Chapter 3

Logarithms

Scientific notation: A positive number is represented as a product of two numbers. First number lies between 1 and 10 & second one is some integral power of 10. i.e. A number x in **scientific notation** is written as $x = a \times 10^m$ where $1 \le a \le 10$ and m is an integer.

Reference Position: The place between the first non-zero digit and its next digit on the left side of a given number is called the **reference position**. It is represented by the symbol "$\wedge$". For example, in 323.5 is the first non-zero digit on the left is 3. so the reference position between 3 and 2 is represented as $3_\wedge 23.5$. We count the number of digits between the reference position and the decimal point, to take the characteristics with the help of reference position. If the decimal point is on the **right side** of the reference position, then the **characteristic** will be **positive** and the decimal point is on the **left side** of the reference position, then the **characteristic** will be **negative**.

Example: convert the following into scientific notation:

i). 7800

Solution: given number 7800

$$7_\wedge 800. = 7.8 \times 10^3$$

ii). 0.00729

Solution: given number 0.00729

$$0.007_\wedge 29 = 7.29 \times 10^{-3}$$

iii). 674 000 000

Solution: given number 674 000 000

$$6_\wedge 74\ 000\ 000 = 6.74 \times 10^8$$

iv). 0.000 003 27

Solution: given number 0.000 003 27

$$0.000\ 003_\wedge 27 = 3.27 \times 10^{-6}$$

v). 0.25 x 10^{-2}

Solution: given number 0.25 x 10^{-2}

$$0.2_\wedge 5 \times 10^{-2} = 2.5 \times 10^{-1} \times 10^{-2}$$

$$= 2.5 \times 10^{-1-2}$$

$$= 2.5 \times 10^{-3}$$

iv). 42.5 x 10^2

Solution: given number 42.5 x 10^2

$$4_\wedge 2.5 \times 10^2 = 4.25 \times 10^1 \times 10^2$$

$$= 4.25 \times 10^{1+2}$$

$$= 4.25 \times 10^3$$

Example: convert the following into standard notation:

i). 4.56 x 10^7

Solution: given 4.56 x 10^7 = $4_\wedge 5\ 600\ 000.$

= 45 600 000

ii). 8.92 x 10^{-5}

Solution: given 8.92 x 10^{-5} = $0.000\ 08_\wedge 9\ 2$

= 0.0000892

Example: how many mile does light travel in 1 day? Speed of light is 186 000mi/sec. write the answer in scientific notation.

Solution: here speed v = 186 000 mi/sec

Time = 1 day =24 hours

=1 x 24 hours since 1 hour = 60 minutes

= 24 x 60 minutes

= 1440 minutes since 1 minute = 60 seconds

= 1440 x 60 seconds

= 86400 seconds

Distance travel S = v t

S = 186 000 x 86 400

S = 16 070 400 000

S = $1_\wedge 6\ 070\ 400\ 000.$

S = 1.607 04 x 10^{10}Miles in 1 day

Exercise 3.1

Q1. Write each number in Scientific Notation.

i). 405 000

Solution: given number 405 000

$4_\wedge 05\ 000$ = 4.05 x 10^5

ii). 1 670 000

Solution: given number 1 670 000

$1_\wedge 670\ 000$ = 1.67 x 10^6

iii). 0.000 000 39

Solution: given number 0.000 000 39

$0.000\ 000\ 3_\wedge 9$ = 3.9 x 10^{-7}

iv). 0.000 92

Solution: given number 0.000 92

$0.000\ 9_\wedge 2$ = 9.2 x 10^{-4}

v). 234 600 000 000

Solution: given number 234 600 000 000

$2_\wedge 34\ 600\ 000\ 000$ = 2.346 x 10^{11}

vi). 8 904 000 000

Solution: given number 8 904 000 000

$8_\wedge 904\ 000\ 000$ = 8.904 x 10^9

vii). 0.001 04

Solution: given number 0.001 04

0.001$_\wedge$04 = 1.04 x 10^{-3}

viii). 0.000 000 005 14

Solution: given number 0.000 000 005 14

0.000 000 005$_\wedge$14= 5.14 x 10^{-9}

ix). 0.05 x 10^{-3}

Solution: given number 0.05 x 10^{-3}

0.05$_\wedge$ x 10^{-3}= 5. x 10^{-2} x 10^{-3}

0.05 x 10^{-3}= 5. x 10^{-2-3}

0.05 x 10^{-3}= 5. x 10^{-5}

Q2. Write each number in standard notation

i). 8.3 x 10^{-5}

Solution: Here 8.3 x 10^{-5}

= 0.00008$_\wedge$3

ii). 4.1 x 10^{6}

Solution: Here 4.1 x 10^{6}

= 4$_\wedge$100 000.

iii). 2.07 x 10^{7}

Solution: Here 2.07 x 10^{7}

2$_\wedge$0 700 000.

iv). 3.15 x 10^{-6}

Solution: Here 3.15 x 10^{-6}

= 0. 000 003$_\wedge$1 5

v). 6.27 x 10^{-10}

Solution: Here 6.27 x 10^{-10}

= 0. 000 000 000 6$_\wedge$27

vi). 5.41 x 10^{-8}

Solution: Here 5.41 x 10^{-8}

= 0. 000 000 05$_\wedge$4 1

vii). 7.632 x 10^{-4}

Solution: Here 7.632 x 10^{-4}

= 0.000 7$_\wedge$63 2

viii). 9.4 x 10^{5}

Solution: Here 9.4 x 10^{5}

= 9$_\wedge$40 000.

ix). -2.6 x 10^{9}

Solution: Here -2.6 x 10^{9}

= -2$_\wedge$600 000 000.

