Lecture No.8

LECTURE OUTLINE

5–1 Basic Combinational Logic Circuits
5–2 Implementing Combinational Logic
5–3 The Universal Property of NAND and NOR Gates

5–1 Basic Combinational Logic Circuits

In Chapter 4, you learned that SOP expressions are implemented with an AND gate for each product term and one OR gate for summing all of the product terms. As you know, this SOP implementation is called AND-OR logic and is the basic form for realizing standard Boolean functions. In this section, the AND-OR and the AND-OR-Invert are examined; the exclusive-OR and exclusive-NOR gates, which are actually a form of AND-OR logic, are also covered.

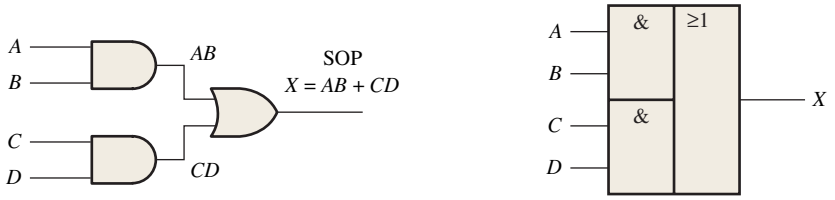
After completing this section, you should be able to

- Analyze and apply AND-OR circuits
- Analyze and apply AND-OR-Invert circuits
- Analyze and apply exclusive-OR gates
- Analyze and apply exclusive-NOR gates

AND-OR Logic

Figure 5–1(a) shows an AND-OR circuit consisting of two 2-input AND gates and one 2-input OR gate; Figure 5–1(b) is the ANSI standard rectangular outline symbol. The Boolean expressions for the AND gate outputs and the resulting SOP expression for the output *X* are shown on the diagram. In general, an AND-OR circuit can have any number of AND gates, each with any number of inputs.

The truth table for a 4-input AND-OR logic circuit is shown in Table 5–1. The intermediate AND gate outputs (the *AB* and *CD* columns) are also shown in the table.



(a) Logic diagram (ANSI standard distinctive shape symbols)

(b) ANSI standard rectangular outline symbol

FIGURE 5–1 An example of AND-OR logic. Open file F05-01 to verify the operation. *A Multisim tutorial is available on the website.*

TABLE 5–1

Truth table for the AND-OR logic in Figure 5–1.

Inputs						Output
A	*B*	*C*	*D*	*AB*	*CD*	*X*
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	1	0	0	0	0
0	0	1	1	0	1	1
0	1	0	0	0	0	0
0	1	0	1	0	0	0
0	1	1	0	0	0	0
0	1	1	1	0	1	1
1	0	0	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	0	0	0
1	0	1	1	0	1	1
1	1	0	0	1	0	1
1	1	0	1	1	0	1
1	1	1	0	1	0	1
1	1	1	1	1	1	1

An AND-OR circuit directly implements an SOP expression, assuming the complements (if any) of the variables are available. The operation of the AND-OR circuit in Figure 5–1 is stated as follows:

For a 4-input AND-OR logic circuit, the output *X* is HIGH (1) if both input *A* and input *B* are HIGH (1) or both input *C* and input *D* are HIGH (1).

EXAMPLE 5–1

In a certain chemical-processing plant, a liquid chemical is used in a manufacturing process. The chemical is stored in three different tanks. A level sensor in each tank produces a HIGH voltage when the level of chemical in the tank drops below a specified point.

Design a circuit that monitors the chemical level in each tank and indicates when the level in any two of the tanks drops below the specified point.

Solution

The AND-OR circuit in Figure 5–2 has inputs from the sensors on tanks *A*, *B*, and *C* as shown. The AND gate G_1 checks the levels in tanks *A* and *B*, gate G_2 checks tanks *A* and *C*, and gate G_3 checks tanks *B* and *C*. When the chemical level in any two of the tanks gets too low, one of the AND gates will have HIGHs on both of its inputs, causing its output to be HIGH; and so the final output *X* from the OR gate is HIGH. This HIGH input is then used to activate an indicator such as a lamp or audible alarm, as shown in the figure.

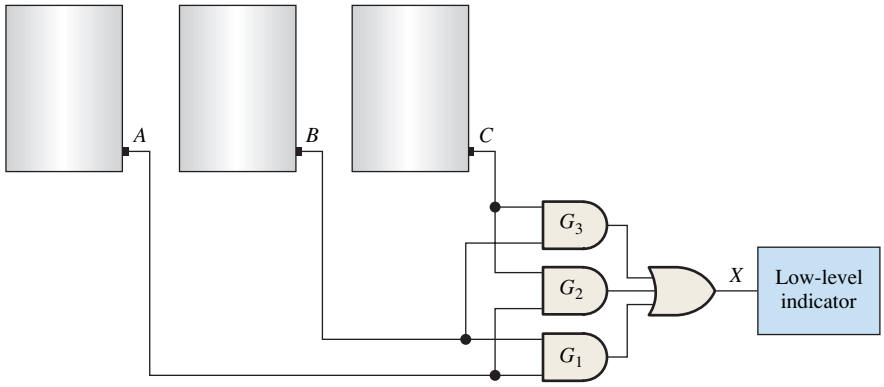


FIGURE 5–2

Related Problem

Write the Boolean SOP expression for the AND-OR logic in Figure 5–2.

