}4 & -2\\ -2 & 1\end{bmatrix}$

Now $|A|=\begin{vmatrix}4 & -2\\ -2 & 1\end{vmatrix}$

$$|A|=(4)(1)-(-2)(-2)$$

$$|A|=4-4=0$$

Hence A is a singular matrix

Example 19 : if $P=\begin{bmatrix}-4 & 2\\ 3 & -7\end{bmatrix}$ check whether P is a singular or non-singular matrix

Solution: Given $P=\begin{bmatrix}-4 & 2\\ 3 & -7\end{bmatrix}$ then

$$|P|=\begin{vmatrix}-4 & 2\\ 3 & -7\end{vmatrix}$$

$$|P|=(-4)(-7)-(2)(3)$$

$$|P|=28-6=22\neq 0$$

Since $|P|\neq 0$ therefore P is non-singular matrix.

Adjoint of Matrix:Let a matrix of order 2×2. Then the matrix obtained by interchanging the elements of diagonals of (i.e. a and d) and changing the sign of the other elements of the other elements (i.e. b and c) is called the adjoint of matrix A. the adjoint of the matrix is denoted by adj (A).For example, if

$$A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\text{, then } adjA=\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$$

Example 20: find adjoint of the following matrices

i). $A=\begin{bmatrix}3 & -2\\ 1 & 4\end{bmatrix}$ ii) $B=\begin{bmatrix}4 & -2\\ -3 & 1\end{bmatrix}$

solution: i). Given $A=\begin{bmatrix}3 & -2\\ 1 & 4\end{bmatrix}$

then adj $A=\begin{bmatrix}4 & 2\\ -1 & 3\end{bmatrix}$

solution;ii) we have $B=\begin{bmatrix}4 & -2\\ -3 & 1\end{bmatrix}$

then adj $B=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$

Exmaple 21: show that $\begin{bmatrix}3 & 2\\ 4 & 3\end{bmatrix}$ is a multiplicative inverse of $\begin{bmatrix}3 & -2\\ -4 & 3\end{bmatrix}$

Solution: To show that $\begin{bmatrix}3 & 2\\ 4 & 3\end{bmatrix}$ and $\begin{bmatrix}3 & -2\\ -4 & 3\end{bmatrix}$ are multiplicative inverse of each other, so

$$\begin{bmatrix}3 & 2\\ 4 & 3\end{bmatrix}\begin{bmatrix}3 & -2\\ -4 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(3)(3)+(2)(-4) & (3)(-2)+(2)(3)\\ (4)(3)+(3)(-4) & (4)(-2)+(3)(3)\end{bmatrix}$$

$$=\begin{bmatrix}9-8 & -6+6\\ 12-12 & -8+9\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Now $\begin{bmatrix}3 & -2\\ -4 & 3\end{bmatrix}\begin{bmatrix}3 & 2\\ 4 & 3\end{bmatrix}$

$=\begin{bmatrix}(3)(3)+(-2)(4) & (3)(2)+(-2)(3)\\ (-4)(3)+(3)(4) & (-4)(2)+(3)(3)\end{bmatrix}$

$=\begin{bmatrix}9-8 & 6-6\\ -12+12 & -8+9\end{bmatrix}$

$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$

Therefore that $\begin{bmatrix}3 & 2\\ 4 & 3\end{bmatrix}$ and $\begin{bmatrix}3 & -2\\ -4 & 3\end{bmatrix}$ are multiplicative inverse of each other.

Multiplicative inverse of matrix: Multiplicative inverse A^{-1}, of any non-singular matrix A is given by relation

$$A^{-1}=\frac{1}{|A|}adjA=\frac{1}{ad-bc}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$$

Example22: Find the inverse $A=\begin{bmatrix}-2 & -1\\ 3 & 4\end{bmatrix}$, using the adjoint method

Solution: Given $A=\begin{bmatrix}-2 & -1\\ 3 & 4\end{bmatrix}$ then

$|A|=\begin{vmatrix}-2 & -1\\ 3 & 4\end{vmatrix}$

$|A|=(-2)(4)-(-1)(3)$

$|A|=-8+3$

$|A|=-5\neq 0$

Therefore A is non-singular, so we find can A^{-1}

Now $adj\ A=\begin{bmatrix}4 & 1\\ -3 & -2\end{bmatrix}$ and using formula

$A^{-1}=\frac{1}{|A|}adjA$ putting the values

$A^{-1}=\frac{1}{-5}\begin{bmatrix}4 & 1\\ -3 & -2\end{bmatrix}$

Example 22: Let $A=\begin{bmatrix}-2 & 1\\ 1 & 1\end{bmatrix}$ & $B=\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}$ then show that $(AB)^{-1}=B^{-1}A^{-1}$

Solution: Given $A=\begin{bmatrix}-2 & 1\\ 1 & 1\end{bmatrix}$ & $B=\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}$

$AB=\begin{bmatrix}-2 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}$

$AB=\begin{bmatrix}(-2)(2)+(1)(3) & (-2)(1)+(1)(2)\\ (1)(2)+(1)(3) & (1)(1)+(1)(2)\end{bmatrix}$

$AB=\begin{bmatrix}-4+3 & -2+2\\ 2+3 & 1+2\end{bmatrix}$

$AB=\begin{bmatrix}-1 & 0\\ 5 & 3\end{bmatrix}$

Now $|AB|=\begin{vmatrix}-1 & 0\\ 5 & 3\end{vmatrix}$

$|AB|=(-1)(3)-(0)(5)$

$|AB|=-3\neq 0$

$\therefore (AB)^{-1}$ exists, now $adj(AB)=\begin{bmatrix}3 & 0\\ -5 & -1\end{bmatrix}$

Since $(AB)^{-1}=\frac{1}{|AB|}adj(AB)$ putting values

$(AB)^{-1}=\frac{1}{-3}\begin{bmatrix}3 & 0\\ -5 & -1\end{bmatrix}$............(1)

Now $|B|=\begin{vmatrix}2 & 1\\ 3 & 2\end{vmatrix}$ $\quad |A|=\begin{vmatrix}-2 & 1\\ 1 & 1\end{vmatrix}$

$|B|=4-3$ $\quad |A|=-2-1$

$|B|=1\neq 0$ $\quad |A|=-3\neq 0$

B and A are non- singular so inverse exists

$adj\ B=\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}$ $\quad adj\ A=\begin{bmatrix}1 & -1\\ -1 & -2\end{bmatrix}$ so,

$B^{-1}=\frac{1}{|B|}adj\ B$ $\quad A^{-1}=\frac{1}{|A|}adj\ A$ putting

$B^{-1}=\frac{1}{1}\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}$ $\quad A^{-1}=\frac{1}{-3}\begin{bmatrix}1 & -1\\ -1 & -2\end{bmatrix}$

Now $B^{-1}A^{-1}=\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}\left(\frac{1}{-3}\begin{bmatrix}1 & -1\\ -1 & -2\end{bmatrix}\right)$

$B^{-1}A^{-1}=\frac{1}{-3}\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}\begin{bmatrix}1 & -1\\ -1 & -2\end{bmatrix}$ $\therefore A(aB)=a(AB)$

$B^{-1}A^{-1}=\frac{1}{-3}\begin{bmatrix}(2)(1)+(-1)(-1) & (2)(-1)+(-1)(-2)\\ (-3)(1)+(2)(-1) & (-3)(-1)+(2)(-2)\end{bmatrix}$

$B^{-1}A^{-1}=\frac{1}{-3}\begin{bmatrix}2+1 & -2+2\\ -3-2 & 3-4\end{bmatrix}$

$B^{-1}A^{-1}=\frac{1}{-3}\begin{bmatrix}3 & 0\\ -5 & -1\end{bmatrix}$.................(2)

From eq (1) and (2) we get $(AB)^{-1}=B^{-1}A^{-1}$

Exercise # 1.5

Q1. Find the determinant of the following matrices and evaluate them.

i). $A=\begin{bmatrix}5 & 6\\ -4 & 1\end{bmatrix}$

Solution: if $A=\begin{bmatrix}5 & 6\\ -4 & 1\end{bmatrix}$, then

$|A|=\begin{vmatrix}5 & 6\\ -4 & 1\end{vmatrix}=(5)(1)-(6)(-4)$

$|A|=5+24=29$

ii). $B=\begin{bmatrix}4 & -2\\ 5 & 13\end{bmatrix}$

Solution: Given $B=\begin{bmatrix}4 & -2\\ 5 & 13\end{bmatrix}$

$|B| = \begin{vmatrix} 4 & -2 \\ 5 & 13 \end{vmatrix} = (4)(13) - (-2)(5)$

$|B| = 52 + 10 = 62$

iii). $C = \begin{bmatrix} 11 & 7 \\ -6 & 5 \end{bmatrix}$

Solution: Given $C = \begin{bmatrix} 11 & 7 \\ -6 & 5 \end{bmatrix}$

$|C| = \begin{vmatrix} 11 & 7 \\ -6 & 5 \end{vmatrix} = (11)(5) - (7)(-6)$

$|C| = 55 + 42 = 97$

iv). $D = \begin{bmatrix} 5 & 6 \\ -8 & -9 \end{bmatrix}$

Solution: Given $D = \begin{bmatrix} 5 & 6 \\ -8 & -9 \end{bmatrix}$

$|D| = \begin{vmatrix} 5 & 6 \\ -8 & -9 \end{vmatrix} = (5)(-9) - (6)(-8)$

$|D| = -45 + 48 = 3$

v). $E = \begin{bmatrix} 2p & -3q \\ r & -s \end{bmatrix}$

Solution: if $E = \begin{bmatrix} 2p & -3q \\ r & -s \end{bmatrix}$, then

$|E| = \begin{vmatrix} 2p & -3q \\ r & -s \end{vmatrix} = (2p)(-s) - (-3q)(r)$

$|E| = -2ps + 3qr$

vi). $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Solution: Given $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$|F| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ expanding by row 1