Q3. How long does it takes the light to travel the Erath from the sun? the sun is 9.3 x 10^{7} mil from the Erath. And the light travels 1.86 x 10^{5} mi/sec

Solution: Given distance from sun and Erath S = 9.3 x 10^{7}

Speed of light v = 1.86 x 10^{5} mi/sec

Using formula S = v t

Putting values 9.3 x 10^{7}= 1.86 x 10^{5} t

$$t=\frac{9.3\times10^{7}}{1.86\times10^{5}}$$

$$t=5\times10^{7-5}$$

$$t=5\times10^{2}=500\,\text{sec}$$

$t=\frac{500}{60}=\frac{25}{3}$ minutes

$t=8\frac{1}{3}$ minutes

$t=8$ minutes $\frac{1}{3}\times 60$ seconds

$t=8$ minutes 20 seconds

Logarithm: If $a^{y}=x$ where $a,x,y\in R$, with $a,x>0$ and $a\neq1$, then y is called logarithm of x to he base a . i.e. $a^{y}=x$, $\Rightarrow y=\log_{a}x$ and read as y is equal to $\log x$ to the base a.

Example: write following logarithmic form.

i). $2^{4}=16$

Solution: Given $2^{4}=16$

Logarithmic form $\log_{2}16=4$

ii). $4^{-3}=\frac{1}{64}$

Solution: Given $4^{-3}=\frac{1}{64}$

Logarithmic form $\log_{4}\frac{1}{64}=-3$

Example: Write the following in exponential form.

i). $\log_{8}(64)=2$

Solution: Given $\log_{8}(64)=2$

Exponential form $8^{2}=64$

ii). $\log_{3}\left(\frac{1}{9}\right)=-2$

Solution: Given $\log_{3}\left(\frac{1}{9}\right)=-2$

Exponential form $3^{-2}=\frac{1}{9}$

Example: find x from the following

i). Find x when $\log_{x}(100)=2$

Solution: Given $\log_{x}(100)=2$

Exponential form $x^{2}=100$

$x^{2}=10^{2}$

$\Rightarrow \quad x=10$

ii). Find x when $\log_{12}(12^{3})=x$

Solution: Given $\log_{12}(12^{3})=x$

Exponential form $12^{x}=12^{3}$

$\Rightarrow \quad x=3$

iii). Find x when $\log_x\left(\frac{4}{9}\right)=2$

Solution: Given $\log_x\left(\frac{4}{9}\right)=2$

Exponential form $x^2=\frac{4}{9}$

$$x^2=\left(\frac{2}{3}\right)^2$$

$$\Rightarrow \quad x=\frac{2}{3}$$

Exercise 3.2

Q1. write the following logarithmic form.

i). Convert into logarithmic form $4^4=256$

Solution: Given $4^4=256$

Logarithmic form $\log_4(256)=4$

ii). Convert into logarithmic form $2^{-6}=\frac{1}{64}$

Solution: Given $2^{-6}=\frac{1}{64}$

Logarithmic form $\log_2\left(\frac{1}{64}\right)=-6$

iii). Convert into logarithmic form $10^0=1$

Solution: Given $10^0=1$

Logarithmic form $\log_{10}(1)=0$

iv). Convert into logarithmic form $x^{\frac{3}{4}}=y$

Solution: Given $x^{\frac{3}{4}}=y$

Logarithmic form $\log_x(y)=\frac{3}{4}$

v). Convert into logarithmic form $3^{-4}=\frac{1}{81}$

Solution: Given $3^{-4}=\frac{1}{81}$

Logarithmic form $\log_3\left(\frac{1}{81}\right)=-4$

vi. Convert into logarithmic form $64^{\frac{2}{3}}=16$

Solution: Given $64^{\frac{2}{3}}=16$

Logarithmic form $\log_{64}(16)=\frac{2}{3}$

Q2. Write the following in exponential form.