AND-OR-Invert Logic

When the output of an AND-OR circuit is complemented (inverted), it results in an AND-OR-Invert circuit. Recall that AND-OR logic directly implements SOP expressions. POS expressions can be implemented with AND-OR-Invert logic. This is illustrated as follows, starting with a POS expression and developing the corresponding AND-OR-Invert (AOI) expression.

$$X = (\overline{A} + \overline{B})(\overline{C} + \overline{D}) = (\overline{AB})(\overline{CD}) = \overline{\overline{(\overline{AB})(\overline{CD})}} = \overline{\overline{AB} + \overline{CD}} = \overline{AB + CD}$$

The logic diagram in Figure 5–3 shows an AND-OR-Invert circuit with four inputs and the development of the POS output expression. In general, an AND-OR-Invert circuit can have any number of AND gates, each with any number of inputs.

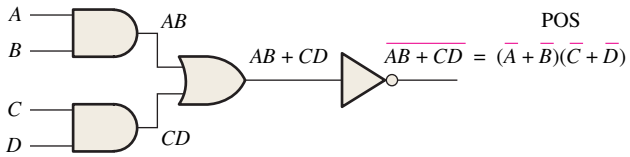


FIGURE 5–3 An AND-OR-Invert circuit produces a POS output. Open file F05-03 to verify the operation.

The operation of the AND-OR-Invert circuit in Figure 5–3 is stated as follows:

For a 4-input AND-OR-Invert logic circuit, the output *X* is LOW (0) if both input *A* and input *B* are HIGH (1) or both input *C* and input *D* are HIGH (1).

A truth table can be developed from the AND-OR truth table in Table 5–1 by simply changing all 1s to 0s and all 0s to 1s in the output column.

EXAMPLE 5–2

The sensors in the chemical tanks of Example 5–1 are being replaced by a new model that produces a LOW voltage instead of a HIGH voltage when the level of the chemical in the tank drops below a critical point.

Modify the circuit in Figure 5–2 to operate with the different input levels and still produce a HIGH output to activate the indicator when the level in any two of the tanks drops below the critical point. Show the logic diagram.

Solution

The AND-OR-Invert circuit in Figure 5–4 has inputs from the sensors on tanks *A*, *B*, and *C* as shown. The AND gate G_1 checks the levels in tanks *A* and *B*, gate G_2 checks tanks *A* and *C,* and gate G_3 checks tanks *B* and *C*. When the chemical level in any two of the tanks gets too low, each AND gate will have a LOW on at least one input, causing its output to be LOW and, thus, the final output *X* from the inverter is HIGH. This HIGH output is then used to activate an indicator.

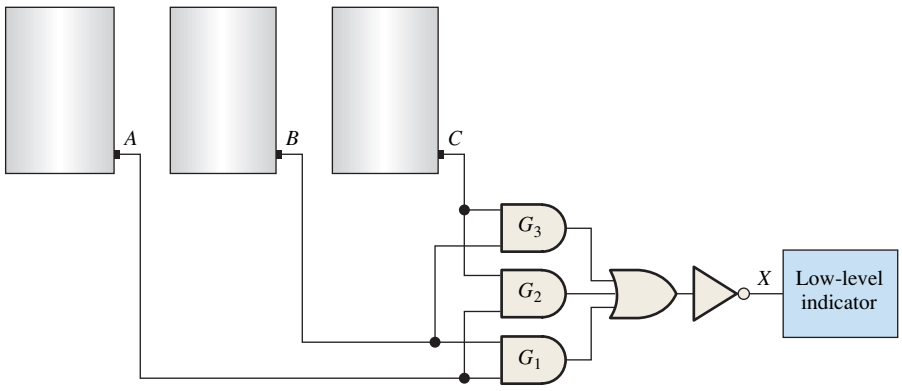


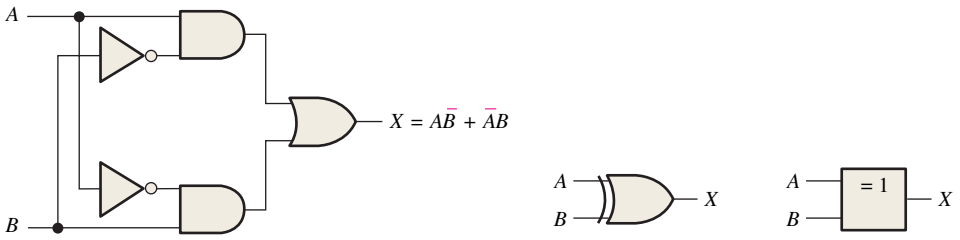
FIGURE 5–4

Related Problem

Write the Boolean expression for the AND-OR-Invert logic in Figure 5–4 and show that the output is HIGH (1) when any two of the inputs *A*, *B*, and *C* are LOW (0).