$|F| = 1\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - 0\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$

$|F| = 1(1-0) - 0(0-0) + 0(0-0)$

$|F| = 1 - 0 - 0$

$|F| = 1$

vii). $G = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ -2 & -3 & 4 \end{bmatrix}$

Solution: Given $G = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ -2 & -3 & 4 \end{bmatrix}$

$|G| = \begin{vmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ -2 & -3 & 4 \end{vmatrix}$ expanding by row 1

$|G| = 1\begin{vmatrix} 2 & 3 \\ -3 & 4 \end{vmatrix} - 2\begin{vmatrix} 3 & 3 \\ -2 & 4 \end{vmatrix} + 2\begin{vmatrix} 3 & 2 \\ -2 & -3 \end{vmatrix}$

$|G| = 1(8+9) - 2(12+6) + 2(-9+4)$

$|G| = 1(17) - 2(18) + 2(-5)$

$|G| = 17 - 36 - 10 = -29$

viii). $H = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

Sol: Now $|H| = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix}$ expanding by row 1

$|H| = a\begin{vmatrix} b & 0 \\ 0 & c \end{vmatrix} - 0\begin{vmatrix} 0 & 0 \\ 0 & c \end{vmatrix} + 0\begin{vmatrix} 0 & b \\ 0 & 0 \end{vmatrix}$

$|H| = a(bc-0) - 0(0-0) + 0(0-0)$

$|H| = abc - 0 - 0$

$|H| = abc$

Q2. Find which of the following matrices are singular and which are non-singular

i). $A = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$

Solution: if $A = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$, then

$|A| = \begin{vmatrix} 5 & 3 \\ 2 & 1 \end{vmatrix} = (5)(1) - (3)(2)$

$|A| = 5 - 6 = -1 \neq 0$

Therefore A is non- singular

ii). $B = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$

Solution: if $B = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$, then

$|B| = \begin{vmatrix} 3 & -6 \\ -2 & 4 \end{vmatrix} = (3)(4) - (-6)(-2)$

$|B| = 12 - 12 = 0$

Therefore B is singular

iii). $C = \begin{bmatrix} 3a & -2b \\ 2a & b \end{bmatrix}$

Solution: if $C = \begin{bmatrix} 3a & -2b \\ 2a & b \end{bmatrix}$, then

$|C| = \begin{vmatrix} 3a & -2b \\ 2a & b \end{vmatrix} = (3a)(b) - (-2b)(2a)$

$|C| = 3ab + 4ab = 7ab$

$|C| = 7ab \neq 0$ $\therefore$ C is non-singular iv). $D=\begin{bmatrix}-3 & 6\\ 2 & -4\end{bmatrix}$

Solution: if $D=\begin{bmatrix}-3 & 6\\ 2 & -4\end{bmatrix}$, then

$|D|=\begin{vmatrix}-3 & 6\\ 2 & -4\end{vmatrix}=(-3)(-4)-(6)(2)$

$|D|=12-12=0$ Therefore D is singular

Q3. Find the adjoint of the following matrices.

i). $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$

Solution: if $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, then $adjA=\begin{bmatrix}4 & -2\\ -3 & 1\end{bmatrix}$

ii). $B=\begin{bmatrix}-3 & -1\\ 2 & 3\end{bmatrix}$

Sol: if $B=\begin{bmatrix}-3 & -1\\ 2 & 3\end{bmatrix}$, then $adjB=\begin{bmatrix}3 & 1\\ -2 & -3\end{bmatrix}$

iii). $C=\begin{bmatrix}2 & -4\\ 3 & 1\end{bmatrix}$

Solution: if $C=\begin{bmatrix}2 & -4\\ 3 & 1\end{bmatrix}$, then $adjC=\begin{bmatrix}1 & 4\\ -3 & 2\end{bmatrix}$

iv). $D=\begin{bmatrix}-3 & 6\\ 2 & -4\end{bmatrix}$

Sol: if $D=\begin{bmatrix}-3 & 6\\ 2 & -4\end{bmatrix}$, then $adjD=\begin{bmatrix}-4 & -6\\ -2 & -3\end{bmatrix}$

Q4: i). Find inverse of $A=\begin{bmatrix}4 & 1\\ 3 & 1\end{bmatrix}$

Sol: if $A=\begin{bmatrix}4 & 1\\ 3 & 1\end{bmatrix}$, then $|A|=\begin{vmatrix}4 & 1\\ 3 & 1\end{vmatrix}=(4)(1)-(1)(3)$

$|A|=4-3=1\neq 0$

Therefore A is non- singular, So we can find A^{-1}

Now $adjA=\begin{bmatrix}1 & -1\\ -3 & 4\end{bmatrix}$ As $A^{-1}=\frac{1}{|A|}adjA$

$A^{-1}=\frac{1}{1}\begin{bmatrix}1 & -1\\ -3 & 4\end{bmatrix}=\begin{bmatrix}1 & -1\\ -3 & 4\end{bmatrix}$

Q4: ii). Find the inverse of $B=\begin{bmatrix}3 & 4\\ 1 & 2\end{bmatrix}$

Solution: if $B=\begin{bmatrix}3 & 4\\ 1 & 2\end{bmatrix}$, then

$|B|=\begin{vmatrix}3 & 4\\ 1 & 2\end{vmatrix}=(3)(2)-(4)(1)$

$|B|=6-4=2\neq 0$

Therefore B is non- singular, So we can find B^{-1}

Now $adjB=\begin{bmatrix}2 & -4\\ -1 & 3\end{bmatrix}$

As $B^{-1}=\frac{1}{|B|}adjB$

$B^{-1}=\frac{1}{2}\begin{bmatrix}2 & -4\\ -1 & 3\end{bmatrix}$

Q4: iii). Find the inverse of $C=\begin{bmatrix}4 & -3\\ -1 & 2\end{bmatrix}$

Solution: if $C=\begin{bmatrix}4 & -3\\ -1 & 2\end{bmatrix}$, then

$|C|=\begin{vmatrix}4 & -3\\ -1 & 2\end{vmatrix}=(4)(2)-(-3)(-1)$

$|C|=8-3=5\neq 0$

Therefore C is non- singular, So we can find C^{-1}

Now $adjC=\begin{bmatrix}2 & 3\\ 1 & 4\end{bmatrix}$

As $C^{-1}=\frac{1}{|C|}adjC$

$C^{-1}=\frac{1}{5}\begin{bmatrix}2 & 3\\ 1 & 4\end{bmatrix}=\begin{bmatrix}\frac{2}{5} & \frac{3}{5}\\ \frac{1}{5} & \frac{4}{5}\end{bmatrix}$

Q4: iv). Find the inverse of $D=\begin{bmatrix}0 & -3\\ 2 & 4\end{bmatrix}$

Solution: if $D=\begin{bmatrix}0 & -3\\ 2 & 4\end{bmatrix}$, then

$|D|=\begin{vmatrix}0 & -3\\ 2 & 4\end{vmatrix}=(0)(4)-(-3)(2)$

$|D|=0+6=6\neq 0$

Therefore D is non- singular, So we can find D^{-1}

Now $adjD=\begin{bmatrix}4 & 3\\ -2 & 0\end{bmatrix}$

As $D^{-1}=\frac{1}{|D|}adjD$

$D^{-1}=\frac{1}{6}\begin{bmatrix}4 & 3\\ -2 & 0\end{bmatrix}$

Q4: v). Find the inverse of $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

solution: Given $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$|I|=\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix}=(1)(0)-(0)(0)$

$|I|=1-0=1\neq 0$

Therefore I is non- singular, So we can find I^{-1}

Now $adj\ I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

As $I^{-1}=\frac{1}{|I|}adj\ \ I$

$I^{-1}=\frac{1}{1}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$I^{-1}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

Q5: i). If $A=\begin{bmatrix}2 & 0\\-3 & 1\end{bmatrix}, B=\begin{bmatrix}1 & -1\\-1 & 3\end{bmatrix}$ Find AB

Solution: we have $A=\begin{bmatrix}2 & 0\\-3 & 1\end{bmatrix}, B=\begin{bmatrix}1 & -1\\-1 & 3\end{bmatrix}$, then