i). Convert into exponential form $\log_a\left(\frac{1}{a^2}\right)=-2$

Solution: Given $\log_a\left(\frac{1}{a^2}\right)=-2$

Exponential form $a^{-2}=\frac{1}{a^2}$

ii).Convert into exponential form $\log_2\left(\frac{1}{128}\right)=-7$

Solution: Given $\log_2\left(\frac{1}{128}\right)=-7$

Exponential form $2^{-7}=\frac{1}{128}$

iii). Convert into exponential form $\log_b(3)=64$

Solution: Given $\log_b(3)=64$

Exponential form $b^{64}=3$

iv). Convert into exponential form $\log_a(a)=1$

Solution: Given $\log_a(a)=1$

Exponential form $a^1=a$

v). Convert into exponential form $\log_a(1)=0$

Solution: Given $\log_a(1)=0$

Exponential form $a^0=1$

vi). Convert into exponential form $\log_4\left(\frac{1}{8}\right)=\frac{-3}{2}$

Solution: Given $\log_4\left(\frac{1}{8}\right)=\frac{-3}{2}$

Exponential form $4^{\frac{-3}{2}}=\frac{1}{8}$

Q3:i). Find x when $\log_{\sqrt{5}}(125)=x$

Solution: Given $\log_{\sqrt{5}}(125)=x$

Exponential form $\left(\sqrt{5}\right)^x=125$

$$5^{\frac{x}{2}}=5^3$$

$$\Rightarrow \quad \frac{x}{2}=3$$

$$x=6$$

Q3:ii). Find x when $\log_4(x)=-3$

Solution: Given $\log_4(x)=-3$

Exponential form $4^{-3}=x$

$$x=\frac{1}{4^3}$$

$$\Rightarrow \quad x=\frac{1}{64}$$

Q3:iii). Find x when $\log_{81}(9)=x$

Solution: Given $\log_{81}(9)=x$

Exponential form $81^x=9$

$$9^{2x}=9^1$$

$$\Rightarrow \quad 2x=1$$

$$x=\frac{1}{2}$$

Q3:iv). Find x when $\log_3(5x+1)=2$

Solution: Given $\log_3(5x+1)=2$

Exponential form $3^2=5x+1$

$$9-1=5x$$

$$8=5x$$

$$\Rightarrow \qquad x=\frac{8}{5}$$

Q3: v). Find x when $\log_2(x)=7$

Solution: Given $\log_2(x)=7$

Exponential form $2^7=x$

$$\Rightarrow \qquad x=128$$

Q3:vi). Find x when $\log_x(0.25)=2$

Solution: Given $\log_x(0.25)=2$

Exponential form $x^2=0.25$

$$x^2=0.5^2$$

$$\Rightarrow \qquad x=0.5$$

Q3:vii). Find x when $\log_x(0.001)=-3$

Solution: Given $\log_x(0.001)=-3$

Exponential form $x^{-3}=0.001$

$$\frac{1}{x^3}=\frac{1}{1000}$$

$$\frac{1}{x^3}=\frac{1}{10^3}$$

$$\Rightarrow \qquad x=10$$

Q3:viii). Find x when $\log_x\left(\frac{1}{64}\right)=-2$

Solution: Given $\log_x\left(\frac{1}{64}\right)=-2$

Exponential form $x^{-2}=\frac{1}{64}$

$$\frac{1}{x^2}=\frac{1}{8^2}$$

$$\Rightarrow \qquad x=8$$

Q3:ix). Find x when $\log_{\sqrt{3}}(x)=16$

Solution: Given $\log_{\sqrt{3}}(x)=16$

Exponential form $\left(\sqrt{3}\right)^{16}=x$

$$x=3^{\frac{16}{2}}=3^8$$

$$\Rightarrow \qquad x=6561$$

Common Logarithm: This logarithm was invented by a British Mathematician Prof. Henry Biggs (1560-1631)

The **logarithm to the base 10** is called the **Common logarithm or Briggs logarithm**. At the time of writing common logarithm, we will not mention the base and it will be considered equal to 10.

Characteristic and Mantissa: As we know that any **positive number x** can be written in scientific notation as $x=a\times10^m$ where $1\le a\le10$ and m is an integer. Thus **logarithm of any positive number** x can be written as sum of **two parts**. **One part is m**, an integer and the **second part** is log, the logarithm of a number **between 1 and 10**.

The integer **m** is called the **characteristic** of logarithm and number log is called **mantissa**.

Example: Write the characteristics of the following logarithms

i). $\log$ 4350

Solution: Given $\log 4_{\wedge}350$.

characteristic is 3

ii). $\log$ 435

Solution: Given $\log 4_{\wedge}35$.

characteristic is 2

iii). $\log$ 43.5

Solution: Given $\log\ 4_{\wedge}3.5$

characteristic is 1

iv). $\log$ 4.35

Solution: Given $\log 4_{\wedge}.35$

characteristic is 0

v). $\log$ 0.435

Solution: Given $\log 0.4_{\wedge}35$

characteristic is -1

vi). $\log$ 0.0435

Solution: Given $\log 0.04_{\wedge}35$

characteristic is -2

vii). $\log$ 0.00435

Solution: Given $\log 0.004_{\wedge}35$

characteristic is -3

viii). $\log$ 0.000435

Solution: Given $\log 0.0004_{\wedge}35$

characteristic is -4

Example: Find logarithm of 763.5

Sol: Suppose that x=763.5

Taking log on both sides

Log x = log763.5

Characteristic =2

Mantissa =0.8828

So Log x =2.8828

Scientific form of 763.5= 7.635 x 10^2

with calculator

log 7.635 =0.8828

Exercise No 3.3

Q1. Find characteristics of the common logarithm of each of the following

i). 57

Solution: Given $5_{\wedge}7$

characteristic is 1

ii). 7.4

Solution: Given $7_{\wedge}.4$

characteristic is 0

iii). 56.3

Solution: Given $5_{\wedge}6.3$

characteristic is 1

iv). 5.63

Solution: Given 5 $_\wedge$ .63

characteristic is 0

v). 982.5

Solution: Given 9 $_\wedge$ 82.5

characteristic is 2

vi). 7824

Solution: Given 7 $_\wedge$ 824

characteristic is 3

vii). 186 000

Solution: Given 1 $_\wedge$ 86 000

characteristic is 5

viii). 0.71

Solution: Given 0.7 $_\wedge$ 1

characteristic is -1

Q2. Find the following.

i). log 87.2

Sol: Suppose that x=87.2
Taking log on both sides
Log x = log 87.2
Characteristic = 1
Mantissa =0.9405
So Log x =1.9405

Scientific form of 87.2= 8.72 x 10^1 with calculator log 87.2 =0.9405

ii). Log 373 00

Sol: Let x=37300
Taking log on both sides
Log x = log 37 300
Characteristic = 4
Mantissa =0.5717
So Log x = 4.5717

Scientific form of 37300=3.73 x 10^4 with calculator log 3.73 =0.5717

iii). Log 573

Sol: Suppose that x= 573
Taking log on both sides
Log x = log 573
Characteristic = 2
Mantissa =0.7582
So Log x = 2.7582

Scientific form of 573 = 5.73 x 10^2 with calculator log 5.73 =0.7582

iv). Log 9.21

Sol: Suppose that x= 9.21
Taking log on both sides
Log x = log 9.21
Characteristic = 0
Mantissa = 0.9643
So Log x = 0.9643

Scientific form of 9.21= 9.21 x 10^0 with calculator log 9.21 = 0.9643

v). log 0.00159

Sol: Let x= 0.00159
Taking log on both sides
Log x = log 0.00159
Characteristic = - 3
Mantissa = 0.2014
So Log x = $\bar{3}$.2014