Exclusive-OR Logic

The exclusive-OR gate was introduced in Chapter 3. Although this circuit is considered a type of logic gate with its own unique symbol, it is actually a combination of two AND gates, one OR gate, and two inverters, as shown in Figure 5–5(a). The two ANSI standard exclusive-OR logic symbols are shown in parts (b) and (c).



(a) Logic diagram (b) ANSI distinctive shape symbol (c) ANSI rectangular outline symbol

FIGURE 5–5 Exclusive-OR logic diagram and symbols. Open file F05-05 to verify the operation.

The output expression for the circuit in Figure 5–5 is

$$X = A\overline{B} + \overline{A}B$$

Evaluation of this expression results in the truth table in Table 5–2. Notice that the output is HIGH only when the two inputs are at opposite levels. A special exclusive-OR operator $\oplus$ is often used, so the expression $X = A\overline{B} + \overline{A}B$ can be stated as "X is equal to A exclusive-OR B" and can be written as

$$X = A \oplus B$$

TABLE 5–2
Truth table for an exclusive-OR.

A	*B*	*X*
0	0	0
0	1	1
1	0	1
1	1	0

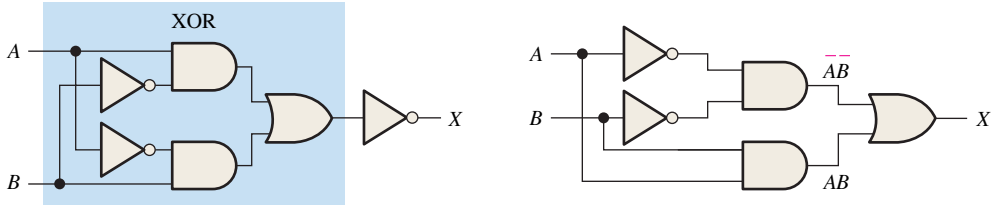
Exclusive-NOR Logic

As you know, the complement of the exclusive-OR function is the exclusive-NOR, which is derived as follows:

$$X = \overline{A\overline{B} + \overline{A}B} = \overline{(A\overline{B})}\,\overline{(\overline{A}B)} = (\overline{A} + B)(A + \overline{B}) = \overline{A}\,\overline{B} + AB$$

Notice that the output X is HIGH only when the two inputs, A and B, are at the same level.

The exclusive-NOR can be implemented by simply inverting the output of an exclusive-OR, as shown in Figure 5–6(a), or by directly implementing the expression $\overline{A}\,\overline{B} + AB$, as shown in part (b).



(a) $X = \overline{A\overline{B} + \overline{A}B}$ (b) $X = \overline{A}\,\overline{B} + AB$

FIGURE 5–6 Two equivalent ways of implementing the exclusive-NOR. Open files F05-06 (a) and (b) to verify the operation.

EXAMPLE 5–3

Use exclusive-OR gates to implement an even-parity code generator for an original 4-bit code.

Solution

Recall from Chapter 2 that a parity bit is added to a binary code in order to provide error detection. For even parity, a parity bit is added to the original code to make the total number of 1s in the code even. The circuit in Figure 5–7 produces a 1 output when there is an odd number of 1s on the inputs in order to make the total number of 1s in the output code even. A 0 output is produced when there is an even number of 1s on the inputs.

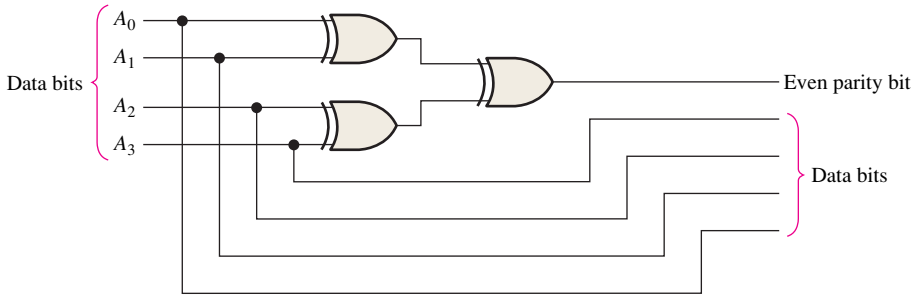


FIGURE 5–7 Even-parity generator.