$AB=\begin{bmatrix}2 & 0\\-3 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\-1 & 3\end{bmatrix}$

$AB=\begin{bmatrix}(2)(1)+(0)(-1) & (2)(-1)+(0)(3)\\(-3)(1)+(1)(-1) & (-3)(-1)+(1)(3)\end{bmatrix}$

$AB=\begin{bmatrix}2+0 & -2+0\\-3-1 & 3+3\end{bmatrix}=\begin{bmatrix}2 & -2\\-4 & 6\end{bmatrix}$

Q5: ii). If $A=\begin{bmatrix}2 & 0\\-3 & 1\end{bmatrix}, B=\begin{bmatrix}1 & -1\\-1 & 3\end{bmatrix}$ find BA

Solution: Given $A=\begin{bmatrix}2 & 0\\-3 & 1\end{bmatrix}, B=\begin{bmatrix}1 & -1\\-1 & 3\end{bmatrix}$

Now $BA=\begin{bmatrix}1 & -1\\-1 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\-3 & 1\end{bmatrix}$

$BA=\begin{bmatrix}(1)(2)+(-1)(-3) & (1)(0)+(-1)(1)\\(-1)(2)+(3)(-3) & (-1)(0)+(3)(1)\end{bmatrix}$

$BA=\begin{bmatrix}2+3 & 0-1\\-2-9 & 0+3\end{bmatrix}$

$BA=\begin{bmatrix}5 & -1\\-11 & 3\end{bmatrix}$

Q5: iii). If $A=\begin{bmatrix}2 & 0\\-3 & 1\end{bmatrix}, B=\begin{bmatrix}1 & -1\\-1 & 3\end{bmatrix}$ find $A^{-1}\ B^{-1}$

Solution; First we find A^{-1}, Where $A=\begin{bmatrix}2 & 0\\-3 & 1\end{bmatrix}$

$|A|=\begin{vmatrix}2 & 0\\-3 & 1\end{vmatrix}=(2)(1)-(0)(-3)$

$|A|=2-0=2\neq 0$

Therefore A is non- singular, So we can find A^{-1}

Now $adjA=\begin{bmatrix}1 & 0\\3 & 2\end{bmatrix}$

As $A^{-1}=\frac{1}{|A|}adjA$

$A^{-1}=\frac{1}{2}\begin{bmatrix}1 & 0\\3 & 2\end{bmatrix}\cdots(1)$

Now we will find B^{-1}, where $B=\begin{bmatrix}1 & -1\\-1 & 3\end{bmatrix}$

$|B|=\begin{vmatrix}1 & -1\\-1 & 3\end{vmatrix}=(1)(3)-(-1)(-1)$

$|B|=3-1=2\neq 0$

Therefore B is non- singular, So we can find B^{-1}

Now $adjB=\begin{bmatrix}3 & 1\\1 & 1\end{bmatrix}$

As $B^{-1}=\frac{1}{|B|}adjB$

$B^{-1}=\frac{1}{2}\begin{bmatrix}3 & 1\\1 & 1\end{bmatrix}\cdots(2)$

Q5. If $A=\begin{bmatrix}2 & 0\\-3 & 1\end{bmatrix}, B=\begin{bmatrix}1 & -1\\-1 & 3\end{bmatrix}$ Show that $(AB)^{-1}=B^{-1}A^{-1}$

Sol: Given $A=\begin{bmatrix}2 & 0\\-3 & 1\end{bmatrix}, B=\begin{bmatrix}1 & -1\\-1 & 3\end{bmatrix}$, then

$AB=\begin{bmatrix}2 & 0\\-3 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\-1 & 3\end{bmatrix}$

$AB=\begin{bmatrix}(2)(1)+(0)(-1) & (2)(-1)+(0)(3)\\(-3)(1)+(1)(-1) & (-3)(-1)+(1)(3)\end{bmatrix}$

$AB=\begin{bmatrix}2+0 & -2+0\\-3-1 & 3+3\end{bmatrix}=\begin{bmatrix}2 & -2\\-4 & 6\end{bmatrix}$

$AB=\begin{bmatrix}2 & -2\\-4 & 6\end{bmatrix}$ Now

$|AB|=\begin{vmatrix}2 & -2\\-4 & 6\end{vmatrix}=(2)(6)-(-2)(-4)$

$|AB|=12-8=4\neq 0$

$\therefore$ AB is non- singular, So we can find $(AB)^{-1}$

Now $adjAB=\begin{bmatrix}6 & 2\\4 & 2\end{bmatrix}$

As $(AB)^{-1}=\frac{1}{|AB|}adjAB$

$(AB)^{-1}=\frac{1}{4}\begin{bmatrix}6 & 2\\4 & 2\end{bmatrix}\cdots(3)$

Now RHS $B^{-1}A^{-1}$ Using equations (2) and (1)

$B^{-1}A^{-1}=\frac{1}{2}.\frac{1}{2}\begin{bmatrix}3 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\3 & 2\end{bmatrix}$

$B^{-1}A^{-1}=\frac{1}{4}\begin{bmatrix}(3)(1)+(1)(3) & (3)(0)+(1)(2)\\(1)(1)+(1)(3) & (1)(0)+(1)(2)\end{bmatrix}$

$B^{-1}A^{-1}=\frac{1}{4}\begin{bmatrix}3+3 & 0+2\\1+3 & 0+2\end{bmatrix}=\frac{1}{4}\begin{bmatrix}6 & 2\\4 & 2\end{bmatrix}\cdots(4)$

From equations (3) and (4) we get $(AB)^{-1}=B^{-1}A^{-1}$

Q5. If $A=\begin{bmatrix}2 & 0\\-3 & 1\end{bmatrix}, B=\begin{bmatrix}1 & -1\\-1 & 3\end{bmatrix}$

Show that $(BA)^{-1}=A^{-1}B^{-1}$

Solution: Given $A=\begin{bmatrix}2 & 0\\-3 & 1\end{bmatrix}, B=\begin{bmatrix}1 & -1\\-1 & 3\end{bmatrix}$

Now $BA=\begin{bmatrix}1 & -1\\-1 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\-3 & 1\end{bmatrix}$

$BA=\begin{bmatrix}(1)(2)+(-1)(-3) & (1)(0)+(-1)(1)\\(-1)(2)+(3)(-3) & (-1)(0)+(3)(1)\end{bmatrix}$

$$BA=\begin{bmatrix}2+3 & 0-1\\ -2-9 & 0+3\end{bmatrix}$$

$$BA=\begin{bmatrix}5 & -1\\ -11 & 3\end{bmatrix}$$

Now $|BA|=\begin{vmatrix}5 & -1\\ -11 & 3\end{vmatrix}$ and $adj(BA)=\begin{bmatrix}3 & 1\\ 11 & 5\end{bmatrix}$

$|BA|=15-11$

$|BA|=4\neq 0$

Since $(BA)^{-1}=\dfrac{1}{|BA|}adj(BA)$ putting the values

$$(BA)^{-1}=\frac{1}{4}\begin{bmatrix}3 & 1\\ 11 & 5\end{bmatrix}\text{....................}(1)$$

Now for RHS

$|A|=\begin{vmatrix}2 & 0\\ -3 & 1\end{vmatrix}$ $\qquad$ $|B|=\begin{vmatrix}1 & -1\\ -1 & 3\end{vmatrix}$

$|A|=(2)(1)-(0)(-3)$ $\qquad$ $|B|=(1)(3)-(-1)(-1)$

$|A|=2-0=2\neq 0$ $\qquad$ $|B|=3-1=2\neq 0$

A and B are non-singular so, adjoint

$adjA=\begin{bmatrix}1 & 0\\ 3 & 2\end{bmatrix}$ $\qquad$ $adjB=\begin{bmatrix}3 & 1\\ 1 & 1\end{bmatrix}$

As $A^{-1}=\dfrac{1}{|A|}adjA$ $\qquad$ As $B^{-1}=\dfrac{1}{|B|}adjB$

$A^{-1}=\dfrac{1}{2}\begin{bmatrix}1 & 0\\ 3 & 2\end{bmatrix}$ $\qquad$ $B^{-1}=\dfrac{1}{2}\begin{bmatrix}3 & 1\\ 1 & 1\end{bmatrix}$

Now $A^{-1}B^{-1}=\left(\dfrac{1}{2}\begin{bmatrix}1 & 0\\ 3 & 2\end{bmatrix}\right)\left(\dfrac{1}{2}\begin{bmatrix}3 & 1\\ 1 & 1\end{bmatrix}\right)$

$$A^{-1}B^{-1}=\frac{1}{2}\frac{1}{2}\begin{bmatrix}1 & 0\\ 3 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ 1 & 1\end{bmatrix}\quad\therefore\ (aA)(bB)=ab(AB)$$

$$A^{-1}B^{-1}=\frac{1}{4}\begin{bmatrix}(1)(3)+(0)(1) & (1)(1)+(0)(1)\\ (3)(3)+(2)(1) & (3)(1)+(2)(1)\end{bmatrix}$$

$$A^{-1}B^{-1}=\frac{1}{4}\begin{bmatrix}3+0 & 1+0\\ 9+2 & 3+2\end{bmatrix}$$

$$A^{-1}B^{-1}=\frac{1}{4}\begin{bmatrix}3 & 1\\ 11 & 5\end{bmatrix}\text{..........................}(2)$$