Scientific form of 0.00159=1.59 x 10^{-3} with calculator log 1.59 = 0.2014

vi). Log 0.0256

Sol: Let x= 0.0256
Taking log on both sides
Log x = log 0.0256
Characteristic = - 2
Mantissa = 0.4082
So Log x = $\bar{2}$.4082

Scientific form of 0.0256=2.56 x 10^{-2} with calculator log 2.56 = 0.4082

vii). Log 6.753

Sol: Suppose that x=6.753
Taking log on both sides
Log x = log 6.753
Characteristic = 0
Mantissa = 0.8295
So Log x = 0.8295

Scientific form of 6.753= 6.753 x 10^0 with calculator log 6.753 = 0.8295

Q3. Find logarithm of the following numbers.

i). 2476

Sol: Suppose that x=2476
Taking log on both sides
Log x = log 2476
Characteristic = 3
Mantissa = 0.3938
So Log x = 3.3938

Scientific form of 2476=2.476 x 10^3 with calculator log 2.476 = 0.3938

ii). 2.4

Sol: Suppose that x=2.4
Taking log on both sides
Log x = log 2.4
Characteristic = 0
Mantissa = 0.3802
So Log x = 0. 3802

Scientific form of 2.4 is 2.4 x 10^0 with calculator log 2.4 = 0. 3802

iii). 92.5

Sol: Suppose that x=92.5
Taking log on both sides
Log x = log 92.5
Characteristic = 1
Mantissa = 0. 9661
So Log x =1.9661

Scientific form of 92.5 is 9.25 x 10^1 with calculator log 9.25 = 0.9661

iv). 482.7

Sol: Suppose that x=482.7
Taking log on both sides
Log x = log 482.7
Characteristic = 2
Mantissa = 0.6837
So Log x =2.6837

Scientific form of 482.7 =4.827 x 10^2 with calculator log 4.827 = 0.6837

v). 0.783

Sol: Suppose that x=0.783
Taking log on both sides
Log x = log 0.783
Characteristic = -1
Mantissa = 0.8938
So Log x = $\bar{1}$.8938

Scientific form of 0.783 =7.83 x 10^{-1} with calculator log 7.83 = 0.8938

vi). 0.09566

Sol: Let x=0.09566

Scientific form of 0.09566=9.566 x 10^{-2}

Taking log on both sides
Log x = log 0.09566
Characteristic = -2
Mantissa = 0. 9807
So Log x = $\overline{2}$ .9807

with calculator
log 9.566 = 0.9807

vii). 0.006735
Sol: Let x=0.006735
Taking log on both sides
Log x = log 0.006735
Characteristic = -3
Mantissa = 0. 8283
So Log x = $\overline{3}$ .8283

Scientific form of 0.006735 is 6.735×10^{-3}
with calculator
log 6.735 = 0.8283

viii). 700
Sol: Suppose that x=700
Taking log on both sides
Log x = log 700
Characteristic = 2
Mantissa = 0. 8451
So Log x =2.8451

Scientific form of 700 is 7.0×10^2
with calculator
log 7 = 0.8451

Anti logarithm:
If $\log x = y$ then x is called the **anti-logarithm** of y and it is written as $x = anti\log y$

Example: Find the number whose logarithms
i). 2.3456
Sol: Let x=2.3456
Taking anti-log on b s
Antilog x = antilog 2.3456
Characteristic = 2
Mantissa = 2.216
Anti-Log x = 2.216×10^2
= 221.6

with calculator
anti-log 0.3456
= 2.216

ii). $\overline{2}.1576$
Sol: Let x= $\overline{2}.1576$
Taking anti-log on b s
Antilog x = antilog $\overline{2}.1576$
Characteristic = -2
Mantissa = 1.438
Anti-Log x = 1.438×10^{-2}
= 0.01438

with calculator
anti-log 0.1576
=1.438

Exercise No 3.4

Q1. Find anti-logarithm of the following numbers.
i). 1.2508
Sol: Let x=1.2508
Taking anti-log on b s
Antilog x = antilog 1.2508
Characteristic = 1
Mantissa = 1.782
Anti-Log x =1.782×10^1
=17.82

with calculator
anti-log 0.2508
=1.782

ii). 8401
Sol: Let x=0.8401
Taking anti-log on b s
Antilog x = anti-log 0.8401
Characteristic = 0
Mantissa = 6.920
Anti-Log x =6.920×10^0
=6.920

with calculator
anti-log 0.8401
=6.920

iii). 2.540
Sol: Let x=2.540
Taking anti-log on b s
Antilogx = antilog 2.540
Characteristic = 2
Mantissa = 3.467
Anti-Log x =3.467×10^2
=346.7

with calculator
anti-log 0.540
=3.467

iv). $\overline{2}$ .2508
Sol: Let x= $\overline{2}$ .2508
Taking anti-log on b s
Antilogx =antilog $\overline{2}$ .2508
Characteristic = -2
Mantissa = 1.782
Anti-Log x =1.782×10^{-2}
=0.01782

with calculator
anti-log 0.2508
=1.782

v). $\overline{1}$ .5463
Sol: Let x= $\overline{1}$ .5463
Taking anti-log on b s
Antilog x=antilog $\overline{1}$ .5463
Characteristic = -1
Mantissa = 3.518
Anti-Log x =3.518×10^{-1}
=0.3518

with calculator
anti-log 0.5463
=3.518

vi). 3.5526
Sol: Let x=3.5526
Taking anti-log on b s
Antilog x = antilog3.5526
Characteristic = 3
Mantissa = 3.569
Anti-Log x =3.569×10^3
=3569.