Related Problem

How would you verify that a correct even-parity bit is generated for each combination of the four data bits?

EXAMPLE 5–4

Use exlusive-OR gates to implement an even-parity checker for the 5-bit code generated by the circuit in Example 5–3.

Solution

The circuit in Figure 5–8 produces a 1 output when there is an error in the five-bit code and a 0 when there is no error.

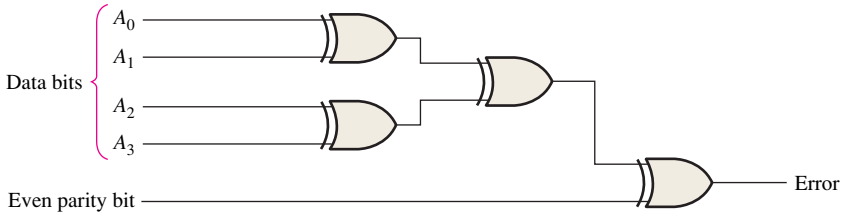


FIGURE 5–8 Even-parity checker.

Related Problem

How would you verify that an error is indicated when the input code is incorrect?

SECTION 5–1 CHECKUP

Answers are at the end of the chapter.

1. Determine the output (1 or 0) of a 4-variable AND-OR-Invert circuit for each of the following input conditions:
   (a) $A = 1, B = 0, C = 1, D = 0$ **(b)** $A = 1, B = 1, C = 0, D = 1$
   (c) $A = 0, B = 1, C = 1, D = 1$
2. Determine the output (1 or 0) of an exclusive-OR gate for each of the following input conditions:
   (a) $A = 1, B = 0$ **(b)** $A = 1, B = 1$
   (c) $A = 0, B = 1$ **(d)** $A = 0, B = 0$
3. Develop the truth table for a certain 3-input logic circuit with the output expression $X = A\overline{B}C + \overline{A}BC + \overline{A}\,\overline{B}\,\overline{C} + AB\overline{C} + ABC$.
4. Draw the logic diagram for an exclusive-NOR circuit.

5–2 Implementing Combinational Logic

In this section, examples are used to illustrate how to implement a logic circuit from a Boolean expression or a truth table. Minimization of a logic circuit using the methods covered in Chapter 4 is also included.

After completing this section, you should be able to

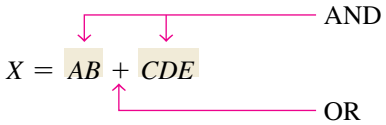
- Implement a logic circuit from a Boolean expression
- Implement a logic circuit from a truth table
- Minimize a logic circuit

From a Boolean Expression to a Logic Circuit

Let's examine the following Boolean expression:

$$X = AB + CDE$$

A brief inspection shows that this expression is composed of two terms, AB and CDE, with a domain of five variables. The first term is formed by ANDing A with B, and the second term is formed by ANDing C, D, and E. The two terms are then ORed to form the output X. These operations are indicated in the structure of the expression as follows:



Note that in this particular expression, the AND operations forming the two individual terms, AB and CDE, must be performed *before* the terms can be ORed.

To implement this Boolean expression, a 2-input AND gate is required to form the term AB, and a 3-input AND gate is needed to form the term CDE. A 2-input OR gate is then required to combine the two AND terms. The resulting logic circuit is shown in Figure 5–9.

As another example, let's implement the following expression:

$$X = AB(C\overline{D} + EF)$$

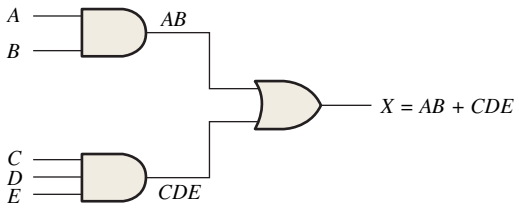
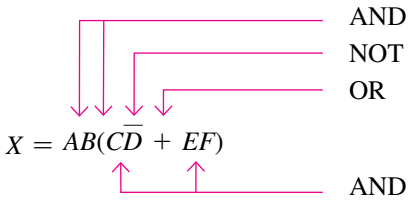


FIGURE 5–9 Logic circuit for $X = AB + CDE$.

A breakdown of this expression shows that the terms AB and $(C\overline{D} + EF)$ are ANDed. The term $C\overline{D} + EF$ is formed by first ANDing C and $\overline{D}$ and ANDing E and F, and then ORing these two terms. This structure is indicated in relation to the expression as follows:



Before you can implement the final expression, you must create the sum term $C\overline{D} + EF$; but before you can get this term; you must create the product terms $C\overline{D}$ and EF; but before you can get the term $C\overline{D}$, you must create $\overline{D}$. So, as you can see, the logic operations must be done in the proper order.