From eq (1) and eq (2) we get $(BA)^{-1}=A^{-1}B^{-1}$

Q6: If $A=\begin{bmatrix}0 & -1\\ 2 & 1\end{bmatrix},B=\begin{bmatrix}2 & 3\\ 1 & 0\end{bmatrix}$ show that $(AB)^{-1}=B^{-1}A^{-1}$

Sol: Given $A=\begin{bmatrix}0 & -1\\ 2 & 1\end{bmatrix},B=\begin{bmatrix}2 & 3\\ 1 & 0\end{bmatrix}$

LHS $(AB)^{-1}$, First we find AB

$$AB=\begin{bmatrix}0 & -1\\ 2 & 1\end{bmatrix}\begin{bmatrix}2 & 3\\ 1 & 0\end{bmatrix}$$

$$AB=\begin{bmatrix}(0)(2)+(-1)(1) & (0)(3)+(-1)(0)\\ (2)(2)+(1)(1) & (2)(3)+(1)(0)\end{bmatrix}$$

$$AB=\begin{bmatrix}0-1 & 0-0\\ 4+1 & 6+0\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 5 & 6\end{bmatrix}$$

then $|AB|=\begin{vmatrix}-1 & 0\\ 5 & 6\end{vmatrix}=(-1)(6)-(0)(5)$

$|AB|=-6-0=-6\neq 0$

Therefore AB is non- singular, So we can find $(AB)^{-1}$

Now $adj\ AB=\begin{bmatrix}6 & 0\\ -5 & -1\end{bmatrix}$

As $(AB)^{-1}=\dfrac{1}{|AB|}adj\ AB$

$$(AB)^{-1}=\frac{1}{-6}\begin{bmatrix}6 & 0\\ -5 & -1\end{bmatrix}\cdots(1)$$

RHS $B^{-1}A^{-1}$

First we find B^{-1} and A^{-1} separately

$|A|=\begin{vmatrix}0 & -1\\ 2 & 1\end{vmatrix}$ $\qquad$ $|B|=\begin{vmatrix}2 & 3\\ 1 & 0\end{vmatrix}$

$|A|=(0)(1)-(-1)(2)$ $\qquad$ $|B|=(2)(0)-(3)(1)$

$|A|=0+2=2\neq 0$ $\qquad$ $|B|=0-3=-3\neq 0$

$\therefore$ A is non- singular, so we can find A^{-1} $\qquad$ $\therefore$ B is non- singular, so we can find B^{-1}

Now $adjA=\begin{bmatrix}1 & 1\\ -2 & 0\end{bmatrix}$ $\qquad$ Now $adjB=\begin{bmatrix}0 & -3\\ -1 & 2\end{bmatrix}$

As $A^{-1}=\dfrac{1}{|A|}adjA$ $\qquad$ As $B^{-1}=\dfrac{1}{|B|}adjB$

$A^{-1}=\dfrac{1}{2}\begin{bmatrix}1 & 1\\ -2 & 0\end{bmatrix}...(i)$ $\qquad$ $B^{-1}=\dfrac{1}{-3}\begin{bmatrix}0 & -3\\ -1 & 2\end{bmatrix}...(ii)$

So RHS $B^{-1}A^{-1}=\dfrac{1}{-3}\dfrac{1}{2}\begin{bmatrix}0 & -3\\ -1 & 2\end{bmatrix}\begin{bmatrix}1 & 1\\ -2 & 0\end{bmatrix}$

$$B^{-1}A^{-1}=\frac{1}{-6}\begin{bmatrix}(0)(1)+(-3)(-2) & (0)(1)+(-3)(0)\\ (-1)(1)+(2)(-2) & (-1)(1)+(2)(0)\end{bmatrix}$$

$$B^{-1}A^{-1}=\frac{1}{-6}\begin{bmatrix}0+6 & 0+0\\ -1-4 & -1+0\end{bmatrix}$$

$$B^{-1}A^{-1}=\frac{1}{-6}\begin{bmatrix}6 & 0\\ -5 & -1\end{bmatrix}\cdots(2)$$

From equations (1) and (2) we get $(AB)^{-1}=B^{-1}A^{-1}$

Q6: If $A=\begin{bmatrix}0 & -1\\ 2 & 1\end{bmatrix},B=\begin{bmatrix}2 & 3\\ 1 & 0\end{bmatrix}$ show that $(BA)^{-1}=A^{-1}B^{-1}$

Sol: Given $A=\begin{bmatrix}0 & -1\\ 2 & 1\end{bmatrix},B=\begin{bmatrix}2 & 3\\ 1 & 0\end{bmatrix}$

LHS $(BA)^{-1}$, First we find BA

$$BA=\begin{bmatrix}2 & 3\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & -1\\ 2 & 1\end{bmatrix}$$

$$BA=\begin{bmatrix}(2)(0)+(3)(2) & (2)(-1)+(3)(1)\\ (1)(0)+(0)(2) & (1)(-1)+(0)(1)\end{bmatrix}$$

$$BA=\begin{bmatrix}0+6 & -2+3\\ 0+0 & -1+0\end{bmatrix}=\begin{bmatrix}6 & 1\\ 0 & -1\end{bmatrix}$$

then $|BA|=\begin{vmatrix}6 & 1\\ 0 & -1\end{vmatrix}=(6)(-1)-(1)(0)$

$|BA|=-6-0=-6\neq 0$

Therefore BA is non- singular, So we can find $(BA)^{-1}$

Now $adj\ BA=\begin{bmatrix}-1 & -1\\ 0 & 6\end{bmatrix}$

As $(BA)^{-1}=\frac{1}{|BA|}adj\ BA$

$(BA)^{-1}=\frac{1}{-6}\begin{bmatrix}-1 & -1\\ 0 & 6\end{bmatrix}\cdots(1)$

RHS $A^{-1}B^{-1}$

First we find B^{-1} and A^{-1} separately

$|A|=\begin{vmatrix}0 & -1\\ 2 & 1\end{vmatrix}$ $\quad |B|=\begin{vmatrix}2 & 3\\ 1 & 0\end{vmatrix}$

$|A|=(0)(1)-(-1)(2)$ $\quad |B|=(2)(0)-(3)(1)$

$|A|=0+2=2\neq 0$ $\quad |B|=0-3=-3\neq 0$

$\therefore$ A is non- singular, so we can find A^{-1} $\quad \therefore$ B is non- singular, so we can find B^{-1}

Now $adjA=\begin{bmatrix}1 & 1\\ -2 & 0\end{bmatrix}$ $\quad$ Now $adjB=\begin{bmatrix}0 & -3\\ -1 & 2\end{bmatrix}$

As $A^{-1}=\frac{1}{|A|}adjA$ $\quad$ As $B^{-1}=\frac{1}{|B|}adjB$

$A^{-1}=\frac{1}{2}\begin{bmatrix}1 & 1\\ -2 & 0\end{bmatrix}\ldots(i)$ $\quad B^{-1}=\frac{1}{-3}\begin{bmatrix}0 & -3\\ -1 & 2\end{bmatrix}\ldots(ii)$

So RHS $B^{-1}A^{-1}=\frac{1}{2}\times\frac{1}{-3}\begin{bmatrix}1 & 1\\ -2 & 0\end{bmatrix}\begin{bmatrix}0 & -3\\ -1 & 2\end{bmatrix}$

$$B^{-1}A^{-1}=\frac{1}{-6}\begin{bmatrix}(1)(0)+(1)(-1) & (1)(-3)+(1)(2)\\ (-2)(0)+(0)(-1) & (-2)(-3)+(0)(2)\end{bmatrix}$$

$$B^{-1}A^{-1}=\frac{1}{-6}\begin{bmatrix}0-1 & -3+2\\ 0+0 & 6+0\end{bmatrix}$$

$$B^{-1}A^{-1}=\frac{1}{-6}\begin{bmatrix}-1 & -1\\ 0 & 6\end{bmatrix}\cdots(2)$$

From equations (1) and (2) $(BA)^{-1}=A^{-1}B^{-1}$

Simultaneous linear Equation:

Let $\quad a_1x+b_1y=c_1$

And $\quad a_2x+b_2y=c_2$

are called two simultaneous linear equations.