with calculator
anti-log 0.5526
=3.569

Q2.Find value of x from following equations:
i). Log x = $\overline{1}$ .8401
Solution: Suppose that
Log x= $\overline{1}$ .8401
Taking anti-log on b s
x=anti-log $\overline{1}$ .8401
Characteristic = -1
Mantissa = 6.920
So x = 6.920×10^{-1}
x = 0.6920

with calculator
anti-log .8401
=6.920

ii). Log x = 2.1931

Sol: Log x= 2.1931 with calculator
Taking anti-log on b s anti-log .1931
x=anti-log 2.1931 =1.560
Characteristic = 2
Mantissa = 1.560 So
x = 1.560×10^2
x = 156.0

iii). Log x = 4.5911

Sol: Log x= 4.5911 with calculator
Taking anti-log on b s anti-log .5911
x=anti-log 4.5911 =3.900
Characteristic = 4
Mantissa = 3.900 So
x = 3.900×10^4
x = 39000.

iv). Log x = $\bar{3}.0253$

Sol: Log x= $\bar{3}.0253$ with calculator
Taking anti-log on b s anti-log .0253
x=anti-log $\bar{3}.0253$ =1.060
Characteristic = -3
Mantissa = 1.060 So
x = 1.060×10^{-3}
x = 0.001060

v). Log x = 1.8716

Sol: Log x= 1.8716 with calculator
Taking anti-log on b sides anti-log .8716
x=anti-log 1.8716 =7.440
Characteristic = 1
Mantissa = 7.440 So
x = 7.440×10^1
x = 74.40

vi). Log x = $\bar{2}.8370$

Sol: Log x= $\bar{2}.8370$ with calculator
Taking anti-log on b sides anti-log .8370
x=anti-log $\bar{2}.8370$ =6.871
Characteristic = -2
Mantissa = 6.871 So
x = 6.871×10^{-2}
x = 0.06871

Natural or Naperian logarithm: logarithm to the base e are called Mathematician **John Napier invented logarithm** to the base $e \cong 2.71828$.

Relation b/w Common & Natural Logarithm

$$\log_{10}(x) = \frac{\log_e x}{\log_e 10} = \frac{\ln x}{\ln 10}$$

$$\therefore \log_{10}(x) = \frac{\ln x}{2.30258}$$

Laws of Logarithm: As we know that the lengthy processes of multiplication and division can be converted into easier processes of addition and subtraction by the use of logarithm.

First Law: The logarithm of a number is equal to the Addition of logarithms of its factors.

First Law: $\log_a mn = \log_a m + \log_a n$

Proof: Let $\log_a m = x$ & $\log_a n = y$

Exponential form $a^x = m$, $a^y = n$

Multiplying $a^x.a^y = mn$

$a^{x+y} = mn$

Logarithmic form $\log_a mn = x + y$

Putting back $\log_a mn = \log_a m + \log_a n$

Second Law: The logarithm of a fraction is equal to the difference of logarithm of the numerator from the logarithm of the denominator.

Second Law: $\log_a \frac{m}{n} = \log_a m - \log_a n$

Proof: Let $\log_a m = x$ & $\log_a n = y$

Exponential form $a^x = m$, $a^y = n$

Dividing $\frac{a^x}{a^y} = \frac{m}{v}$

$a^{x-y} = \frac{m}{n}$

Logarithmic form $\log_a \frac{m}{n} = x - y$

Putting back $\log_a \frac{m}{n} = \log_a m - \log_a n$

Third Law: logarithm of the power of the number is equal to the product of the power and logarithm of the number.

Third Law: $\log_a (m)^n = n.\log_a m$

Proof: Let $\log_a m = x$

Exponential form $a^x = m$

Taking power n on both sides

$(a^x)^n = m^n$

$a^{xn} = m^n$

Logarithmic form $\log_a m^n = nx$

Putting back $\log_a (m)^n = n.\log_a m$

Fourth Law: $\log_a b.\log_b m = \log_a m$

Proof: Let $\log_a b = x$ & $\log_b m = y$

Exponential form $a^x = b$, $b^y = m$ ....(1)

Putting the value of b in eq (1)

$(a^x)^y = m$

$a^{xy} = m$

Logarithmic form $\log_a m = xy$

Putting back $\log_a b.\log_b m = \log_a m$

Example: Express each of the following as single logarithm.

i). $\log 2+\log 3$

Solution: Given $\log 2+\log 3$

$=\log 2\times 3$

$=\log 6$

ii). $\log 6-\log 2$

Solution: Given $\log 6-\log 2$

$=\log \frac{6}{2}$

$=\log 3$

iii). $-1+\log y$

Solution: Given $-1+\log y$

$=-\log 10+\log y$

$=\log \frac{y}{10}$

$=\log 0.1y$

iv). $\log_2 3.\log_3 5$

Solution: Given $\log_2 3.\log_3 5$

$=\log_2 5$

v). $\log_2 8+\log_2 32$

Solution: Given $\log_2 8+\log_2 32$

$=\log_2 8\times 32$

$=\log_2 256$

vi). $\log 3+\log 5+\log 6-\log 25$

Solution: Given $\log 3+\log 5+\log 6-\log 25$

$=\log \frac{3\times 5\times 6}{25}$

$=\log \frac{18}{5}$

Exercise 3.5

Q1. Use logarithm properties to simplify the expressions.