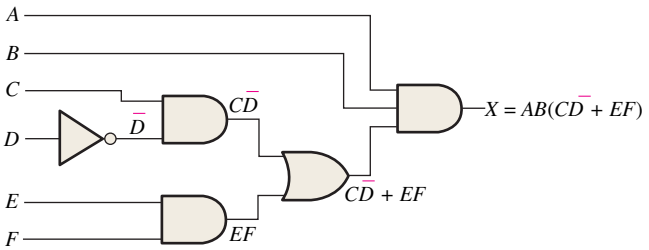
The logic gates required to implement $X = AB(C\overline{D} + EF)$ are as follows:

1. One inverter to form $\overline{D}$
2. Two 2-input AND gates to form $C\overline{D}$ and EF
3. One 2-input OR gate to form $C\overline{D} + EF$
4. One 3-input AND gate to form X

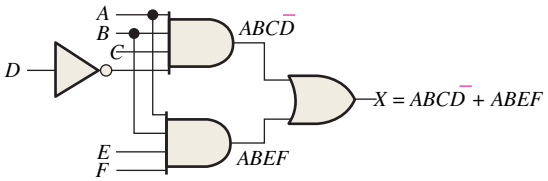
The logic circuit for this expression is shown in Figure 5–10(a). Notice that there is a maximum of four gates and an inverter between an input and output in this circuit (from input D to output). Often the total propagation delay time through a logic circuit is a major consideration. Propagation delays are additive, so the more gates or inverters between input and output, the greater the propagation delay time.

Unless an intermediate term, such as $C\overline{D} + EF$ in Figure 5–10(a), is required as an output for some other purpose, it is usually best to reduce a circuit to its SOP form in order to reduce the overall propagation delay time. The expression is converted to SOP as follows, and the resulting circuit is shown in Figure 5–10(b).

$$AB(C\overline{D} + EF) = ABC\overline{D} + ABEF$$



(a)



(b) Sum-of-products implementation of the circuit in part (a)

FIGURE 5–10 Logic circuits for $X = AB(C\overline{D} + EF) = ABC\overline{D} + ABEF$.

TABLE 5–3

Inputs			Output	
A	*B*	*C*	*X*	Product Term
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	$\overline{A}BC$
1	0	0	1	$A\overline{B}\,\overline{C}$
1	0	1	0	
1	1	0	0	
1	1	1	0	

From a Truth Table to a Logic Circuit

If you begin with a truth table instead of an expression, you can write the SOP expression from the truth table and then implement the logic circuit. Table 5–3 specifies a logic function.

The Boolean SOP expression obtained from the truth table by ORing the product terms for which $X = 1$ is

$$X = \overline{A}BC + A\overline{B}\,\overline{C}$$

The first term in the expression is formed by ANDing the three variables $\overline{A}$, B, and C. The second term is formed by ANDing the three variables A, $\overline{B}$, and $\overline{C}$.

The logic gates required to implement this expression are as follows: three inverters to form the $\overline{A}$, $\overline{B}$, and $\overline{C}$ variables; two 3-input AND gates to form the terms $\overline{A}BC$ and $A\overline{B}\,\overline{C}$; and one 2-input OR gate to form the final output function, $\overline{A}BC + A\overline{B}\,\overline{C}$.

The implementation of this logic function is illustrated in Figure 5–11.

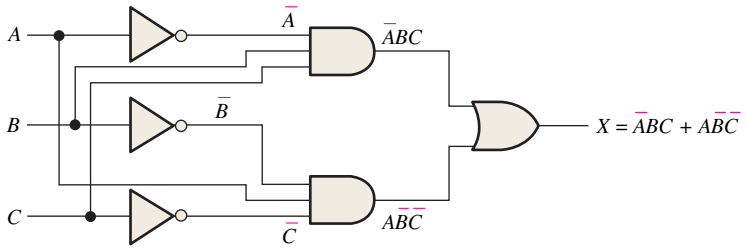


FIGURE 5–11 Logic circuit for $X = \overline{A}BC + A\overline{B}\,\overline{C}$. Open file F05-11 to verify the operation.

EXAMPLE 5–5

Design a logic circuit to implement the operation specified in the truth table of Table 5–4.

TABLE 5–4

Inputs			Output	
A	*B*	*C*	*X*	Product Term
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	$\overline{A}BC$
1	0	0	0	
1	0	1	1	$A\overline{B}C$
1	1	0	1	$AB\overline{C}$
1	1	1	0	

Solution

Notice that $X = 1$ for only three of the input conditions. Therefore, the logic expression is

$$X = \overline{A}BC + A\overline{B}C + AB\overline{C}$$

The logic gates required are three inverters, three 3-input AND gates and one 3-input OR gate. The logic circuit is shown in Figure 5–12.