Solution of Simultaneous linear Equation by Matrices:

Simultaneous linear equations can be written in matrix form

$$\begin{bmatrix}a_1 & b_1\\ a_2 & b_2\end{bmatrix}.\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}c_1\\ c_2\end{bmatrix}$$ or

$AX=B$ where

$A=\begin{bmatrix}a_1 & b_1\\ a_2 & b_2\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\end{bmatrix}$ and $B=\begin{bmatrix}c_1\\ c_2\end{bmatrix}$

And A is non-singular, To find values of variables x & y,

$\because AX=B$

$\Rightarrow \therefore A^{-1}AX=A^{-1}B$

$\Rightarrow IX=A^{-1}B \qquad (\because A^{-1}A=I)$

$X=A^{-1}B \qquad (\because IX=X)$

or $X=\frac{1}{|A|}\begin{bmatrix}b_2 & -b_1\\ -a_2 & a_1\end{bmatrix}\begin{bmatrix}c_1\\ c_2\end{bmatrix}$ or

$$X=\frac{1}{a_1b_2-a_2b_1}\begin{bmatrix}b_2 & -b_1\\ -a_2 & a_1\end{bmatrix}\begin{bmatrix}c_1\\ c_2\end{bmatrix}$$ or

$$X=\frac{1}{a_1b_2-a_2b_1}\begin{bmatrix}b_2c_1-b_1c_2\\ -a_2c_1+a_1c_2\end{bmatrix}$$

Note: If A is singular matrix i.e. $|A|=0$, then it is not possible to find solution of the given equations.

Example 24: solve the system of equation with the help of matrices. $x-3y=0,\quad 2x+y=7$

Solution: Given
$$x-3y=0$$
$$2x+y=7$$

These equations can be written in the fro of matrices as $\begin{bmatrix}1 & -3\\ 2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 7\end{bmatrix}$

Let $A=\begin{bmatrix}1 & -3\\ 2 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}0\\ 7\end{bmatrix}$

And $|A|=\begin{vmatrix}1 & -3\\ 2 & 1\end{vmatrix}$

$|A|=(1)(1)-(-3)(2)$

$|A|=1+6$

$|A|=7\neq 0$

$\therefore A^{-1}$ exists, so $adj\ A=\begin{bmatrix}1 & 3\\ -2 & 1\end{bmatrix}$ using

$\because AX=B$

$\Rightarrow \therefore A^{-1}AX=A^{-1}B$

$\Rightarrow IX=A^{-1}B \qquad (\because A^{-1}A=I)$

$X=A^{-1}B \qquad (\because IX=X)$

Putting the values $X=\frac{1}{7}\begin{bmatrix}1 & 3\\ -2 & 1\end{bmatrix}\begin{bmatrix}0\\ 7\end{bmatrix}$

$$X=\frac{1}{7}\begin{bmatrix}(1)(0)+(3)(7)\\ (-2)(0)+(1)(7)\end{bmatrix}$$

$$X=\frac{1}{7}\begin{bmatrix}0+21\\ 0+7\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{7}\begin{bmatrix}21\\ 7\end{bmatrix}=\begin{bmatrix}3\\ 1\end{bmatrix}$$

By definition of equal matrices their corresponding elements are equal

$\Rightarrow x=3,\qquad y=1$

Solution set $=\{(3,1)\}$

Example25: Is the following system of equations solvable? $\quad 3x-6y=9,\quad 2x-4y=-3$

Solution: Given
$$3x-6y=9$$
$$2x-4y=-3$$

These equations can be written in the fro of matrices as $\begin{bmatrix}3 & -6\\ 2 & -4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}9\\ -3\end{bmatrix}$

Let $A=\begin{bmatrix}3 & -6\\ 2 & -4\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}9\\ -3\end{bmatrix}$

And $|A|=\begin{vmatrix}3 & -6\\ 2 & -4\end{vmatrix}$

$|A|=(3)(-4)-(-6)(2)$

$|A|=-12+12$

$|A|=0$

Hence the given equations are non-solvable

Cramer's Rule: Simultaneous linear equations can also solved by Cramer's Rule Let $a_1x+b_1y=c_1$

And $a_2x+b_2y=c_2$

Simultaneous linear equations can be written in matrix form

$$\begin{bmatrix}a_1 & b_1\\ a_2 & b_2\end{bmatrix}.\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}c_1\\ c_2\end{bmatrix} \text{ or } AX=B$$

where $A=\begin{bmatrix}a_1 & b_1\\ a_2 & b_2\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\end{bmatrix}$ and $B=\begin{bmatrix}c_1\\ c_2\end{bmatrix}$

And A is non-singular, To find the value of the variables x and y by Cramer's rule

$\because AX=B$

or $X=A^{-1}B$

or $\begin{bmatrix}x\\ y\end{bmatrix}=\dfrac{1}{|A|}\begin{bmatrix}b_2 & -b_1\\ -a_2 & a_1\end{bmatrix}\begin{bmatrix}c_1\\ c_2\end{bmatrix}$

$=\dfrac{1}{|A|}\begin{bmatrix}b_2c_1-b_1c_2\\ -a_2c_1+a_1c_2\end{bmatrix}$

$\Rightarrow x=\dfrac{b_2c_1-b_1c_2}{|A|}=\dfrac{|A_x|}{|A|}$ and $\Rightarrow y=\dfrac{a_1c_2-a_2c_1}{|A|}=\dfrac{|A_y|}{|A|}$

where $|A_x|=\begin{vmatrix}c_1 & b_1\\ c_2 & b_2\end{vmatrix}$ and $|A_y|=\begin{vmatrix}a_1 & c_1\\ a_2 & c_2\end{vmatrix}$

Exmaple 26: Solve the following system of equations by using cramer's rule $x-2y=1, \quad 3x+y=10$

Solution: Given $x-2y=1$
$3x+y=10$

in terms of matrices we can write the above system as

$$\begin{bmatrix}1 & -2\\ 3 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1\\ 10\end{bmatrix}$$

Where $A=\begin{bmatrix}1 & -2\\ 3 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}1\\ 10\end{bmatrix}$

Now $|A|=\begin{vmatrix}1 & -2\\ 3 & 1\end{vmatrix}=(1)(1)-(-2)(3)$

$|A|=1+6=7$

Replaceing coefficients of x in A of B &taking determinant

$|A_x|=\begin{vmatrix}1 & -2\\ 10 & 1\end{vmatrix}=(1)(1)-(-2)(10)$

$|A_x|=1+20=21$

Replaceing coefficients of y in A of B &taking determinant

$|A_y|=\begin{vmatrix}1 & 1\\ 3 & 10\end{vmatrix}=(1)(10)-(1)(3)$

$|A_y|=10-3=7$

$\therefore x=\dfrac{|A_x|}{|A|} \qquad \therefore y=\dfrac{|A_y|}{|A|}$ putting values

$x=\dfrac{21}{7} \qquad y=\dfrac{7}{7}$

$x=3 \qquad y=1$

Solution set $=\{(3,1)\}$

Example 27: My friend asked me this question. There are two numbers such that the sum of the first and three times the second is 53. While the difference between 4 times the first and twice the second is 2. Can you help me out in finding the numbers?

Solution: Let one number = x and second number = y

Then from the first set of facts

$x+3y=53$

From the second set of facts

$4x-2y=2$

These equations can be written as in the form of matrices

$$\begin{bmatrix}1 & 3\\ 4 & -2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}53\\ 2\end{bmatrix}$$

Let $A=\begin{bmatrix}1 & 3\\ 4 & -2\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}53\\ 2\end{bmatrix}$

Now $|A|=\begin{vmatrix}1 & 3\\ 4 & -2\end{vmatrix}=(1)(-2)-(3)(4)$

$|A|=-2-12=-14\neq 0 \quad \therefore A^{-1}$ exists

Now $adj\ A=\begin{bmatrix}-2 & -3\\ -4 & 1\end{bmatrix}$ using

$\because AX=B$

$\Rightarrow \therefore A^{-1}AX=A^{-1}B$

$\Rightarrow IX=A^{-1}B \qquad (\because A^{-1}A=I)$

$X=A^{-1}B \qquad (\because IX=X)$

Putting the values

$X=\dfrac{1}{-14}\begin{bmatrix}-2 & -3\\ -4 & 1\end{bmatrix}\begin{bmatrix}53\\ 2\end{bmatrix}$

$X=\dfrac{1}{-14}\begin{bmatrix}(-2)(53)+(-3)(2)\\ (-4)(53)+(1)(2)\end{bmatrix}$

$X=\dfrac{1}{-14}\begin{bmatrix}-106-6\\ -212+2\end{bmatrix}$

$\begin{bmatrix}x\\ y\end{bmatrix}=\dfrac{1}{-14}\begin{bmatrix}-112\\ -210\end{bmatrix}=\begin{bmatrix}8\\ 15\end{bmatrix}$

By definition of equal matrices their corresponding elements are equal

$\Rightarrow x=8, \qquad y=15$

therefore the numbers are 8 and 15

Example 28: the cost of 1 rubber and 7 sharpeners are 15 rupees, while that of 3 rubbers and 1 sharpeners are 5 ruppes. What are the prices of a rubber and a sharpener respectively.