i). $\log_7 \sqrt{7}$

Solution: Let $x=\log_7 \sqrt{7}$

$7^x=\sqrt{7}$

$7^x=7^{\frac{1}{2}}$

$\Rightarrow \quad x=\frac{1}{2}$

ii). $\log_8\left(\frac{1}{2}\right)$

Solution: Let $x=\log_8\left(\frac{1}{2}\right)$

$8^x=\frac{1}{2}$

$2^{3x}=2^{-1}$

$\Rightarrow \quad 3x=-1$

$x=\frac{-1}{3}$

iii). $\log_{10}\sqrt{1000}$

Solution: Let $x=\log_{10}\sqrt{1000}$

$10^x=\sqrt{1000}$

$10^x=10^{\frac{3}{2}}$

$\Rightarrow \quad x=\frac{3}{2}$

iv). $\log_9 3+\log_9 27$

Solution: Let $x=\log_9 3+\log_9 27$

$x=\log_9 3\times 27$

$x=\log_9 81$

$\Rightarrow 9^x=81$

$9^x=9^2$

$\Rightarrow \quad x=2$

v). $\log \frac{1}{(0.0035)^{-4}}$

Solution: Let $x=\log \frac{1}{(0.0035)^{-4}}$

$x=\log(0.0035)^4$

$x=4\log(0.0035)$

$x=4(\bar{3}.5441)$

$x=4(-3+0.5441)$

$x=4(-2.4559)$

$x=-9.8237$

vi). $\log 45$

Solution: Let $x=\log 45$

$=\log 9\times 5=2\log 3+\log 5$

$x=1.6532$

Q2. Express each of the following as a single logarithm;

i). $3\log 2-4\log 3$

Solution: Given $3\log 2-4\log 3$

$=\log 2^3-\log 3^4$

$=\log 8-\log 81$

$=\log \frac{8}{81}$

ii). $2\log 3+4\log 2-3$

Solution: Given $2\log 3+4\log 2-3$

$=\log 3^2+\log 2^4-3\log 10$

$=\log 9+\log 16-\log 10^3$

$=\log \frac{9\times 16}{10^3}=\log \frac{144}{1000}$

$=\log 0.144$

iii). $\log 5-1$

Solution: Given $\log 5-1$

$=\log 5-\log 10$

$=\log\frac{5}{10}$

$=\log 0.5$

iv). $\frac{1}{2}\log x-2\log 2y+3\log z$

Solution: Given $\frac{1}{2}\log x-2\log 2y+3\log z$

$=\log x^{\frac{1}{2}}-\log(2y)^2+\log z^3$

$=\log\sqrt{x}-\log 4y^2+\log z^3$

$=\log\frac{z^3\sqrt{x}}{4y^2}$

Q3. Find value of 'a' from the following equations.

i). $\log_2 6+\log_2 7=\log_2 a$

Solution: Given $\log_2 6+\log_2 7=\log_2 a$

$\log_2 6\times 7=\log_2 a$

$\log_2 42=\log_2 a$

$\Rightarrow a=42$

ii). $\log_{\sqrt{3}} a=\log_{\sqrt{3}} 5+\log_{\sqrt{3}} 8-\log_{\sqrt{3}} 2$

Sol: Given $\log_{\sqrt{3}} a=\log_{\sqrt{3}} 5+\log_{\sqrt{3}} 8-\log_{\sqrt{3}} 2$

$\log_{\sqrt{3}} a=\log_{\sqrt{3}} 5+\log_{\sqrt{3}} 8-\log_{\sqrt{3}} 2$

$\log_{\sqrt{3}} a=\log_{\sqrt{3}}\frac{5\times 8}{2}$

$\log_{\sqrt{3}} a=\log_{\sqrt{3}} 20$

$\Rightarrow a=20$

iii). $\frac{\log_7 r}{\log_7 t}=\log_a r$

Solution: Given $\frac{\log_7 r}{\log_7 t}=\log_a r$

$\log_t r=\log_a r \qquad \Rightarrow a=t$

iv). $\log_6 25-\log_6 5=\log_6 a$

Solution: Given $\log_6 25-\log_6 5=\log_6 a$

$\log_6\frac{25}{5}=\log_6 a$

$\log_6 5=\log_6 a$

$\Rightarrow a=5$

Q4.Find $\log_2 3.\log_3 4.\log_4 5.\log_5 6\log_6 7.\log_7 8$

Sol: Let $x=\log_2 3.\log_3 4.\log_4 5.\log_5 6\log_6 7.\log_7 8$

$x=\log_2 4.\log_4 5.\log_5 6\log_6 7.\log_7 8$

$x=\log_2 5.\log_5 6\log_6 7.\log_7 8$

$x=\log_2 6\log_6 7.\log_7 8$

$x=\log_2 7.\log_7 8$

$x=\log_2 8$

$\Rightarrow \quad 2^x=8$

$2^x=2^3$

$\Rightarrow \quad x=3$

Example 1: simplify (238.2)(9.506) with the help of logarithm.

R . Work

238.2 =2.382×10^2 9.506 = 9.506×10^0

with calculator

log 2.382 =0.3769 log 2.382 =0.9780

Solution: Let x = (238.2)(9.506)

Taking log on both sides

Log x = log (238.2)(9.506)

Log x = log 238.2 + log 9.506

Ch= 2 ch= 0

M= 0.3769 M=0.9780

So Log x = 2.3769+0.9780

Log x = 3.3549

Taking anti-log on both sides

anti-log (Log x) = anti-log 3.3549

x = anti-log 3.3549

Ch = 3 M= 2.264 With calculator

Then X= 2.264×10^3 Anti-log .3549

X = 2264 =2.264

Example 2: simplify $\frac{2.83}{(6.52)^2}$ with logarithm.

R . Work

2.83=2.83×10^0 6.52=6.52×10^0

with calculator with calculator

log 2.83=0.4518 log 6.52 =0.8142

Solution: Let $x=\frac{2.83}{(6.52)^2}$

Taking log on both sides

Log x = $\log\frac{2.83}{(6.52)^2}$

Log x = log 2.83 -2 log 6.52

Ch = 0 ch= 0

M = 0.4518 M=0.8142

So Log x = 0.4518-2(0.8142)

Log x = 0.4518-1.6284

Log x = -1.1766

Log x = -2 +2 -1.1766

Log x = -2 +0.8234= $\bar{2}.8234$

Taking anti-log on both sides

Anti-log Log x = anti-log $\bar{2}.8234$

Ch= -2, M=6.659 with calculator

Then X= 6.659×10^{-2} Anti-log

X = 0.06659 0.8234=6.659

Exercise 3.6

Q1. Simplify with the help of logarithm.

i). Simplify 3.81×43.4 with help of logarithm.