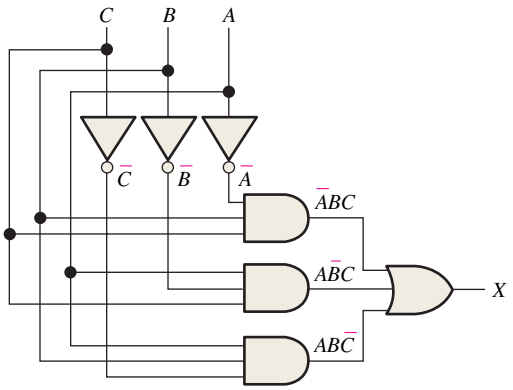


FIGURE 5–12 Open file F05-12 to verify the operation.

Related Problem

Determine if the logic circuit of Figure 5–12 can be simplified.

EXAMPLE 5–6

Develop a logic circuit with four input variables that will only produce a 1 output when exactly three input variables are 1s.

Solution

Out of sixteen possible combinations of four variables, the combinations in which there are exactly three 1s are listed in Table 5–5, along with the corresponding product term for each.

TABLE 5–5

A	*B*	*C*	*D*	Product Term
0	1	1	1	$\overline{A}BCD$
1	0	1	1	$A\overline{B}CD$
1	1	0	1	$AB\overline{C}D$
1	1	1	0	$ABC\overline{D}$

The product terms are ORed to get the following expression:

$$X = \overline{A}BCD + A\overline{B}CD + AB\overline{C}D + ABC\overline{D}$$

This expression is implemented in Figure 5–13 with AND-OR logic.

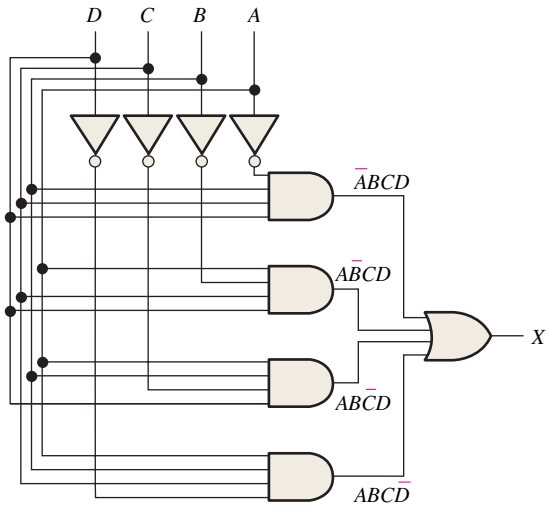


FIGURE 5–13 Open file F05-13 to verify the operation.

Related Problem

Determine if the logic circuit of Figure 5–13 can be simplified.

EXAMPLE 5–7

Reduce the combinational logic circuit in Figure 5–14 to a minimum form.

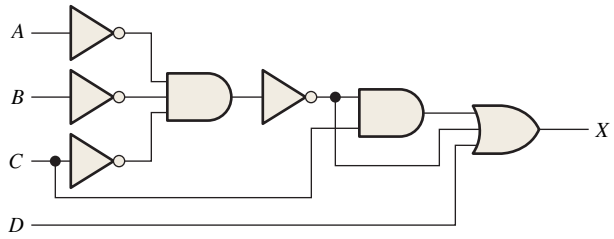


FIGURE 5–14
Open file F05-14 to verify that this circuit is equivalent to the gate in Figure 5–15.

Solution

The expression for the output of the circuit is

$$X = (\overline{\overline{A}\,\overline{B}\,\overline{C}})C + \overline{\overline{A}\,\overline{B}\,\overline{C}} + D$$

Applying DeMorgan's theorem and Boolean algebra,

$$\begin{aligned} X &= (\overline{\overline{A}} + \overline{\overline{B}} + \overline{\overline{C}})C + \overline{\overline{A}} + \overline{\overline{B}} + \overline{\overline{C}} + D \\ &= AC + BC + CC + A + B + C + D \\ &= AC + BC + C + A + B + \cancel{C} + D \\ &= C(A + B + 1) + A + B + D \\ X &= A + B + C + D \end{aligned}$$

The simplified circuit is a 4-input OR gate as shown in Figure 5–15.

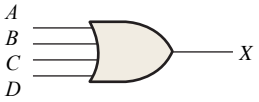


FIGURE 5–15

Related Problem

Verify the minimized expression $A + B + C + D$ using a Karnaugh map.

EXAMPLE 5–8

Minimize the combinational logic circuit in Figure 5–16. Inverters for the complemented variables are not shown.