Solution: Let cost of 1 rubber = x

Cost of 1 sharpener = y

From the first set of facts

$$x+7y=8$$

From the second set of facts

$$3x+y=5$$

These equations can be written as matrices form

$$\begin{bmatrix}1&7\\3&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}15\\5\end{bmatrix}$$

Let $A=\begin{bmatrix}1&7\\3&1\end{bmatrix}, X=\begin{bmatrix}x\\y\end{bmatrix}, B=\begin{bmatrix}15\\5\end{bmatrix}$

$$|A|=\begin{vmatrix}1&7\\3&1\end{vmatrix}=(1)(1)-(7)(3)$$

$|A|=1-21=-20\neq 0 \quad \therefore A^{-1}$ exists

Now $adj\ A=\begin{bmatrix}1&-7\\-3&1\end{bmatrix}$ using

$\because AX=B$

$\Rightarrow \therefore A^{-1}AX=A^{-1}B$

$\Rightarrow IX=A^{-1}B \qquad (\because A^{-1}A=I)$

$X=A^{-1}B \qquad (\because IX=X)$

Putting the values

$$X=\frac{1}{-20}\begin{bmatrix}1&-7\\-3&1\end{bmatrix}\begin{bmatrix}15\\5\end{bmatrix}$$

$$X=\frac{1}{-20}\begin{bmatrix}(1)(15)+(-7)(5)\\(-3)(15)+(1)(5)\end{bmatrix}$$

$$X=\frac{1}{-20}\begin{bmatrix}15-35\\-45+5\end{bmatrix}$$

$$X=\frac{1}{-20}\begin{bmatrix}-20\\-40\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}$$

By definition of equal matrices their corresponding elements are equal

$\Rightarrow x=1, \qquad y=2$

Therefore cost of 1 rubber $x=1$ rupee

Cost of 1 sharpener $y=2$ rupees

Exercise # 1.6

Q1. Solve the following system of linear equations using inversion method.

i). $2x+3y=-1, \quad x-y=2$

Solution: Giventhe system of linear equations

$$2x+3y=-1$$

$$x-y=2$$

These equations can be written in form of matrices as;

$$\begin{bmatrix}2&3\\1&-1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-1\\2\end{bmatrix}$$

Let $AX=B$ pre multiply by A^{-1}

$$A^{-1}AX=A^{-1}B$$

$$IX=A^{-1}B$$

$$X=A^{-1}B$$

Where $A=\begin{bmatrix}2&3\\1&-1\end{bmatrix}, \quad X=\begin{bmatrix}x\\y\end{bmatrix}, \qquad B=\begin{bmatrix}-1\\2\end{bmatrix}$

$$|A|=\begin{vmatrix}2&3\\1&-1\end{vmatrix}=(2)(-1)-(3)(1)$$

$$|A|=-2-3=-5\neq 0$$

Therefore A is non- singular, so we can find A^{-1}

Now $adjA=\begin{bmatrix}-1&-3\\-1&2\end{bmatrix}$

As $A^{-1}=\frac{1}{|A|}adjA$

$$A^{-1}=\frac{1}{-5}\begin{bmatrix}-1&-3\\-1&2\end{bmatrix}\cdots(1)$$

Now $AX=B$

$\Rightarrow X=A^{-1}B$

$$X=\frac{1}{-5}\begin{bmatrix}-1&-3\\-1&2\end{bmatrix}\begin{bmatrix}-1\\2\end{bmatrix}$$

$$X=\frac{1}{-5}\begin{bmatrix}(-1)(-1)+(-3)(2)\\(-1)(-1)+(2)(2)\end{bmatrix}$$

$$X=\frac{1}{-5}\begin{bmatrix}1-6\\1+4\end{bmatrix}=\begin{bmatrix}\frac{-5}{-5}\\\frac{5}{-5}\end{bmatrix}=\begin{bmatrix}1\\-1\end{bmatrix}$$

$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1\\-1\end{bmatrix} \qquad \because X=\begin{bmatrix}x\\y\end{bmatrix}$

By definition of equal matrices their corresponding elements are equal

So, $x=1, \quad y=-1$

Hence the solution set $=\{(1,-1)\}$

ii). $x+2y=-13, \quad 3x+6y=11$

Solution: Giventhe system of linear equations

$$x+2y=-13$$

$$3x+6y=11$$

These equations can be written in form of matrices as;

$$\begin{bmatrix}1&2\\3&6\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-13\\11\end{bmatrix}$$

Let $A=\begin{bmatrix}1&2\\3&6\end{bmatrix}, \quad X=\begin{bmatrix}x\\y\end{bmatrix}, \qquad B=\begin{bmatrix}-13\\11\end{bmatrix}$

$$|A|=\begin{vmatrix}1&2\\3&6\end{vmatrix}=(1)(6)-(2)(3)$$

$$|A|=6-6=0$$

Therefore A is singular, so A^{-1} does not exists

Or the system of linear equation are parallel

iii). $x+2y=1, \qquad 2x+3y=\frac{5}{2}$

Solution: Giventhe system of linear equations

$x+2y=1$

$2x+3y=\frac{5}{2}$

These equations can be written in form of matrices as;

$$\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1\\ \frac{5}{2}\end{bmatrix}$$

Let $AX=B$ pre multiply by A^{-1}

$$A^{-1}AX=A^{-1}B$$
$$IX=A^{-1}B$$
$$X=A^{-1}B$$

where $A=\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\end{bmatrix}$, $B=\begin{bmatrix}1\\ \frac{5}{2}\end{bmatrix}$

$$|A|=\begin{vmatrix}1 & 2\\ 2 & 3\end{vmatrix}=(1)(3)-(2)(2)$$

$$|A|=3-4=-1\neq 0$$

Therefore A is non- singular, so we can find A^{-1}

Now $adjA=\begin{bmatrix}3 & -2\\ -2 & 1\end{bmatrix}$

As $A^{-1}=\frac{1}{|A|}adjA$

$$A^{-1}=\frac{1}{-1}\begin{bmatrix}3 & -2\\ -2 & 1\end{bmatrix}\cdots(1)$$

Now $AX=B$

$\Rightarrow X=A^{-1}B$

$$X=\frac{1}{-1}\begin{bmatrix}3 & -2\\ -2 & 1\end{bmatrix}\begin{bmatrix}1\\ \frac{5}{2}\end{bmatrix}$$

$$X=\frac{1}{-1}\begin{bmatrix}(3)(1)+(-2)(\frac{5}{2})\\ (-2)(1)+(1)(\frac{5}{2})\end{bmatrix}$$

$$X=\frac{1}{-1}\begin{bmatrix}3-5\\ -2+\frac{5}{2}\end{bmatrix}=-1\begin{bmatrix}-2\\ \frac{1}{2}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}2\\ \frac{-1}{2}\end{bmatrix} \qquad \because X=\begin{bmatrix}x\\ y\end{bmatrix}$$

By definition of equal matrices their corresponding elements are equal

So, $x=2,\quad y=\frac{-1}{2}$

Hence the solution set $=\left\{\left(2,\frac{-1}{2}\right)\right\}$

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iv). $x-2y-1=0,\quad 2x+y+3=0$

Solution: Giventhe system of linear equations

$$\begin{cases}x-2y-1=0\\ 2x+y+3=0\end{cases} \Rightarrow \begin{cases}x-2y=1\\ 2x+y=-3\end{cases}$$

These equations can be written in form of matrices as;

$$\begin{bmatrix}1 & -2\\ 2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1\\ -3\end{bmatrix}$$

Let $AX=B$ pre multiply by A^{-1}

$$A^{-1}AX=A^{-1}B$$
$$IX=A^{-1}B$$
$$X=A^{-1}B$$

Where $A=\begin{bmatrix}1 & -2\\ 2 & 1\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\end{bmatrix}$, $B=\begin{bmatrix}1\\ -3\end{bmatrix}$

$$|A|=\begin{vmatrix}1 & -2\\ 2 & 1\end{vmatrix}=(1)(1)-(-2)(2)$$

$$|A|=1+4=5\neq 0$$

Therefore A is non- singular, so we can find A^{-1}

Now $adjA=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}$

As $A^{-1}=\frac{1}{|A|}adjA$

$$A^{-1}=\frac{1}{5}\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}\cdots(1)$$

Now $AX=B$

$\Rightarrow X=A^{-1}B$

$$X=\frac{1}{5}\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}\begin{bmatrix}1\\ -3\end{bmatrix}$$

$$X=\frac{1}{5}\begin{bmatrix}(1)(1)+(2)(-3)\\ (-2)(1)+(1)(-3)\end{bmatrix}$$

$$X=\frac{1}{5}\begin{bmatrix}1-6\\ -2-3\end{bmatrix}=\begin{bmatrix}\frac{-5}{5}\\ \frac{-5}{5}\end{bmatrix}=\begin{bmatrix}-1\\ -1\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-1\\ -1\end{bmatrix} \qquad \because X=\begin{bmatrix}x\\ y\end{bmatrix}$$

By definition of equal matrices their corresponding elements are equal

So, $x=-1,\ y=-1$

Hence the solution set $=\{(-1,-1)\}$

Q2. Solve the following system of linear equations using Cramer's rule.

i). $x-2y=5,\qquad 2x-y=6$

Solution: Giventhe system of linear equations

$$x-2y=5$$
$$2x-y=6$$

These equations can be written in form of matrices as;

$$\begin{bmatrix}1 & -2\\ 2 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}5\\ 6\end{bmatrix}$$