R . Work

3.81=3.81×10^0 43.4=4.34×10^1

with calculator

log 3.81=0.5809 log 4.34 = 0.6375

Solution: Let $x=3.81\times 43.4$

Taking log on both sides

$\log x =$ log 3.81 + log 43.4

Ch= 0 ch=1

M=0.5809 M=0.6375

$\log x =$ 0.5809+1.6375

$\log x =$ 2.2184

Taking anti-log on both sides

x = anti-log 2.2184

Ch =2 M= With calculator

Then X= 1.654 x 10^2 Anti-log0.2184

X = 165.4 =1.654

ii). Simplify $73.42 \times 0.00462 \times 0.5143$ with the help of logarithm.

R . Work

73.42=7.342x10^1 0.00462=4.62x10^{-3}

with calculator

log 7.342=0.8658 log 4.62 = 0.6646

with calculator

0.5143=5.143x10^{-1} log 5.143 = 0.7112

Solution: Let $x = 73.42 \times 0.00462 \times 0.5143$

Taking log on both sides

$\log x =$ log 73.42 + log 0.00462 + log 0.5143

Ch=1 ch= - 3 Ch= - 1

M=0.8658 M=0.6646 M= 0.7112

$\log x =$ 1.8658 + (-3 + 0.6646) + (-1+0.7112)

$\log x =$ 1.8658 – 2.3354 – 0.2888

$\log x =$ - 0.7584

$\log x =$ - 1 + 1 – 0.7584

$\log x =$ - 1 +0.2416

$\log x = \bar{1}.2416$

Taking anti-log on both sides

x = anti-log $\bar{1}.2416$

Ch = - 1 M= 1.744 With calculator

Then X= 1.744 x 10^{-1} Anti-log0.2416

X = 0.1744 =1.744

iii). Simplify $\frac{784.6 \times 0.0431}{28.23}$ with help of logarithm.

R . Work

784.6=7.846x10^2 0.0431=4.31x10^{-2}

with calculator

log 7.846=0.8946 log 4.31 = 0.6345

with calculator

28.23=2.823x10^1 log 2.823=0.4507

Solution: Let $x = \frac{784.6 \times 0.0431}{28.23}$

Taking log on both sides

$\log x =$ log 784.6 + log 0.0431 – log 28.23

Ch=2 ch= - 2 ch= 1

M=0.8946 M=0.6345 M=0.4507

$\log x =$ 2.8946 + (-2+0.6345) -1.4507

$\log x =$ 2.8946 – 1.3655 – 1.4507

$\log x =$ 0.0784

Taking anti-log on both sides

x = anti-log 0.0784

Ch = 0 M=1.198 With calculator

Then X= 1.198 x 10^0 Anti-log0.0784

X = 1.198 =1.198

iv). Simplify $\frac{0.4932 \times 653.7}{0.07213 \times 8456}$ with help of logarithm.

R . Work

0.4932=4.932x10^{-1} 653.7=6.537x10^2

with calculator

log 4.932=0.6930 log 6.537 = 0.8154

0.07213=7.213x10^{-2} 8456=8.456x10^3

with calculator

log 7.213=0.8581 log 8.456 = 0.9272

Solution: Let $x = \frac{0.4932 \times 653.7}{0.07213 \times 8456}$

Taking log on both sides

$\log x =$ log 0.4932 + log 653.7 – log 0.07213 –log 8456

Ch=-1 ch=2 Ch=-2 ch=3

M=.6930 M=.8154 M=.8581 M=9272

$\log x =$ -1+0.6930+2.8154–(-2+0.8581)-3.9272

$\log x =$ -0.2769

$\log x =$ -1 + 1-0.2769

$\log x = \bar{1}.7231$

Taking anti-log on both sides

x = anti-log $\bar{1}.7231$

Ch =-1 M=5.286 With calculator

Then X= 5.286 x 10^{-1} Anti-log0.7231

X = 0.5286 =5.286

v). Simplify $\frac{(78.41)^3 \sqrt{142.3}}{\sqrt[4]{0.1562}}$ with help of logarithm.

R . Work

78.41=7.841x10^1 142.3=1.423x10^2

with calculator

log 7.841=0.8944 log 1.423 = 0.1532

And with calculator

0.1562=1.562x10^{-1} log 1.562=0. 1937

Solution: Let $x = \frac{(78.41)^3 \sqrt{142.3}}{\sqrt[4]{0.1562}}$

Taking log on both sides

$\log x = \log \frac{(78.41)^3 \sqrt{142.3}}{\sqrt[4]{0.1562}}$

$\log x =$ 3 log 78.41 + $\frac{1}{2}$ log 142.3- $\frac{1}{4}$ log 0.1562

Ch=1 ch=2 ch= -1

M=0.8944 M=0.1532 M=0.1937

$\log x =$ 3(1.8944) + $\frac{1}{2}$ (2.1532) - $\frac{1}{4}$ (-1+0.1937)

$\log x =$ 5.6832 + 1.0766- 0.2016

$\log x = 6.5582$
Taking anti-log on both sides
x = anti-log 6.5582
Ch = 6 M=3.616 With calculator
Then X= 3.616×10^6 Anti-log0.5582
X = 3 616 000 =3.616

Q2. Find the following If log 2 = 0.3010, log 3 = 0.4771 , log 5 = 0.6990 & log 7= 0.8450.