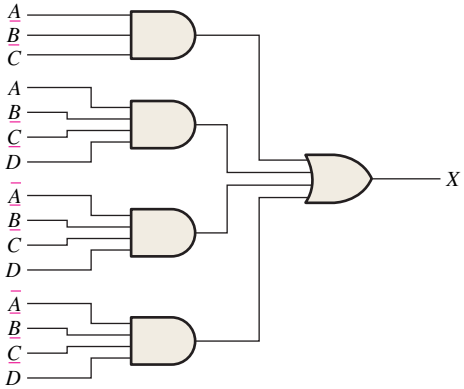


FIGURE 5–16

Solution

The output expression is

$$X = A\overline{B}\,\overline{C} + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$$

Expanding the first term to include the missing variables D and $\overline{D}$,

$$X = A\overline{B}\,\overline{C}(D + \overline{D}) + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$$
$$= A\overline{B}\,\overline{C}D + A\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$$

This expanded SOP expression is mapped and simplified on the Karnaugh map in Figure 5–17(a). The simplified implementation is shown in part (b). Inverters are not shown.

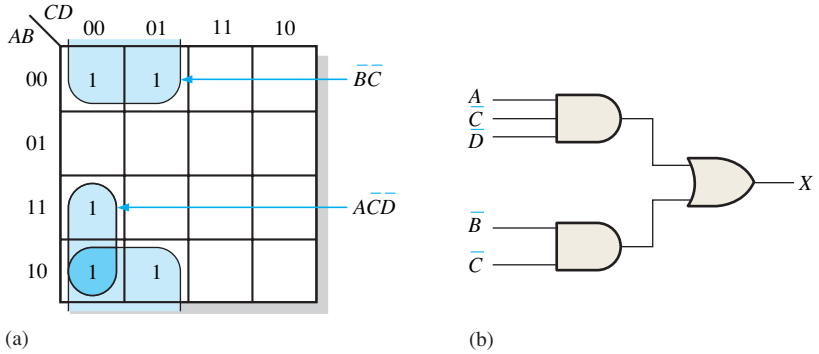


FIGURE 5–17

Related Problem

Develop the POS equivalent of the circuit in Figure 5–17(b). See Section 4–10.

SECTION 5–2 CHECKUP

1. Implement the following Boolean expressions as they are stated:
   (a) $X = ABC + AB + AC$ **(b)** $X = AB(C + DE)$
2. Develop a logic circuit that will produce a 1 on its output only when all three inputs are 1s or when all three inputs are 0s.
3. Reduce the circuits in Question 1 to minimum SOP form.

5–3 The Universal Property of NAND and NOR Gates

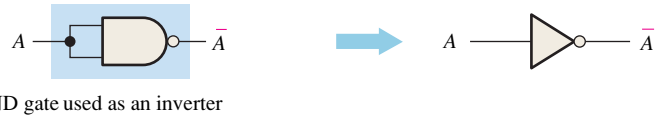
Up to this point, you have studied combinational circuits implemented with AND gates, OR gates, and inverters. In this section, the universal property of the NAND gate and the NOR gate is discussed. The universality of the NAND gate means that it can be used as an inverter and that combinations of NAND gates can be used to implement the AND, OR, and NOR operations. Similarly, the NOR gate can be used to implement the inverter (NOT), AND, OR, and NAND operations.

After completing this section, you should be able to

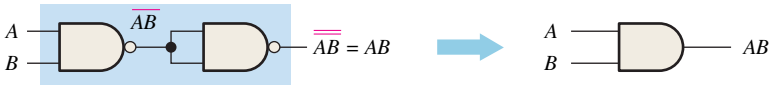
- Use NAND gates to implement the inverter, the AND gate, the OR gate, and the NOR gate
- Use NOR gates to implement the inverter, the AND gate, the OR gate, and the NAND gate

The NAND Gate as a Universal Logic Element

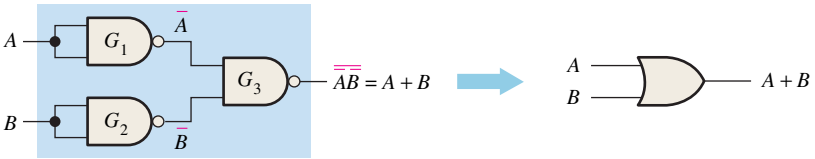
The NAND gate is a **universal gate** because it can be used to produce the NOT, the AND, the OR, and the NOR functions. An inverter can be made from a NAND gate by connecting all of the inputs together and creating, in effect, a single input, as shown in Figure 5–18(a) for a 2-input gate. An AND function can be generated by the use of NAND gates alone, as shown in Figure 5–18(b). An OR function can be produced with only NAND gates, as illustrated in part (c). Finally, a NOR function is produced as shown in part (d).