Let $A=\begin{bmatrix}1 & -2\\ 2 & -1\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\end{bmatrix}$, $B=\begin{bmatrix}5\\ 6\end{bmatrix}$

$$|A|=\begin{vmatrix}1 & -2\\ 2 & -1\end{vmatrix}=(1)(-1)-(-2)(2)$$

$$|A|=-1+4=3\neq 0$$

Therefore A is non- singular, so solution exists

Replacing coefficients of x in A by the matrix B

$$A_X=\begin{bmatrix}5 & -2\\ 6 & -1\end{bmatrix}$$

$$|A_X|=\begin{vmatrix}5 & -2\\ 6 & -1\end{vmatrix}=(5)(-1)-(-2)(6)$$

$$|A_X|=-5+12=7$$

Replacing coefficients of y in A by the matrix B

$$A_Y=\begin{bmatrix}1 & 5\\ 2 & 6\end{bmatrix}$$

$|A_Y| = \begin{vmatrix} 1 & 5 \\ 2 & 6 \end{vmatrix} = (1)(6)-(2)(5)$

$|A_Y| = 6-10 = -4$

Now $\therefore x = \dfrac{|A_X|}{|A|} = \dfrac{7}{3}$ *and* $\therefore y = \dfrac{|A_Y|}{|A|} = \dfrac{-4}{3}$

$x = \frac{7}{3}$ $\quad$ $y = \frac{-4}{3}$

Hence the solution set $= \left\{\left(\frac{7}{3}, -\frac{4}{3}\right)\right\}$

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ii). $4x+3y=-2, \quad x-2y=5$

Solution: Giventhe system of linear equations

$4x+3y=-2$

$x-2y=5$

These equations can be written in form of matrices as;

$\begin{bmatrix} 4 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$

Let $A = \begin{bmatrix} 4 & 3 \\ 1 & -2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$

$|A| = \begin{vmatrix} 4 & 3 \\ 1 & -2 \end{vmatrix} = (4)(-2)-(1)(3)$

$|A| = -8-3 = -11 \neq 0$

Therefore A is non- singular, so solution exists

Replacing coefficients of x in A by the matrix B

$A_X = \begin{bmatrix} -2 & 3 \\ 5 & -2 \end{bmatrix}$

$|A_X| = \begin{vmatrix} -2 & 3 \\ 5 & -2 \end{vmatrix} = (-2)(-2)-(3)(5)$

$|A_X| = 4-15 = -11$

Replacing coefficients of y in A by the matrix B

$A_Y = \begin{bmatrix} 4 & -2 \\ 1 & 5 \end{bmatrix}$

$|A_Y| = \begin{vmatrix} 4 & -2 \\ 1 & 5 \end{vmatrix} = (4)(5)-(-2)(1)$

$|A_Y| = 20+2 = 22$

Now

$\therefore x = \dfrac{|A_X|}{|A|} = \dfrac{-11}{-11}$ *and* $\therefore y = \dfrac{|A_Y|}{|A|} = \dfrac{22}{-11}$

$x = 1$ $\quad$ $y = -2$

Hence the solution set $= \{(1,-2)\}$

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iii). $5x+7y=3, \quad 3x+y=5$

Solution: Giventhe system of linear equations

$5x+7y=3$

$3x+y=5$

These equations can be written in form of matrices as;

$\begin{bmatrix} 5 & 7 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

Let $A = \begin{bmatrix} 5 & 7 \\ 3 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$|A| = \begin{vmatrix} 5 & 7 \\ 3 & 1 \end{vmatrix} = (5)(1)-(7)(3)$

$|A| = 5-21 = -16 \neq 0$

Therefore A is non- singular, so solution exists

Replacing coefficients of x in A by the matrix B

$A_X = \begin{bmatrix} 3 & 7 \\ 5 & 1 \end{bmatrix}$

$|A_X| = \begin{vmatrix} 3 & 7 \\ 5 & 1 \end{vmatrix} = (3)(1)-(7)(5)$

$|A_X| = 3-35 = -32$

Replacing coefficients of y in A by the matrix B

$A_Y = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$

$|A_Y| = \begin{vmatrix} 5 & 3 \\ 3 & 5 \end{vmatrix} = (5)(5)-(3)(3)$

$|A_Y| = 25-9 = 16$

Now

$\therefore x = \dfrac{|A_X|}{|A|} = \dfrac{-32}{-16}$ *and* $\therefore y = \dfrac{|A_Y|}{|A|} = \dfrac{16}{-16}$

$x = 2$ $\quad$ $y = -1$

Hence the solution set $= \{(2,-1)\}$

Q3. Amjad thought of two numbers whose sum is 12 and whose difference is 4. Find the numbers

Solution: Let the first number = x

And the second number = y

From the first fact $x+y=12$

From the second fact $x-y=4$

These equations can be written as matrices from

$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 4 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 12 \\ 4 \end{bmatrix}$ Now

$|A| = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = (1)(-1)-(1)(1)$

$|A| = -1-1$

$|A| = -2 \neq 0$

$\therefore A^{-1}$ exist and using $adj\ A = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$

$\because AX = B$

$\Rightarrow \therefore A^{-1}AX = A^{-1}B$

$\Rightarrow IX = A^{-1}B \quad (\because A^{-1}A = I)$

$X = A^{-1}B \quad (\because IX = X)$

Putting the values

$X = \dfrac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 4 \end{bmatrix}$

$X = \dfrac{1}{-2}\begin{bmatrix} (-1)(12)+(-1)(4) \\ (-1)(12)+(1)(4) \end{bmatrix}$

$$X=\frac{1}{-2}\begin{bmatrix}-12-4\\-12+4\end{bmatrix}=\frac{1}{-2}\begin{bmatrix}-16\\-8\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}8\\4\end{bmatrix}$$

By definition of equal matrices their corresponding elements are equal

So, $x=8,\quad y=4$

the first number $x=8$

And the second number $y=4$

Q4. The length of rectangular playground is twice its width. The perimeter is 30 find its dimensions

Solution: Let the width of rectangle = x

And the Length of rectangle = y

From the first fact $y=2x$

$$2x-y=0 \quad ....(1)$$

From the second fact $2(x+y)=30$

$$x+y=15 \quad ....(2)$$

These equations can be written as matrices from

$$\begin{bmatrix}2&-1\\1&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\15\end{bmatrix}$$

Let $A=\begin{bmatrix}2&-1\\1&1\end{bmatrix}, X=\begin{bmatrix}x\\y\end{bmatrix}, B=\begin{bmatrix}0\\15\end{bmatrix}$

$$|A|=\begin{vmatrix}2&-1\\1&1\end{vmatrix}=(2)(1)-(-1)(1)$$

$$|A|=2+1$$

$$|A|=3\neq 0$$

$\therefore A^{-1}$ exist and using $adj\ A=\begin{bmatrix}1&1\\-1&2\end{bmatrix}$

$\because AX=B$

$\Rightarrow \therefore A^{-1}AX=A^{-1}B$

$\Rightarrow\ IX=A^{-1}B \qquad (\because A^{-1}A=I)$

$X=A^{-1}B \qquad (\because IX=X)$

Putting the values

$$X=\frac{1}{3}\begin{bmatrix}1&1\\-1&2\end{bmatrix}\begin{bmatrix}0\\15\end{bmatrix}$$

$$X=\frac{1}{3}\begin{bmatrix}(1)(0)+(1)(15)\\(-1)(0)+(2)(15)\end{bmatrix}$$

$$X=\frac{1}{3}\begin{bmatrix}0+15\\0+30\end{bmatrix}=\frac{1}{3}\begin{bmatrix}15\\30\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}5\\10\end{bmatrix}$$

By definition of equal matrices their corresponding elements are equal So, $x=5,\quad y=10$

the width of rectangle $x=5$

And the Length of rectangle $y=10$

Q5. 3 bags and 4 pens together cost 257 rupees whereas 4 bags and 3 pens together cost 324 rupees. Find the cost of a bag and 10 pens

Solution: Let cost of Bag = x

And cost of Pen = y

From the first fact $3x+4y=257$

From the second fact $4x+3y=324$

These equations can be written as matrices from

$$\begin{bmatrix}3&4\\4&3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}257\\324\end{bmatrix}$$

Let $A=\begin{bmatrix}3&4\\4&3\end{bmatrix}, X=\begin{bmatrix}x\\y\end{bmatrix}, B=\begin{bmatrix}257\\324\end{bmatrix}$

$$|A|=\begin{vmatrix}3&4\\4&3\end{vmatrix}=(3)(3)-(4)(4)$$

$$|A|=9-16$$

$$|A|=-7\neq 0$$

$\therefore A^{-1}$ exist and using $adj\ A=\begin{bmatrix}3&-4\\-4&3\end{bmatrix}$

$\because AX=B$

$\Rightarrow \therefore A^{-1}AX=A^{-1}B$

$\Rightarrow\ IX=A^{-1}B \qquad (\because A^{-1}A=I)$

$X=A^{-1}B \qquad (\because IX=X)$

Putting the values

$$X=\frac{1}{-7}\begin{bmatrix}3&-4\\-4&3\end{bmatrix}\begin{bmatrix}257\\324\end{bmatrix}$$

$$X=\frac{1}{-7}\begin{bmatrix}(3)(257)+(-4)(324)\\(-4)(257)+(3)(324)\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-7}\begin{bmatrix}-525\\-56\end{bmatrix}=\begin{bmatrix}75\\8\end{bmatrix}$$

By definition of equal matrices their corresponding elements are equal So, $x=75,\ y=8$

cost of Bag $x=75$

And cost of Pen $y=8$

Q6.If twice the son's age in years is added to the father's age, sum is 70 but if twice the father's age is added to the son's age the sum is 55. Find the ages of the father and son.