i). log 105
Solution: Given log 105
= log 3x5x7
= log 3 + log 5 + log 7
= 0.4771 +0.6990+0.8450
= 2.0211

ii). Log 108
Solution: Given log 108
= log 3x3x3x2x2
=log $3^3 x 2^2$
=3 log 3 + 2 log 2
= 3(0.4771) +2(0.3010)
= 2.0333

iii). $\log\sqrt[3]{72}$
Solution: Given $\log\sqrt[3]{72}$

$= \log[8\times 9]^{\frac{1}{3}}$

$= \frac{1}{3}\log\left[2^3\times 3^2\right]$

$= \frac{1}{3}[3\log 2 + 2\log 3]$

$= \frac{1}{3}$[3x0.3010+2x0.4771]

$= \frac{1}{3}$[1.8572]

=0.6191

iv). $\log 2.4$
Solution: Given log 2.4

$= \log\frac{24}{10}$

$= \log\frac{2^3\times 3}{10}$

= 3 log 2 + log 3 –log 10
= 3(0.3010) +0.4771 -1
= 0.9030 +0.4771 – 1
= 0.3801

v). log 0.0081
Solution: Given log 0.0081

$= \log\frac{81}{10000}$

$= \log\frac{3^4}{10^4}$

$= \log\left(\frac{3}{10}\right)^4$

$= 4\log\frac{3}{10}$

$= 4[\log 3 - \log 10]$

= 4[0.4771 -1]
= 4[- 0.5229]
= -2.0916

Review Exercise 3

Q1. Select the correct answer

i). $\log_9\left(\frac{1}{81}\right) =$
a). -1 b). -2
c). 2 d). Does not exists

ii). If $\log_2 8 = x$ then $x =$
a). 64 b). 3^2
c). 3 d). 2^8

iii). Base of common log is
a). 10 b) e
c). π d). 5

iv). $\log\sqrt{10} =$
a). -1 b). $-\frac{1}{2}$
c). $\frac{1}{2}$ d). 2

v). For any non-zero x , $x^0 =$
a). 2 b). 1
c). 0 d). 10

vi). Rewrite $t = \log_b m$ as exponential equation
a). $t = m^b$ b). $b^m = t$
c). $m = b^t$ d). $m^t = b$

vii). $\log_{10}(10) =$
a). 2 b). 3
c). 0 d). 1

viii). Characteristic of log 0.000 059
a). - 5 b). 5
c). - 4 d). 4

ix). Evaluate $\log_7\left(\frac{1}{\sqrt{7}}\right) =$
a). -1 b). $-\frac{1}{2}$
c). $\frac{1}{2}$ d). 2

x). Base of natural log is
a). 10 b). e
c). π d). 1

xi). $\log m + \log n =$
a). $\log m \log n$ b). $\log m - \log n$
c). $\log mn$ d). $\log\frac{m}{n}$

xii). 0.069 can be written as in scientific notation as

a). 6.9×10^3 b). 6.9×10^{-2}
c). 0.69×10^3 d). 69×10^2

xiii). $\ln x - 2\ln y$

a). $\ln \frac{x}{y}$ b). $\ln xy^2$
c). $\ln \frac{x^2}{y}$ d). $\ln \frac{x}{y^2}$

Q2. Write 9473.2 in scientific notation

Solution: given number 9473.2

$9_{\wedge}473.2 = 9.4732 \times 10^3$

Q3. Write 5.4×10^6 in standard notation

Solution: Here 5.4×10^6

$5_{\wedge}400\ 000.$

Q4. Write in logarithmic form; $3^{-3} = \frac{1}{27}$

Solution: Given $3^{-3} = \frac{1}{27}$

Logarithmic form $\log_3\left(\frac{1}{27}\right) = -3$

Q5. Write in exponential form $\log_5 1 = 0$

Solution: Given $\log_5 1 = 0$

Exponential form $5^0 = 1$

Q6. Solve for x $\log_4 16 = x$

Solution: Given $\log_4 16 = x$

Exponential form $4^x = 16$

$4^x = 4^2$

$\Rightarrow x = 2$

Q7. Find the characteristic of the common logarithm 0.0083

Solution: Given $0.008_{\wedge}3$

characteristic is -3

Q8. Find log 12.4

Solution: Given log 12.4

Here Ch=1 M= 0.0934

= 1.0934

Q9. Find the value of a

$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 9 + \log_{\sqrt{5}} 2 - \log_{\sqrt{5}} 3$

Sol: Given $\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 9 + \log_{\sqrt{5}} 2 - \log_{\sqrt{5}} 3$

$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}}\left(\frac{9 \times 2}{3}\right)$

$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 6$

$\Rightarrow 3a = 6$

$a = 2$

Q10. Solve with the help of logarithm

$$\frac{(63.28)^3(0.00843)^2(0.4623)}{(412.3)(2.184)^5}$$

R.W

$63.28 = 6.328 \times 10^1$ $0.4623 = 4.623 \times 10^{-1}$

with calculator

log 6.328=0.8013 log 4.623 = 0.6649

$0.00843 = 8.43 \times 10^{-3}$ $2.184 = 2.184 \times 10^0$

with calculator

log 8.43=0.9258 log 2.184 = 0.3393

with calculator

$412.3 = 4.123 \times 10^2$ log 4.123=0.6152

Solution: Let $x = \frac{(63.28)^3(0.00843)^2(0.4623)}{(412.3)(2.184)^5}$

Taking log on both sides

$\log x = 3\log(63.28) + 2\log(0.00843)$
$+ \log 0.4623 - \log 412.3 - 5\log 2.184$

$\log x = 3(1.8013) + 2(-3 + 0.9258)$
$+ (-1 + 0.6649) - 2.6152 - 5(0.3393)$

$\log x = -3.3913$

$\log x = -4 + 4 - 3.3913$

$\log x = \bar{4}.6087$

Taking anti-log on both sides

$x = anti\log \bar{4}.6087$

Ch = -4 M=4.062

Then $X = 4.062 \times 10^{-4}$

X = 0. 000 4062

With calculator
Anti-log0.5582
=3.616