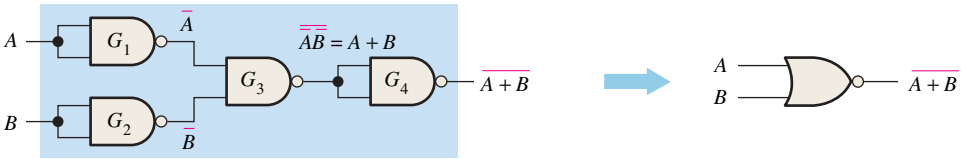
(a) One NAND gate used as an inverter



(b) Two NAND gates used as an AND gate



(c) Three NAND gates used as an OR gate



(d) Four NAND gates used as a NOR gate

FIGURE 5–18 Universal application of NAND gates. Open files F05-18(a), (b), (c), and (d) to verify each of the equivalencies.

In Figure 5–18(b), a NAND gate is used to invert (complement) a NAND output to form the AND function, as indicated in the following equation:

$$X = \overline{\overline{AB}} = AB$$

In Figure 5–18(c), NAND gates G_1 and G_2 are used to invert the two input variables before they are applied to NAND gate G_3. The final OR output is derived as follows by application of DeMorgan's theorem:

$$X = \overline{\overline{A}\,\overline{B}} = A + B$$

In Figure 5–18(d), NAND gate G_4 is used as an inverter connected to the circuit of part (c) to produce the NOR operation $\overline{A + B}$.

The NOR Gate as a Universal Logic Element

Like the NAND gate, the NOR gate can be used to produce the NOT, AND, OR, and NAND functions. A NOT circuit, or inverter, can be made from a NOR gate by connecting all of the inputs together to effectively create a single input, as shown in Figure 5–19(a) with a 2-input example. Also, an OR gate can be produced from NOR gates, as illustrated in Figure 5–19(b). An AND gate can be constructed by the use of NOR gates, as shown in

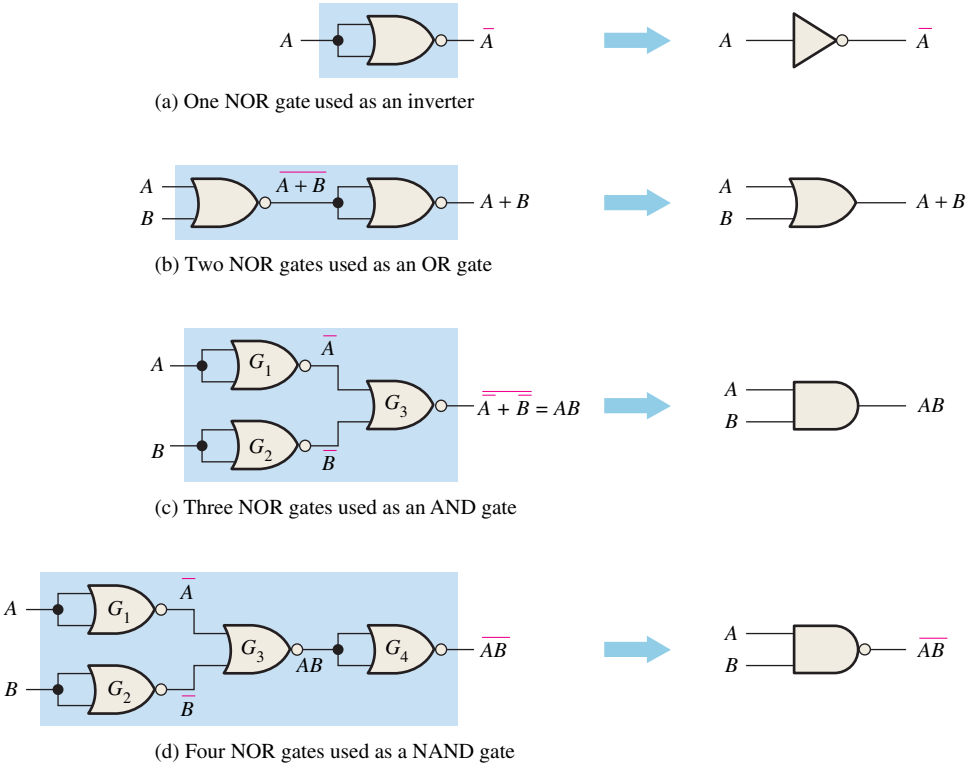


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FIGURE 5–19 Universal application of NOR gates. Open files F05-19(a), (b), (c), and (d) to verify each of the equivalencies.

Figure 5–19(c). In this case the NOR gates G_1 and G_2 are used as inverters, and the final output is derived by the use of DeMorgan's theorem as follows:

$$X = \overline{\overline{A} + \overline{B}} = AB$$

Figure 5–19(d) shows how NOR gates are used to form a NAND function.

SECTION 5–3 CHECKUP

1. Use NAND gates to implement each expression: **(a)** $X = A + B$ **(b)** $X = \overline{AB}$
2. Use NOR gates to implement each expression: **(a)** $X = A + B$ **(b)** $X = \overline{AB}$