Solution: Let Son's Age = x

And Father age = y

From the first fact $2x+y=70$

From the second fact $x+2y=95$

These equations can be written as matrices from

$$\begin{bmatrix}2&1\\1&2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}70\\95\end{bmatrix}$$

Let $A=\begin{bmatrix}2&1\\1&2\end{bmatrix}, X=\begin{bmatrix}x\\y\end{bmatrix}, B=\begin{bmatrix}70\\95\end{bmatrix}$

$$|A|=\begin{vmatrix}2&1\\1&2\end{vmatrix}=(2)(2)-(1)(1)$$

$$|A|=4-1$$

$$|A|=3\neq 0$$

$\therefore A^{-1}$ exist and using $adj\ A=\begin{bmatrix}2&-1\\-1&2\end{bmatrix}$

$\because AX=B$

$\Rightarrow \therefore A^{-1}AX = A^{-1}B$

$\Rightarrow IX = A^{-1}B \qquad (\because A^{-1}A = I)$

$X = A^{-1}B \qquad (\because IX = X)$

Putting the values

$$X = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 70 \\ 95 \end{bmatrix}$$

$$X = \frac{1}{3}\begin{bmatrix} (2)(70)+(-1)(95) \\ (-1)(70)+(2)(95) \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 140-95 \\ -70+190 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 45 \\ 120 \end{bmatrix} = \begin{bmatrix} 15 \\ 40 \end{bmatrix}$$

By definition of equal matrices their corresponding elements are equal So, $x = 15,\ y = 40$

Son's Age $x = 15$

And Father age $y = 40$

Review Exercise # 1

Q1. Choose the correct answer in each of the following problems.

i). $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is

a). an identity matrix w.r.t. multiplication
b). an identity matrix w.r.t addition
c). a column matrix d). a row matrix

ii). The matrix $\begin{bmatrix} 4 & 0 \\ 0 & 12 \end{bmatrix}$ is

a). Scalar b). 2 by 3
c). Diagonal d). None of these

iii). If $A = \begin{bmatrix} -1 & -2 \\ 3 & 1 \end{bmatrix}$ then adj A is equal to

a). $\begin{bmatrix} -1 & -2 \\ 3 & 1 \end{bmatrix}$ b). $\begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix}$

c). $\begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$ d). $\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$

iv). If $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ then A^{-1} equals

a). $\begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix}$ b). $\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}$

c). $\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$ d). $\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$

v). For what value of d is the 2×2 matrix $\begin{bmatrix} 5 & 1.5 \\ 2 & d \end{bmatrix}$ not invertible?

a). -0.6 b). 0
c). 0.6 c). 3

vi). Suppose A and B are 2×5 matrices. What of the are the dimensions of the matrx $A+B$?

a). 2×5 b). 10×10
c). 7×1 d). 7×7

viii). which of following is multiplicative inverse of $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

a). $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ b). $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

c). $\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$ d). $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$

viii). Evaluate the determinant of $\begin{bmatrix} 4 & -1 \\ -9 & 2 \end{bmatrix}$

a). 17 b). 1
c). -1 c). -17

Q2. Find x and y when

$$\begin{bmatrix} x-1 & 4 \\ y+3 & -7 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -2 & -7 \end{bmatrix}$$

Solution: Given $\begin{bmatrix} x-1 & 4 \\ y+3 & -7 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -2 & -7 \end{bmatrix}$

By definition of equal matrices their corresponding elements are equal

$x-1=0 \qquad y+3=-2$

$x=0+1 \qquad y=-2-3$

$x=1 \qquad y=-5$

Q3. Find the product if possible

$$\begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}\begin{bmatrix} -6 & 5 & 8 \\ 0 & 4 & -1 \end{bmatrix}$$

Solution: Given $\begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}\begin{bmatrix} -6 & 5 & 8 \\ 0 & 4 & -1 \end{bmatrix}$

First matrix have 1 columns $\neq$ second matrix have 2 rows
Therefore product is not possible

Q4. Find the inverse of the matrix

$$A = \begin{bmatrix} 6 & -3 \\ 5 & -2 \end{bmatrix}$$

Solution: Given $A = \begin{bmatrix} 6 & -3 \\ 5 & -2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 6 & -3 \\ 5 & -2 \end{vmatrix} = (6)(-2)-(-3)(5)$$

$|A| = -12+15$

$|A| = 3 \neq 0$

$\therefore A^{-1}$ exists

Now $adj\ A = \begin{bmatrix} -2 & 3 \\ -5 & 6 \end{bmatrix}$ using $A^{-1} = \frac{1}{|A|}adj\ A$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} -2 & 3 \\ -5 & 6 \end{bmatrix}$$

Q5. Solve the system $\begin{matrix} 2x+5y=9 \\ 5x-2y=8 \end{matrix}$

Solution: Given $\begin{matrix} 2x+5y=9 \\ 5x-2y=8 \end{matrix}$ given system can be written as matrices form $\begin{bmatrix} 2 & 5 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \end{bmatrix}$

Let $A=\begin{bmatrix}2 & 5\\ 5 & -2\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}9\\ 8\end{bmatrix}$

Now $|A|=\begin{vmatrix}2 & 5\\ 5 & -2\end{vmatrix}=(2)(-2)-(5)(5)$

$|A|=-4-25$

$|A|=-29\neq 0$

$\therefore A^{-1}$ exist and using $adj\ A=\begin{bmatrix}-2 & -5\\ -5 & 2\end{bmatrix}$

$\because AX=B$

$\Rightarrow \therefore A^{-1}AX=A^{-1}B$

$\Rightarrow IX=A^{-1}B \qquad (\because A^{-1}A=I)$

$X=A^{-1}B \qquad (\because IX=X)$

Putting the values

$$X=\frac{1}{-29}\begin{bmatrix}-2 & -5\\ -5 & 2\end{bmatrix}\begin{bmatrix}9\\ 8\end{bmatrix}$$

$$X=\frac{1}{-29}\begin{bmatrix}(-2)(9)+(-5)(8)\\ (-5)(9)+(2)(8)\end{bmatrix}=\frac{1}{-29}\begin{bmatrix}-18-40\\ -45+16\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-29}\begin{bmatrix}-58\\ -29\end{bmatrix}=\begin{bmatrix}2\\ 1\end{bmatrix}$$

By definition of equal matrices their corresponding elements are equal So, $x=2, \quad y=1$

Solution set $=\{(2,1)\}$

Q6. Qasim and farzana are selling fruit for a school fundraiser. Customers can buy small boxes of oranges and large boxes of oranges. Qasim sold 3 small boxes of oranges and 14 large boxes of oranges for a total of Rs 203. Farzana sold 11 small boxes of oranges and 11 large boxes of oranges for a total of Rs 220. Find the cost of each one small box of oranges and one large box of oranges..

Solution: Let the cost of one small box of oranges = x
and one large box of oranges = y

From the first fact $3x+14y=203$

From the second fact $11x+11y=220$

These equations can be written as matrices from

$$\begin{bmatrix}3 & 14\\ 11 & 11\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}203\\ 220\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & 14\\ 11 & 11\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}203\\ 220\end{bmatrix}$

$|A|=\begin{vmatrix}3 & 14\\ 11 & 11\end{vmatrix}=(3)(11)-(14)(11)$

$|A|=33-154$

$|A|=-121$

$\therefore A^{-1}$ exist and using $adj\ A=\begin{bmatrix}11 & -14\\ -11 & 3\end{bmatrix}$

$\because AX=B$

$\Rightarrow \therefore A^{-1}AX=A^{-1}B$

$\Rightarrow IX=A^{-1}B \qquad (\because A^{-1}A=I)$

$X=A^{-1}B \qquad (\because IX=X)$

Putting the values

$$X=\frac{1}{-121}\begin{bmatrix}11 & -14\\ -11 & 3\end{bmatrix}\begin{bmatrix}203\\ 220\end{bmatrix}$$

$$X=\frac{1}{-121}\begin{bmatrix}(11)(203)+(-14)(220)\\ (-11)(203)+(3)(220)\end{bmatrix}$$

$$X=\frac{1}{-121}\begin{bmatrix}2233-3080\\ 2233+660\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-121}\begin{bmatrix}-847\\ -1573\end{bmatrix}=\begin{bmatrix}7\\ 13\end{bmatrix}$$

By definition of equal matrices their corresponding elements are equal So, $x=7, \quad y=13$

cost of one small box of oranges $x=7$ rupees
and one large box of oranges $y=13$ rupees